INTELLIGENT TRACKING CONTROL USING PSO-BASED INTERVAL TYPE-2 FUZZY LOGIC FOR A MIMO MANEUVERING SYSTEM

Asst. Prof. Dr Mohammed Y. Hassan,

University of Technology, Control and Systems Department, Baghdad, Iraq.

E- mail: 60003@uotechnology.edu.iq

Abstract: Air vehicle modeling like the helicopter is very challenging assignment because of the highly nonlinear effects, effective cross-coupling between its axes, and the uncertainties and complexity in its aerodynamics. The Twin Rotor Mutli-Input Multi-Output System (TRMS) represents in its behavior a helicopter. TRMS has been widely used as an apparatus in Laboratories for experiments of control applications. The system consists of two degrees of freedom (DOF) model; that is yawing and pitching.

This paper discusses the design of Four Interval Type-2 fuzzy logic controllers (IT2FLC) for yaw and pitch axes and their cross-couplings of a twin rotor MIMO system. The objectives of the designed controllers are to maintain the TRMS position within the pre-defined desired trajectories when exposed to changes during its maneuver. This must be achieved under uncertain or unknown dynamics of the system and due to external disturbances applied on the yaw and pitch angles. The coupling effects are determined as the uncertainties in the nonlinear TRMS. A PSO algorithm is used to tune the Inputs and output gains of the four Proportional-Derivative (PD) Like IT2FLCs to enhance the tracking characteristics of the TRMS model.

Simulation results show the substantial enhancement in the performance using PSO-Based Interval Type-2 fuzzy logic controllers compared with that of using IT2FLCs only. The maximum percentage of enhancements reaches about 33% and the average percentage of enhancements is about 17.1%. They also show the proposed controller effectiveness improving time domain characteristics and the simplicity of the controllers.

Nomenclatures

Symbol	Description	Unit	Symbol	Description	Unit
a_1, a2	Static Characteristic parameters		T_P	Cross reaction momentum parameter	
$B_{1\Psi}$, $B_{2\Psi}$	Parameters of friction momentum function	N.m.sec^2/rad	$u(k)$	Output of controller	V

$B_{1\Phi}$, $B_{2\Phi}$	Parameters of friction momentum function	N.m.sec^2/rad	u1, u2	Motor voltages	V
b_1, b_2	Static characteristic parameters		W	The constriction coefficient in PSO	
c_1, c_2	Acceleration coefficients in PSO		X	The universe of discourse	
e(k)	Error		x_i	The position of the ith particle in PSO	
$e_\Psi(k)$, $e_\Phi(k)$	Error in Pitch and Yaw angles	Rad	$X_g$best	The previous global best position of particles in PSO	
I_1, I_2	Moment of inertia for the rotors in vertical and horizontal directions	Kg.m^2	$X_p$best$_i$	The previous best ith position in PSO	
L	Left switch point		Y	Center of Sets Type Reduction	
K_1, K_2	Motor 1 and Motor 2 gains		y_l	Left end point	
K_c	Cross reaction momentum gain		y_{out}	Output of IT2FLC	
K_{gy}	Gyroscopic momentum parameter		y_r	Right end point	
K	Proportional gain for each controller (P for pitch and Y for yaw).		$\underline{y}^{\,n}, \overline{y}^{\,n}$	Lower and upper end point	
KD	Derivative gain for each controller (P for pitch and Y for yaw).		$\tilde{A}$	Type-2 Fuzzy Set	
KO	Output gain for each controller (P for pitch and Y for yaw).		$\Delta e(k)$	Rate of change of error	
M_g	Gravity momentum	N.m	$\Delta e_\Psi(k), \Delta e_\Phi(k)$	Rate of change of error in pitch and Yaw angles	rad
N	Number of iterations		Ψ	Pitch angle	rad
R	Right switch point		Ψ_{ref}	Pitch angle	rad
r_1, r_2	Random numbers in PSO		Φ	Yaw angle	rad
T_o	Cross reaction momentum parameter		Φ_{ref}	Yaw angle	rad
T_{10}, T_{11}	Motor 1 denominators parameters		$u_{\tilde{A}}(x, u)$	Type-2 membership function	
T_{21}, T_{20}	Motor 2 denominators parameters		$\overline{\mu_{\tilde{A}}}(x), \underline{\mu_{\tilde{A}}}(x)$	Upper and Lower membership functions	

INTRODUCTION

TRMS, Twin Rotor MIMO System, has been widely used as an apparatus in Laboratories for experiments of control applications [1]. Since the model is of nonlinear type with significant coupling between the two axes (yaw and pitch) and complex aerodynamics, the controlling design using conventional, hybrid and intelligent methods is researchers challenge [2-5]. Fuzzy Logic Control (FLC) is a technique to control through the investigation and description of model behavior in terms of linguistic variables formalizing the rule base [6]. Different control method strategies combining FLC with conventional controllers (Like PD and PID), Neural Networks, sliding mode control and Self-Tuning algorithm have been used widely to control the axes of TRMS and track the desired trajectories efficiently [7-9]. Furthermore, Evolution algorithms like Differential Evolution (DE), Genetic algorithm

(GA), Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) have been used to tune the FLC parameters in order to enhance the response of tracking and minimize the steady state error [10-13]. To eliminate the effect of uncertainty, achieve robustness, and enhance the performance of the controlled system in recent time, an Interval Type 2 FLC (IT2FLC) was introduced as a new generation of Type 1 FLC. The difference in structure, mainly in the defuzzifier block, is the addition of the type reduction block during defuzzification [14]. Different researches have dealt with the use of IT2FLC and adaptive IT2FLC to control the TRMS [14-17].

In this paper, the design of Four Type-2 fuzzy logic controllers for the yaw and pitch axes with their couplings of a twin rotor MIMO system is discussed. The objectives of the designed controllers are to reduce overshoot and chattering, exist by the effect of external disturbances, in the yaw and pitch angles during when the TRMS system is exposed to changes during its maneuver. A PSO algorithm is used to tune the inputs and output gains of the Proportional-Derivative (PD) Like IT2FLCs to improve the tracking characteristics of the TRMS model.

The remainder sections of this paper are as follows: section 1 describes the detailed TRMS model. Section 2 illustrates the structure of the Type-2 and Interval Type-2 Fuzzy Logic Control (IT2FLC). The detailed steps for the design of the four PD-Like Interval Type-2 FLCs and tuning the inputs and output gains of the mentioned controllers using PSO algorithm are explained in section 3 and section 4 respectively. Simulation Results are presented in section 5. Finally, concluding remarks are provided in the section of conclusion.

1. TWIN ROTOR MIMO SYSTEM MODEL

The helicopter as one of flight vehicles consists of many elastic parts like rotor, control surfaces and engine. This vehicle is acted by nonlinear aerodynamics forces and gravity, and complexity increases because of flexible surfaces structures which make a realistic analysis difficult [16]. To study the control of this aerodynamics model, the TRMS, a Lab. Setup, is designed by Feedback Company for control experiments [1]. The main parts of TRMS are the beam pivoted on its base which rotates in horizontal and vertical planes freely. Two rotors driven by two Direct Current (DC) motors located at such end of the beam. Aerodynamic force through the blades and coupling effect are produced by both motors. This produces non-linear and high order system with cross coupling [18]. However, there are many differences between the TRMS and helicopter. The pivot point location in the helicopter is located in the main rotor head while it is located in midway between two rotors of TRMS. Moreover, the lift generation of vertical axis in helicopter is by collective pitch control while it is generated in TRMS by speed control of the main rotor. Finally, the yaw is controlled in helicopter by pitch angle of tail rotor blades while is controlled in TRMS by tail rotor speed [18]. The setup of TRMS is shown in Figure 1 [9].

The mathematical model of the TRMS consists of electrical and mechanical parts where the electro-mechanical diagram is depicted in Figure 2 and the TRMS schematic diagram is shown in Figure 3.
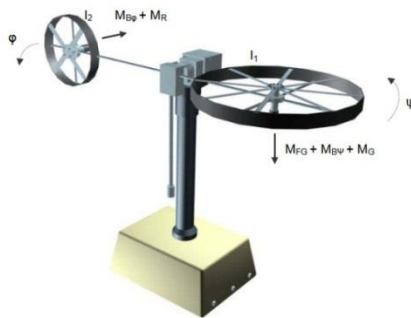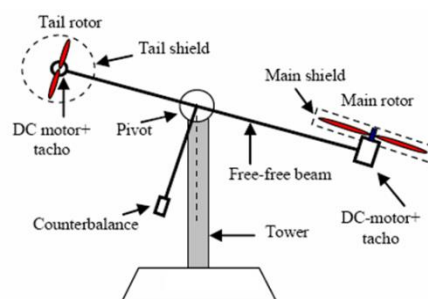

Figure1.Twin rotor MIMO system model (TRMS) [9].
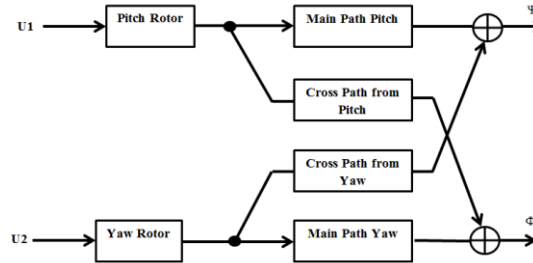
Figure 2. TRMS electro-mechanical model [1].

Figure 3. TRMS schematic

The horizontal motion of the beam is described with the following equation:

$$I_2.\ddot{\phi} = M_2 - M_{B\phi} - M_R \quad (1) \ [1]$$

where M_2 is the tail propeller thrust which is a nonlinear static function of the DC motor momentum and described by:

$$M_2 = a_2.\tau_2^2 + b_2.\tau_2 \quad (2)$$

$M_{B\emptyset}$ is the friction forces momentum represented by:

$$M_{B\emptyset} = B_{1\emptyset}.\dot{\emptyset} + B_{2\emptyset}.sign(\dot{\emptyset}) \quad (3)$$

and M_R is the momentum of cross reaction approximated by:

$$M_R = \frac{K_C.(T_o.s+1)}{T_p.s+1}.\tau_1 \quad (4)$$

The electrical circuit with the DC motor is approximated by a transfer function of first or and given in Laplace transform by:

$$\tau_2 = \frac{K_2}{T_{21}.s+T_{20}}.u_2 \quad (5)$$

where the input voltage of the DC motor is u_2, K_2 is the static gain of DC motor and T_{21} is the main rotor time constant.

Moreover, the momentum equations for the vertical movement are described by:

$$I_1.\ddot{\psi} = M_1 - M_{FG} - M_{B\psi} - M_G \quad (6) \ [1]$$

where M_1 is the main propeller thrust which is a nonlinear static function of the DC motor momentum and described by:

$$M_1 = a_1.\tau_1^2 + b_1.\tau_1 \quad (7)$$

and M_{FG} is the gravity momentum represented by:

$$M_{FG} = M_g.sign(\psi) \quad (8)$$

The friction forces momentum is described by:

$$M_{B\psi} = B_{1\psi}.\dot{\psi} + B_{2\psi}.sign(\dot{\psi}) \quad (9)$$

and the gyroscopic momentum is given by:

$$M_G = K_{gy}.M_1.\dot{\phi}.cos(\psi) \quad (10)$$

The electrical circuit with the DC motor is approximated by a first order transfer function and the motor momentum is given in Laplace transform by:

$$\tau_1 = \frac{K_1}{T_{11}.s+T_{10}}.u_1 \quad (11)$$

where the input voltage of the DC motor is u_1, K_1 is the static gain of DC motor and T_{11} is the time constant of the main rotor.

In this paper, the physical parameters of the TRMS model are listed in table 1 [1].

Table 1. TRMS physical parameters [1]

Symbol	Value	Symbol	Value	Symbol	Value
I_1	$6.8*10^{-2}$ Kg.m^2	K_{gy}	0.05 sec/rad	$B_{2\Psi}$	$1*10^{-3}$ N.m.sec^2/rad
I_2	$2*10^{-2}$ Kg.m^2	K_1	1.1	$B_{1\Phi}$	$1*10^{-1}$ N.m.sec^2/rad
a_1	0.0135	K_2	0.8	$B_{2\Phi}$	$1*10^{-2}$ N.m.sec^2/rad
b_1	0.0924	T_{11}	1.1	T_o	3.5
a_2	0.02	T_{10}	1	K_c	-0.2
b_2	0.09	T_{21}	1	u1, u2	±2.5 V
M_g	0.32 N.m	T_{20}	1		

2. INTERVAL TYPE-2 FUZZY LOGIC CONTROL (IT2FLC)

Fuzzy sets theory was introduced by **Lotfi A. Zadeh** in 1965 as a method to describe non-probabilistic uncertainties. In 1975, the idea of Type-2 FLC (T2FLC) as an expansion of Type-1 FLC (T1FLC) was proposed by **Zadeh** too. The uncertainties in Fuzzy sets of membership functions (MFs) of T2FLC are in three dimensions while the ones in T1FLC are in two dimensions, that is the typical memberships of Type-2 consists of two Type-1 MFs. Fuzzy memberships in Type-2 have the Footprint Of Uncertainty (FOU) which is a bounded region of a fuzzy set ($\tilde{A}$) that can handle the uncertainties, nonlinearities and linguistics related with inputs and outputs of FLC and reducing them [19]. It represents the union of all primary membership functions, where:

$$FOU(\tilde{A}) = U_{x \in X} J_X \quad (12) \, [21]$$

where $\tilde{A}$ is characterized by Type-2 MF $u_{\tilde{A}}(x, u)$, where $x \subset X, X$ is the universe of discourse and $u \in J_x \subseteq [0, 1]$, then:

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | x \in X, u \in J_x \subseteq [0, 1] \} \quad (13)$$

in which $0 \leq u_{\tilde{A}}(x, u) \leq 1$. It can also be represented by:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} J_x \subseteq [0.1] \quad (14)$$

where $\iint$ denotes union over all admissible x and u.

The upper and lower membership functions are defined by $\overline{\mu_{\tilde{A}}}(x) \; x \in X$ and $\underline{\mu_{\tilde{A}}}(x) \; x \in X$ respectively, as follows:

$$\overline{\mu_{\tilde{A}}}(x) = \overline{FOU(\tilde{A})} \quad (15) \, [21]$$

and

$$\underline{\mu_{\tilde{A}}}(x) = \underline{FOU(\tilde{A})} \quad (16)$$

The secondary memberships functions (MFs) domain is within [0, 1]. Moreover, the two dimension plane whose axes are u and $u_{\tilde{A}}(x, u)$ is known as the vertical slice of $u_{\tilde{A}}(x, u)$ and represented as follows:

$$\mu_{\tilde{A}}(x = x_1, u) = \mu_{\tilde{A}}(x_1) = \int_{u \in J_x} \frac{f_{x_1}(u)}{u} J_{x1} \subseteq [0, 1] \quad (17) \, [21]$$

where $0 \leq f_{x_1}(u) \leq 1$ and the secondary membership function is represented by $\mu_{\tilde{A}}(X_1)$. It is the Type-1 fuzzy set where the primary membership function of x_1 is J_{x1}, It is secondary membership domain where $J_{x1} \subseteq [0, 1]$ for all x_1 in X. Now, the interval set is defined when the secondary membership function is $f_{x_1}(u) = 1 \; u \in J_{x1} \subseteq [0, 1]$. An Interval Type-2 (IT2) membership function is obtained when this it is true for $x_1 \in X$. The uniform uncertainty at the primary membership of x is represented by secondary MF of Type-2. The membership function A of Type-2 with its secondary memberships is shown in Figure 4 [20].
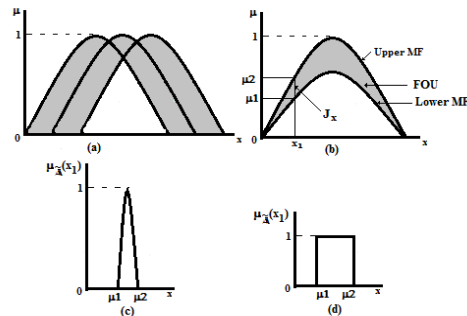
Figure 4. Type-2 MF with its MFs [20]: a) Fuzzy set of type-2 representing fuzzy set of type-1 with uncertain mean b) A sample type-2 fuzzy set for FOU c) The secondary MF for type-2 fuzzy set d) IT2FLC secondary MF.

Type-2 FLC is divided into two types; that is Mamdani type where the output membership functions are fuzzy sets and the Takagi-Sugeno-Kang (TSK) type where the output membership functions are either linear or constants. Figure 5 illustrates the structure of the T2FLC [19].
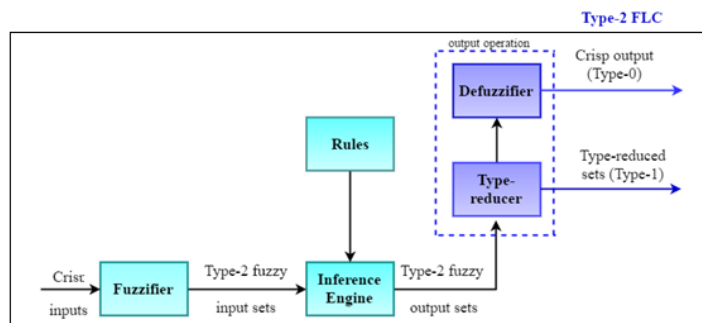

Figure 5. Structure of IT2FLC [1].

The difference between Type-1 and Type-2 FLC is in the nature of the membership functions used. The main blocks of T2FLC are [1]:
   a. Fuzzifier: It makes the inference engine works by crisp inputs into type-2 fuzzy sets mapping.
   b. Rule base: The difference between the rules in T2FLC and the rules in T1FLC is in the antecedents and consequents that are represented by the interval Type-2 fuzzy sets.
   c. Inference engine: The fuzzy inputs to fuzzy outputs are assigned in the inference engine block using the operators such as the intersection and union operators and the rule base.
   d. Type-reduction: Type-reduced sets are the outputs of Type-2 fuzzy sets for the inference engine when converted into fuzzy sets of Type-1. In Interval Type-2 FLC three methods for type-reduction operation. That are, Karnik-Mendel (KM) iteration method, Enhanced Karnik-Mendel (EKM) iteration method, and Wu-Mendel Uncertainty Bounds method.
In this paper, Modified Karnick Mendel is used to design the controller. It is an enhancement of the original KM algorithm with three improvements. First, reducing the number of iterations, better initialization is used. Second, one unnecessary iteration is removed by changing the termination condition. Third, reducing the cost of computation for each iteration, a subtle computing technique is used. The detailed algorithm is given in table 2 [22]**.**
   e. Defuzzification: The input to the defuzzification block is the type-reduction output block. This is done through two steps: first, by transforming the fuzzy sets of Type-2 into the fuzzy sets of Type-1. The left and right end points are used to calculate the type reduction sets. Second, by calculating the average

of the points. Crisp value (Type-0) is produced by defuzzify the Type-1 fuzzy generated set using the fuzzy logic control known techniques. The calculations of type-reduction operations are very complex. To simplify calculations, the Interval Type-2 Fuzzy set is used [20]. In this paper, the Centroid method is used to calculate the defuzzified values as follows:

$$y_{out} = \frac{y_l + y_r}{2} \quad (18)$$

Table 2. EKM Algorithm [22]

Step	For computing y_1	For computing y_r
1.	Set $l = \left[\frac{N}{2.4}\right]$ (the nearest integer to $\frac{N}{2.4}$). and compute $$a = \sum_{n=1}^{l} y^n \overline{\mu^n} + \sum_{n=l+1}^{N} y^n \underline{\mu^n}$$ $$b = \sum_{n=1}^{l} \overline{\mu^n} + \sum_{n=l+1}^{N} \underline{\mu^n}$$ $$y = a/b$$	Set $r = \left[\frac{N}{1.7}\right]$ $\left(\begin{array}{c}\text{the nearest} \\ \text{integer to } \frac{N}{1.7}\end{array}\right)$. and compute $$a = \sum_{n=1}^{r} y^n \overline{\mu^n} + \sum_{n=r+1}^{N} y^n \underline{\mu^n}$$ $$b = \sum_{n=1}^{r} \overline{\mu^n} + \sum_{n=r+1}^{N} \underline{\mu^n}$$ $$y = a/b$$
2.	FIND $l` \in [1. N-1]$ such that $\underline{y^{l`}} < y \le \underline{y^{l`+1}}$	FIND $r` \in [1. N-1]$ such that $\underline{y^{r`}} < y \le \overline{y^{r`+1}}$
3.	IF $l` = l$. stop and $set\ y_1 = y$ and $L = l$; otherwise. continue.	IF $r` = r$. stop and $set\ y_1 = y$ and $R = r$; otherwise. continue.
4.	Compute $s = \text{sign}(l` - l)$. and $$a` = a + s \sum_{n=\min(l.l`)+1}^{\max(l.l`)} y^n \overline{\mu^n} - \underline{\mu^n})$$ $$b` = b + s \sum_{n=\min(l.l`)+1}^{\max(l.l`)} (\overline{\mu^n} - \underline{\mu^n})$$ $$y` = \frac{a`}{b`}$$	Compute $s = \text{sign}(r` - r)$. and $$a` = a - s \sum_{n=\min(r.r`)+1}^{\max(r.r`)} y^n (\overline{\mu^n} - \underline{\mu^n})$$ $$b` = b - s \sum_{n=\min(r.r`)+1}^{\max(r.r`)} (\overline{\mu^n} - \underline{\mu^n})$$ $$y` = \frac{a`}{b`}$$
5.	Set $y = y`. a = a`. b = b`$ and $l = l`$. Go to step 2.	set $y = y`. a = a`. b = b`$ and $r = r`$. Go to step 2.

3. DESIGN OF PD-LIKE IT2FLC FOR TRMS MODEL

The objective of Fuzzy controllers is to maintain the TRMS position within the pre-defined desired trajectory. This must be achieved under uncertain or unknown dynamics of the system. The

MATLAB\Simulink of the PD Like IT2FLCs controlled TRMS system is shown in Figure 6 where the TRMS model explained in section 1 is simulated using MATLAB\Simulink. In order to take the effect of cross coupling between the pitch and yaw channels into consideration, four controllers are designed to control the Pitch (P), Pitch-Yaw (PW), Yaw-Pitch (YP) and Yaw (Y).
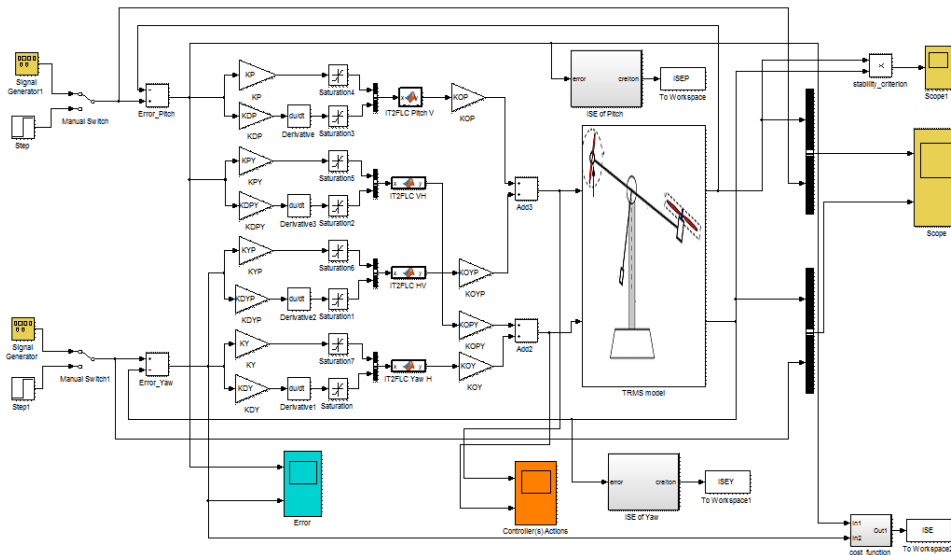

Figure 6. Simulink of TRMS controlled by IT2FLCs

Two controlled signals are generated from the outputs of the above controllers to control the pitch and yaw angles. The Pitch channel is controlled signal is generated by summing the outputs of the (P) and (YP) controllers. While the controlled signal of the yaw channel is generated by summing the outputs of the (Y) and (PY) controllers. The inputs to the (P) and (PY) controllers are the error ($e_\Psi(k)$) and rate of error which are calculated in discrete time domain as follows:

$$e_\Psi(k)=\Psi_{ref}-\Psi \quad (19)$$
$$\Delta e_\Psi(k)= e_\Psi(k)- e_\Psi(k-1) \quad (20)$$

where k is the sampling instant.

Moreover, the inputs to the (Y) and (YP) controllers are the error ($e_\Phi(k)$) and rate of error in discrete time domain is calculated as follows:

$$e_\Phi(k)=\Phi_{ref}-\Phi \quad (21)$$
$$\Delta e_{\Phi\Psi}(k)= e_\Phi (k)- e_\Phi (k-1) \quad (22)$$

The inputs and output scaling factors of the four PD Like IT2FLCs are K_P, KD_P, KO_P, K_{PY}, KD_{PY}, KO_{PY}, K_{YP}, KD_{YP}, KO_{YP}, K_Y, KD_Y, and KO_Y where K is the proportional gain, KD is the derivative gain and KO is the output gain for each controller. These gains will be tuned manually to reach the TRMS position within the pre-defined desired trajectory in the pitch and yaw axes when exposed to changes during its maneuver. This must be achieved under uncertainty due to the axes coupling effects and due to external disturbances that are represented by noise signal.

Each controller is of Mamdani type where each input and output has two Trapezoid shaped Type-2 membership functions within the range of (-1.5,1.5) for the inputs and (-1, 1) for the outputs, see Figure 7, where the linguistic variables (N) and (P) represent Negative and Positive respectively.
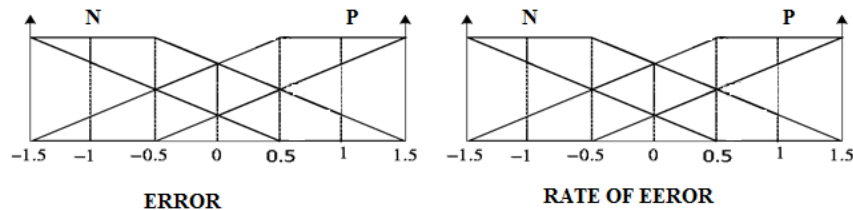
Figure 7. Error and rate of error MFs

The rule base and the corresponding consequents of output membership functions for controller are listed in table 3.

Table 3. Rule base and the corresponding consequents of PD Like IT2FLC

$e/\dot{e}$	N	P
N	N Y^1 [-1, -0.9]	N Y^2 [-0.6 -0.4]
P	P Y^3 [0.4 0.6]	P Y^4 [0.9 1]

The rules are chosen to achieve minimum error in angles in both negative and positive directions. The controller equation for each controller in discrete time domain is:

$$u(k) = K.\,e(k) + KD.\,\Delta e(k) \quad (23)$$

The output of each controller is multiplied by the output gain (KO).

- **ANALYSIS OF SATBILITY**

The guarantee of robustness and stability of IT2FLC is very big challenge because of the complexity in its structure. A Bounded Input Bounded Output (BIBO) is one of the approaches to realize the stability of IT2FLC [17]. Assume G1 and G2 are representing T2FLC and the controlled plant model respectively, see Figure 8.
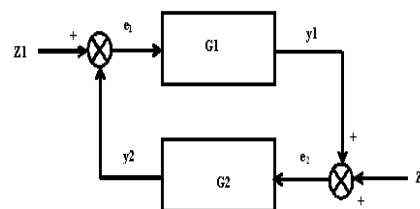

Figure 8. Closed loop subsystem

Assume the gains of G1 and G2 are λ1 and λ2 respectively, where λ1 > 0 and λ2 > 0, and Γ1 and Γ2 are constants, then:

$$\|y1\| = \|G1.\,e_1\| \leq \lambda1.\,\|e_1\| + \Gamma1 \quad (24)\ [17]$$
$$\|y2\| = \|G2.\,e_2\| \leq \lambda2.\,\|e_2\| + \Gamma2 \quad (25)$$

According to the theorem of small Gain, which illustrates that any bounded output pair (y1, y2) is generated by any bounded input pair (Z1, Z2), and to the stability conditions in equations (24 and 25), the system is BIBO stable if y1.y2 <1 [23].

4. DESIGN OF PSO-BASED IT2FLC FOR TRMS MODEL

Particle Swarm Optimization (PSO) is one of the heuristic search methods which is inspired by the swarming which is introduced by **Kenndy** and **Ebrhart** in 1995. It has the advantages of; converging towards an optimum solution, computation is simple, and easy in implementation as

compared with other evolution algorithms, like Genetic Algorithm. Each population member in PSO algorithm is named as "Particle". Each particle (x_i(k)) "files" around the multidimensional search space with a velocity (v_i(k)) of that is updated by the own experience of the neighbors of particle in the swarm [24].

In this paper, the PSO algorithm with the constriction coefficient formula instead of weight is used; it's a good method that gives faster convergence ability with minimum number of iterations to reach a goal [24].The inputs and output gains (12 Gains) of the four PD-Like IT2FLCs are tuned to reach the best values depending on minimizing the following objective functions for pitch and yaw angles:

$$ISE_\Psi = \sum_{i=1}^{N} e_\psi{}^2(i) \quad (26)$$

$$ISE_\phi = \sum_{i=1}^{N} e_\phi{}^2(i) \quad (27)$$

It is the overall performance index (PI) of Integral Square of Error (ISE) for the Pitch and Yaw motions, as follows:

$$ISE = \sum_{i=1}^{N} (e_\psi{}^2(i) + e_\phi{}^2(i)) \quad (28)$$

The minimization of this (PI) means that the TRMS model will follow the desired trajectories in both yaw and pitch motions in spite of the appearances of uncertainties in the model or the disturbances affecting them. The velocity of ith particle will be calculated as:

$$v_i(k+1)=w(v_i(k)+c_1r_1(X_pbest_i(k) –x_i(k)) +c_2r_2 (X_gbest –x_i(k))) \quad (29) [25]$$

where for the ith particle in the kth iteration, (x_i) is the position, (X_pbest_i) is the previous best position, (X_gbest) is the previous global best position of particles, (c_1) and (c_2) are the acceleration coefficients namely the cognitive and social scaling parameters, (r_1) and (r_2) are two random numbers in the range of [0 1] and (w) is a constriction coefficient given by:

$$w = \frac{2}{\left|4-\phi-\sqrt{\phi^2+4\phi}\right|} \quad (30) [25]$$

Where (ϕ =c_1+c_2, ϕ>4). The convergence of the particle is controlling the constriction coefficient. As a result, it prevents explosion and ensures convergence. A new position of the ith particle is then calculated as:

$$x_i(k+1)= x_i(k) + v_i(k+1) \quad (31) [25]$$

The PSO algorithm is repeated until the goal is achieved.

5. SIMULATION RESULTS

The physical parameters of the TRMS model simulated in this section are listed in table 1. The PD-Like IT2FLC controlled system has been simulated for 100 seconds with zero initial conditions for both; pitch and yaw angles. In this simulation, the reference signals of Sinusoidal wave and Saw tooth with amplitude of 0.2 rad and frequency of 0.02Hz and step input of 0.2 rad are applied to both angles [16].

To investigate the robustness of both controllers with respect to the measurement noise and parametric variations, a signal noise with is added to the measured variables. The measured signals from sensors are in general subject to noise in spite of the output of systems are measured using adequate sensors [16,17]. In the following simulations, a uniformly distributed random signal with amplitude of (0.01) is added to the measured pitch and yaw signals, see Figure 9.
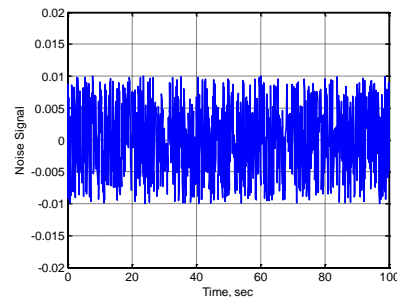
Figure 9. Uniformly distributed random noise signal

To measure the best response among all the simulations, equations (26, 27 and 28) are used. The minimization of this (ISE) means that the TRMS model follows the desired trajectories in both yaw and pitch motions in spite of the appearances of disturbances and uncertainties applied. The inputs and output gains of the four controllers has been tuned to reach the best time response, minimum ISE, and table 4 lists the best values of gains.

Table 4. Gains of controllers obtained manually

K_P	KD_P	KO_P	K_{PY}	KD_{PY}	KO_{PY}	K_{YP}	KD_{YP}	KO_{YP}	K_Y	KD_Y	KO_Y
0.4	0.8	1	1	4	1	0.1	0.5	0.1	0.15	0.7	5

The time responses for the above three reference signals and the actual signals for the pitch and yaw angles without and with applying noise are shown in Figures 10-15. The control signals for controlling the pitch and yaw motors are also shown on the same previous figures.
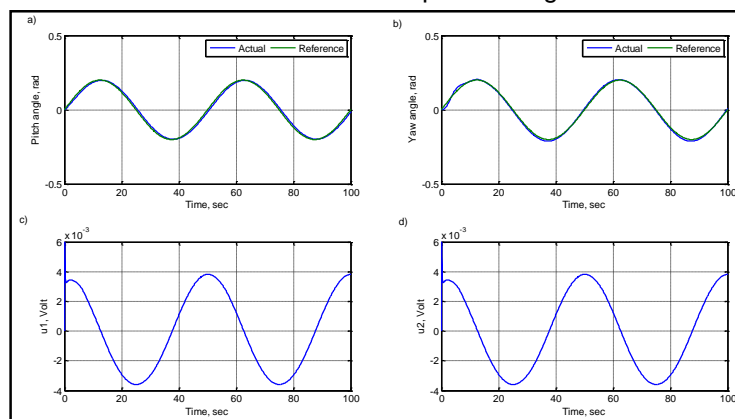

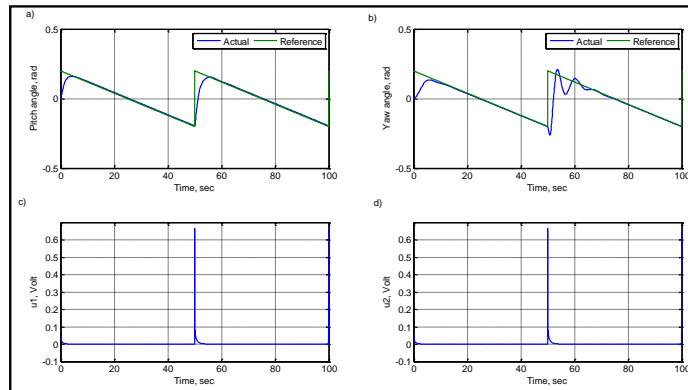**Figure 10**. Sinewave response of the IT2FLC system without applying noise

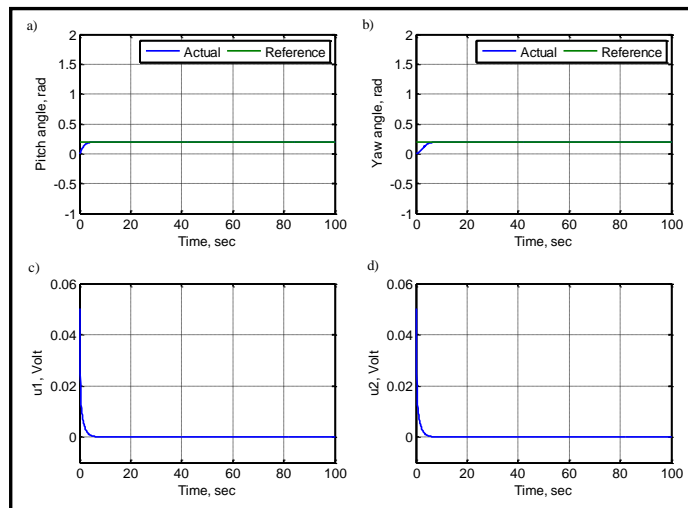**Figure 11**. Sawtooth response of the IT2FLC system without applying noise

Figure 12. Unit step response of the IT2FLC system without applying noise
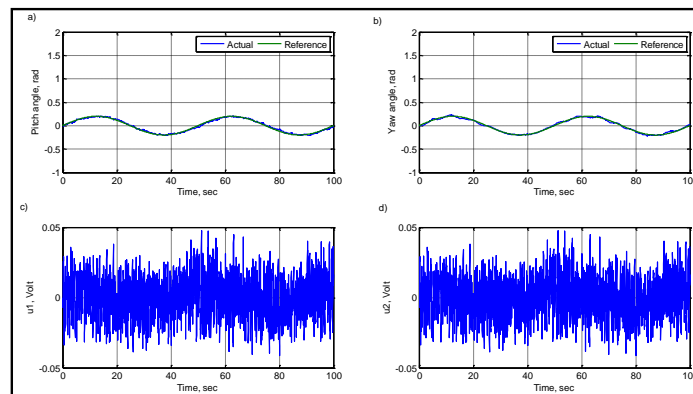

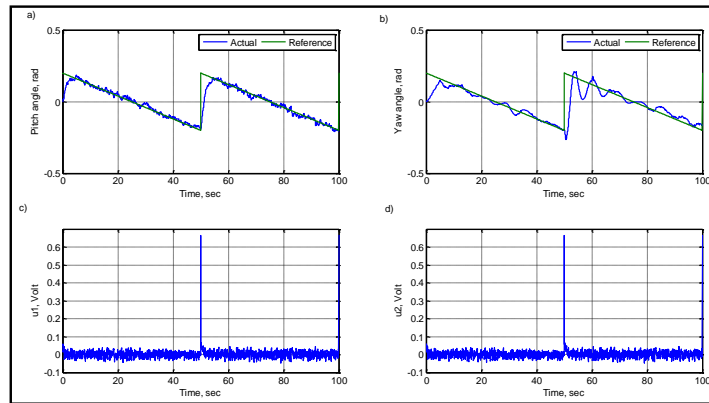**Figure 13**. Sinewave response of the IT2FLC system with applying noise

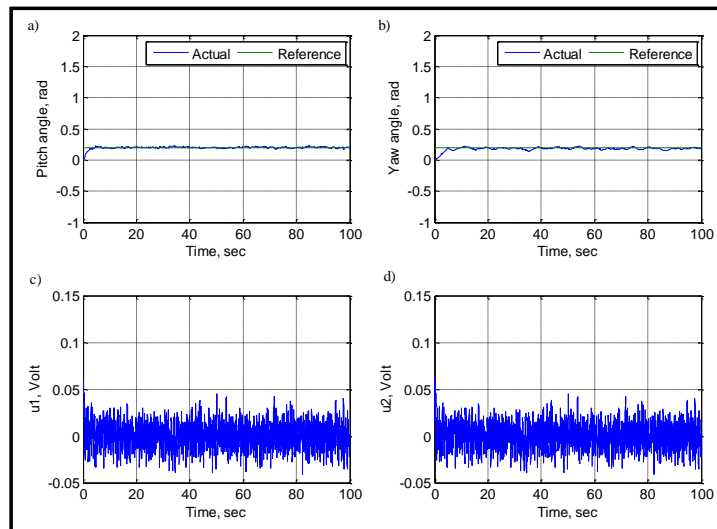**Figure 14**. Sawtooth response of the IT2FLC system with applying noise

Figure 15. Unit step response of the IT2FLC system with applying noise

In order to enhance the time response of the pitch and yaw angles with the appearance of uncertainties without and with applying noise, PSO algorithm explained in section 4 is used to find the best gains of the controllers. The parameters of PSO are selected as: the dimension of the swarm is 12 (number of tuned gains of the controllers), the number of birds (n=40), and (c1=c2=4). The best values of gains (global best birds) are listed in table 5 and the global best fitness is 8.8188 where the number of iterations is 25. The time responses for the same above three reference and actual signals for the pitch and yaw angles without and with applying noise are shown in Figures 16-21. The control signals for controlling the pitch and yaw motors are also shown on the same previous figures. The ISE for the Pitch and Yaw motions for both controllers with the overall ISE are listed in table 6. The maximum percentage of enhancements reaches about 33% and the average percentage of enhancements is about 17.1%.
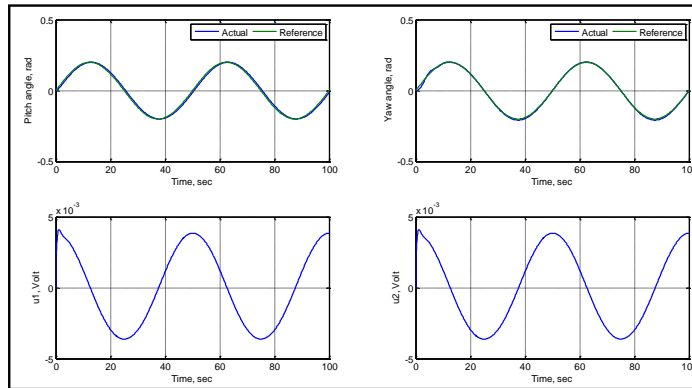
Figure 16. Sinewave response of the PSO based IT2FLC system without applying noise
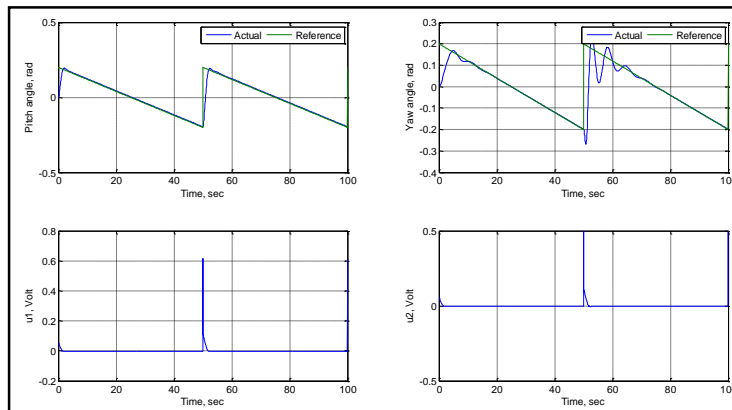

Figure 17. Sawtooth response of the PSO based IT2FLC system without applying noise
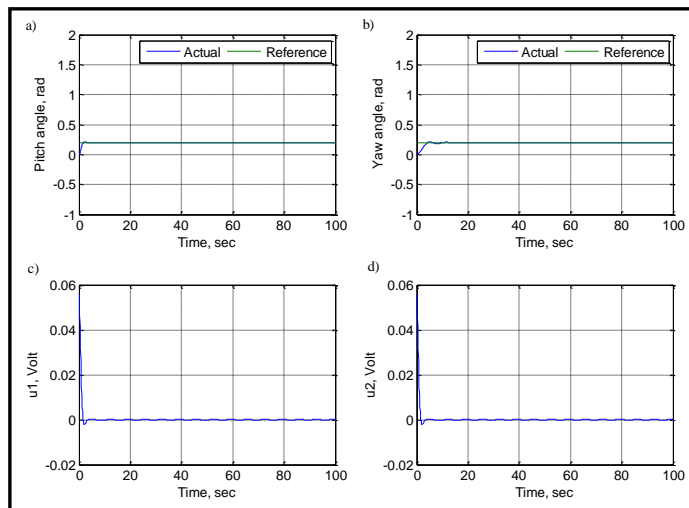

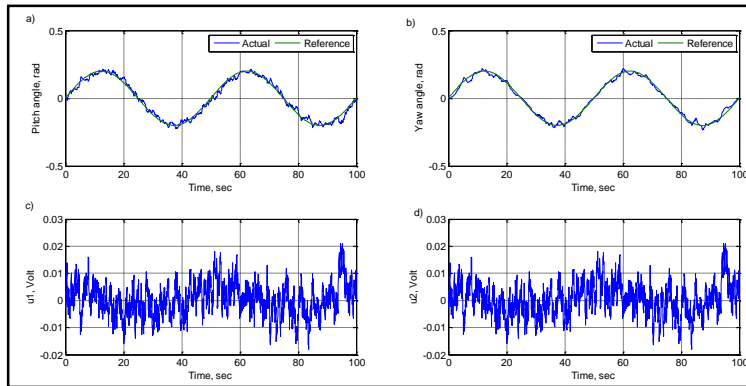**Figure 18**. Unit step response of the PSO based IT2FLC system without applying noise

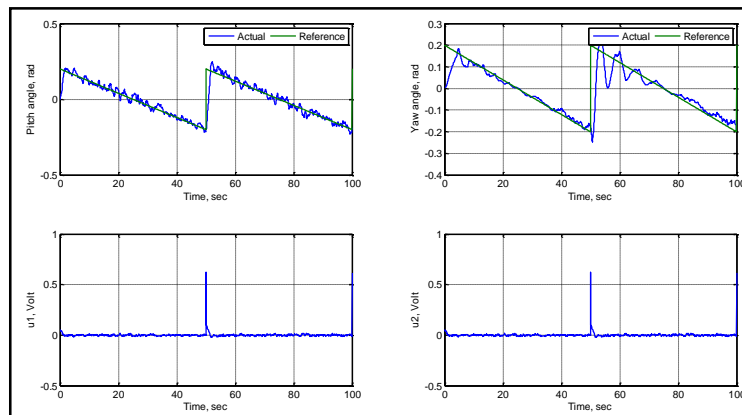**Figure 19**. Sinewave response of the PSO based IT2FLC system with applying noise

Figure 20. Sawtooth response of the PSO based IT2FLC system with applying noise
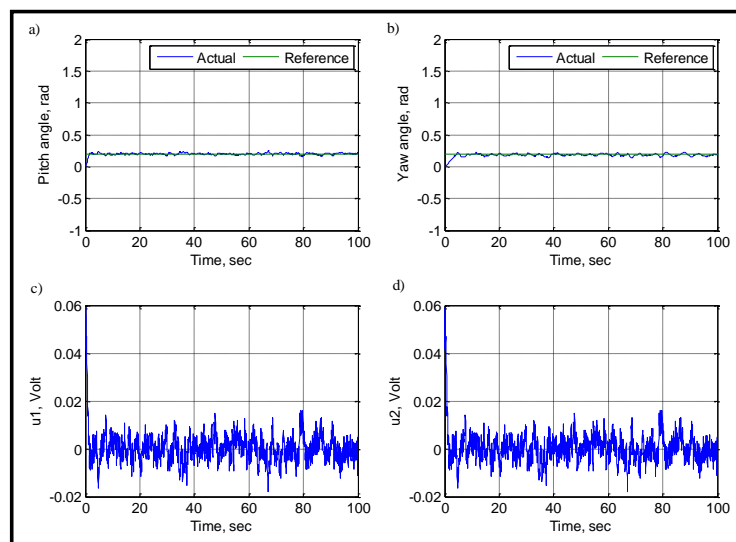

Figure 21. Unit step response of the PSO based T2FLC system with applying noise

As illustrated from results and from table 6 that the PSO-Based PD-Like IT2FLCs provide better performance than using PD-Like IT2FLCs. The proposed PSO based controller presents better performance and good convergence in both pitch and yaw channels with smaller oscillations. The control signals of the pitch and yaw channels for the PD-Like IT2FLCs contain higher oscillations which causes significant error in angles than using PSO Based PD-Like IT2FLCs. The less control efforts cause less consumption in power.

Simulation results also show the effectiveness of the proposed controller in terms of the simplicity of the controller and improving time domain characteristics. The proposed controller uses two input membership function which reduces the rules into 4 as compared with the designed ones in references (9, 13, 16, 17) which uses Type-1and Typ-2 FLC.

Both controllers are BIBO stable for all reference trajectories applied without and with the application of noise, see equations (24 and 25).

Table 5. Gains of controllers obtained by PSO

K_P	KD_P	KO_P	K_{PY}	KD_{PY}	KO_{PY}	K_{YP}	KD_{YP}	KO_{YP}	K_Y	KD_Y	KO_Y
0.487	0.168	0.922	0.787	0.561	0.897	0.534	0.496	0.027	0.121	0.535	7.745

Table 6. ISE Performance Index of TRMS motion

Trajectory	Addition of Noise	Manual Tuning of controller gains			PSO Tuning of Controller gains			Percentage of Overall Improvements (%)
		Pitch	Yaw	Overall	Pitch	Yaw	Overall	
Sine wave	No	0.011	0.007	0.018	0.009	0.003	0.012	33.3
	Yes	0.026	0.029	0.055	0.033	0.021	0.054	1.81
Sawtooth wave	No	0.176	0.444	0.62	0.125	0.386	0.511	17.58
	Yes	0.18	0.479	0.659	0.155	0.385	0.54	18.06
Unit step	No	0.035	0.086	0.121	0.024	0.066	0.09	25.62
	Yes	1.103	1.483	2.586	0.663	1.787	2.45	6.29

CONCLUSIONS

Type-2 FLC is a highly sensitive and robust controller through perturbations and uncertainties in the controlled system as compared with Type-1 FLC for the same class of systems. Type-1 FLC has higher tracking errors especially when disturbances exist.

In this paper, Four PSO-Based IT2FLCs were designed for trajectory tracking for yaw and pitch axes and their cross-couplings of the 2DOF TRMS nonlinear model using MATLAB/Simulink. The PSO algorithm is used to tune the Inputs and output gains of the four Proportional-Derivative (PD) Like IT2FLCs to cancel high nonlinearities and to solve high the effect of coupling. Simulation results show that the PSO-Based IT2FLCs produce better stable tracking than IT2FLCs in terms of maintaining the TRMS position within the pre-defined desired trajectory, when exposed to changes during its maneuver without and with the presence of noise. The maximum percentage of enhancements reaches about 33% and the average percentage of enhancements is about 17.1%. They also show the effectiveness of the proposed controller in terms of improving time domain characteristics and the simplicity of the controllers compared with the designed ones proposed in previous published works.

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