

TRANSIENT RESPONSE OF ROTARY REGENERATIVE HEAT EXCHANGER BY USING FINITE DIFFERENCE METHOD

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Abstract:

A mathematical model for the unsteady state heat and mass transfer in regenerative rotary heat exchanger is derived and solved numerically by using finite difference method. Triangular shape of the rotary matrix with silica gel (SiO_2) that used as desiccant material is taken here. The heat transfer coefficient and the mass transfer coefficient that used in the present study are suitable for different types of the matrix shapes. The unsteady state time effect, inlet air temperature, Number of Transfer Unit (NTU_t), heat exchanger length, inlet air temperature and the desiccant material volume on the latent, sensible and total effectiveness are discussed. It has been notes that the increasing of the heat exchanger length, inlet air temperature, desiccant material volume and the (NTU_t) will increase the sensible, latent and total effectiveness. The increasing of the inlet air mass flow rate will decrease the total, latent and sensible effectiveness. The steady state time for the latent effectiveness is lower than the steady state time for the sensible and total effectiveness.

Keywords: Heat Exchanger, Rotary, Enthalpy Exchange, Latent and Sensible Heat Transfer, Finite Difference, Regenerative, Counterflow.

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الخلاصة

لقد تم اشتقاق النموذج الرياضي لانتقال الحرارة والكتلة الغير مستقرة داخل مبادل حراري دوّار متجدد وتم حلّ هذه المعادلات بشكل عددي باستعمال طريقة الفروقات المحدودة (Finite Difference). لقد تم اخذ الشكل المثلثي للمصفوفة الدوّارة مع مادة السيليكا (SiO_2) التي تستعمل كمادة مجففة. إنّ معامل انتقال الحرارة ومعامل انتقال أكتله الذي أستعمل خلال هذه الدراسة مناسباً لأنواع مختلفة من أشكال المصفوفة. أنّ تأثير الوقت اللازم للوصول إلى حالة الاستقرار، درجة حرارة الدخول للهواء، عدد وحدات النقل (NTU_t)، طول المبادل الحراري، درجة حرارة الهواء الداخل وحجم المادة المجففة على فاعلية المبادل الحراري المحسوسة والكامنة والكلية تم دراستها. لقد لوحظ ان تأثير زيادة طول المبادل الحراري، درجة حرارة الهواء الداخل، حجم المادة المجففة وعدد وحدات النقل سيّزياً فاعلية الكلية والمحسوسة والكامنة. زيادة كمية الهواء الداخل يؤدي إلى نقصان الفاعلية الكلية والمحسوسة والكامنة. أنّ الوقت اللازم للوصول إلى حالة الاستقرار بالنسبة للفاعلية الكامنة يكون اقل من الوقت اللازم للوصول إلى حالة الاستقرار بالنسبة إلى الفاعلية المحسوسة والكلية.

Nomenclature

A	Area (m ²)	NTU _m	Mass Number of Transfer Unit
m	Mass flow rate (kg/sec)	NTU _t	Thermal Number of Transfer Unit
C	Fluid capacitance (W / °C)	Nu	Nusselt Number
C _p	Constant pressure mass heat capacity (kJ/kg.K)	P	Periphery area for equivalent diameter (m ²)
C _r	Fluid capacitance ratio	Pr	Prandtle number
D	Diffusion coefficient (cm ² /s)	Re	Reynolds number
d _{eq}	Equivalent diameter (m)	Sc	Schmidt Number
f	Cross-sectional area for equivalent diameter (m ²)	Sh	Sherwood number
H	Enthalpy (kJ/kg.K)	T	Temperature (°C)
h _a	Heat transfer coefficient (W/m ² .K)	u	Fluid velocity in the flow channel
h _m	Mass transfer coefficient (m/s)	w	Moisture content (kg air/ kg water)
k	Thermal conductivity (W/m.K)	x	Longitudinal distance in x-direction (m)
L	Length of the heat exchanger (m)	z	Dimensionless longitudinal distance in x-direction
Le	Lewis number		
M	Mass (kg)		

Greek

θ	Rotational speed direction (s)	Y	Kinematics viscosity (m ² /sec)
λ	Time interval	Ψ	Shape factor
Δ	Change	Γ	Constant factor in heat transfer equations
ε	Heat exchanger effectiveness	Φ	Heat transfer factor
μ	Dynamic viscosity (Pa.sec)	Z	Effectiveness constant
τ	Dimensionless time	P	Density (Kg/m ³)
Γ	Dimensionless time constant		

Subscript

a	Air	sa	Saturated air
in	Inlet	S	Supply
x	x- direction	E	Exit
θ	θ- direction	R	Ratio
m	Matrix desiccant material	eq	Equivalent
ut	Out	J	Interval
∞	Limiting case	T	Thermal

Superscript

i	Refer to distance	J	Refer to time
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Introduction

Energy recovery can substantially reduce the mechanical heating and cooling requirements associated with conditioning ventilation air in most locations and application. Heating and cooling systems can be downsized and down cost when energy recovery is used, because energy recovery systems reduce peak heating and cooling requirements, [ASHRAE, 2005].

Rotary regenerative heat exchangers have certain advantages over other types of heat exchangers for the recovery of heat from exhaust air [WorsÁe-Schmidt, 1991]. Two distinct advantages are that a considerably larger heat transfer area per unit volume and relatively less expensive, and this type of heat exchanger has higher effectiveness compared with other recovery heat exchanger. Effectiveness of plate-type heat exchangers is between 45% and 65%, as compared to 60% and 80% for rotary regenerators [Heinrich, and Franzke, 1993]. Besides these, the speed of the rotation of the matrix can be used to regulate the amount of the heat transferred. Thus, rotary regenerative heat exchangers combine both compactness and high performance [Skiępko, 1989]. Rotary regenerators find increasingly frequent use in air-conditioning systems and gas turbine engines. They have a long history as air pre-heaters in steam power plants [Romie, 1988].

In many researches considered, rotary heat exchangers that transfer sensible heat only and steady state operating conditions [Klein et al., 1990, Ciepliski et al., 1998, Simonson et al., 1999 a and b].

Kays and London, 1984, presented an “Effectiveness-NTU” (ε-NTU) method for predicting the effectiveness of sensible recuperative and rotary regenerative heat exchangers as a function of dimensionless numbers. They showed that the effectiveness depends on two dimensionless numbers for sensible heat exchangers (NTU and C_r) and four dimensionless numbers for rotary heat exchangers (NTU, C_r, C_r^{*}, h_sA_s / h_eA_e). Bowlen, 1993, stated that the energy recovery in the cooling/dehumidifying mode can be up to 2.5 times greater than for an equally sized sensible heat

exchanger and approximately 40% greater in the heating/humidifying mode. **Gunnar Stiesch, 1994**, investigated experimentally the performance of the rotary regenerative heat and mass exchangers for different types of desiccants material to allow energy savings in the heating and cooling of ventilated buildings by recovering energy from the exhaust air and transferring it to the supply air stream for different types of the desiccants. **JoAnna Christen Staton, 1998**, discussed different types of polymeric desiccant materials and the ultimate goal of his study was to develop a material that, when applied to an existing air-to-air heat exchanger would achieve the necessary heat and mass transfer in a single process, thus transforming a sensible heat exchanger into a total enthalpy exchanger. His study focused on the development and determination of appropriate polymeric desiccant materials for use in different heat and mass transfer applications. Various candidate materials were initially studied. **Buyukalaca, and Yilmaz, 2002**, presented a correction factor for the speed of the rotary heat exchanger effectiveness, this factor is used with traditional "Effectiveness-NTU" method to estimate corrected value of rotary heat exchanger effectiveness. **Yilmaz and Ukalaca, 2003**, presented a calculation method for the design of rotary regenerators having different flow channel geometries. This method is dependent to correct the tradition heat exchanger calculation method "Effectiveness-NTU" by introducing two factors for correction, one for rotational correction and other for the purge area. They showed this method is valid for rotational speeds between 0.05 and 7 rev/min. **Oyetope, 2005**, studied the energy wheel effectiveness by a selected international laboratory for many types and sizes of wheel. His study used a new, low-cost, transient method to predict the effectiveness using only data obtained from transient measurements. In his study, an analytical model was presented for predicting the effectiveness of rotating energy wheels using only the characteristics measured on the same non-rotating wheels exposed to a step change in temperature and humidity. A relationship between the step response and the periodic response of an energy wheel is developed using first order linear system design theory. **Wang et al., 2005**, developed a new experimental facility to study the transient response of sensors and equipment. They studied the transient response of a humidity sensor and used the same sensor to measure the relative humidity downstream of a non-rotating energy wheel following a step change in inlet air relative humidity.

In the present study, a mathematical model is derived to generate the governing equations for the unsteady state countercurrent flow regenerative heat exchanger with mass and energy conservation, because these equations are impossible to solved analytically, then a numerical techniques is used to solve these equations by using finite difference method. Triangular shape of the matrix and silica gel (SiO_2) is used as desiccant material in the present study because the silica gel is most popular desiccant material. It can selectively absorbs compounds of different molecular weights.

Mathematical Formulation

The assumptions that will be used in the present study are (**Figure (1)**):

- 1- The inlet and outlet state properties for the both fluids are uniform and not change with time.
- 2- The thermodynamic properties of the air and the matrix are not affected by the pressure drop in axial direction of the matrix which is small compared to the total pressure.
- 3- There is no mixing or carry out between the two fluid streams.
- 4- The fluids and matrix states are considered to be uniform in radial direction.
- 5- Angular and axial heat conduction and vapor diffusion due to temperature and concentration gradient respectively are neglected.
- 6- The matrix is considered to be a homogenous solid with constant matrix characteristics.
- 7- The convective heat and mass transfer coefficient between the air streams and the matrix are constant throughout the system.
- 8- The rotary regenerative heat exchanger operates adiabatically.
- 9- The working fluids (inlet and exhaust) are air.

The mathematical formulation of combined heat and mass transfer in unsteady state regenerative rotary heat exchanger can be derived by applying the mass and energy conservation on the rotary regenerative heat exchanger (**Figure (2)**) compound with the previous assumptions, [**JoAnna Christen Staton, 1998**]:

Σ (the inlet air mass) – Σ (the outlet air mass) = mass change of the desiccant matrix material with time

$$M_{a,in,x} + M_{a,in,\theta} - M_{a,out,x} - M_{a,out,\theta} = \text{mass change of the desiccant matrix material with time} \quad (1)$$

$$M_{a,out,x} = M_{a,in,x} - \frac{\partial M_{a,in,x}}{\partial x} \Delta x \quad \text{and} \quad M_{a,out,\theta} = M_{a,in,\theta} - \frac{\partial M_{a,in,\theta}}{\partial \theta} \Delta \theta \quad (2)$$

$$-\dot{m}_a \frac{\partial w_a}{\partial x} \Delta x - M_a \frac{\partial w_a}{\partial \theta} \Delta \theta = M_m \frac{\partial w_m}{\partial \theta} \Delta \theta \quad (3)$$

For the time interval λ_j

$$-\dot{m}_a \frac{\partial w_a}{\partial x} \frac{\Delta x}{\Delta \theta} - M_a \frac{\partial w_a}{\partial \theta} \frac{\lambda_j}{\lambda} = M_m \frac{\partial w_m}{\partial \theta} \frac{\lambda_j}{\lambda} \quad (4)$$

$$\dot{m}_a \frac{1}{L} = \dot{m}_{a,j} \frac{\Delta \theta}{\Delta x} \Rightarrow \dot{m}_a \frac{\Delta x}{\Delta \theta} = \dot{m}_{a,j} L \quad (5)$$

Then, the final form can express as following:

$$\dot{m}_{a,j} L \frac{\partial w_a}{\partial x} + M_a \frac{\partial w_a}{\partial \theta} \frac{\lambda_j}{\lambda} + M_m \frac{\partial w_m}{\partial \theta} \frac{\lambda_j}{\lambda} = 0 \quad (6)$$

For the same procedures mentioned previously. Applying the heat transfer balance for the system in order to get the energy equation, [**JoAnna Christen Staton, 1998**]:

The advective heat transfer in the air = heat transfer to the desiccant matrix material wheel

$$\dot{m}_{a,j} L \frac{\partial H_a}{\partial x} + M_a \frac{\partial H_a}{\partial \theta} \frac{\lambda_j}{\lambda} + M_m \frac{\partial H_m}{\partial \theta} \frac{\lambda_j}{\lambda} = 0 \quad (7)$$

For the same procedures, applying the mass balance for the air and desiccant matrix material wheel, [**Gunnar Stiesch, 1994**]:

$$\frac{\dot{m}_a}{\rho_a A} \frac{\partial w_m}{\partial \tau} = h_m (w_a - w_m) \Rightarrow \frac{\partial w_m}{\partial \tau} = \frac{h_m A}{\dot{m}_a / \rho_a} (w_a - w_m) \quad (8)$$

$$\frac{\partial H_m}{\partial \tau} = \frac{hA}{\dot{m}_a C_{pa}} \frac{\partial H_a}{\partial T_a} (T_a - T_m) + \frac{h_m A}{\dot{m}_a / \rho_a} (w_a - w_m) H_m \quad (9)$$

The final heat and mass transfer governing equations based on mass and energy conservation combined with the assumptions for the rotary regenerative heat exchanger as follow, [Gunnar Stiesch, 1994]:

$$1- \text{ Mass conservation: } \frac{\partial w_a}{\partial z} + M_r \frac{\partial w_a}{\partial \tau} + \frac{\partial w_m}{\partial \tau} = 0 \quad (10)$$

$$2- \text{ Energy conservation: } \frac{\partial H_a}{\partial z} + M_r \frac{\partial H_a}{\partial \tau} + \frac{\partial H_m}{\partial \tau} = 0 \quad (11)$$

$$3- \text{ Mass transfer rate: } \frac{\partial w_m}{\partial \tau} = NTU_m (w_a - w_m) \quad (12)$$

$$4- \text{ Thermal energy transfer rate: } \frac{\partial H_m}{\partial \tau} = NTU_t \frac{\partial H_a}{\partial T_a} (T_a - T_m) + NTU_m (w_a - w_m) H_m \quad (13)$$

The dimensionless parameters in Eqs. (10 -13) as following, [Gunnar Stiesch, 1994]:

$$\tau = \frac{\theta}{\lambda_j \Gamma_j} \quad \text{for} \quad 0 \leq \tau \leq \frac{1}{\Gamma_j}, \quad \text{where } \Gamma_j = \frac{M_m}{\lambda \dot{m}_{a,j}}, \quad \text{and } \tau = \frac{\theta}{\lambda_j} \frac{\dot{m}_{a,j}}{M_m} \quad (14)$$

$$z = \frac{x}{L}, \quad NTU_t = \frac{h_a A}{\dot{m}_a C_{pa}}, \quad NTU_m = \frac{h_m A}{\dot{m}_a / \rho_a}, \quad M_r = \frac{M_a}{M_m} \quad (15)$$

The enthalpy of the moist air can expressed as following, [Al-Judi, 1983]:

$$H_a = H_{aa} + H_w = H_{aa} + w_a H_{sa} \quad (16)$$

Where

$$H_{aa} = \begin{cases} 1.007T_a - 0.026 & 0 < T_a < 60 \text{ }^\circ\text{C} \\ 1.005T_a & 0 > T_a > -10 \text{ }^\circ\text{C} \end{cases} \quad (17)$$

$$H_{sa} = 2501 + 1.84T_a \quad (18)$$

Then, Eq. (16) can be written as following:

$$H_a = 1.007T_a - 0.026 + w_a (2501 + 1.84T_a) \quad (19)$$

$$T_a = \frac{H_a - 2501w_a + 0.026}{1.007 + 1.84w_a} \quad \text{and} \quad w_a = \frac{H_a - 1.007T_a + 0.026}{2501 + 1.84T_a} \quad (20)$$

The enthalpy of the matrix desiccant can expressed as following, [JoAnna Christen Staton, 1998]:

$$H_m = (C_{pm} + w_m \times 4.18 \frac{\text{kJ}}{\text{kg.K}}) T_m \quad (21)$$

The specific heat of desiccant (silica gel) known as a function of temperature, and the average value of the specific heat at the room temperature is [JoAnna Christen Staton, 1998].

$$C_{p,m} = 0.95645 \tag{22}$$

$$H_m = (0.95645 + 4180 w_m) T_m \tag{23}$$

Due to the enthalpy and the temperature of the air are dependent on the longitudinal distance and the time, then:

$$H_a = H_a(z, \tau) \text{ and } T_a = T_a(z, \tau) \tag{24}$$

The chain rule is used to find the derivative of the air enthalpy with temperature, as following:

$$\frac{\partial H_a}{\partial T_a} = \frac{\partial H_a}{\partial z} \frac{\partial z}{\partial T_a} + \frac{\partial H_a}{\partial \tau} \frac{\partial \tau}{\partial T_a} \tag{25}$$

Finite Difference Techniques:

Because of the analytical solution for the coupled partial equations with variable parameters is very complicated and difficult, the numerical solution is used. A finite difference technique is used to solve the Eqs. (10-13), as following, [Kreyszig, 1999]:

1- Mass conservation:
$$\frac{w_a^{i,j} - w_a^{i+1,j}}{\Delta z} + M_r \frac{w_a^{i,j+1} - w_a^{i,j}}{\Delta \tau} + \frac{w_m^{i,j+1} - w_m^{i,j}}{\Delta \tau} = 0 \tag{26}$$

2- Energy conservation:
$$\frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} + M_r \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta \tau} + \frac{H_m^{i,j+1} - H_m^{i,j}}{\Delta \tau} = 0 \tag{27}$$

3- Mass transfer rate:
$$\frac{w_m^{i,j+1} - w_m^{i,j}}{\Delta \tau} = NTU_m (w_a^{i,j} - w_m^{i,j}) \tag{28}$$

4- Thermal energy transfer rate:

$$\frac{H_m^{i,j+1} - H_m^{i,j}}{\Delta \tau} = NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{\partial H_a}{\partial z} \frac{\partial z}{\partial T_a} + \frac{\partial H_a}{\partial \tau} \frac{\partial \tau}{\partial T_a} \right) + NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} \tag{29}$$

In order to solve the above equations, firstly determine the relative humidity for the air. Substitution the Eq. (28) in Eq. (26),

$$\frac{w_a^{i,j} - w_a^{i+1,j}}{\Delta z} + M_r \frac{w_a^{i,j+1} - w_a^{i,j}}{\Delta \tau} + NTU_m (w_a^{i,j} - w_m^{i,j}) = 0 \tag{30}$$

$$w_a^{i,j+1} = w_a^{i,j} - \frac{\Delta \tau}{M_r} \left(NTU_m (w_a^{i,j} - w_m^{i,j}) + \frac{w_a^{i,j} - w_a^{i+1,j}}{\Delta z} \right) \tag{31}$$

$$w_a^{i,j+1} = \left(1 - \frac{\Delta \tau}{M_r} NTU_m - \frac{\Delta \tau}{\Delta z M_r} \right) w_a^{i,j} + \frac{\Delta \tau}{M_r} NTU_m w_m^{i,j} + \frac{\Delta \tau}{\Delta z M_r} w_a^{i+1,j} \tag{32}$$

Then from Eq.(28) determine the relative humidity of the desiccant matrix material.

$$w_m^{i,j+1} = w_m^{i,j} + \Delta\tau NTU_m (w_a^{i,j} - w_m^{i,j}) \tag{33}$$

$$w_m^{i,j+1} = (1 - \Delta\tau NTU_m) w_m^{i,j} + \Delta\tau NTU_m w_a^{i,j} \tag{34}$$

For the enthalpy, to determine the air enthalpy, substitution Eq.(29) in Eq.(27),

$$\left\{ \begin{aligned} & \frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} + M_r \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta\tau} + NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} \frac{\Delta z}{T_a^{i+1,j} - T_a^{i,j}} + \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta\tau} \frac{\Delta\tau}{T_a^{i,j+1} - T_a^{i,j}} \right) \\ & + NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} = 0 \end{aligned} \right\} \tag{35}$$

$$\left\{ \begin{aligned} & M_r \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta\tau} + NTU_t (T_a^{i,j} - T_m^{i,j}) \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta\tau} \frac{\Delta\tau}{T_a^{i,j+1} - T_a^{i,j}} \\ & + \frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} + NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} \frac{\Delta z}{T_a^{i+1,j} - T_a^{i,j}} \right) + NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} = 0 \end{aligned} \right\} \tag{36}$$

$$\left\{ \begin{aligned} & M_r \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta\tau} + NTU_t (T_a^{i,j} - T_m^{i,j}) \frac{H_a^{i,j+1} - H_a^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} + \frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} + NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{H_a^{i+1,j} - H_a^{i,j}}{T_a^{i+1,j} - T_a^{i,j}} \right) \\ & + NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} = 0 \end{aligned} \right\} \tag{37}$$

$$(H_a^{i,j+1} - H_a^{i,j}) \left[\frac{M_r}{\Delta\tau} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right] = -(H_a^{i+1,j} - H_a^{i,j}) \left[\frac{1}{\Delta z} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i+1,j} - T_a^{i,j}} \right] - NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} \tag{38}$$

$$H_a^{i,j+1} = H_a^{i,j} - (H_a^{i+1,j} - H_a^{i,j}) \frac{\left[\frac{1}{\Delta z} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i+1,j} - T_a^{i,j}} \right]}{\left[\frac{M_r}{\Delta\tau} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right]} - \frac{NTU_m (w_a^{i,j} - w_m^{i,j})}{\left[\frac{M_r}{\Delta\tau} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right]} H_m^{i,j} \tag{39}$$

$$H_a^{i,j+1} = \left(1 - \frac{\left[\frac{1}{\Delta z} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i+1,j} - T_a^{i,j}} \right]}{\left[\frac{M_r}{\Delta\tau} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right]} \right) H_a^{i,j} + \frac{\left[\frac{1}{\Delta z} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i+1,j} - T_a^{i,j}} \right]}{\left[\frac{M_r}{\Delta\tau} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right]} H_a^{i+1,j} - \frac{NTU_m (w_a^{i,j} - w_m^{i,j})}{\left[\frac{M_r}{\Delta\tau} + NTU_t \frac{T_a^{i,j} - T_m^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right]} H_m^{i,j} \tag{40}$$

Where:

$$H_a^{i,j} = 1.007 T_a^{i,j} - 0.026 + w_a^{i,j} (2501 + 1.84 T_a^{i,j}) \Rightarrow T_a^{i,j} = \frac{H_a^{i,j} + 0.026 - 2501 w_a^{i,j}}{1.007 + 1.84 w_a^{i,j}} \tag{41}$$

$$H_m^{i,j} = (0.95645 + 4180 w_m^{i,j}) T_m^{i,j} \Rightarrow T_m^{i,j} = \frac{H_m^{i,j}}{0.95645 + 4180 w_m^{i,j}} \tag{42}$$

These equations are solved by using trail and error (firstly determined the H_a from Eq. (40) at new time, then determined the T_a from Eq. (41) at a new time and replay the procedures to get the approached values of H_a and T_a with good accuracy.

Finally, the desiccant matrix material enthalpy is determined as following:

$$\frac{H_m^{i,j+1} - H_m^{i,j}}{\Delta\tau} = NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{\partial H_a}{\partial z} \frac{\partial z}{\partial T_a} + \frac{\partial H_a}{\partial \tau} \frac{\partial \tau}{\partial T_a} \right) + NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} \quad (43)$$

$$\left\{ \begin{aligned} \frac{H_m^{i,j+1} - H_m^{i,j}}{\Delta\tau} &= NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{H_a^{i+1,j} - H_a^{i,j}}{\Delta z} \frac{\Delta z}{T_a^{i+1,j} - T_a^{i,j}} + \frac{H_a^{i,j+1} - H_a^{i,j}}{\Delta\tau} \frac{\Delta\tau}{T_a^{i,j+1} - T_a^{i,j}} \right) \\ &+ NTU_m (w_a^{i,j} - w_m^{i,j}) H_m^{i,j} \end{aligned} \right\} \quad (44)$$

$$H_m^{i,j+1} = (1 + \Delta\tau NTU_m (w_a^{i,j} - w_m^{i,j})) H_m^{i,j} + \Delta\tau NTU_t (T_a^{i,j} - T_m^{i,j}) \left(\frac{H_a^{i+1,j} - H_a^{i,j}}{T_a^{i+1,j} - T_a^{i,j}} + \frac{H_a^{i,j+1} - H_a^{i,j}}{T_a^{i,j+1} - T_a^{i,j}} \right) \quad (45)$$

The time interval that used in the present analysis to avoid the negative fluctuation term in the above equations is derived from the Eq. (32) and Eq. (34) as following:

$$1 - \frac{\Delta\tau}{M_r} NTU_m - \frac{\Delta\tau}{\Delta z M_r} \geq 0 \quad \Rightarrow \quad \Delta\tau \leq \frac{M_r \Delta z}{NTU_m \Delta z + 1} \quad (46)$$

or

$$1 - \Delta\tau NTU_m \geq 0 \quad \Rightarrow \quad \Delta\tau \leq \frac{1}{NTU_m} \quad (47)$$

The lower value of the $(\Delta\tau)$ from the Eq. (46) or Eq. (47) will be selected in the computer program. A Quick-Basic computer program is written to calculate the values of the enthalpy, temperature and moisture content for the air and the matrix.

Boundary and Initial Conditions

The conditions in the following analysis are taken as the summer conditions because this condition is very common through the year for Arab countries (**Table-1**), [JoAnna Christen Staton, 1998]:

The boundary and initial conditions that used during the solution process can be described as following.

Two sets of boundary conditions were required to allow for different, independent air conditions of the supply and exhaust air streams, different supply and exhaust boundary conditions must be imposed. For most realistic applications the exiting supply and inlet exhaust conditions would be dependent in some manner.

When the passage of supply air over the surface of the desiccant channel is modeled, the solution satisfies the supply air boundary conditions:

$$T_a(0, \tau) = T_{ai} , \text{ and } w_a(0, \tau) = w_{ai} \quad (48)$$

When the passage of exhaust air over the surface of the desiccant channel is modeled, the solution satisfies the exhaust air boundary conditions:

$$T_a(L, \tau) = T_{ae} , \text{ and } w_a(L, \tau) = w_{ae} \quad (49)$$

The initial conditions applied to the governing equations, regardless of which air flow is being analyzed first, are:

$$T_a(z,0) = T_{ai}, \text{ and } w_a(z,0) = w_{ai} \quad (50)$$

The area of the hot and cold air flow are taken as following (hot air passage area = cold air passage area = $(165^\circ/360^\circ)$ of the total area of the wheel, and the purge passage area = $(30^\circ/360^\circ)$ of the total area of the wheel).

Then after applying the boundary and initial conditions, we calculate the humidity and enthalpy of the air and the matrix are calculated from the Eq.(32), Eq. (34), Eq. (40) and Eq. (45), respectively.

Heat Transfer Coefficient

Yılmaz and Cihan [1993] developed a method to calculate the convective heat transfer coefficient for flow in the channels of various cross-sections for a constant wall temperature boundary condition and showed the Nusselt number can be calculated by using the following equation:

$$Nu_a = Nu_\infty \left[1 + \frac{4.212 \psi \Phi^3}{\gamma Nu_\infty^3} - 0.8 \left(\frac{\psi \Phi^3}{\gamma Nu_\infty^3} \right)^{2/3} \right]^{1/3} \quad (51)$$

In which Nu_∞ is the Nusselt number for the limiting case of $Z \rightarrow \infty$ and can be obtained as follows:

$$Nu_\infty = 3.657 \Phi \quad (52)$$

Where Φ is a heat transfer factor taking the shape of the cross-section of the channel into account for developed velocity and temperature profiles at a constant wall temperature. It is given by [Yılmaz and Cihan, 1993]:

$$\Phi = 1 + \frac{\Phi_\infty - 1}{1 + 1/(n-1)} + \Delta\Phi \quad (53)$$

The parameters Φ_∞ and $\Delta\Phi$ can be calculated by using the following equations:

$$\Phi_\infty = 0.5155 \frac{d^{*2}}{3 - d^*} \quad (54)$$

$$\Delta\Phi = \Delta\Phi_{\max} + \frac{0.95(n-1)^{0.5}}{1 + 0.038(n-1)^3} \quad (55)$$

Where

$$\Delta\Phi_{\max} = \frac{0.007 d^{*8}}{(1 + 10 d^{*-28})(1 + 64 * 10^{-8} d^{*28})^{1/2}} \quad (56)$$

The parameters (d^*) and (n) used in the equations above describe the shape of the channel [Yılmaz and Cihan, 1994]:

$$d^* = \frac{d_{eq}}{d_{max}} \quad \text{and} \quad n = \frac{F}{F_{eq}} = \frac{P}{P_{eq}} \quad (57)$$

Where (F_{eq}) and (P_{eq}) are, respectively, the cross-sectional area and periphery of the equivalent diameter of the flow channel. The corresponding values for (d^*) and (n) are given in **Fig. (3)**. Equivalent diameter (d_{eq}) is defined as [Yilmaz and Cihan, 1994]:

$$d_{eq} = \frac{4F}{P} \quad (58)$$

Where (d_{max}) is the diameter of the largest circle that can be inscribed in the cross-section of the flow channel. The shape factor (ψ), which is required for the calculation of Nusselt number, is given as [Yilmaz and Cihan, 1994]:

$$\psi = 1 + \frac{\psi_{\infty} - 1}{1 + 0.33 d^{*2.25} / (n - 1)} \quad (59)$$

Where

$$\psi_{\infty} = \frac{3}{8} d^{*2} (3 - d^*) \quad (60)$$

The other parameters that used in calculation of the Nusselt number as following [Yilmaz and Cihan, 1994]:

$$\Phi = 1 + \frac{[3(d^*/2)^{7/8} / (1 + d^*)] - 1}{1 + 0.25 / (n - 1)} \quad \text{and} \quad \gamma = \frac{L}{d_{eq} \text{ Re Pr}} \quad (61)$$

Where

$$\text{Re} = \frac{u d_{eq}}{\nu} \quad (62)$$

$$\text{Nu}_a = \frac{h_a d_{eq}}{k_a} \quad (63)$$

Mass Transfer Coefficient

The common equation that used to calculate the mass transfer coefficient is, [Incoropera and DeWitt, 2002]:

$$\text{Sh} = 0.023 \text{Re}^{0.83} \text{Sc}^{0.44} \quad (64)$$

Where Sh is called Sherwood number ($\text{Sh} = \frac{h_m d_{eq}}{D}$), Sc is called Schmidt number ($\text{Sc} = \frac{\nu}{D} = \frac{\mu}{\rho D}$) and D is diffusion coefficient (cm^2/s)

Rotary Regenerative Heat Exchanger Performance:

The performance of the rotary regenerative heat exchanger is calculated by three types of the effectiveness: the sensible (ϵ_s), latent (ϵ_L) and total (ϵ_t) effectiveness.

Energy from rotary heat exchanger is transferred both sensible and latent energy between the air streams due to the desiccant attached to the aluminium substrate of the wheel matrix. Sensible energy transfer is caused by the temperature differences between the two airstreams and latent energy transfer is caused by the water vapour pressure differences between the two airstreams. The total energy transfer rate in energy wheels is the sum of both sensible and latent energy transfer rates. The effectiveness is described as following:

$$\varepsilon = \frac{\dot{m}_s (\zeta_{in} - \zeta_{out})|_s}{\dot{m}_{min} (\zeta_{s,in} - \zeta_{e,in})} \quad (67)$$

Where

ζ = is equal either the dry bulb temperature, the humidity ratio, or the enthalpy of the moisture-carrying air stream. When temperature is used, a sensible effectiveness is obtained, reflecting the wheel's ability to transfer heat. When humidity ratio is used, a latent effectiveness which reflects the wheel's moisture transfer performance is obtained. When enthalpy is used, a total effectiveness is obtained, reflecting the wheel's ability to transfer both heat and moisture.

\dot{m}_{min} = is the minimum value of either \dot{m}_s or \dot{m}_{exit}

Also, in the present analysis a dimensionless parameter used with the enthalpy heat exchanger is called Lewis number (Le), [Schultz , 1987], it is defined as the ratio of the number of transfer units for heat transfer to the number of transfer units for mass (humidity) transfer, as follows:

$$Le = \frac{NTU_T}{NTU_w} \quad (68)$$

In the rotary regenerative heat exchanger, Lewis number is very affected factor. The Lewis number depends on the type of desiccant used and the thickness of the desiccant coating. When the Lewis number greater than one, this means decreasing in the humidity transfer effectiveness compared with the thermal effectiveness for a fixed conditions of inlet states.

Results and Discussion

Before discussion the results, the accuracy of the Q. Basic computer program is checked with the other results [Gunnar Stiesch, 1994], for the data (Le =1, $T_{s,in}=25^\circ\text{C}$, $T_{e,in}=35^\circ\text{C}$, triangular matrix duct, matrix length =0.15 m, matrix diameter =2 m, Silica Gel). The comparison is plotted in Fig. (4), the results of the current program for the sensible heat exchanger effectiveness are very acceptable with the results of the Gunnar Stiesch, 1994.

The variation of the temperature and enthalpy of the inlet air against the dimensionless time is plotted in Fig. (5). The increase of the dimensionless time will increase the temperature and enthalpy due to increase the time for the energy transfer from the hot to cold fluid (air).

The effect of the dimensionless time on the rotary regenerative heat exchanger effectiveness for ($NTU_t=4$ and $Le=2$) is plotted in Figure (6). The increase of the time will increase the sensible, latent and total effectiveness. The sensible effectiveness is increased as the time increased due to transfer of the sensible heat (temperature) from the hot fluid to cold fluid by the desiccant material. The latent effectiveness is increased due to transfer of the relative humidity from the exhaust humid air to the inlet fresh air and then reaches to steady state faster than the sensible heat effectiveness due to the mass transfer by relative humidity which need less time compared with heat transfer by temperature to reach the steady state value. The total effectiveness is also increases because it is the summation effect of the sensible and latent effectiveness. The steady state time for the sensible and total effectiveness is approximately identical and it's greater than the steady state time for the latent effectiveness.

In Fig. (7), the variation of the sensible heat effectiveness is plotted against the dimensionless time constant for different values of the Number of Transfer Unit (NTU_t). The increase of the time

will increase the sensible effectiveness. The increase of the NTU value will increase the sensible effectiveness due to increase of the quantity of the sensible energy that transferred from the hot fluid to the cold fluid. The difference in the steady state sensible effectiveness value for larger values of the NTU_t is become not noticeable due to the system nearly reach to the steady state faster (higher effectiveness) than when the NTU_t value is small (lower effectiveness). The increasing of the NTU_t will increase the steady state value due to increase of the heat transfer between the hot fluid and cold fluid. The same effect is appearing for the total effectiveness with dimensionless time for different values of the NTU_t (as shown **Figure (8)**). Due to the steady state, the total effectiveness is directly proportional to the steady state sensible effectiveness.

The effect of dimensionless time on the sensible effectiveness for different values of the inlet air temperature is plotted in **Figure (9)**. The increasing of the inlet air temperature will increase the sensible heat effectiveness value due to reduce of the maximum heat transfer and in same time increase the energy of the inlet air. Also, the effect of the number of transfer unit on the sensible effectiveness for different values of the inlet air temperature is plotted in **Fig. (10)**. The increasing of the inlet temperature and the number of the transfer unit increases the sensible effectiveness for the same reasons mentioned before.

The effect of the heat exchanger length (matrix wheel length) against the steady state sensible, latent and total effectiveness is plotted in the **Figure (11)** for ($NTU_t=7$ and $Le=2$). The increase of the heat exchanger length will increase the steady state sensible, latent and total effectiveness due to increase the exchange time for the energy and mass transfer between the fluids. The effect of the heat exchanger length on the steady state value of the latent effectiveness is not noticeable compared with the steady state value of the sensible and total effectiveness because the latent effectiveness needs less time to reach the steady state value compared with sensible and total effectiveness.

The effect of the inlet air mass flow rate on the steady state sensible, latent and total effectiveness is plotted in **Figure (12)**. The increase of the inlet air mass flow rate decreased the steady state sensible, latent and total effectiveness due to decrease the number of transfer unit (NTU) for heat and mass transfer. The increase of the mass flow rate will increase the inlet air storage energy and then the inlet air needs more thermal and mass energy in order to rise its temperature and its relative humidity. The effectiveness decreased firstly and this decreasing becomes sharply at high values of the inlet air mass flow rate the reason is the maximum heat transfer and maximum mass transfer become large compared with the actual mass and energy transferring.

The effect of the desiccants material mass on the sensible, latent and total effectiveness is plotted in **Figure (13)**. The increasing of the desiccants material mass (volume) will increase the energy storage inside the desiccant material (internal energy) and then increases the transferred energy from the hot fluid to the cold fluid through the desiccant material.

CONCLUSIONS:

The heat and mass transfer from a rotary regenerative heat exchanger is solved numerically by using finite difference techniques. The following conclusions that drawn from the results of the present work:

- 1- The increasing of the time will increase the sensible, latent and total effectiveness.
- 2- The steady state point for the latent effectiveness is less than the sensible and total effectiveness.
- 3- The increasing of the NTU will increase the effectiveness and reduce the time for reaching the steady state value.
- 4- The increasing of heat exchanger length will increase the sensible and total effectiveness sharply compared with latent heat effectiveness (the increase of the heat exchanger length not affected largely on the latent effectiveness).

- 5- The increasing of the inlet air flow rate will decrease the sensible, latent and total effectiveness.
- 6- The increase of the desiccant material mass (thickness) will increase the total, sensible and latent effectiveness.
- 7- Neglecting the latent heat transfer is not acceptable because the difference between the sensible and the latent effectiveness is noticeable.
- 8- The using of traditional ϵ - NTU method and the LMTD method for this system without any corrections may be lead to mistake in the calculations.

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Table (1): Specifications of the Case Study

Standard	Supply	Return
dry bulb temperature (°C)	35	21
wet bulb temperature(°C)	26	16
relative humidity (%)	47	60
humidity ratio (kg H ₂ O / kg Air)	0.017	0.009
mass flow rate (kg/s)	0.22	0.4
Silica gel density = 1200 kg/m ³		
Aluminum foil density = 2700 kg/m ³ and the specific heat (C _{pa})= 903 J/kg·K		
Rotary heat exchange wheel length = 0.1 m		
Rotary heat exchange wheel diameter = 1 m		
The other needed air properties is determined from the psychometric chart		

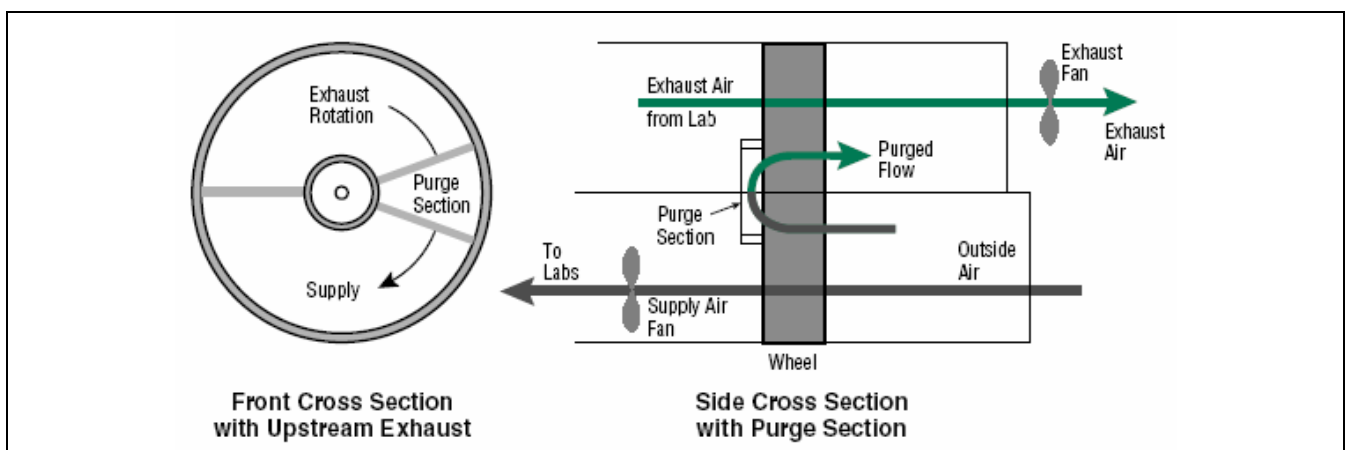


Figure (1) : Cross Sections of Enthalpy Regenerative Heat Exchanger, [Reilly and Van Geet, 2003]

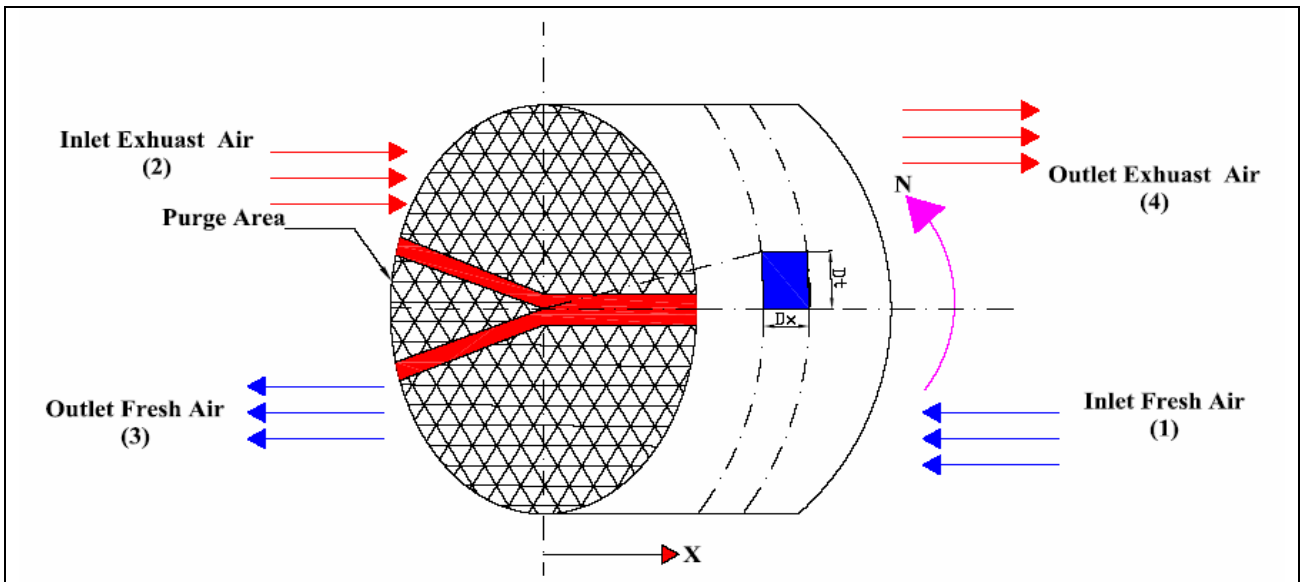


Figure (2) : Schematic of Enthalpy Regenerative Heat Exchanger, [Gunnar Stiesch, 1994]

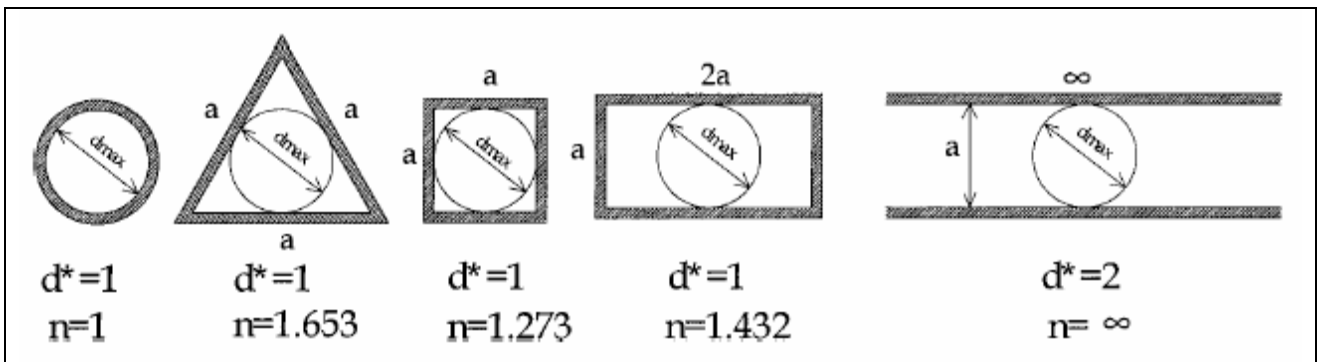
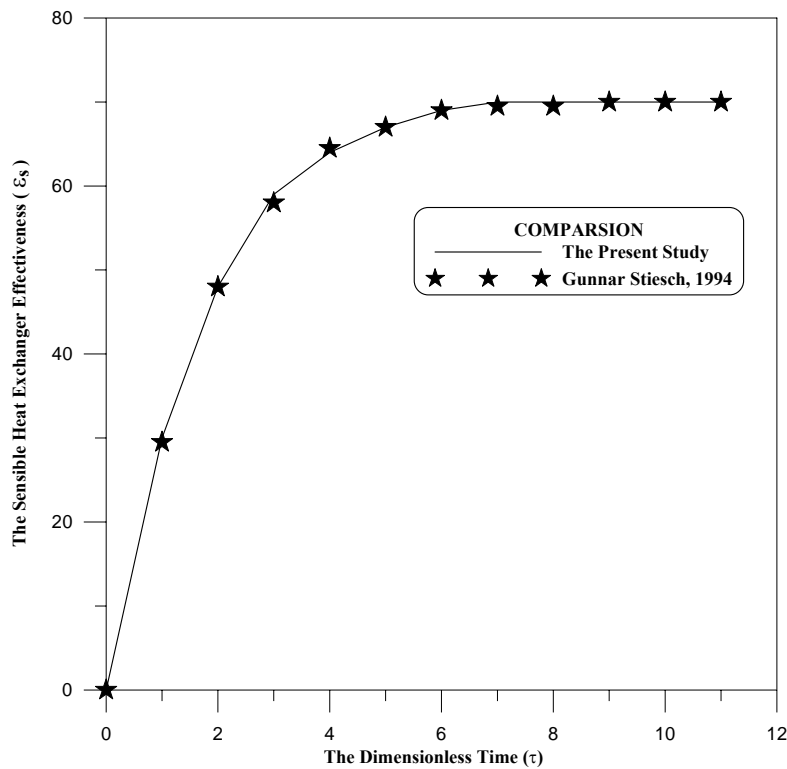
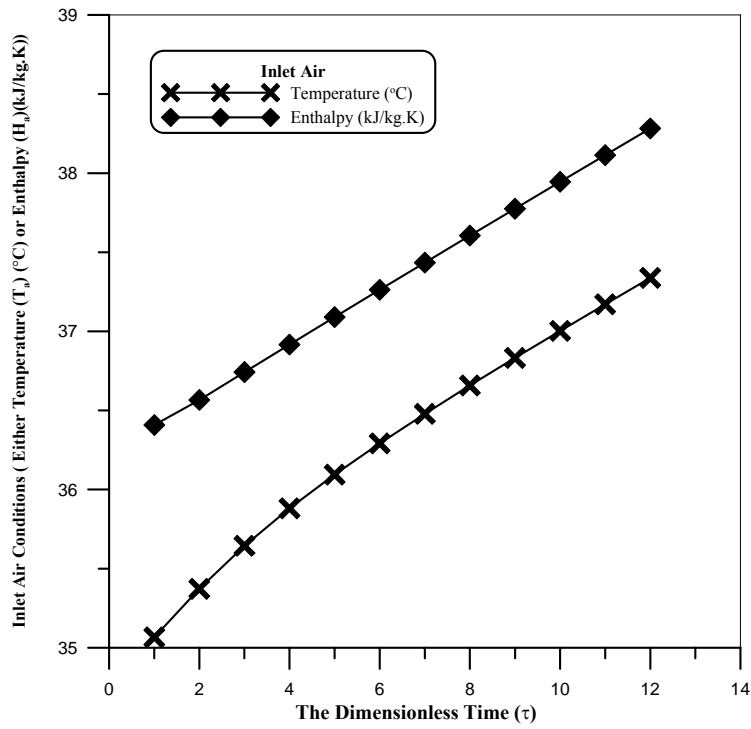


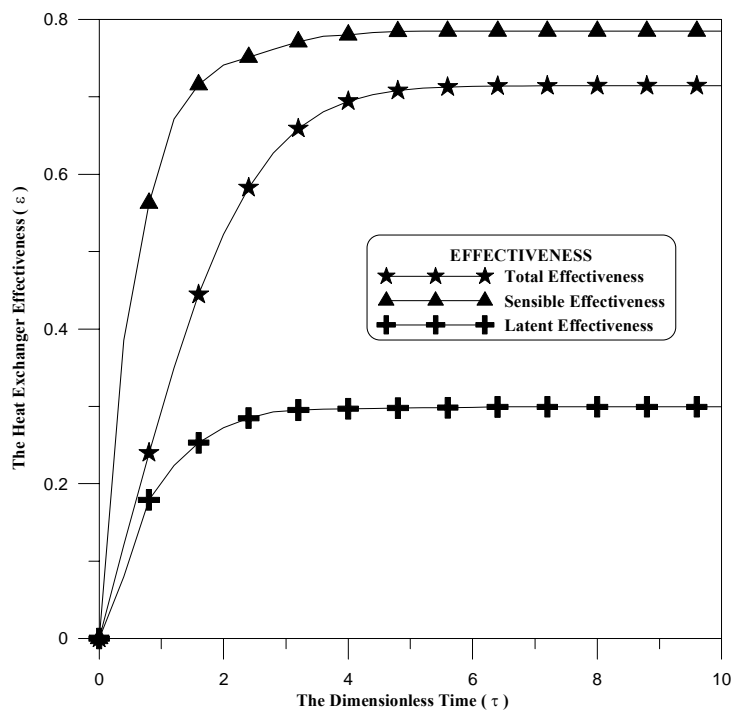
Figure (3) : The Parameters d^* and n for Different Channel Shape [Yilmaz and Ukalaca, 2003]



Figure(4): Comparison between the present study and Gunnar Stiesch, 1994, ($Le = 1$, $T_{s,in} = 25^\circ C$, $T_{e,in} = 35^\circ C$, triangular matrix duct, matrix length = 0.15 m, matrix diameter = 2 m, Silica Gel)



Figure(5): The variation of the temperature and enthalpy of the inlet air against the dimensionless time



Figure(6): The total, sensible and latent effectiveness for the heat exchanger against the dimensionless time for ($NTU_t = 4$ and $Le = 2$)

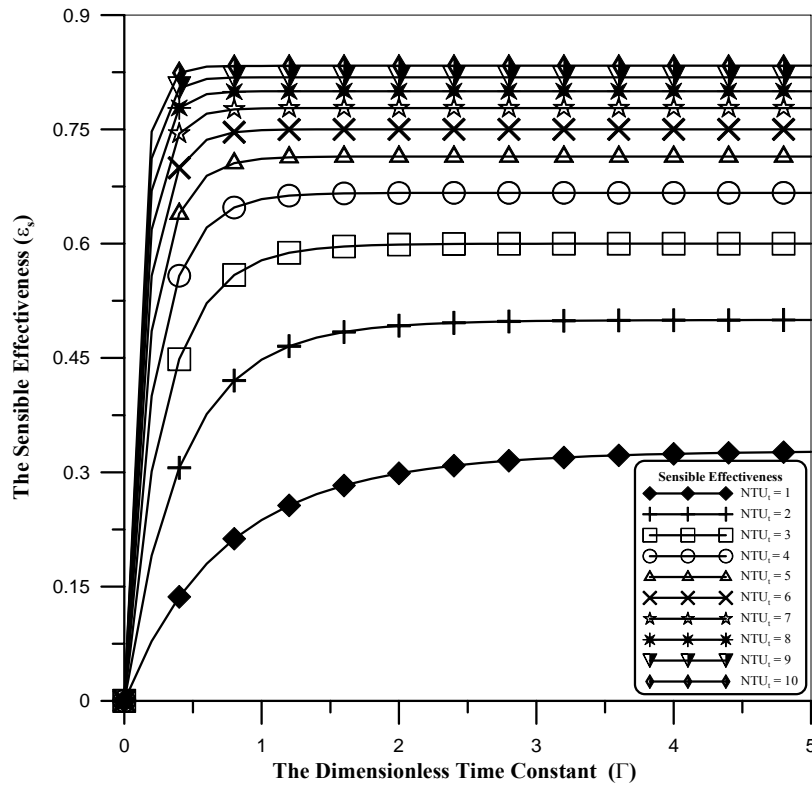


Figure.(7): The effect of the dimensionless time constant (Γ) on the sensible effectiveness for different values of the NTU_t

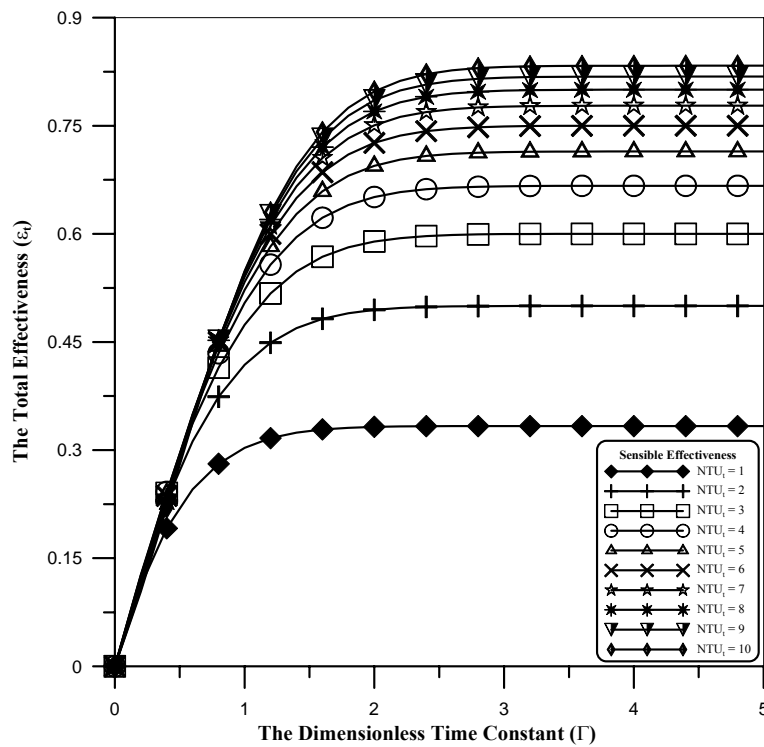


Figure (8): The effect of the dimensionless time constant (Γ) on the total effectiveness for different values of the NTU_t

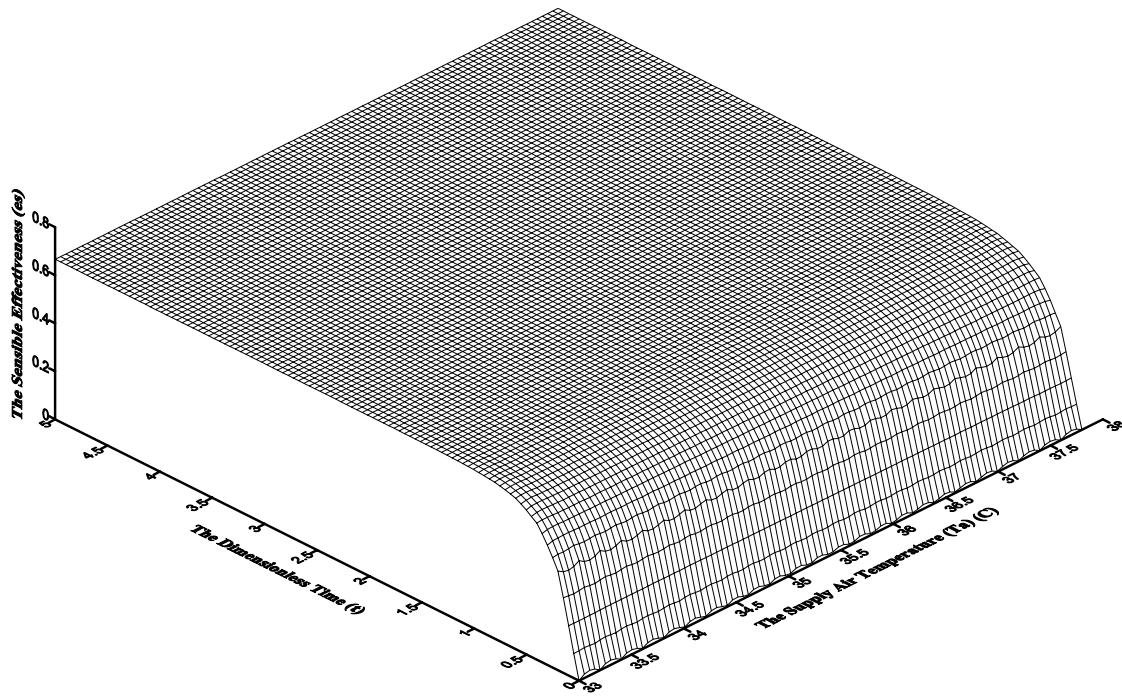


Figure (9): The effect of dimensionless time (τ) on the sensible effectiveness for different values of the inlet air temperature

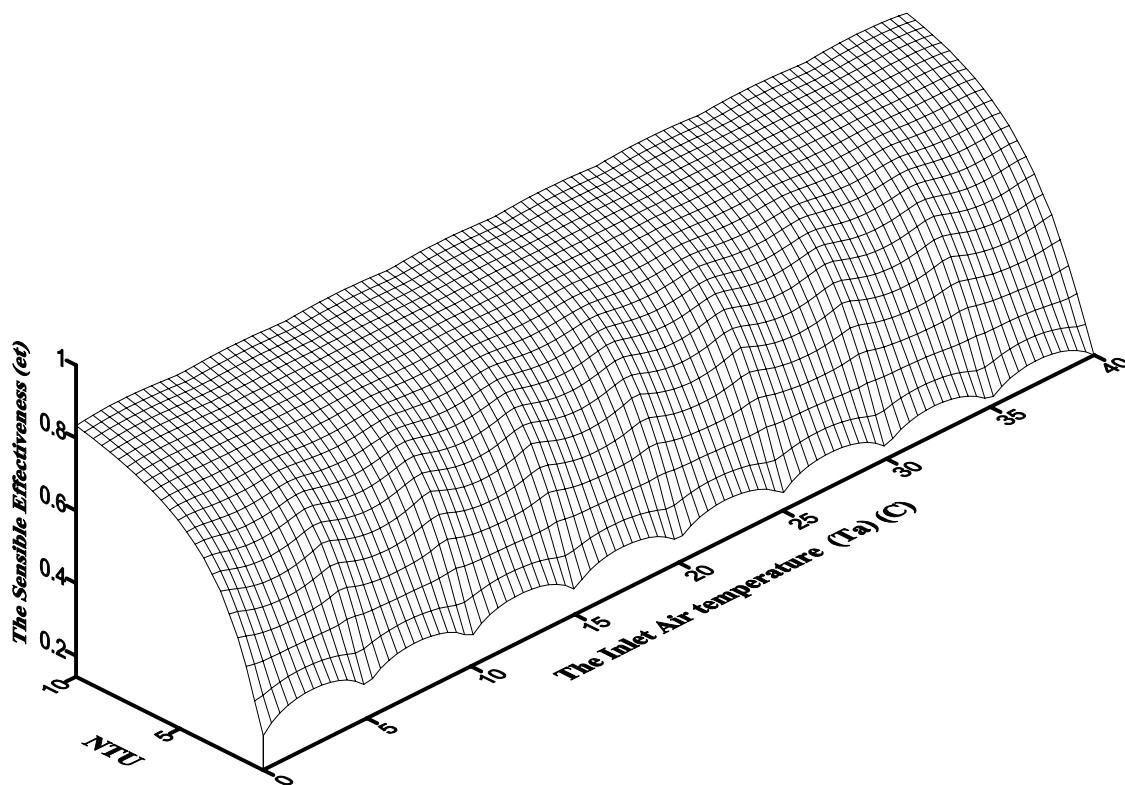


Figure (10): The effect of the number of transfer unit on the sensible effectiveness for different values of the inlet air temperature

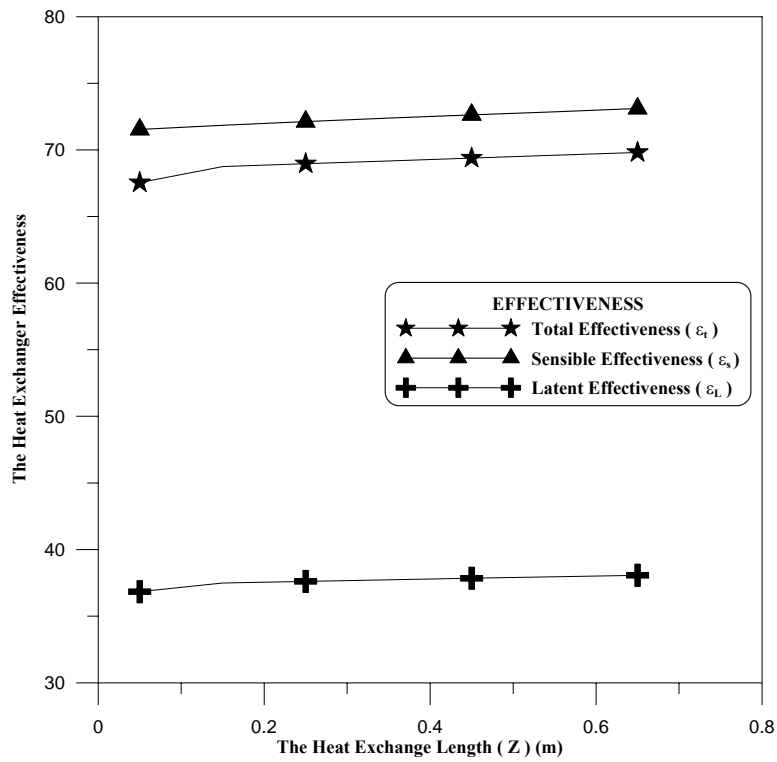


Figure (11): The effect of the heat exchanger length on the total, sensible and latent effectiveness ($NTU_t = 7$ and $Le = 2$)

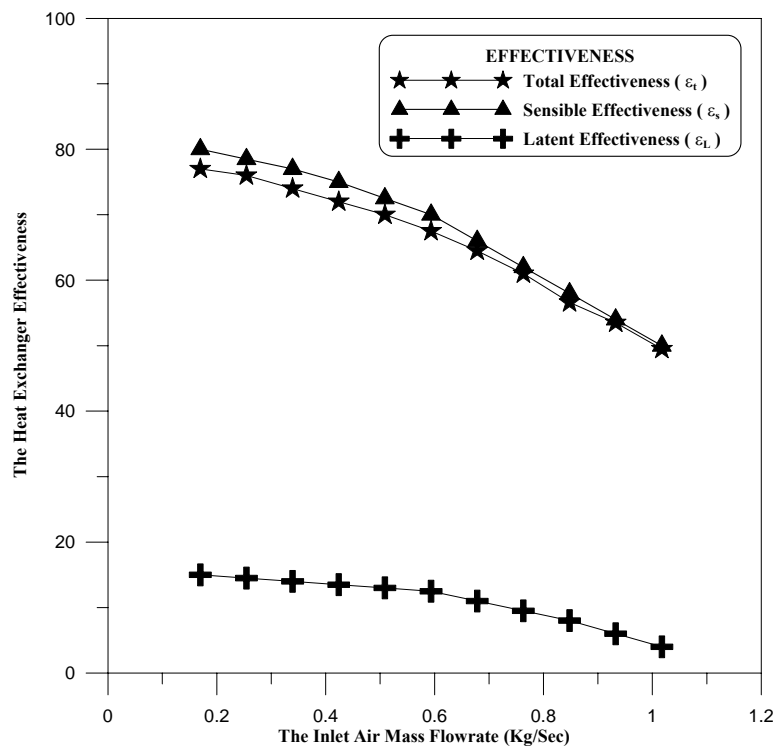
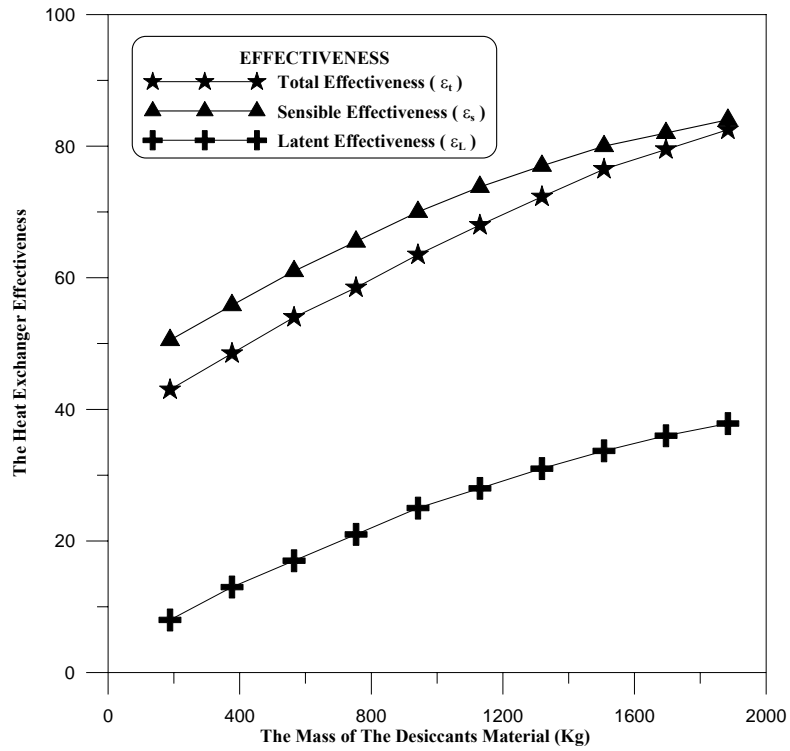


Figure (12): The effect of the inlet air mass flow rate on the sensible, latent and total effectiveness



Figure(13): The effect of the desiccants material mass on the sensible, latent and total effectiveness