Published by
NIGERIAN SOCIETY OF PHYSICAL SCIENCES
Available online @ https://journal.nsps.org.ng/index.php/jnsps

J. Nig. Soc. Phys. Sci. 5 (2023) 1512

Journal of the Nigerian Society of Physical Sciences

Simulation of the Movement of Groundwater in an Aquifer

Suha Ibrahim Salih AL-Ali, Nihad Jalal Kadhem AL-Awsi

Computer Science and Mathematics Department, College of Computer Science and Mathematics, Tikrit University, Tikrit, Iraq

Abstract

This study investigates the impact of extracting fresh water from areas where salt water and fresh water meet in tropical regions. Traditionally, fresh water is expected to be found above salt water in the ocean or underground. To carry out the investigation, Green's Function method is used, and a numerical chart is presented that includes an equation derived from Green's II matching. The study computes the shape of the interface during water withdrawal and flows through the basins and sources of the line. In addition, this study obtains an analytical solution to the linear problem for the non-withdrawal scenario. Finally, the study identifies the maximum rate of water withdrawal before the initial breakthrough of salt water for different density ratios.

DOI:10.46481/jnsps.2023.1512

Keywords: Free Surface, Tropical Island, Porous Medium, Freshwater Lens, Island Tub, Green's Function.

Article History :
Received: 21 April 2023
Received in revised form: 19 May 2023
Accepted for publication: 22 May 2023
Published: 22 June 2023

Communicated by: B. J. Falaye

1. Introduction

The provision of drinking water in tropical islands often relies on a freshwater lens trapped in the soil beneath the surface. However, this supply is threatened by various factors, such as the interface between freshwater and saltwater layers, surface channels, and pumping practices. To ensure continued access to freshwater, it is crucial to conserve the freshwater lens and use efficient extraction practices to prevent saltwater intrusion and depletion of the freshwater supply [1]. Figure (1) provides a visual representation of this process. According to the United Nations, around 2.2 billion people lack access to safe drinking water, and this number is expected to increase due to population growth and climate change [2]. Small island developing states (SIDS) are particularly vulnerable to water scarcity because of their limited resources, geographic isolation, and susceptibility to natural disasters [3]. Climate change exacerbates the water scarcity crisis in SIDS by increasing the frequency and intensity of extreme weather events, causing saltwater intrusion, and reducing precipitation [4]. For example, the Pacific Island nation of Tuvalu faces severe water shortages due to droughts and over-extraction from the freshwater lens, which has led to soil and groundwater salinization [5]. Some island communities are responding to these difficulties by implementing creative solutions such as rainwater collection, desalination, and water conservation techniques. These solutions, however, necessitate money and technical skill, both of which may be limited in SIDS [6, 7]. Furthermore, policies that promote water security and sustainability in these vulnerable areas are required [8, 9]. The interface between low-density top liquids and higher-density liquids in a two-layer water object changes shape when

Email address: suhaibrahim3@tu.edu.iq (Suha Ibrahim Salih AL-Ali)

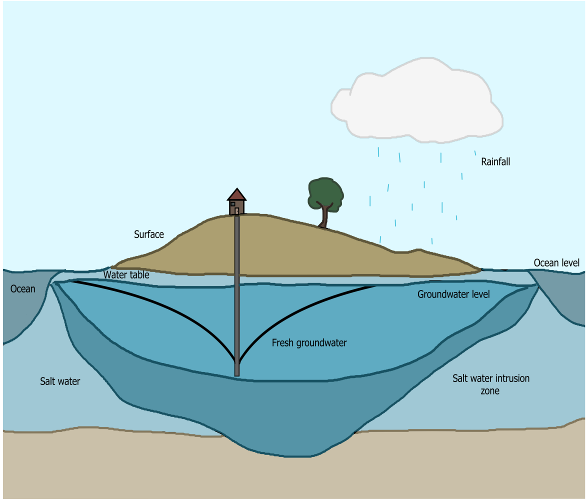

Figure 1. A diagram indicate the focus of research-pulling freshwater over the underground seawater on an island

liquid is preferentially removed from one layer, according to research [10]. One of the primary goals of this research is to validate the deformed interface shape.

The goal of this study is to look into the variables that could cause the interface to advance towards the extraction port, resulting in liquid extraction from both levels and mixing. The research focuses on a single island with defined top left and right bounds that contains a freshwater lens atop a saltwater layer [11]. To address this issue, academics employ the Darcy Act as a foundation, which contains boundary conditions on both sides and the island's bottom. This study is in the field of hydrology, and it focuses on saltwater intrusion in coastal aquifers, which can cause a variety of environmental and economic problems [12]. The authors assume the presence of a freshwater lens above a saltwater layer, which is a common condition in real-world scenarios like Aruba. The findings of the study can be used to develop policies and management methods to reduce saltwater intrusion in coastal aquifers [13].

Section 3 of the study employs an analytical technique [14] to address the issue raised in Section 2. The Fourier series and its properties are used as the principal tool for problem analysis [15, 16]. It was explained in detail how the series' resulting factors are implemented in MATLAB to generate a solution. The Fourier series is a mathematical approach that uses sine and cosine functions to describe periodic functions as a sum of the sin and cos functions. It is frequently employed in a variety of domains, including signal processing, communication systems, and control theory [17]. The Fourier series' handling qualities are critical to solving the problem stated in Section 2, as they allow the authors to analyse the system's periodic nature.

2. Formulation of free surface case problem

On the island, there is a porous medium with dimensions of $-L < x < L$, which contains fluid of varying densities, specifically two layers of freshwater and saltwater. The properties of these fluids can be described using the following equations.

$$\Omega_i = p_i + \rho_i g y, \quad y = \xi(x), \tag{1}$$

where $i = 1, 2$ and $y = \xi(x)$. Equation (1) represents the sum of total pressures, which remains constant even when gravity changes. This is called the compression of heads in the two layers. The variables, ρ_i andp_i represent the density and pressure of each layer of fluid, respectively. The interface between the two fluids is denoted by $y = \xi(x)$, and the surface of the less dense fluid is assumed to be in contact with air.

Since the air is assumed to be in a stable state, the pressure at the horizontal surface of the less dense fluid must be equal to the atmospheric pressure, which is denoted by $p_1 = 0$. Additionally, we assume that there is no flow through the fluid surface interface and no flow in the lower layer, i.e.,

$$\Omega_i = \rho_1 g y. \tag{2}$$

When dealing with a non-persistent pressure situation, it is physically impossible to obtain a fixed solution. Thus, the pressure along the interface of two fluid layers must match, specifically at the point where $y = \xi(x)$. This means that the pressures of both fluid layers, denoted by $p1$ and $p2$, must be equal i.e.,

$$p_1 = p_2, \tag{3}$$

at this interface point. This condition ensures that the fluids are in hydrostatic equilibrium and that there is no net force acting on the interface. Assuming no fluid flow across the surface interface and in the lower layer (i.e., $\Omega_2 = 0$) and with $y = \xi(x)$, we can obtain:

$$p_2 = -\rho_2 g y. \tag{4}$$

By matching the pressures at the fluid interface, we can conclude the following:

$$\Omega_1 = (\rho_1 - \rho_2)\, g y, \tag{5}$$

and by dividing (5) by $\rho_1 g$ to be $\Omega_1 = \rho_1 g z \omega_1$ we will have the following equation:

$$\omega_1 = (1 - \psi). \tag{6}$$

In order to simulate the process of extracting liquids from a freshwater lens on a tropical island, a tub was positioned at coordinates $(x_s,\ y_s)$. To achieve this, we applied the following formula to Ω_1:

$$\Omega_1 \to \frac{m}{4\pi} ln[(x - x_s)^2 + (y + y_s)^2]. \tag{7}$$

We then applied the equation $\Omega_1 = p_1 g z \omega_1$, taking into account that $\mu = \frac{z}{p_1 g} as\,(x,\ y)$ approaches $(x_s,\ y_s)$.

$$\Omega_1 \to \frac{\mu}{4\pi} ln[(x - x_s)^2 + (y + y_s)^2]. \tag{8}$$

Assuming that the tub pressure force and density ratio between the two layers are represented by the balanced ratios of L and Ψ, and the island's aspect ratio is represented by the balanced ratio of μ, we can rewrite the given equations as follows: Using the expression $\nabla\Omega_1 = \frac{\rho_1 g z}{z}\nabla\omega_1$, where $q = -\kappa\nabla\omega_1$ represents the speed, we can use Darcy's law to find the relation between q, z, and $\nabla\omega_1$:

$$q\frac{z}{T} = -\kappa\rho_1 g\nabla\omega_1. \quad (9)$$

Since $T = \frac{z}{\rho_1 g}$, we can simplify equation (9) as follows:

$$q = -\nabla\omega_1. \quad (10)$$

Equation (10) shows that q is directly proportional to $-\nabla\omega_1$. The parameters L, ψ, and μ have balanced ratios that represent the tub pressure force, the density ratio between the two layers, and the island's aspect ratio, respectively.

3. Freshwater lens on an island with No-withdrawal

To solve the linear problem, Laplace's equation is used with the conditions specified in equation

$$\nabla^2\omega_1 = \frac{\partial^2\omega_1}{\partial x^2}+\frac{\partial^2\omega_1}{\partial y^2} = 0 \quad -L < x < L,\ \zeta(x) < y < 1, (11)$$

which is subject to a range of conditions:

$$\begin{cases}\omega_1 = 1, y = 1 & -L < x < L\\ \omega_1 = y, x = L & 0 < y < 1.\end{cases}$$

We assume $\omega_1 = \omega$ and use the formula given in equation

$$\omega(x,\ y) = y+\sum_{k=0}^{N} S_k \sinh\left[\left(\frac{k}{L}\right)\pi(y-1)\right]\ \sin\left[\left(\frac{k}{L}\right)\pi k\right].(12)$$

and equation

$$\begin{cases}\omega = (1-\psi)\,y = 1 & y = \zeta(x)\\ \omega_y - \zeta'(x)\,\omega_x = 0,\end{cases} \quad (13)$$

defines additional conditions that must be met, including $\omega = (1-\psi)y = 1$ and $\omega_y = 0$ at $y = 0$, where we take a small value of ζ assume a large value of ψ.

$$\omega_y = 1+\sum_{k=0}^{N} S_k\left(\frac{k}{L}\right)\pi\cosh\left[\left(\frac{k}{L}\right)\pi(y-1)\right]\ \sin\left[\left(\frac{k}{L}\right)\pi x\right].(14)$$

At $y = 0$, equation (14) becomes:

$$-1 = \sum_{k=0}^{N} S_k\left(\frac{k}{L}\right)\pi\cosh\left(-\frac{k}{L}\right)\pi\sin\left[\left(\frac{k}{L}\right)\pi x\right]. \quad (15)$$

To apply Orthogonal, we multiply both sides of (15) by $\sin\left(\frac{j}{L}\right)\pi x$, integrate over the range$-L$ to L, and get:

$$\int_{-L}^{L} sin\left(\frac{j}{L}\right)\pi x\, dx = \int_{-L}^{L}\sum_{k=0}^{N} -S_k\left(\frac{k}{L}\right)\pi\cosh\left(-\frac{k}{L}\right)\pi\sin^2\left(\frac{j}{L}\right)\pi x\, dx, \quad (16)$$

assume that $A_k = S_k\left(\frac{k}{L}\right)\pi\cosh\left(-\frac{k}{L}\right)\pi$, i.e.

$$S_k = \frac{2[(-1)^k-1]}{k\pi}.$$

From which we can conclude that:

$$A_k = \frac{2L[(-1)^k-1]}{(k\pi)^2\cosh(\frac{k}{L})\pi}. \quad (17)$$

Therefore, the Fourier series approximation is:

$$\Omega_1(x,\ y) = y+\sum_{k=0}^{N} S_k\left[\left(\frac{k}{L}\right)\pi(y-1)\right]\sin\left[\left(\frac{k}{L}\right)\pi k\right]. \quad (18)$$

Using (13) and evaluating it at $y = 0$, we get an approximation of $\zeta = \frac{\omega}{1-\psi}$:

$$\zeta = \frac{\omega}{1-\psi}\left[\sum_{k=0}^{N} S_k shin\left(\frac{k}{L}\right)\pi\ \ sin\left[\left(\frac{k}{L}\right)\pi x\right]\right]. \quad (19)$$

The linear island problem involves the interface between freshwater and saltwater, and the depth of this interface is determined by the density rate. The Ghyben-Herzberg relation, which is based on hydrological principles, provides a formula for determining the depth of the interface in a system that is in a constant state of balance. The depth of the interface, according to this relationship, is proportional to the ratio of the freshwater head to the total head, where the total head is the sum of the freshwater head and the depth of the interface below sea level [18]. Therefore, if the freshwater head increases, the depth of the interface will also increase, and if the freshwater head decreases, the depth of the interface will decrease as well. The resulting approximation of ζ is given in Figure (2) shows the resulting approximation of ζ for an island with width L=100 and a density ratio of 1.01. The measurements were conducted with varying intensity ratios, and it was discovered that the best measurement was achieved at a density value of $\psi = 1.01$ with $n = 300$ points. To analyse the interface between freshwater and saltwa-

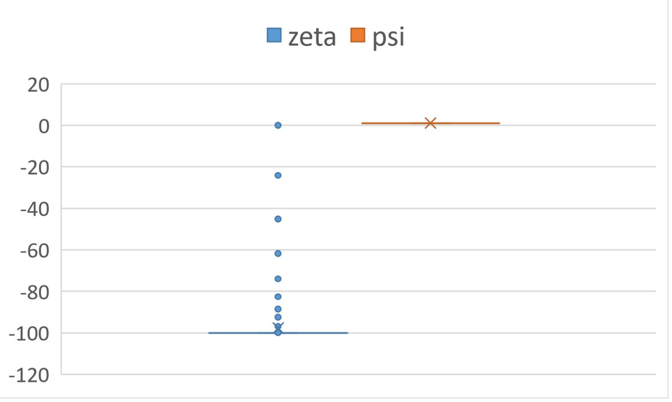

Figure 2. A Diagram determine w-plane and z-plane for free surface condition

ter in the linear island problem, researchers use the Fourier series approximation, a mathematical technique that breaks down complex functions into simpler functions that can be more easily studied. According to the Fourier series approximation, the depth of the interface is inversely proportional to the density rate, which means that as the density rate increases, the interface depth decreases, and vice versa. The Ghyben-Herzberg relation, which states that $\zeta = \frac{\omega}{1-\psi}$, also supports this relationship between the interface depth and the density rate. In a system in constant balance, when freshwater is present above the interface, the depth of the interface above sea level is equal to the depth of the interface below sea level. Both the Fourier series approximation and the Ghyben-Herzberg relation are crucial tools for understanding the interface between freshwater and saltwater in the linear island problem [19, 30]. These principles have practical applications in fields such as hydrology and groundwater management, where researchers can use them to better understand groundwater systems and make informed decisions about resource management and conservation [20].

3.1. Integration of the free surface problem equation

To solve a problem numerically, we need to find an appropriate solution that satisfies the governing equation and boundary conditions. In the case of Laplace's equation, we use Green's function F to find a single solution that meets the necessary boundary requirements for the area of concern [21]. Green's function F can be derived using the free surface condition, which is a solution to the equation:

$$\nabla^2 F(x,\ y,\ x_0,\ y_0) = \delta(x - x_0,\ y - y_0), \tag{20}$$

where the function is subject to the boundary conditions:

$$F(\pm L,\ y,\ x_0,\ y_0) = 0, \quad 0 < y < 1. \tag{21}$$

$$F(x,\ 1,\ x_0,\ y_0) = 0, \quad -L < y < L. \tag{22}$$

To solve this equation using integral equations or finite element methods, we only need to find a solution for on the boundary. This is achieved by using Green's second identity and applying the boundary clauses of equations (21) and (22), which ensures that the integration line of remains on the boundary [22].

The appropriate Green's function F for this problem needs to satisfy the conditions of equations (20), (21), and (22) [23]. One way to find this function is by using the techniques of portfolio conversions of angles in composite variables and applying the solution on the w-plane to the physical (real) plane. We can consider logarithms to be individual at $w = \ w_0$, where $w_0 = \ u_0 - \ iv_0$ and $\bar{w_0} \ = \ u_0 - \ iv_0$. We need a function that evaluates to zero on the real axis, and so we test:

$$F = Re\left[\ln(w - \ w_0) - \ln(w - \bar{w_0})\right]. \tag{23}$$

There are different methods to find the appropriate Green's function F for a particular problem, depending on the boundary conditions and the governing equation. Some examples of these methods include the method of images, separation of variables, and the method of integral equations.

$$f(z) = A\int_0^z (\tau - x_1)^{2L_1}(\tau - x_2)^{2L_2}\dots(\tau - x_n)^{2L_n}\, d\tau + C. \tag{24}$$

To convert from the w-plane to the z-plane for each L_i as shown in Figure (3), the conversion should be applied to the inner corners of the polygon. Functions of form (24) were used to transform the real axis into a polygonal path. Beginning with the assumption that $w = 1$ corresponds to $z = i + L$, the appropriate values were offset in (23) to obtain the desired outcome.

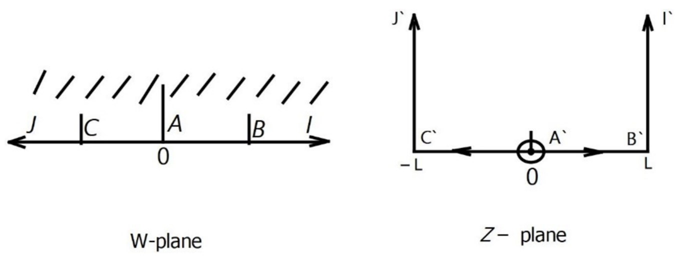

Figure 3. Diagram determine w-plane and z-plane for free surface condition

$$z = A\int_1^w (w-1)^{\frac{-1}{2}}(w+1)^{\frac{-1}{2}}\, dw + i + L$$
$$= A\int_1^w (w-1)^{\frac{-1}{2}}\, dw + i + L. \tag{25}$$

And by using the definitions $\cosh^2\theta - \sinh^2\theta = 1$ we obtain

$$z = A\int_1^w (\sinh^2\theta)^{\frac{-1}{2}}\ \sinh\theta\, d\theta + i + L. \tag{26}$$

To clarify the given expression, let us rewrite it as follows: Let $w = \cosh\left[\frac{z-(i+L)}{A}\right]$. Since $\cosh(D)$ cannot be zero unless D is complex, we can use $\cosh y = \cos iy$ and set $\cos\left(\frac{-iL}{A}\right) = 0$, which yields $A = \frac{-iL}{2\pi}$.
Using this, we can rewrite w as:

$$w = \cosh\left[\frac{z-(i+L)}{-i2L\pi}\right] = \cosh\left[\frac{\pi(z-i)}{2L} - \frac{\pi}{2}\right] = \cos i\left[\frac{\pi(z-i)}{2L} - \frac{\pi}{2}\right]$$
$$= \cos i\left(z - \frac{\pi}{2}\right) = \sin z, \tag{27}$$

and when $z = x + iy$ we will obtain

$$w = \sin\frac{\pi x}{2L} + \cosh\frac{\pi(y-1)}{2L}$$
$$+ i\sinh\frac{\pi(y-1)}{2L}\cos\frac{\pi x}{2L}. \tag{28}$$

Using the Schwarz-Christoffel transformation, we can obtain the following function: [missing function].

$$F = ln\left|\frac{w - \ w_0}{(w - \bar{w_0}}\right|, \tag{29}$$

and now let us say that

$$f = \ \sin\frac{\pi x}{2L}\ \cosh\frac{\pi(y-1)}{2L}, \tag{30}$$

$$g = \ \sinh\frac{\pi(y-1)}{2L}\cos\frac{\pi x}{2L}, \tag{31}$$

We will obtain a

$$\frac{w - w_0}{w - \bar{w_0}} = \frac{(f - f_0) + i(g - g_0)}{(f - f_0) + i(g + g_0)}, \tag{32}$$

and when you take the real part of F, we find that (25) simplified to the follows:

$$F = \frac{1}{2}\, ln \frac{(f - f_0)^2 + (g - g)^2}{(f - f_0)^2 + (g + g)^2}. \tag{33}$$

3.2. Derivation of the free surface problem equation

We will now derive the complementary equation for the undefined interface, which needs to satisfy the following equation, as mentioned earlier:

$$\nabla^2 \omega_1 = \frac{\partial^2 \omega_1}{\partial x^2} + \frac{\partial^2 \omega_1}{\partial y^2} = 0 \quad -L < x < L,\ \zeta(x) < y < 1. \tag{34}$$

These equations are subject to the following boundary conditions:

$$f(x) \begin{cases} \omega_1 = 1, y = 1, -L < x < L \\ \omega_1 = y, x = \pm L, 0 < y < 1. \end{cases} \tag{35}$$

We also impose the condition that the pressure must match along the interface between saltwater and freshwater where $y = \zeta(x)$. Furthermore, we assume that there is no flow through the interface, leading to the following conditions:

$$\begin{cases} \frac{\partial \omega_1}{\partial n}, -L < x < L \\ \omega_1 = y, x = \pm L, 0 < y < 1 \end{cases} \tag{36}$$

When considering fluid dynamics, it is important to keep in mind the effect of negative pressure which results from the pulling of liquids. To address this issue, Green's second identity is commonly used as a tool in solving related equations [24, 225].

By utilizing the condition that the Laplacian of a function $\nabla^2 \omega_1$ is equal to zero ($\nabla^2 \omega_1 = 0$), we can simplify the equation and arrive at the expression below:

$$\omega_1 \to \frac{\mu}{2\pi} ln \sqrt{(x - x_s)^2 + (y - y_s)^2}. \tag{37}$$

Here, F is a function that satisfies Laplace's equation, i.e., $\nabla^2 F = 0$, except at a specific point (x_0, y_0). Along the free surface, $\partial\omega/\partial n = 0$ and similarly, along the other three boundaries (top, right side, and left side), $F = 0$. As a result, the second term in Equation (1) falls away, leading to:

$$\int_\Gamma \omega \frac{\partial F}{\partial n} - \omega \frac{\partial \omega}{\partial n} = 0. \tag{38}$$

However, we must also consider what happens along the boundaries s_1, s_{E_1} s_{E_2}, wheres_1 is the limit along the boundary and the top, s_{E_1} is the ring around the source (i.e., where the liquid passes through), and s_{E_2} is the ring around a single point on the surface. Therefore, we obtain:

$$\int_{S_1} \omega \frac{\partial F}{\partial n} ds = \int_{s_{E_1}} F \frac{\partial \omega}{\partial n} ds = \int_{s_{E_2}} \omega \frac{\partial F}{\partial n} ds = 0. \tag{39}$$

Assuming that a single point is enclosed within the ring s_{E_1}, which causes all the liquid to pass through it, we can consider its specific behaviour.

$$\omega \to \frac{1}{4\pi} ln \left[\frac{(f_s - f_0)^2 + i(g_s - g_0)^2}{(f_s - f_0)^2 + i(g_s + g_0)^2} \right]. \tag{40}$$

We start by considering a function of a point $(x_0,\ y_0)$. Now, let's consider $\frac{\partial \omega}{\partial n}$ as an integral of s_{E_1}. This integral represents the speed of the liquid out of the tub. We can express this as follows:

$$\int_{S_1} -F \frac{\partial \omega}{\partial n} ds = -F \int_{s_{E_1}} \frac{\partial \omega}{\partial n} ds$$

$$= F \int_{s_{E_2}} \frac{\mu}{2\pi} . \frac{1}{r} ds = \frac{F\mu}{2\pi} \int_{2\pi}^{0} r. \frac{1}{r} d\theta = F\mu, \tag{41}$$

where $r = [(x - xs)^2 + (y - ys)^2]^{\frac{1}{2}}$ is the distance from the source point (x_s, y_s) to the point (x, y). In other words, the flow from the tub has force μ, so the contribution from the integration of s_{E_1} is $\mu F(x_s,\ y_s)$. Therefore, we can write:

$$\frac{\mu}{4\pi} ln \left[\frac{(f_0 - f_s)^2 + (g_0 - g_s)^2}{(f_0 - f_s)^2 + (g_0 + g_s)^2} \right] + \int_{s_1} \omega \frac{\partial F}{\partial n} ds = \int_{s_{E_2}} F \frac{\partial \omega}{\partial n} ds = 0, \tag{42}$$

where (f_s, g_s) is the location of the source point, and (f_0, g_0) is the location of the point at which we are evaluating the function. We can think of the integral along the s_{E_1} line as the integration of a function with a constantly changing value along this line where $(x, y) \to (x_0, y_0)$. $\int \frac{\partial F}{\partial n} ds$ is "the flow" of "the source" due to the effect of F. Since the loop around the tub point (x_0, y_0) is a semicircle, the flow volume due to the source the tub F is $Q = \pi$. Thus, the second integration in (42) could be $\omega\pi(x_0, y_0)$. Therefore, we can write:

$$\frac{\mu}{4\pi} ln \left[\frac{(f_0 - f_s)^2 + (g_0 - g_s)^2}{(f_0 - f_s)^2 + (g_0 + g_s)^2} \right] + \int_{s_1} \omega \frac{\partial F}{\partial n} ds - \omega\pi(x_0,\ y_0) = 0. \tag{43}$$

Note that there is a single point of integration on s_1 such as $(x, y) \to (x_0, y_0)$, which is the surface point. To deal with this single point, we extract from the method of collecting and subtracting ω_0 under integration, giving us the following formula:

$$\frac{\mu}{4\pi} ln \left[\frac{(f_0 - f_s)^2 + (g_0 - g_s)^2}{(f_0 - f_s)^2 + (g_0 + g_s)^2} \right] - \omega\pi(x_0,\ y_0) + \int_{s_1} (\omega - \omega_0) \frac{\partial F}{\partial n} ds + \omega_0 \int_{s_{E1}} \frac{\partial F}{\partial n} ds = 0. \tag{44}$$

4. Outcomes And discussion

The study examined the impact of density ratios on the formation of the interface and depth of the freshwater lens in free surface models. It used both analytical and numerical methods to solve the problem of individual free surfaces. The Fourier series and its orthogonal properties were employed to calculate the interface without the need for pumping or pulling on the island. The study also explored the impact of the presence or absence of a source/tub on the island on the maximum pulling rates at different density ratios. With no statistically significant difference between the analytical and numerical solutions, the results demonstrated that the lowest intensity situation resulted in the greatest pulling rate. The findings were consistent with previous research [26, 27]. Finally, the study assessed the efficiency of the numerical chart and found that the execution of the program with a low n-value produced results that were just as accurate as those with a high n-value. The research provides valuable insights into the impact of density ratios on free surface models and offers practical solutions for calculating interfaces and freshwater lens depth. The study's methodology and results can be used in various applications, including environmental management and coastal engineering.

5. Conclusion

The study aimed to examine the impact of withdrawing water from the freshwater layer of an island that has fixed boundaries at its bottom, left, and right. The unknown interface between two layers of fluids of different densities on an island with consistent boundaries was calculated by integrating relevant parameters such as the island's pulling rate and density ratio. The study used an analytical approach based on the Fourier series to calculate a solution to the linear problem, and it was found to provide a good solution to the non-linear problem formulated. The height of the calculated interface through the analytical approach was consistent with the height predicted by the Ghyben-Herzberg relation [29, 30, 31], which describes the equilibrium interface between freshwater and saltwater in a coastal aquifer system. Interestingly, the study also found that there is a maximum pulling rate for different density ratios after which fixed solutions cease to exist. These findings are consistent with the results of previous studies conducted on similar systems [32]. The study highlights the importance of understanding the complex hydrological processes that occur in coastal aquifer systems and the potential impact of human activities on these systems.

References

[1] W. Li, X. Chen, L. Xie, Z. Liu & X. Xiong, " Reply to Comments on: Li et al.(2019)" Bioelectrochemical Systems for Groundwater Remediation: "The Development Trend and Research Front Revealed by Bibliometric Analysis", Water **11** (2020) 1532. https://doi.org/10.3390/w12061603.

[2] World Health Organization (WHO), *United Nations Department of Economic and Social Affairs*, Progress on sanitation and drinking water: 2015 update and MDG assessment. World Health Organization, 2015.

[3] A. E. Urai & C. Kelly, "Rethinking academia in a time of climate crisis", ELife **12** (2023) 84991.

[4] G. Worster, K. Moffatt & G. Batchelor (Eds.) *Perspectives in Fluid Dynamics: A Collective Introduction to Current Research*, Cambridge, UK: Cambridge University Press, 2000.

[5] World Health Organization, *Health and climate change: country profile 2019: small island developing states initiative: Tuvalu* No. WHO/CED/PHE/EPE/19.3. 3. World Health Organization, 2019.

[6] L. Chambers, S. Lui, R. Plotz, D. Hiriasia, P. Malsale, R. Pulehetoa-Mitiepo, M. Natapei, N. Sanau, M. Waiwai, L. Tahani & A. Willy, "Traditional or contemporary weather and climate forecasts: reaching Pacific communities", Regional Environmental Change **19** (2019) 1521.

[7] M. A. Mycoo, *Achieving SDG 6: "water resources sustainability in Caribbean Small Island Developing States through improved water governance*, In Natural Resources Forum (Vol. 42, No. 1, pp. 54-68). Oxford, UK: Blackwell Publishing Ltd.

[8] Pacific Community, *Water Security Toolkit for Pacific Island Countries*, 2019.

[9] H. Canton, Canton, Helen, ed. "The Europa Directory of International Organizations: 2021". Routledge, 2021.

[10] S. Al-Ali, G. C. Hocking, D. E. Farrow & H. Zhang, "A spectral modelling approach for fluid flow into a line sink in a confined aquifer", European Journal of Applied Mathematics **33** (2022) 960.

[11] A. D. Werner, & M. Bakker, "Saltwater intrusion in coastal aquifers", Wiley Interdisciplinary Reviews: Water **6** (2019) 1388.

[12] E. Custodio, "Aquifer overexploitation: what does it mean?". Hydrogeology Journal (2002)10:254–277.

[13] L. Martinez-Perez, L. L. Carrera, J. Marazuela, M. A. Goyetche, T. M. Pool & A. Folch, " A multidisciplinary approach to characterizing coastal alluvial aquifers to improve understanding of seawater intrusion and submarine groundwater discharg", Journal of Hydrology, 607,(2022)127510.

[14] M. O. Ogunniran "A Class of Block Multi-derivative Numerical Techniques for Singular Advection Equations", Journal of the Nigerian Society of Physical Sciences **1** (2019) 62

[15] S. Al-Ali, G. C. Hocking, & D. E. Farrow "Critical surface coning due to a line sink in a vertical drain containing a porous medium", The ANZIAM Journal **61** (2019) 249.

[16] O. O. Olaiya, R. A. Azeez, & M. I. Modebei, "Efficient Hybrid Block Method For The Numerical Solution Of Second-order Partial Differential Problems via the Method of Lines". Journal of the Nigerian Society of Physical Sciences, (2021) 26.

[17] R. N. Bracewell, & R. N. Bracewell, *The Fourier transform and its applications* New York: McGraw-Hill Vol. 31999, 1986.

[18] C. K. Wentworth, "Storage consequences of the Ghyben-Herzberg theor", Eos, Transactions American Geophysical Union **23** (1942) 683.

[19] W. Badon-Ghyben, *Notes on the probable results of well drilling near Amsterdam*, Tijdschrift van het Koninklijk Inst. van Ing. Den Haag, 1888.

[20] D. K. Todd, & L. W. Mays, *Groundwater Hydrology*, Third Edition. John Wiley and Sons, Inc. (2005).

[21] P. K. Kundu, I. M. Cohen & D. R. Dowling, *Fluid mechanics*, Academic press, 2015.

[22] L. N. Trefethen & D. Bau,"Numerical linear algebr", Siam **181** (2022).

[23] H. T. Davis, "Introduction to nonlinear differential and integral equations", US Atomic Energy Commission, 1960.

[24] K. Aziz, & A. Settari, *Petroleum reservoir simulation*, 1979.

[25] C. A. Fletcher, *Computational techniques for fluid dynamics 2: Specific techniques for different flow categories* Springer Science & Business Media, 2012.

[26] Z. Ling, L. Shu, Y. Sun, R. Wang & Y. Li, "Impact of island urbanization on freshwater lenses: A case study on a small coral island", Water **13** (2021) 3272.

[27] A. D. Werner & C. T. Simmons, "Impact of sea-level rise on sea water intrusion in coastal aquifers", Groundwater **47** (2009) 197.

[28] A. D. Werner, M. Bakker, V. E. Post, A. Vandenbohede, C. Lu, B. Ataie-Ashtiani & D. A. Barry, "Seawater intrusion processes, investigation and management: recent advances and future challenges", Advances in water resources **51** (2013) 3.

[29] H. I. Essaid, "A comparison of the coupled fresh water-salt water flow and the Ghyben-Herzberg sharp interface approaches to modeling of transient behavior in coastal aquifer systems", Journal of Hydrology **86** (1986) 169.

[30] H. Bouwer & H. Bouwer, *Groundwater hydrology*, New York: McGraw-Hill, 1978.

[31] A. J. Raudkivi & R. A. Callander, *Analysis of groundwater flow*, No. Monograph, 1976.

[32] W. B. Ghyben, *Nota in verband met de voorgenomen putboring nabij Amsterdam*, Tijdschrift van Let Koninklijk Inst. Van Ing., 1888.