SOLVING TRACKING PROBLEM OF A NONHOLONOMIC WHEEL MOBILE ROBOT USING BACKSTEPPING TECHNIQUE

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ABSTRACT

Nonholonomic system is a mechanical system that is subject to nonholonomic constraints. They are the constraints on the velocity of the system which can not be integrated into position constraints that can be used to reduce the number of generalized coordinates. Mobile robots constitute a typical example of non-holonomic systems. This project attempts to control a nonholonomic mobile robot to track the desired trajectories. In this project, the combination of kinematics and dynamics of the mobile robot are used to control the robot using backstepping technique. Two types of input are presented in this paperwork. From the simulation results, the controller is able to control a non-holonomic mobile robot to track the desired trajectories. All simulations are performed using SIMULINK/MATLAB.

***KEYWORDS**: Backstepping technique, non-holonomic system, mobile robot*

1.0 INTRODUCTION

Mobile robots have the capability to move around in their environment and are not fixed to one physical location. The mobile robot can be broken down into holonomic and nonholonomic mobile robot. Nonholonomic mobile robot means that a mobile robot that cannot move laterally. This is due to the velocity of the mobile robot possess two degree of freedom which cannot be integrated into positioning constraint since it has three degree of freedom.

Several methods of control techniques have been studied and proposed considering the system kinematics and dynamics model. In early years, many researchers have been done using kinematics control. In this technique, the dynamic model is neglected to simplify the work. It is always assumed that the mobile robot systems fulfill the perfect velocity tracking. In general, these controllers have successfully driven the trajectory tracking error to converge to zero asymptotically. Nevertheless, the kinematics control is inadequate to provide good stability, maneuverability and robustness of the mobile robot. However, the simplification is acceptable when the velocities are low, as in most mobile robot applications (T. C. Lee, 2001).

In contrast, the dynamic control technique is approaching closer to the real mobile robot system compared to the kinematics control because it includes dynamic environments of the system such as the mass and inertia factor. The main equation of motion employed in dynamic control model is derived from the Euler-Langrange method. In later years, many researchers have investigated the application of dynamic control incorporate with elements of adaptive, intelligent, robust control and many more.

R. Fierro has proposed a dynamic control that is extended to integrate the kinematics controller with a torque controller using a backstepping method. This method combines both kinematics and torque control laws. It is asymptotically stable and guarantees to converge through the derivative of a Lyapunov function.

This paper will use the controller developed by (R. Fiero, 1995) to observe the performance of the mobile robot with the input of straight line and circular.

2.0 NONHOLONOMIC WHEELED MOBILE ROBOT

The model of a nonholonomic mobile robot is shown in Figure 1. It has two active wheels mounted on the same axis at rear and a passive wheel at front. The active wheels will drive and steer the mobile robot. Previous research shown that linearization control technique failed at point P and a new reference point, C is used to develop the mathematical model (R. Fierro, 1995).

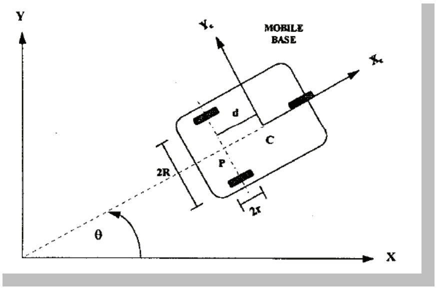

FIGURE 1
Model of a nonholonomic mobile robot

The steering system derived from non-holonomic constraint is known as

$$\dot{q} = S(q)v(t) \tag{1}$$

With S(q) as

$$S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \tag{2}$$

Therefore, (1) can be written as

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{3}$$

The dynamic equation of the mobile robot is

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q)\lambda \tag{4}$$

The definitions of (4) can be obtained in (R. Fierro, 1995) and (Shen Lin, 2000) as:

$$M(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix}$$

$$V(q,\dot{q})\dot{q} = \begin{bmatrix} md\dot{\theta}^2\cos\theta \\ md\dot{\theta}^2\sin\theta \\ 0 \end{bmatrix}$$

$$B(q)=\frac{1}{r}\begin{bmatrix}\cos\theta & \cos\theta\\ \sin\theta & \sin\theta\\ R & -R\end{bmatrix}$$

$$G(q)=0,\tau=\begin{bmatrix}\tau_r\\ \tau_l\end{bmatrix},\lambda=-m(\dot{x}_c\cos\theta+\dot{y}\sin\theta)\dot{\theta} \quad (5)$$

In this case, G(q)=0, because the trajectory of the robot base is constrained to the horizontal plane, since the system cannot change its vertical position (R. Fierro, 1995).

3.0 BACKSTEPPING CONTROL DESIGN

Control theory is a combination of engineering and mathematics that deals with the behaviour of dynamical systems. In control theory, backstepping is a technique (P. V. Kokotovic, 1992) for designing stabilizing controls for a special class of nonlinear dynamical systems. It breaks a design problem for a full system into a sequence of design problems. Because of this recursive structure, the design process can be started at the known-stable system and "back out" new controllers that progressively stabilize each outer subsystem. The process terminates when the final external control is reached. Hence, this process is known as backstepping (H. K. Khalil, 2002).

R. Fierro has proposed to convert the velocity control into a torque control for the actual physical cart. The selection of the torque control is obtained from the dynamic equation of the mobile robot, so that the steering system will behave in the same manner of the desired velocity.

Figure 2 shows the structure of the complete system. It starts with calculating the errors position and then continues with control law that calculates the target velocities. Next, the target velocities will be converted to the desired torque to drive the mobile robot. The current position of the mobile robot is then obtained from the steering system. Since this is a closed loop system, the system tries to nullify the errors position until it is able to follow the desired tracking.

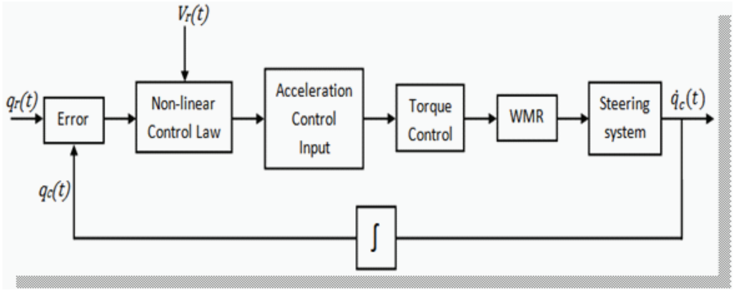

FIGURE 2
The structure of the system

The control law of the system is:

$$v_c = \begin{bmatrix} v_r \cos\theta_e + k_x x_e \\ \omega_r + k_y v_r y_e + k_\theta v_r \sin\theta \end{bmatrix} \tag{6}$$

where k_x, k_y and k_θ are positive consonant.

While the accelaration control input is:

$$u = \dot{v}_c + K(v_c - v) = \dot{v}_c + Ke_c \tag{7}$$

where K is a positive definite, diagonal matrix given by:

$$K = kI \tag{8}$$

To prove it's stability, let a scalar function V be a Lyapunov function candidate as below;

$$V = k_x\left(x_e^{\,2} + \theta_e^{\,2}\right) + \frac{2k_x}{k_y}\left(1 - \cos\theta_e\right) + \frac{1}{2}\left(v_e^{\,2} + \frac{k_x}{k_y k_\theta v_r}\omega_e\right) \tag{9}$$

And the derivative is;

$$\dot{V} = -k_x^{\,2} x_e^{\,2} - \frac{k_x k_\theta}{k_y} v_r \sin^2\theta_e - \left(v_e + k_x x_e\right)^2 - \frac{k_x}{k_y k_\theta v_r}\left(\omega_e + k_\theta v_r \sin\theta_e\right)^2 \tag{10}$$

By considering $e_c \rightarrow 0$ and $[\, x_e \;\; \theta_e \,]^T \rightarrow 0$ as $t \rightarrow \infty$, then;

$$-k_y v_r y_e = 0 \tag{11}$$

By assuming $v_r > 0$, then $y_e \to 0$ as $t \to \infty$. Therefore, the equilibrium point e = 0 is uniformly asymtotically stable. While the torque equation is;

$$\tau = f_c(q, \dot{q}, v, u) = \bar{B}^{-1}(q)\left[\bar{M}(q)u + \bar{V}_m(q, \dot{q})v + \bar{F}(v)\right] \tag{12}$$

4.0 RESULT AND DISCUSSION

In this work, all the simulations are performed using the MATLAB/SIMULINK. The following is the wheel mobile robot's parameters used in this work (Didik, 2003).

m	=	31kg
d	=	0.1m
r	=	0.15m
R	=	0.8m

Initial position coordinates [0,0]

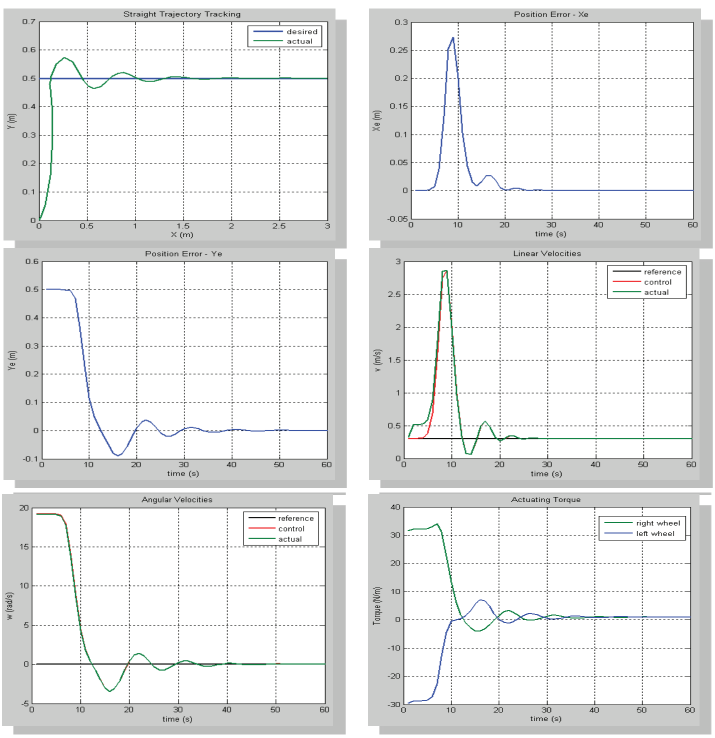

FIGURE 3
The performance of the straight line input

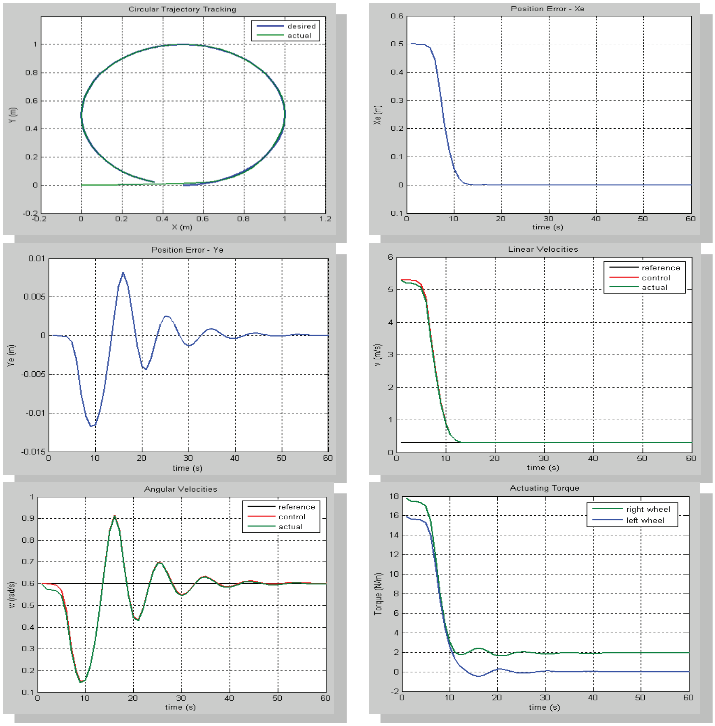

FIGURE 4
The performance of circular input.

There are two types of input presented here, which are straight line and circular input. For all the input, v_r is fixed to 0.03 m/s to keep low system velocity as in most mobile robot applications. The performances of both inputs are depicted in Figure 3 and Figure 4. For the straight line input, the ω_r is set to 0 rad/s while the circular input, the ω_r is set to 0.6 rad/s. Both input show that the system is able to track the trajectory input with all the velocities are converging to the reference values. The torque for both wheels in the straight line trajectory is the same when the mobile robot totally followed the trajectory. Meanwhile, for circular input, the torque for the right wheel is always higher than the left wheel since the input is in circular motion and the mobile robot is moving in counter clockwise direction.

5.0 CONCLUSION

In this paperwork, the performance of a nonholonomic wheel mobile robot has been discussed. All the simulations are performed using SIMULINK/ MATLAB. The results show that the system is able to track the reference trajectories and the stability of the system is proved since all the errors have converged to zero. This shows that the backstepping method can be applied to the system.

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