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Research Article

Binomial Method in Bermudan Option

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Abstract

The Bermudan option allows the contract holders to make and buy a hybrid contract between American and European options. Bermudan option contract can be executed at certain times until the due of the contract. The purpose of this research is to determine the price of the Bermudan option using the binomial method, and then to compare the binomial method result of n steps with the market option price. In determining stock prices at each point, there will be two branches of the binomial method: up and down branches. These branches represent the movement of stock prices in the market. The result shows the price of Bermudan option is convergent at a certain value when the binomial procedure is enlarged. The comparison of the Bermudan option price using a binomial method to the market price shows that the price of Bermudan option is an approach to the market price in certain conditions. Empirically, the price of Bermudan call option is in approach to the market option price or has a minimum error when the exercise price is below the current stock price. The price of Bermudan put option empirically is in approach to the market option price or having a minimum error when the exercise price is above the current stock price.

Keywords: option, bermudan option, binomial method

1. INTRODUCTION

Investment is a capital expenditure in certain assets to gain profit in future times. Investors have many choices in investment. Besides real assets investment (such as land, building, and precious metal), investors can also invest in monetary investment in the financial market (such as securities) or capital market (such as stock, obligation, foreign currency, etc.). As time continues, investment products are developing, and one of the developments is a derivative product. This product aims to minimize loss risk and increase profit opportunity in investment. There are future contracts, forward contracts, swaps, and options [1].

An option contract provides freedom to the buyer of the option contract. When the option contract is due, the option contract buyer has the freedom either to continue or to stop the contract. Thus, this research will examine the challenge of the option [2]. According to the purpose of a derivative product, the option contract is a risk management instrument that protects the contract buyer against stock price movement that either results in loss or profit. Investment in options is more profitable than investing only in stocks [3].

According to the schedule, options consist of two: American type and European type [4]. The American type option is an option contract that has flexible execution time, which is the beginning of the contract until the due date of the contract. The European type is an option contract with execution time only at the due date [5]. This research will examine the arrangement of call and put Bermudan option price. Bermudan option is a hybrid option of American and European options with a certain execution time that begins on the publication date of the options contract until the due date [6,7].

In 1979, John Cox, Stephen Ross, and Mark Rubinstein created a numerical approach to calculate option prices, known as the binomial method. The binomial method is a simple and popular method commonly used to calculate option prices. The binomial method in calculating option prices at each point will issue two branches: up and down branches. These two branches represent stock movement prices in the market where there are two possibilities in every term: the increase and decrease of the prices [8].

There is no definitive solution for determining the price of Bermudan option. Thus, a numerical approach needs to be applied [9-11]. Other several methods for determining the price of Bermudan option are stochastic grid [12], low-discrepancy

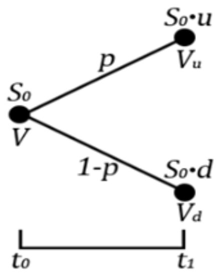

Figure 1. Fluctuation scheme of stock price in one-step binomial method

mesh [13], Lévy Process Models [14], recombining quadratures method [15], jump-diffusion processes [16], Merton jump-diffusion [17], least-squares Monte Carlo [15], neural network regression [18], regression trees/random forests [19], and a pure jump Lévy process method [20].

Besides those methods, there is a binomial method for determining Bermudan option [21-24]. Binomial method can be used to determine the fair price of an option. Binomial method is an easy applicable discreet method for determining option prices [25]. Fahria [21] explains the use of binomial method for determining the price of Bermudan call option has the same value as American and European call options. The fundamental objective of this study is to determine the price of Bermudan call and put option using a binomial method with varying n steps, then to compare the calculation result of binomial method n steps with option price in the market. The focus of this comparison is to analyze the error resulting from the different contract price variable factors. This error analysis is what distinguishes this research from previous studies.

2. MATERIALS AND METHODS

2.1. Materials

2.1.1. Binomial Method

According to Obradovic and Mishra [26], the binomial formula as in equation (1):

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n} a^0 b^n = \sum_{j=0}^{n} \binom{n}{j} a^j b^{n-j} \quad \textbf{(1)}$$

Whereas $\binom{n}{j}$ is the binomial coefficient with the formula as in equation (2) [27,28].

$$\binom{n}{j} = \frac{n!}{j!\,(n-j)!} \quad \textbf{(2)}$$

The binomial formula is the basis for forming a model in calculating option prices. The calculation of option price using a binomial method is based on the fact that stock price always fluctuates, up or down during the time in the free market. Binomial option formula from Cox-Ross-Rubenstein [8] is a formula for a binomial diagram that is used for determining the increase and decrease factor of stock price. Thus, the probability of increasing and/ or decreasing stock price can be anticipated. The value of u, d, p, and q which are used in the binomial method are written below:

$$u = e^{\sigma\sqrt{\Delta t}} \quad \textbf{(3)}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \quad \textbf{(4)}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \quad \textbf{(5)}$$

$$q = 1 - p \quad \textbf{(6)}$$

$$\Delta t = \frac{T}{n} \quad \textbf{(7)}$$

whereas the u parameter states the percentage of stock price increase, d states the percentage of stock price decrease. p states the probability of stock price increase under the assumption of a risk neutral valuation and q states the probability of stock price decrease under the assumption of a risk neutral

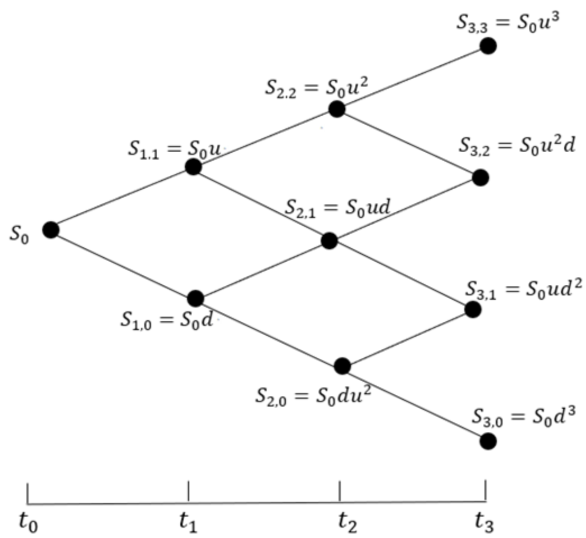

Figure 2. Fluctuation scheme of stock price in n steps binomial method

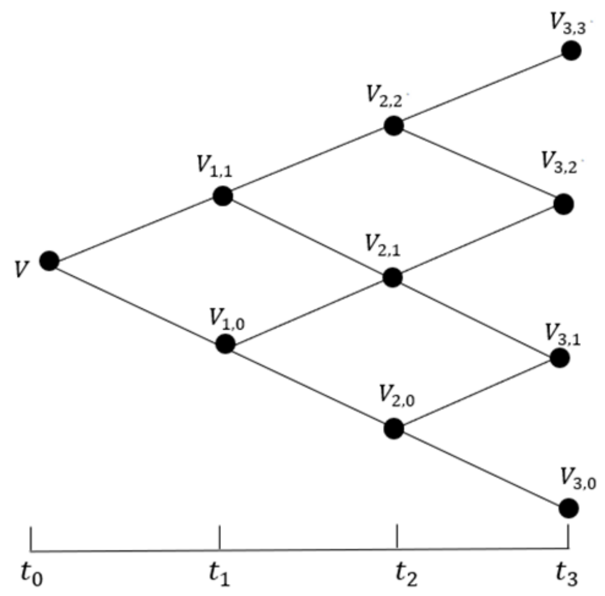

Figure 3. Option values in the binomial model

valuation. The risk neutral valuation assumes that investors do not consider the level of risk when investing. This assumption is used because the movement of assets that are not at risk can be predicted. The risk neutral assumption in calculating the option price states that the current price is equal to the discounted value of the expected future price at a risk-free interest rate. Δt value states the period duration with T states lifetime duration of option and n the number of binomial steps.

For example, stock price in $t = 0$ or (t_0) is S_0 can be predicted in the future. When $t = T$ or (t_1), the stock price increases with the probability of increasing (p) to S_u or the stock price decreases with the probability of decreasing (1 - p) to S_d. Option value in t_0 is V . Besides stock price, option value also has two possibilities: option value if stock price increases are V_u or $V_{1,1}$ and option value if stock price decreases are V_d or $V_{1,0}$ (Figure 1) [29].

As time continues, the stock will move statically, being at rest. The movement of stock price will fluctuate according to the influencing factors. Thus, binomial method does not end with one step, but there are binomial method of n steps, as portrayed below.

Based on Figure 2, it can be confirmed that in the first period t_1, stock price will change to S_0u with p probability or S_0d with 1 - p probability. In second period t_2, there is possibility of change in stock price to S_0u^2 with p^2 probability, S_0ud with $2p$ (1 - p) probability, or S_0d^2 with $(1 - p)^2$ probability [5].

2.1.2. Volatility

One of the factors that influence the option price is a return value and its volatility. Return for one period [30] is notated below.

$$R_t = \ln\frac{S_t}{S_{t-1}} \tag{8}$$

For a description of the risk amount of particular investment, variance equation of return is applied [30]:

$$s^2 = \frac{1}{n-1}\sum_{t=1}^{n}(R_t - \overline{R_t})^2 \tag{9}$$

Table 1. The price of KO in Bermudan option with different n value

n	*Call*	*Put*	Error Call	Error *Put*
3	2.08010	17.0727	0.1044	0.3490
6	1.90495	16.7875	0.0707	0.0638
12	2.01147	16.7797	0.0358	0.0560
24	1.99676	16.7508	0.0211	0.0271
48	1.98618	16.7420	0.0105	0.0183
96	1.97499	16.7179	0.0007	0.0058
192	1.97588	16.7257	0.0002	0.0020
384	1.97757	16.7239	0.0019	0.0002
768	1.97683	16.7249	0.0012	0.0012
1536	1.97566	16.7236	0.0000	0.0001
3072	1.97567	16.7237		

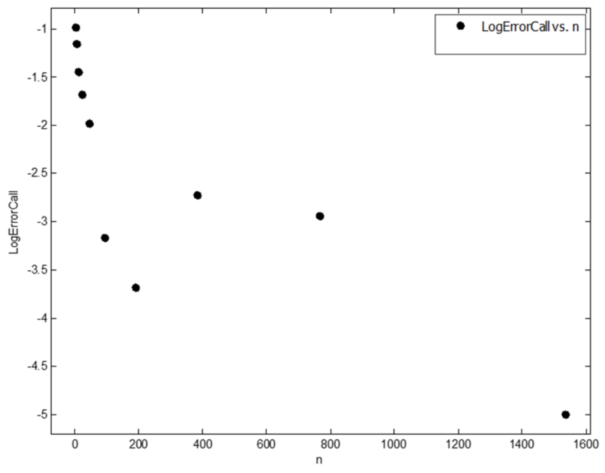

Figure 4. Log plot for error call option

With $\overline{R_t} = \frac{1}{n}\sum_{t=1}^{n} R_t$. R_t value states stock return in t period, while $\overline{R_t}$ states the average return of stock. S_t states stock price in t period and S_{t-1} states stock price before t period. n value states the amount of return day during calculated option lifetime.

Volatility is deviation standard of annual return. Volatility shows the amount of uncertainty or risk on the changing amount of stock value. The S^2 variance formula in equation (9) uses the denominator $n - 1$ because equation (9) is an unbiased estimator for the σ_l^2 variance. By using the assumption that ln stock prices have a normal distribution, the variance of the ln stock price is $\sigma_l^2 = \sigma^2\tau$ with $\tau = \frac{1}{T}$. Volatility is calculated with equation (10) [31]:

$$\sigma = \sqrt{T \times \sigma_l^2}$$

$$\sigma = \sqrt{T \times s^2}$$

$$\sigma = \sqrt{T \times \frac{1}{n-1}\sum_{t=1}^{n}(R_t - \overline{R_t})^2} \quad \textbf{(10)}$$

with σ states volatility and T states the amount of stock trading day in one year, namely 252 days.

2.2. Methods

This research is literature (library research) and applied research. Formulation of Bermudan option uses binomial method that is applied to several company stocks in the exchange of the United States of America, which are the stocks of The Coca-Cola Company (KO), The Walt Disney Company (DIS), Walmart Inc. (WMT), and International Business Machines Corporation (IBM).

Bermudan Options are traded bilaterally in the over-the-counter market. In the over-the-counter market, buyers and sellers are free to bargain over prices. So, there is no fixed price in the over-the-counter market because the contract is customized to the interests of the seller and the buyer. Before there is a price agreement between the seller and the buyer, the seller and buyer are based on the price listed on the stock exchange. So, in this study, the Bermudan option price used is the option price listed on the stock exchange. The source of this research data is obtained from Yahoo! Finance [32]. The stock data from the site are in the form of initial stock price (S_0), exercise price (K), call option data (C Market, C_M), put option (P Market, P_M), and daily closing price of the stock. The daily closing price data is used to determine historical volatility. The daily closing price data is taken from December 9, 2019 to December 7, 2022. The execution time in this research is every four months or the fourth month (t_{k1}) and the eighth month (t_{k2}). The data of risk-free interest rate in December 2022 is 4% or 0.04. This data is obtained from the interest rate of The Fed on Global-rates website [33].

Several analysis steps in this research are below:

a. Deducting the company stock data and risk-free interest rate from the website.
b. Determining the parameters of the initial stock price (S_0), exercise price (K), execution time (t_k)

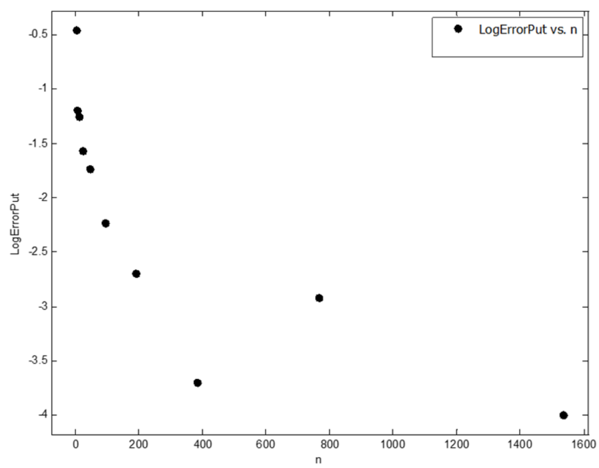

Figure 5. Log plot for error put option

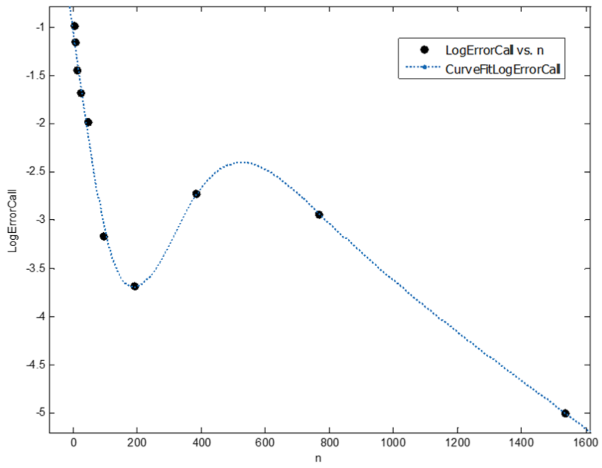

Figure 6. Curve fitting for log error call option

due time (T), and n binomial step

c. Calculating the parameter of volatility value ($\boldsymbol{\sigma}$), the percentage of stock price increase (u), the percentage of stock price decrease (d), and the probability of increasing stock price (p)

d. Calculating Bermudan call and put option using binomial method n step

e. Comparing the result of Bermudan option calculation using the binomial method to option price in the market.

3. RESULTS AND DISCUSSIONS

Executing time of Bermudan option can be written as $t_k \leq T$ with $k = 1, 2, \ldots, n$. The arrangement of executing time of Bermudan option has been agreed by both option seller and buyer, as stated in the contract. In the call option, if $S_T > K$ then the option will be executed so that the option payoff is $S_T - K$. If $S_T \leq K$ then the call option will not be executed so that the option payoff is 0. In a put option, if $S_T < K$ then the option will be executed so that the option payoff is $K - S_T$. If $S_T \geq K$ then the option will not be executed so the option payoff is 0. The payoff of Bermudan call option is $C_T = \max\{S_T - K, 0\}$ and payoff of Bermudan put option is $P_T = \max\{K - S_T, 0\}$.

Payoff function can be perceived as option value at the final point of n steps binomial method, with n as the amount of used binomial steps in calculating the option. The value of the Bermudan option at the end of time t_n can be written as equation (11) for call option and (12) for call option.

$$V_{n,j} = \max\{S_{(n,j)} - K, 0\} \quad \textbf{(11)}$$

$$V_{n,j} = \max\{K - S_{(n,j)}, 0\} \quad \textbf{(12)}$$

The option value at time t_n is used to determine the option value at time t_{n-1}. The option value at time t_{n-1} is used to determine the option value at time t_{n-2}, and so on. At time t_i, the value of the binomial option has several cases: call option in t_k execution time, put option in t_k execution time, and both call and put option at other times t. t_i is the time of the i^{th} partition with $i = 0, 1, 2, \ldots, n$ and $j = 0, 1, 2, \ldots, i$. Call option and put option formulas in t_k execution time and other times t as conducted in equations (13), (14), and (15). Call option in t_k execution time:

$$V_{(i,j)} = \max\{S_{(i,j)} - K, e^{-r\Delta t}[pV_{(i+1,j+1)} + (1-p)V_{(i+1,j)}], 0\} \quad \textbf{(13)}$$

Put option in t_k execution time:

$$V_{(i,j)} = \max\{K - S_{(i,j)}, e^{-r\Delta t}[pV_{(i+1,j+1)} + (1-p)V_{(i+1,j)}], 0\} \quad \textbf{(14)}$$

Put and call option in other time t:

$$V_{(i,j)} = e^{-r\Delta t}[pV_{(i+1,j+1)} + (1-p)V_{(i+1,j)}] \quad \textbf{(15)}$$

In equations (13), (14), and (15), the p value indicates the probability for the stock price to increase under the assumption of a risk neutral valuation. For $S_{(i,j)}$ states the price of stock in t_i time and K value states the agreed price in the option contract where the contract holder has the right to buy or sell the stock with agreed price. Then,

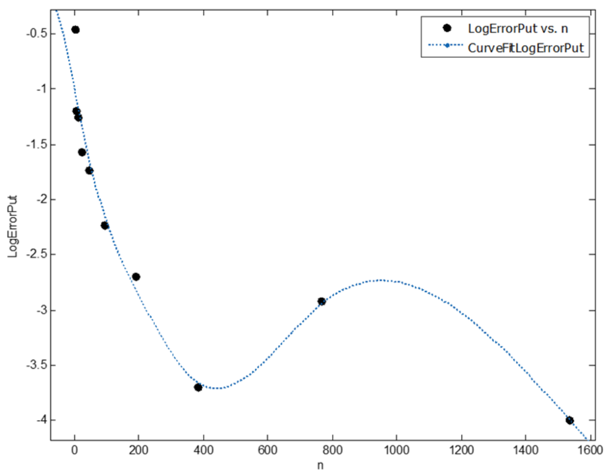

Figure 7. Curve fitting for log error put option

Table 2. Calculation of Bermudan option price using the binomial method on DIS

No	Data Call				Data Put			
	Exercise Price	C Market	C Binomial	% Error	Exercise Price	P Market	P Binomial	% Error
1	65	35.28	34.6016	1.9229	65	3.20	2.14837	32.8634
2	75	26.73	27.3830	2.4429	75	5.35	4.53197	15.2903
3	80	24.47	24.1912	1.1394	80	6.70	6.15526	8.1304
4	85	20.70	21.2940	2.8696	85	8.35	8.08639	3.1570
5	90	18.80	18.6755	0.6622	90	10.15	10.3121	1.5970
6	95	16.20	16.3251	0.7722	95	12.38	12.8248	3.5929
7	100	13.70	14.2301	3.8693	100	14.85	15.6142	5.1461
8	105	11.65	12.3820	6.2833	105	18.00	18.6735	3.7417
9	110	9.75	10.7465	10.2205	110	20.55	21.9696	6.9080
10	115	8.00	9.30498	16.3123	115	24.03	25.4857	6.0578
11	120	6.51	8.05380	23.7143	120	29.15	29.2125	0.2144
12	125	5.20	6.95301	33.7117	125	32.75	33.1157	1.1166

$V_{(i+1,j+1)}$ states option value when the stock price increase, while $V_{(i+1,j)}$ states option value when stock price decrease. r value states the current interest rate according to the Central Bank of America [34].

An illustration of the application of the binomial model to the Bermudan call option is presented in Figure 3.

Figure 3 is 3-step binomial tree for Bermudan options with t_1 and t_2 execution times. The value of the Bermudan call option at time t_3 based on equation (11) is shown below.

$$V_{3,3} = \max\{S_{3,3} - K, 0\}$$

$$V_{3,2} = \max\{S_{3,2} - K, 0\}$$

$$V_{3,1} = \max\{S_{3,1} - K, 0\}$$

$$V_{3,0} = \max\{S_{3,0} - K, 0\}$$

Based on equation (13) the value of the Bermudan call option at the execution time t_2 and t_1 is expressed as below.

$$V_{2,2} = \max\{S_{2,2} - K, e^{-r\Delta t}[pV_{3,3} + (1-p)V_{3,2}], 0\}$$

$$V_{2,1} = \max\{S_{2,1} - K, e^{-r\Delta t}[pV_{3,2} + (1-p)V_{3,1}], 0\}$$

$$V_{2,0} = \max\{S_{2,0} - K, e^{-r\Delta t}[pV_{3,1} + (1-p)V_{3,0}], 0\}$$

$$V_{1,1} = \max\{S_{1,1} - K, e^{-r\Delta t}[pV_{2,2} + (1-p)V_{2,1}], 0\}$$

$$V_{1,0} = \max\{S_{2,2} - K, e^{-r\Delta t}[pV_{2,1} + (1-p)V_{2,0}], 0\}$$

Option values at other times t only occur at t_0, so based on equation (15) the value of $V_{0,0}$ is calculated as below.

3.1. The arrangement of Bermudan option price with many n steps

The use of n steps in this research is based on the execution time of the option. Suppose that the execution time is every 4 months and the maturity time is 1 year, so the execution time of the Bermudan option is the fourth and eighth month. Thus, the applied n steps are $3 \times 2^i; i = 0,1,2,3, ...$ If $n = 3$ then the execution time occurs at t_1 and t_2. If $n = 6$ then the execution time occurs at t_2 and t_4, and so on. The used stock data in arranging Bermudan option prices with many n steps is the stock data of KO. The KO stock price on December 9, 2022 is S_0 = \$63.14. The chosen exercise price is K = \$ 80 and free-risk interest rate (The Fed interest rate) in December is r = 4%. n value = 3, 6, 12, 24, 48, 96, 192, 384, 768, 1536, and 3072. The volatility value of KO stocks calculated using equation (10) is $\boldsymbol{\sigma} = 0.24215$. The Bermudan option price of KO stock with many values is shown in Table 1. Error value obtained by comparing Bermudan option price in each n with Bermudan option price in n = 3072.

Table 3. Calculation of Bermudan option price using the binomial method on WMT

No	Data Call				Data Put			
	Exercise Price	*C* Market	*C* Binomial	% Error	Exercise Price	*P* Market	*P* Binomial	% Error
1	120	35.00	36.2518	3.5766	120	4.15	3.09278	25.4752
2	125	31.05	32.5491	4.8280	125	5.20	4.20200	19.1923
3	130	28.15	29.0760	3.2895	130	6.22	5.55308	10.7222
4	135	25.00	25.8431	3.3724	135	7.60	7.16040	5.7842
5	140	22.05	22.8566	3.6580	140	9.15	9.03410	1.2667
6	145	18.95	20.1231	6.1905	145	10.90	11.1838	2.6037
7	150	16.20	17.6423	8.9031	150	13.00	13.6136	4.7200
8	155	13.60	15.3987	13.2257	155	16.20	16.3144	0.7062
9	160	11.40	13.3783	17.3535	160	18.80	19.2716	2.5085
10	165	9.19	11.5879	26.0925	165	22.71	22.4883	0.9762
11	190	3.02	5.3571	77.3874	190	41.45	41.7545	0.7346

The search for error value is conducted to obtain the convergence of the binomial method.

In Table 1, the error value of the call option and put option in KO stock will be closer to 0 if the amount of binomial n steps is augmented. The value of the Bermudan call option and Bermudan put option are convergent in certain values when applied n steps in the binomial method is enlarged. This result corresponds to the research [32,35-37] which states the binomial method is the prompt convergent numerical method in analytical value.

Error call option and put option in Table 1 can be presented in a log plot as shown in Figures 4 and 5. Based on Figures 4 and 5, log error values of the call option and put option decrease as n increases. When $n > 3$, log error value of the call option or put option has shown a downward trend. The log error value drops faster when $n \geq 96$. In Figure 5, log error call option goes up slightly at $n = 384$ but drops back down after that. When $n = 768$, the log error put option in Figure 6 goes up slightly but after that goes down. A drastic decrease in the log error from call options and put options occurs when $n = 1536$. The decreasing log error value indicates that the error converges to a certain value.

The points of log plot from Figures 4 and 5 form a certain curve. Curve fitting at points on log plot of Figure 4 uses a smoothing spline with the smoothing parameter $p = 5.5633431\text{x}10^{-5}$. The fitted curve is shown in Figure 6. The fitted curve of log error call option has SSE (Sum of Square Error): 0.07177 and RMSE (Root Mean Square Error): 0.1367. The points on the log plot of Figure 5 are fitted using a smoothing spline with the smoothing parameter $p = 7.5295283\text{x}10^{-6}$. The fitted curve of log error put option is shown in Figure 7. The fitted curve has SSE and RMSE of 0.4313 and 0.3011, respectively.

3.2. The comparison of the Bermudan option binomial method calculation result with option price in the market

The simulation of binomial method in determining Bermudan call option and put option price applies several stock data. This simulation is conducted to find out if obtained Bermudan option price is in approach to market option price. Applied stock data in this simulation are DIS, WMT, and IBM. A certain amount of different exercise prices is used on each stock to apply the binomial method in determining Bermudan option price. In binomial method simulation, the selected stock data is taken from December 12, 2022. The option due time is January 19, 2024. Thus, $T = 403$ days. The amount of n partition is 768.

The price data of DIS stock on December 12, 2022, is $S_0 = 94.66$. Based on equation (10), the volatility of DIS is $\sigma = 0.37176$. According to the calculation, the obtained parameters of DIS are $p = 0.4985$, $q = 0.5015$, $u = 1.0142$, and $d = 0.9860$. The price data of WMT stock on December 12, 2022, is $S_0 = 148.02$. The p, q, u, and d values of WMT are

Table 4. Calculation of Bermudan option price using the binomial method on IBM

No	Data Call				Data Put			
	Exercise Price	C Market	C Binomial	% Error	Exercise Price	P Market	P Binomial	% Error
1	120	33.45	39.0019	16.5976	120	5.61	4.66729	16.8041
2	125	29.33	35.5081	21.0641	125	6.78	5.98691	11.6975
3	130	27.38	32.2232	17.6888	130	8.15	7.52577	7.6593
4	135	23.92	29.1610	21.9105	135	9.32	9.29818	0.2341
5	140	19.91	26.3037	32.1130	140	11.15	11.2912	1.2664
6	145	17.00	23.6650	39.2059	145	13.00	13.5198	3.9985
7	150	15.60	21.2328	36.1077	150	15.30	15.9730	4.3987
8	155	12.60	19.0055	50.8373	155	18.50	18.6531	0.8276
9	160	10.95	16.9638	54.9205	160	21.25	21.5429	1.3784
10	170	6.98	13.4423	92.5831	170	30.45	27.9520	8.2036
11	180	4.75	10.5680	122.4842	180	35.50	35.1020	1.1211
12	200	1.82	6.40192	251.7538	200	56.00	51.2226	8.5311

consecutively 0.5006, 0.4994, 1.0096, and 0.9905. The value of historical volatility determined based on equation (10) from December 9, 2019 – December 7, 2022, is $\sigma = 0.25086$. The third stock data which is used in the binomial method simulation is IBM stock data. The price of IBM stock on December 12, 2022, is $S_0 = 149.21$. The historical volatility results from equation (10) is $\sigma = 0.29861$. The obtained parameters are $p = 0.4997$, $q = 0.5003$, $u = 1.0114$, and $d = 0.9887$.

The result of binomial method simulation in determining DIS stock option price with an amount of different exercise prices is displayed in Table 2. Table 3 shows the calculation result of Bermudan option price in each exercise price of WMT. Table 4 shows the calculation result of Bermudan option price in each exercise price of IBM. The obtained Bermudan option price using binomial method is compared to market option price to obtain error percentage. According to error percentage, the distance of Bermudan option price of binomial method can be measured to the price of the market option.

According to the calculation result of Bermudan option price using the binomial method on DIS, WMT, and IBM, it can be concluded that the call option price is constantly decreasing, consequent with the constantly increasing exercise price. On the other, the put option price constantly increases with the increase in the exercise price.

The comparison of Bermudan option price from binomial method to the price of market option obtains different error percentages. On call option, the higher the option exercise price, the error percentage is also higher. Small error percentage on call option is obtained when the exercise price is below current stock price. It means the Bermudan call option price is empirically in approach to market option price when the exercise price is below current stock price. In other words, binomial method is empirically suitable to use on Bermudan call option with a K value below the S_0 value. If exercise price is far above the current stock price, the error percentage will be high.

On put option, if the exercise price is high, the error percentage will be low. On the exercise price of put option that is above the current stock price, the error percentage obtained is low. Empirically, if exercise price is above the current stock price, the put option price is in approach to market option price. It indicates the binomial method is empirically suitable to Bermudan put option with a K value above the S_0 value. If the exercise price is far below the current stock price, the error percentage will be high.

4. CONCLUSIONS

The calculation result of the Bermudan option using the binomial method with certain n step produce option prices that are not much different, in which the error value is closer to 0 if the binomial step is enlarged. The comparison of the Bermudan option price using the binomial method n step to market option price shows the Bermudan call option price empirically has a small error or is in approach to market option price when the exercise price is below the current stock price. The calculated Bermudan put option using the binomial method produces a small error or is in approach to market option price when the exercise price is above the current stock price. Based on the results of this study, the binomial method is suitable for calculating Bermudan option prices, especially if a certain contract price is selected. The results of this study can be used as an alternative for investors in setting the Bermudan option price for both calls and putting in the over-the-counter market (outside the stock market). In future studies, error analysis can be carried out based on the maturity time variable factor.

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Conflicts of Interest

The author(s) declared no conflict of interest

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