Cash Flow Implied Interest Rate: A Unified Approach

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Abstract

One of the most important topics in finance is the concept of the discounted cash flow (DCF) analysis. A recurring problem in introductory finance textbooks requires the search for the cash flow implied interest rate, that is, the implied rate of return/cost on the cash flows. There are numerous instances of such applications - for example, the calculations of the yield to maturity (YTM) and the internal rate of return (IRR). Many students regard these instances as separate, unrelated, and totally context specific exercises, despite the fact that they are the results of the same DCF method and that only the symbols have changed. This paper suggests that the solution to this problem lies in emphasizing the underlying DCF-based unified approach (UA) to the cash flow implied interest rate (CFIIR) calculation which should be explicitly brought to the attention of students in every specific instance. This paper focuses on four related aspects of the discounted cash flow analysis in an attempt to clarify and to unify the threads under the concept of the implied interest rate: namely, first, the discussion of the UA to the CFIIR calculation; second, the step-by-step illustration of the equivalency of the present value and future value calculations; third, the simultaneous consideration of the two sides to a financial contract and the understanding of the corresponding rate of return to be earned by the provider of the funds and the percentage cost to be incurred by the recipient of the funds; and fourth, the explicit definition of the multi-period rate of return to an investor.

Cash Flow Implied Interest Rate: A Unified Approach

A frequent scenario portrayed in current introductory finance textbooks and courses is the situation in which the parties to a financial transaction have enough information about the cash flows expected from their transaction but would like to calculate the expected rate of interest which they will earn or pay over the contractual period. Two well-known instances are the calculation of the yield to maturity (YTM) in the context of bonds and the calculation of the internal rate of return (IRR) in the context of capital budgeting. Other equally important instances include: the calculation of the rate of return on investment in assets, the yield to call, the rate of return on equity, the rate of return on preferred shares, the cost of debt, the cost of common equity, the cost of preferred shares, the modified internal rate of return, the costs of different types of short-term bank loans, the cost of bonds with warrants, and the cost of debt in international financial markets. Many students seem to regard such calculations as separate, unrelated, and totally context specific exercises, despite the fact that they are the results of the same application of the discounted cash flow (DCF) method.

This paper argues that the solution to this problem lies in emphasizing the underlying concept of the DCF-based *unified approach (UA)* to the *cash flow implied interest rate (CFIIR)* calculation which should be brought to the attention of students in every specific instance. In particular, this approach highlights the importance of focusing on the time profile of cash flows, of applying *the unified approach to the cash flow implied interest rate calculation (the UA to CFIIR calculation)*, and of finding the "i," the "CFIIR." The mathematics involved is well-known. The major contribution of the paper is the discussion of the CFIIR calculation by developing a UA common to all instances encountered in the calculation.

At an introductory level, many students seem not have an intuitive understanding of the discounting and compounding processes and their equivalency, given the interest rate, "i." Without such an understanding, the discussions of the calculation of the rate of return, the CFIIR, and the UA to CFIIR calculation face severe difficulties. In regard to this, the paper offers a helpful step-by-step illustration.

At an applied level, when the cost of capital is discussed, even those students who have a better understanding of the discounting and compounding processes are often unable to completely follow the discussion. The paper provides a solution to this problem; it discusses the position of the two major parties to the financial transaction, i.e., the investors/the banks, the providers of the funds, and the corporations, the users of the funds.

Authors of current finance textbooks explicitly define the one-period rate of return on a one-period investment but do not always explicitly define the multi-period rate of return on a multi-period investment. Finally, the paper illustrates an explicit definition of the multi-period rate of return on an investment as the CFIIR.

This paper can be used both as a teaching tool by instructors and as a learning aid by students. It offers an intuitive and clarifying discussion of the concept of the cash flow implied interest rate along with corresponding numerical examples. Because of this discussion it becomes a useful complement to the current finance textbooks: Besley-Brigham (1999), Block-Hirt (2000), Brealey-Myers (2000), Brealey-Myers-Marcus (2001), Brigham-Gapenski (1999), Brigham-Gapenski-Ehrhardt (1999), Brigham-Houston (2001), Chambers-Lacey (1999), Damodaran (2001), Eakins (1999), Emery (1998), Emery-Finnerty (1997), Emery-Finnerty-Stowe (1998), Gallagher-Andrew (2000), Gitman (2000), Keown-Petty-Scott-Martin (2001), Lee-Finnerty-Norton (1997), Levy (1998), Moyer-McGuigan-Kretlow (2001), Ross-Westerfield-

Jaffe (1999), Ross-Westerfield-Jordan (2001), Scott-Martin-Petty-Keown (1999), Van Horne (2002), and Van Horne-Wachowicz (2001).

The extant literature on financial education focuses on specific concepts and involves detailed treatment of these concepts. For instance, Dongsae (1998), Rodriguez (1988), and Tavakkol (1995) deal with the bond YTM; Bey (1998), Plath and Kennedy (1994), and Hittle and Gitman (1992) examine the IRR; and Bali and Hovakimian (1999) discuss stock return. In contrast, this paper emphasizes the intuitive concept of the implied interest rate which is the unifying concept in the discounted cash flow topics discussed in finance textbooks.

Section 1 introduces the UA to the CFIIR calculation. Section 2 provides an intuitive step-by-step illustration and explanation of the relationship between the discounting and compounding processes. Section 3 shows how to find the rate of return on investment in assets by applying the UA to the CFIIR calculation in one-period and multi-period scenarios. Section 4 considers the component costs of the long-term sources of capital to corporations, namely, long-term debt, common equity, and preferred shares. It simultaneously looks at the two major parties involved in each of these financial transactions, i.e., the investors/the banks who provide the funds and the corporations which need the funds for investment. For each source of financing, it applies the UA to the CFIIR calculation and finds the expected rate of return to the investors/the banks and the percentage cost to the corporations. Section 5 concludes the discussion of this unified approach (UA). Further applications of the UA to the CFIIR are contained in the Appendix to this paper. Section A1 is concerned with the investment in real assets and discusses how the IRR and the Modified IRR (MIRR) are found by applying the UA to the CFIIR calculation. Section A2 looks at short-term financing and finds the costs of different types of short-term bank loans by applying the UA to the CFIIR calculation. Section A3 examines the

topic of financing by bonds with warrants and shows how the expected rate of return to the bondholders is found by applying the UA to the CFIIR calculation. Section A4 extends the discussion of the cost of capital to include international financial markets and applies the UA to the CFIIR calculation to find the percentage cost of debt.

1. The UA to CFIIR Calculation

The DCF method defines a mathematical relationship among four parameters: the present value of the cash flows, the future values of the cash flows, the number of time periods associated with each cash flow, and the appropriate interest rate. Based on this relationship, if the values for three of the four parameters are known, the value for the fourth parameter can always be calculated. When the three known parameters are the present value of the cash flows, the future values of the cash flows, and the number of time periods associated with each cash flow, and the unknown parameter is the appropriate interest rate, the UA to the CFIIR calculation is applicable. The time profile of cash flows for the general case is depicted in Exhibit 1.
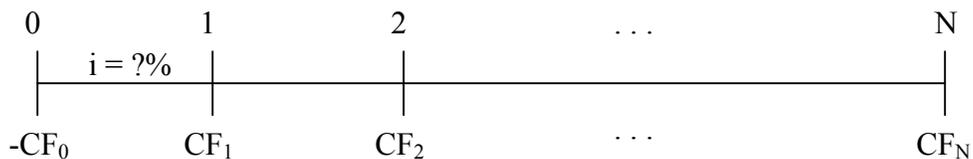

Exhibit 1. The UA to the CFIIR Calculation: The General Case.

Algebraically, the UA to the CFIIR calculation requires one first to set up the mathematical relationship among the present value of the cash flows, the future values of the

cash flows, the number of time periods associated with cash flow, and the interest rate (the "i" or

the CFIIR), as defined by the DCF method:

$$CF_0 = PV(CF_1) + PV(CF_2) + \ldots + PV(CF_N) \tag{1}$$

which is solved by

$$CF_0 = CF_1 \left[1/(1+i)^1\right] + CF_2 \left[1/(1+i)^2\right] + \ldots + CF_N \left[1/(1+i)^N\right]$$
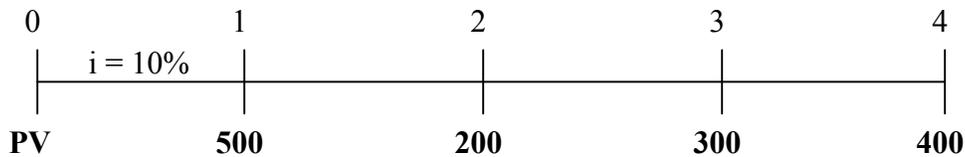
or

$$CF_0 = CF_1 (PVIF_{i,1}) + CF_2 (PVIF_{i,2}) + \ldots + CF_N (PVIF_{i,N})$$

where CF designates a *cash flow*, CF_0 represents a *cash outflow at t=0* which is considered to be

equal to the algebraic sum of the present values of all the expected future cash flows, PV

designates *present value*, i designates the appropriate periodic *interest rate*, the CFIIR (or the

"i"), t, a subscript for a cash flow, designates the *time period associated with a cash flow*, and

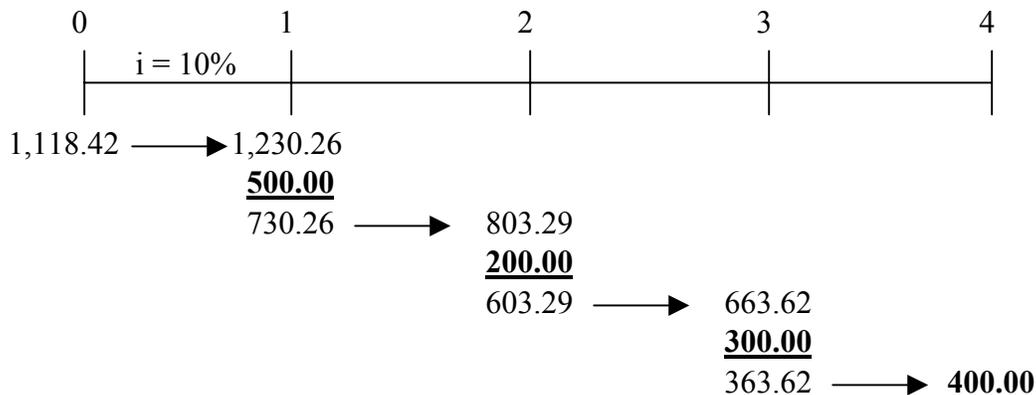PVIF designates the *present value interest factor*.

Then, given the values for the first three parameters, one can calculate the value of the

final parameter, i.e., the interest rate, the CFIIR.[1] In other words, the time profile of cash flows

implies the interest rate, hence the *cash flow implied interest rate (CFIIR)*. This scenario is

common to all the situations listed in the first paragraph of the paper and, therefore, the same

approach can be applied to calculate the interest rate from the time profile of cash flows, hence

the unified approach to cash flow implied interest rate calculation (the UA to CFIIR

calculation). In fact, Sections 3 and 4 and the sections included in the Appendix illustrate the UA

to CFIIR calculation for these different situations.

2. The Equivalency of Discounting and Compounding

It is useful for students to have an intuitive and step-by-step understanding of the relationship between the discounting and compounding processes. Exhibit 2 shows the equivalency of the present value with the future value of cash flows given the interest rate. It shows that the present value of cash flows, PV, can be replicated from the future cash flows, CF_1 ... CF_N, and vice versa.

```
0              1              2              3              4
|  i = 10%      |              |              |              |
PV           500            200            300            400
```

Panel a. Discounting.

```
0              1              2              3              4
|  i = 10%      |              |              |              |

1,118.42 ──────▶ 1,230.26
                 500.00
                 730.26 ──────▶ 803.29
                                200.00
                                603.29 ──────▶ 663.62
                                               300.00
                                               363.62 ──────▶ 400.00
```

Panel b. Compounding.

Exhibit 2. The Intuitive Explanation.

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[1] The "i" is found by a search - by the use of a spreadsheet, a financial calculator, or by trial and error.

To see that the present value can be replicated by the future cash flows, consider the time profile of cash flows and the interest rate given in Exhibit 2, Panel a. The present value of the future cash flow stream is equal to the sum of the present values of the future cash flows:[2]

$$PV = PV(CF_1) + PV(CF_2) + \ldots + PV(CF_N) \qquad (2)$$

which is solved by

$$PV = CF_1 [1/(1+i)^1] + CF_2 [1/(1+i)^2] + \ldots + CF_N [1/(1+i)^N]$$

or

$$PV = CF_1 (PVIF_{i,1}) + CF_2 (PVIF_{i,2}) + \ldots + CF_N (PVIF_{i,N})$$

$$PV = 500 (PVIF_{10\%,1}) + 200 (PVIF_{10\%,2}) + 300 (PVIF_{10\%,3}) + 400 (PVIF_{10\%,4})$$

$$PV = 500 (0.9091) + 200 (0.8264) + 300 (0.7513) + 400 (0.6830)$$

$$PV = \$1,118.42$$

That is, given the time profile of cash flows and the interest rate, the equivalent value of the four future cash flows at t = 0 is $1,118.42.

In Exhibit 2, Panel b, the future cash flows are replicated from the present value of the cash flows. The present value is $1,118.42, the interest rate is 10%, and the total number of periods is four (4). As the $1,118.42 is compounded period-by-period into the future, at the end of each period the cash flow which exists at that point in time in Panel a is set aside, replicated.

Exhibit 2, Panel b, starts with a cash flow of $1,118,42 at t = 0. At the end of the first period, with the interest rate of 10%, the principal and interest will be $1,230.26. At that point in time, $500.00 is set aside to replicate the cash flow presented at the end of the first period in Panel a. Therefore, there will be $730.26 left, which is used as the principal over the second period. At the end of the second period, the principal and interest will be $803.29. To replicate

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[2] In what follows, the solutions can be obtained by solving the algebraic equations, by using tables for the present value interest factors (as above), by using a financial calculator, or by using a spreadsheet.

the cash flow which exists at the end of the second period in Panel a, $200 is set aside. The remainder, $603.29, is the principal for the third period. At the end of the third period, the principal and interest will be $663.62. Out of this amount $300 needs to be set aside in order to replicate the third cash flow presented in Panel a. The balance, $363.62, constitutes the principal for the fourth period. By the end of that period, the principal and interest will be $400, which is exactly equal to the cash flow presented at the end of the fourth period in Panel a. In other words, from the present value of $1,118.42 the future cash flows are replicated, i.e., they are equivalent to each other. Therefore, one would be indifferent to receiving the PV at t = 0 or the future cash flows along the time line.

In summary, the present value and future values of cash flows are equivalent given the interest rate. This equivalency is the cornerstone of the DCF method.

3. The Rate of Return on Investments in Assets

This section shows how to find the rate of return on investments in assets by applying the UA to CFIIR calculation in one-period and multi-period scenarios, the latter being the general case.

Consider the following one-period (year) scenario. A current investment of $1,000 in a bank certificate will be worth $1,100 at the end of one period. The time profile of the cash flows is depicted in Exhibit 3, Panel a. The rate of return on this investment is found by applying the UA to the CFIIR calculation: algebraically,

$$FV_1 = PV (1 + i)^1 \qquad\qquad (3)$$
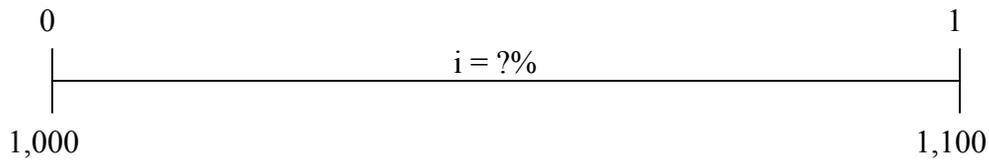$$(1 + i) = FV_1/PV$$

$i = FV_1/PV - 1$

or solving with the tables,

$FV_1 = PV\ (FVIF_{i\%,1})$, where FVIF designates the *future value interest factor*

$1,100 = 1,000\ (FVIF_{i\%,1})$

$1.10 = FVIF_{i\%,1}$

$i = 10\%$

```
0                          i = ?%                          1
|——————————————————————————————————————————————————————————|
1,000                                                    1,100
```

Panel a. One Period.

```
0          1          2        . . .              10
|—————————|——————————|——————————————————————————————|
   i = ?%
1,000                                            2,593.70
```

Panel b. Multi Periods.

Exhibit 3. The Rate of Return on Investment in Assets.

Consider the following multi-period scenario. An investment of $1,000 in a bank certificate will be worth $2,593.70 at the end of the 10th period. The time profile of cash flows is depicted in Exhibit 3, Panel b. The rate of return on this investment is found by applying the UA to the CFIIR calculation:

$$FV_{10} = PV\ (1 + i)^{10} \tag{4}$$

or

$$FV_{10} = PV \ (FVIF_{i\%,10})$$

$$2,593.70 = 1,000 \ (FVIF_{i\%,10})$$

$$2.5937 = FVIF_{i\%,10}$$

$$i = 10\%$$

In summary, the UA to the CFIIR calculation is applied to one-period and to multi-period investments to find the expected interest rate.
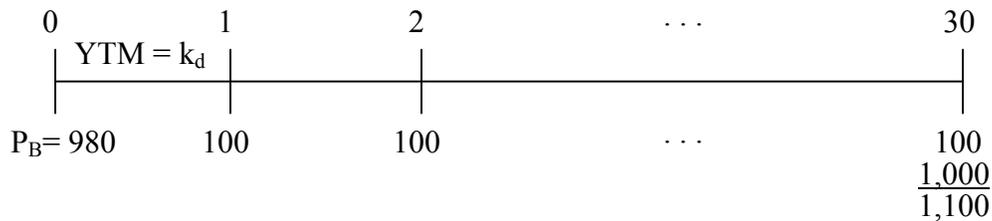
4. The Sources of Long-Term Capital: The Return to Investors and The Cost to Corporations

This section considers the component cost of the long-term sources of capital for corporations, namely, long-term debt, common equity, and preferred shares. It looks at the two major parties involved in each of these financial transactions, i.e., the investors/the banks who provide the funds and the corporations. For each source of financing, it applies the UA to the CFIIR calculation to find the "i," the expected/required rate of return to the investors/the banks and the percentage cost of capital to the corporations.
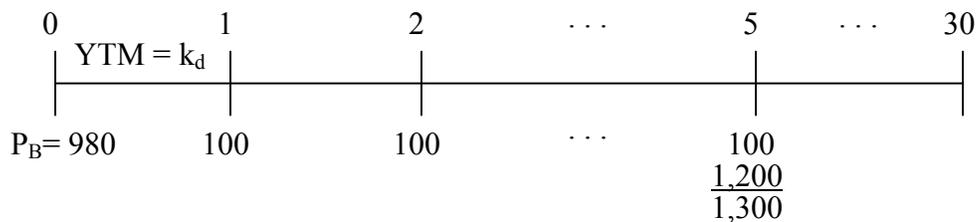
4.A. Long-Term Debt

Consider the case in which there are only two parties to the debt contract: the bondholder, who lends, and the corporation, which borrows. Note that there is no third party, no government, and therefore no taxes in this scenario. Suppose the corporation borrows $980,000,000 to finance its projects for the coming year. If the annual coupon rate is 10% and the price of each bond is

$980.00 (i.e., P_B = $980.00), the corporation is promising to pay each bondholder $100.00 per year in interest for thirty years and at the end of 30th year the par value of $1,000. The time profile of cash flows is depicted in Exhibit 4, Panel a. Given that all of the cash flows to be paid and received by the two parties are identical, the "i," the implied rate of interest to be earned by the bondholder (YTM - the yield to maturity) and the cost of debt capital to be incurred by the corporation (k_d) are identical.

```
0          1          2         . . .              30
| YTM = kd |          |                            |
|          |          |                            |

PB= 980      100        100        . . .           100
                                                  1,000
                                                  1,100
```

Panel a. Regular Bond.

```
0          1          2      . . .     5      . . .    30
| YTM = kd |          |               |               |
|          |          |               |               |

PB= 980      100        100    . . .    100
                                       1,200
                                       1,300
```

Panel b. Callable Bond.

Exhibit 4. Debt Financing.

To find the interest rate, the UA to the CFIIR calculation is used. In this case, the price of the bond is known and the mathematical relationship follows:

$$P_B = PV(CF_1) + PV(CF_2) + \ldots + PV(CF_{30}) \tag{5}$$

$$P_B = CF_1[1/(1+i)^1] + CF_2[1/(1+i)^2] + \ldots + CF_{30}[1/(1+i)^{30}]$$

$$980 = 100[1/(1+i)^1] + 100[1/(1+i)^2] + \ldots + 1,100[1/(1+i)^{30}]$$

Then, the "i" is found by a search, using a financial calculator or a spreadsheet,

$i = 10.2160\% = YTM = k_d$

The situation for a callable bond is very similar to the above except that the time line has been shortened and the final cash flow because of the call premium will be different, but the same principle applies. For example, suppose the above bond will be called at the call price of $1,200.00 after 5 years when market interest rates are expected to have fallen. The time profile of cash flows is depicted in Exhibit 4, Panel b. Here, the bondholder expects to earn the yield to call (YTC) and the corporation expects to incur a percentage cost of debt k_d. Again, since the same cash flows to be paid by one party are to be received by the other, the implied interest rates for both parties are the same, i.e., $YTC = k_d$. The UA to the CFIIR calculation is used to find the interest rate, the "i":

$$P_B = PV(CF_1) + PV(CF_2) + \ldots + PV(CF_5) \tag{6}$$

$$P_B = CF_1 [1/(1+i)^1] + CF_2 [1/(1+i)^2] + \ldots + CF_5 [1/(1+i)^5]$$

$$980 = 100 [1/(1+i)^1] + 100 [1/(1+i)^2] + \ldots + 1,300 [1/(1+i)^5]$$

$i = 13.6256\% = YTC = k_d$

In summary, the UA to the CFIIR calculation is applied to find the YTM, the YTC, and the k_d. When there are only two parties to the transaction, i.e., no government taxes, the expected rate of interest to be earned by the bondholder is equal to the cost of debt to be incurred by the corporation, the "i" is the same for both.

When there are more than the two major parties involved, e.g., the government with its taxes or the investment bankers with their underwriting fees, then the cash flows paid by one major party to the debt contract will not be totally received by the other major party and therefore the equality of the rates will not hold anymore. The UA to the CFIIR calculation is

applied to the time profile of cash flows, which are expected to be paid or received by a specific major party to the debt contract, in order to find the interest rate applicable to that specific party. For instance, when government taxes are present, the UA to the CFIIR calculation should be applied to the time profile of after-tax cash flows for the corporation in order to find its expected after-tax cost of debt.

4.B. Common Equity

There are two sources of common equity, retained earnings and new common shares. The shareholders and the corporation are the two major parties involved in this financial transaction. The corporation is expected to pay dividends to the shareholders and the time profile of these cash flows for the general case is depicted in Exhibit 5, Panel a.
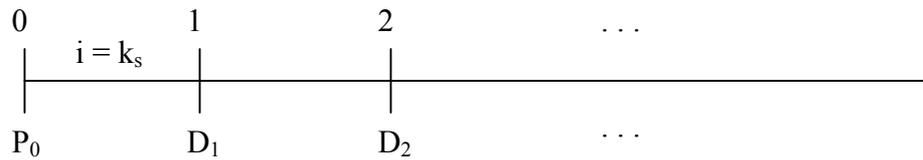
Panel a. The General Case.
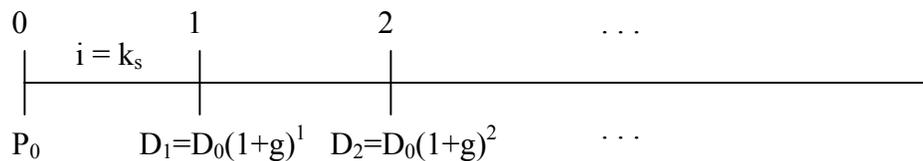
Panel b. The Constant Growth Case.

Exhibit 5. Common Equity Financing: Retained Earnings.

The internally generated funds, called retained earnings, actually belong to the current shareholders and are invested by the management into projects. On these funds the shareholders expect a rate of return equal to the opportunity cost of the funds if the funds were invested in a stock of a firm with similar risk. Theoretically, the funds would be invested in a firm with a similar share price of P_0 and with a similar expected stream of dividends of $D_1, D_2, \ldots$.

The cost of retained earnings to the corporation, k_s, equals the required rate of return expected by the shareholders, i.e., the cost of funds raised at a share price of P_0 with the expected dividend stream of $D_1, D_2, \ldots$. The time profile of cash flows, which is the same for both parties, is depicted in Exhibit 5, Panel a. Therefore, the implied expected/required rate of return to the shareholders and the expected cost of retained earnings to the corporation are equal. In theory the UA to the CFIIR calculation would be used to find the "i":

$$P_0 = PV\ (D_1) + PV\ (D_2) + \ldots + PV\ (D_N) + \ldots \tag{7}$$

$$P_0 = D_1\ [1/(1+k_s)^1] + D_2\ [1/(1+k_s)^2] + \ldots + D_N\ [1/(1+k_s)^N] + \ldots$$

$$P_0 = D_1\ (PVIF_{ks,1}) + D_2\ (PVIF_{ks,2}) + \ldots D_N\ (PVIF_{ks,N}) + \ldots$$

and $k_s =$ "i".

In practice if the dividends are expected to grow at a constant rate over time (the constant growth model), Exhibit 5, Panel b, would show the time profile of cash flows and the UA to the CFIIR calculation would be used to find the interest rate, the "i":

$$P_0 = D_1/(k_s - g) \tag{8}$$

where g designates the *expected and constant rate of growth in dividends*, $D_1 = D_0(1 + g)$, and D_0 is the current dividend. Given the values for P_0, D_1, and g, one can find the cost of retained earnings, k_s, which is the "i" in our discussion, by solving

$$i = k_s = (D_1/P_0) + g \tag{9}$$

The external source of common equity, new common shares, is dealt with in a similar manner. The only difference is that when the firm issues new common shares, it incurs flotation costs. Now a third party, the investment banker, in addition to the two major parties, the new shareholders and the corporation, is involved in bringing the new shares to the market. As in the case of the bonds, since the third party collects a portion of the initial cash flow, the expected cost of new common shares, k_e, and the expected rate of return to new common shareholders, k_s, will no longer be equal. However, the UA to the CFIIR calculation is applied to the cash flows applicable to each party in order to find the "i" corresponding to that party.

The time profile of cash flows for the general case is presented in Exhibit 6, Panel a. The corporation incurs per share flotation costs equal to F and, therefore, receives P_0-F at t = 0. The other cash flows remain the same as in the case of financing with retained earnings. The UA to the CFIIR calculation would be used to find the cost of new common shares:

$$P_0 - F = PV (D_1) + PV (D_2) + \ldots + PV (D_N) + \ldots \tag{10}$$

$$P_0 - F = D_1 [1/(1+k_e)^1] + D_2 [1/(1+ k_e)^2] + \ldots + D_N [1/(1+ k_e)^N] + \ldots$$

$$P_0 - F = D_1 (PVIF_{ke,1}) + D_2 (PVIF_{ke,2}) + \ldots D_N (PVIF_{ke,N}) + \ldots$$

where k_e designates the *cost of newly issued common shares* and is not equal to k_s. Note that since the new shareholders do not incur the flotation cost, their expected rate of return is found by solving the above equation with F = 0.
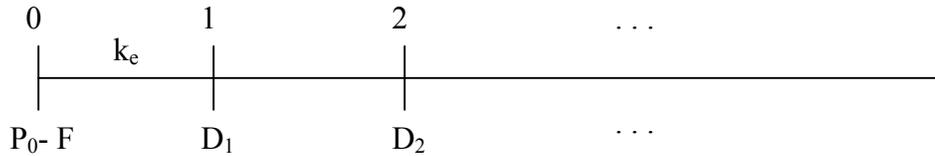
Exhibit 6, Panel b, shows the time profile of cash flows for the corporation whose dividend and earnings are assumed to grow at a constant rate (the constant growth model). The UA to the CFIIR calculation is used to find the expected cost of new shares to the corporation:
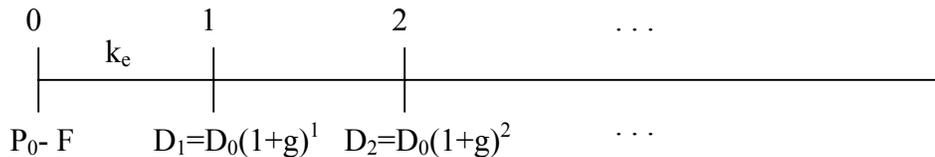
$$P_0 – F = D_1/(k_e - g) \tag{11}$$

Then, given P_0 - F, D_1, and g,

$$k_e = [D_1/(P_0 - F)] + g \tag{12}$$

Again, since the new shareholders do not incur the flotation cost, their expected rate of return, k_s, its "i," is found by solving the above equation with $F = 0$.

```
 0            1            2            . . .
 |     k_e    |            |
 |------------|------------|--------------------------
 |            |            |
P_0- F       D_1          D_2          . . .
```

Panel a. The General Case.

```
 0            1            2            . . .
 |     k_e    |            |
 |------------|------------|--------------------------
 |            |            |
P_0- F    D_1=D_0(1+g)^1  D_2=D_0(1+g)^2    . . .
```

Panel b. The Constant Growth Case.

Exhibit 6. Common Equity Financing: New Common Shares.

In summary, the UA to the CFIIR calculation is applied to obtain the expected/required rate of return to shareholders and the expected cost of common equity to the corporation, whether there are two or three parties involved in a single financial transaction.

4.C. Preferred Share

In this long-term method of financing there are two major parties to the financial transaction: the preferred shareholders and the corporation. The preferred shareholders provide long-term funds to the corporation, which promises to pay a constant dividend per period to the preferred shareholders. The time profile of cash flows is the same for both parties and, therefore,

the expected/required rate of return to the preferred shareholders, its "i," and the expected cost of preferred shares to the corporation, k_{ps}, its "i," are equal. The time profile of cash flows is depicted in Exhibit 7.
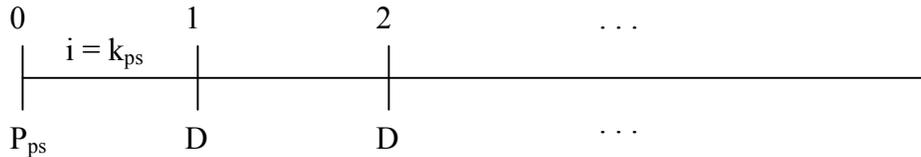

Exhibit 7. Preferred Share Financing.

Usually, the information available to the investors/preferred shareholders and the corporation consists of the market price for the preferred shares, P_{ps}, and the expected dividend per period, D. What is not known is the rate of return to the preferred shareholders, the "i," or the cost of preferred shares to the issuing corporation, k_{ps}. This is the usual scenario in financial markets where market prices and expected cash flows are known, and the "i" must be calculated. This is the situation that applies to all the cases considered in the paper.

The UA to the CFIIR calculation is applied to the time profile of cash flows to find the "i". Since the traditional preferred stock is considered a perpetuity,

$$P_{ps} = D/k_{ps} \tag{13}$$

$$k_{ps} = D/P_{ps} \tag{14}$$

Note that since the time profiles of cash flows for the two major parties are the same, then "i" = k_{ps}.

In summary, the UA to the CFIIR calculation is applied to find the expected/required rate of return to investors/preferred shareholders and the expected cost to the issuing corporation.

5. Conclusion

The purpose of this paper was to emphasize the concept of the underlying implied interest rate in the discounted cash flow analysis in financial education. It focused on the discussion of the UA to the CFIIR calculation, the step-by-step illustration of the equivalency of the present value and future value calculations, the simultaneous consideration of the two sides to a financial contract and the understanding of the corresponding rate of return to be earned by the provider of the funds and the percentage cost to be incurred by the recipient of the funds, and the explicit definition of the multi-period rate of return to an investor.

Most importantly, this paper emphasized on the UA to the CFIIR calculation, showed how it is the thread in many of the applications of the discounted cash flow analysis in introductory finance textbooks, and provided a comprehensive list of such applications with numerical examples. Often some students regard these instances as separate, unrelated, and totally context specific exercises, despite the fact that they are the results of the same DCF method and that only the symbols have changed. Teachers need to reinforce the UA to the CFIIR calculation in each instance so that the students clearly see the connection and have a better intuitive grasp of the underlying method. In the end, students, when confronted with a new situation, will be able to apply the underlying method independently.

This paper can be used both as a teaching tool by instructors and as a learning aid by students. It discusses and emphasizes concepts which make it a useful complement to the current finance textbooks.

Appendix

A1. The Internal Rate of Return (IRR) and The Modified Internal Rate of Return (MIRR)

This section is concerned with the investment in real assets, i.e., capital budgeting, and discusses how the IRR and the MIRR are found by applying the UA to the CFIIR calculation.

The IRR is the interest rate which equates the present value of cash inflows with the present value of cash outflows. The time profile of cash flows for a standard capital budgeting situation is depicted in Exhibit A1, Panel a. The UA to the CFIIR calculation can be applied to find the IRR, the "i":

$$CF_0 = PV\,(CF_1) + PV\,(CF_2) + \ldots + PV\,(CF_N) \tag{A1}$$

$$CF_0 = CF_1\,[1/(1+IRR)^1] + CF_2\,[1/(1+IRR)^2] + \ldots + CF_N\,[1/(1+IRR)^N]$$

$$1{,}000 = 400\,[1/(1+IRR)^1] + 400\,[1/(1+IRR)^2] + 300\,[1/(1+IRR)^3] + 200\,[1/(1+IRR)^4]$$

$$IRR = i = 12.88\%$$

The MIRR is the interest rate which equates the present value of cash outflows with the terminal value, the sum of the future values of the cash inflows, where the market-determined cost of capital is used in finding the present and future values. Once the present value of the outflows and the terminal value of the inflows are determined, the UA to the CFIIR calculation can be applied to find the MIRR, the "i."

Consider a project with a time profile of cash flows as depicted in Exhibit A1, Panel b, where the market-determined opportunity cost of capital is 8%. At this rate the present value of the only cash outflow is $1,000 and the sum of the future values of inflows, the terminal value, equals $1,494.45. The time profile of these two cash flows is depicted in Exhibit A1, Panel b.

The MIRR is the rate at which the present value of outflows equals the present value of the terminal value. The UA to the CFIIR calculation is used to find the MIRR, the "i":
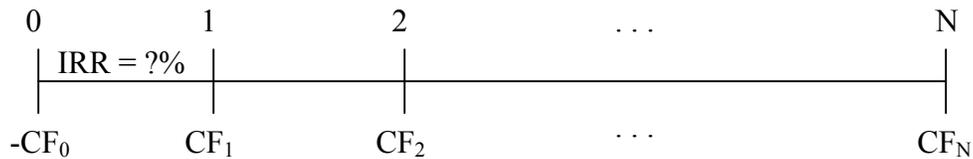
$$FV_4 = PV_0 (1 + MIRR)^4 \qquad\qquad (A2)$$

$$FV_4 = PV_0 (FVIF_{MIRR,4})$$
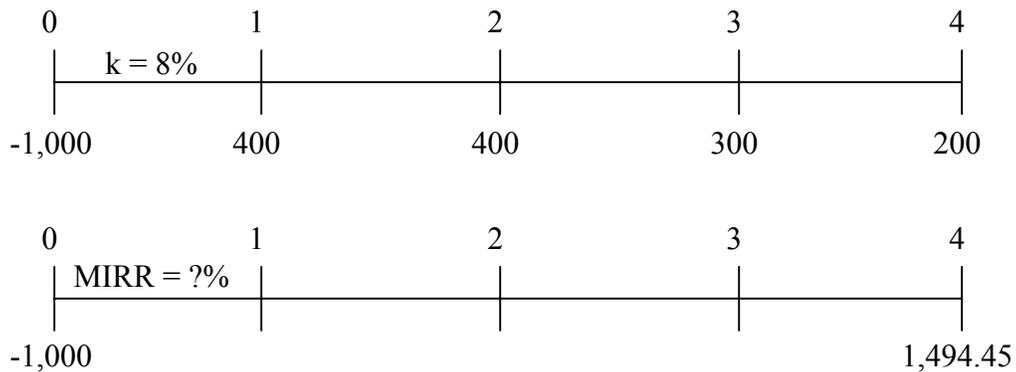
$$1{,}494.45 = 1{,}000 (FVIF_{MIRR,4})$$

$$MIRR = 10.57\%$$

Since 10.57%>8%, then, according to the MIRR capital budgeting decision rule, the investment in the project should be made.

```
0            1          2        . . .             N
|  IRR = ?%  |          |                          |
|            |          |                          |
-CF₀         CF₁        CF₂       . . .            CFN
```

$$
\begin{array}{cccccc}
0 & & 1 & 2 & \cdots & N \\
| & IRR = ?\% & | & | & & | \\
-CF_0 & & CF_1 & CF_2 & \cdots & CF_N
\end{array}
$$

Panel a. The IRR.

$$
\begin{array}{ccccc}
0 & 1 & 2 & 3 & 4 \\
| \quad k = 8\% & | & | & | & | \\
-1{,}000 & 400 & 400 & 300 & 200
\end{array}
$$

$$
\begin{array}{ccccc}
0 & 1 & 2 & 3 & 4 \\
| \quad MIRR = ?\% & | & | & | & | \\
-1{,}000 & & & & 1{,}494.45
\end{array}
$$

Panel b. The MIRR.

Exhibit A1. Capital Budgeting.

In summary, the UA to the CFIIR calculation is applied to capital budgeting problems in order to find the IRR and the MIRR in the decision-making process.

A2. The Cost of Short-Term Financing:

This section looks at the short-term financing of corporations, bank borrowings, and illustrates the underlying concept of the implied interest rate in calculating the cost of different types of short-term bank loans: Simple Interest, Discount Interest, Loans with Compensating Balances, and Add-On Interest. In each case, the interest rate, the "i," is found by applying the UA to the CFIIR calculation. In all of the cases considered in this section, there are only two parties to the financial transaction and the time profiles of cash flows are identical for each. Therefore, the expected interest rate earned by the bank equals the expected cost of short-term funds incurred by the corporation.

A2.A. Simple Interest

In this type of short-term bank loan arrangement, in which there is no compounding of interest, first, the nominal interest is divided by the number of days in the year to obtain the daily rate. Then, the daily rate is multiplied by the number of days in the specific payment period and by the amount of the loan to find the applicable interest payment for that period.

For example, a one-year loan of $100,000 requires an interest payment of $9,000 at the end of the year (annual interest). The expected rate of return to be earned by the lender and the expected percentage cost to be incurred by the borrower both equal the "i." The UA to the CFIIR

calculation is used to find the interest rate, the "i." The time profile of the cash flows appears in Exhibit A2.

$$PV = PV\ (CF_1),\ \text{where } CF_1 = FV_1 \tag{A3}$$

$$PV = FV_1\ [1/(1+i)]$$

or

$$PV = FV_1\ (PVIF_{i,1})$$
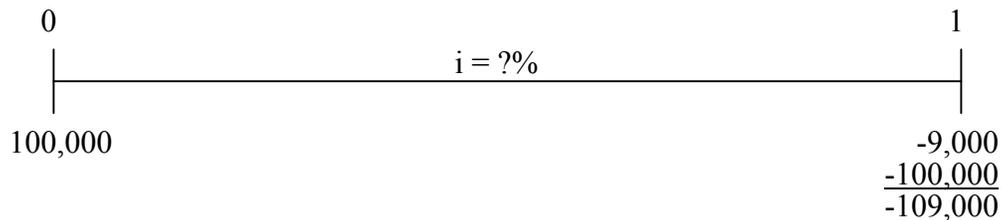
$$100,000 = 109,000\ (PVIF_{i,1})$$

$$i = 9\%$$

Exhibit A2. Short-Term Bank Loan: Simple Interest, Paid Annually.

A2.B. Discount Interest

In this type of short-term bank loan arrangement, the bank deducts the interest in advance, i.e., at the beginning of the loan period, and the borrower promises to pay back the principal at the end of the loan period.

For example, if the corporation borrows $100,000 for one year from the bank at the rate of 8%, the discount interest, then the time profile of cash flows will be as depicted in Exhibit A3. The expected rate of return to be earned by the lender and the expected percentage cost to be incurred by the borrower both equal the "i." The UA to the CFIIR calculation is applied to find the interest rate, the "i":

$$PV = PV~(CF_1), \text{ where } CF_1 = FV_1 \qquad\qquad\qquad (A4)$$

$$PV = FV_1~[1/(1+i)]$$

or

$$PV = FV_1~(PVIF_{i,1})$$

$$92{,}000 = 100{,}000~(PVIF_{i,1})$$

$$i = 8.7\%$$

```
0                                                              1
         i = ?%
|------------------------------------------------------------|
100,000                                                  -100,000
 -8,000
 92,000
```
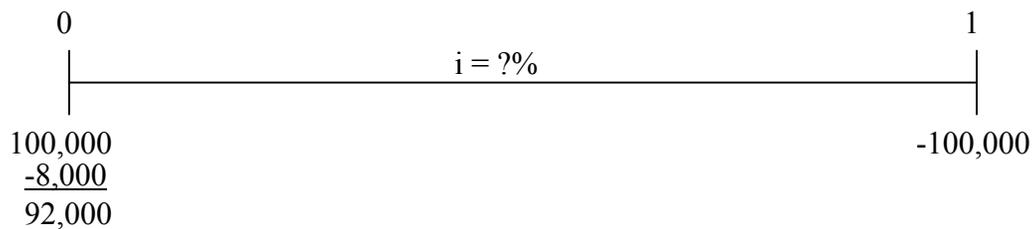
Exhibit A3. Short-Term Bank Loan: Discount Interest.

A2.C. Loan with Compensating Balance

Banks often require corporations to hold compensating balances in their demand deposit accounts. If the amount of the compensating balance for a particular loan exceeds the amount the corporation would normally hold on deposit with the bank, then a portion of the loan must be held back to fulfill the required compensating balance, i.e., is deducted at $t = 0$ from the loan, and then added back when the loan matures.

Given the previous discount interest loan example, if the bank also requires a 10% compensating balance, then the time profile of cash flows will be as depicted in Exhibit A4. The expected rate of return to be earned by the lender and the expected percentage cost to be incurred by the borrower both equal the "i." The UA to the CFIIR calculation is used to find the interest rate, the "i":

$$PV = PV\ (CF_1), \text{ where } CF_1 = FV_1 \tag{A5}$$

$$PV = FV_1\ [1/(1+i)]$$

or

$$PV = FV_1\ (PVIF_{i,1})$$

$$82{,}000 = 90{,}000\ (PVIF_{i,1})$$

$$i = 9.8\%$$

It should be noted that this rate is higher than the rate for a similar loan with no compensating balances.

```
       0                                                           1
       |                      i = ?%                               |
       |-----------------------------------------------------------|
       |                                                           |

   100,000                                                   -100,000
    -8,000                                                     10,000
   -10,000                                                    -90,000
    82,000
```

Exhibit A4. Short-Term Bank Loan: Discount Interest with Compensating Balance.

A2.D. Add-On Interest

In this type of short-term bank loan arrangement, the interest amount is added to the principal amount borrowed and then the total is divided by the number of periods over which the principal and interest are to be repaid.

For example, consider a loan of $10,000 on an add-on basis at a nominal rate of 10% to be repaid in 12 equal monthly installments. The interest amount is $1,000, the principal and interest are $11,000, and the monthly payment is $916.67. The time profile of cash flows is

depicted in Exhibit A5. The expected rate of return to be earned by the lender and the expected percentage cost to be incurred by the borrower both equal the "i." The UA to the CFIIR calculation is used to find the interest rate, the "i":

$$PV = PV(CF_1) + PV(CF_2) + \ldots + PV(CF_N) \tag{A6}$$

$$PV = CF_1 [1/(1+i)^1] + CF_2 [1/(1+i)^2] + \ldots + CF_N [1/(1+i)^N]$$

$$PV = CF_1 (PVIF_{i,1}) + CF_2 (PVIF_{i,2}) + \ldots + CF_N (PVIF_{i,N})$$

$PV = PMT (PVIFA_{i,N})$, where PMT is the *monthly payment* made to the bank and PVIFA is the *present value interest factor annuity*.

$$10,000 = 916.67 (PVIFA_{i,12}) \tag{A7}$$

$$i = 1.4977\%$$

which is the rate of interest per month, and in annual terms the *equivalent annual rate*, EAR, would be:

$$EAR = (1 + 0.014977)^{12} - 1 = 19.53\% \tag{A8}$$

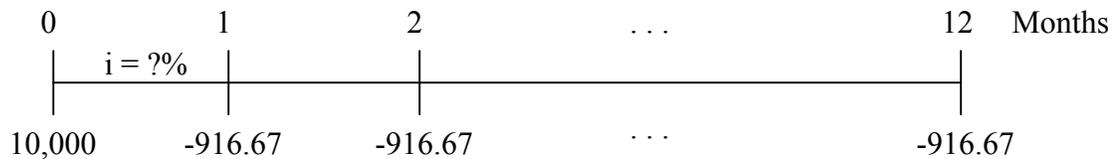This rate in much higher than the stated nominal rate of 10%.

Exhibit A5. Short-Term Bank Loan: Add-On Interest.

A3. Financing with Bonds and Warrants

This section refers to corporations financing with bonds and attached warrants and shows how to find the expected rate of return to the bondholders, the "i," by applying the UA to the CFIIR calculation. To reduce what would be a higher interest rate on a new issue of debt, corporations often attach a "sweetener" for the bondholder, that is, a warrant. The bondholder accepts a lower required rate of return based on the risk of the bond, but has an opportunity to participate in the expected growth of the firm by being able to purchase a stated number of shares later at a specified price (much like a call option).

For example, a corporation issues 30-year bonds with warrants. Each bond is sold for $1,000 and promises to pay $90 in interest annually. Each bond has 25 warrants, each of which entitles its holder to buy one share of stock for $30. The stock price, currently at $25, is expected to grow at 10 percent per year. When the warrants expire ten years from now, the stock price would be $25(1 + 0.10)^{10} = 64.84. Assuming the warrants had not been exercised during the ten-year period, the corporation would then have to issue one share of stock worth $64.84 for each warrant exercised and, in turn, would receive only the exercise price of $30. If the bondholders immediately sold these shares in the market, their profit would be $64.84 − 30 = $34.84 for each share. Since each bond has 25 warrants attached, the bondholders would have a gain of 25($34.84) = $871 per bond at the end of the tenth year. The time profile of the cash flows is depicted in Exhibit A6. The expected rate of return to be earned by the lender and the expected percentage cost to be incurred by the borrower both equal the "i." The UA to the CRIIR calculation is used to find the "i" by means of a search:

$$PV = PV(CF_1) + PV(CF_2) + \ldots + PV(CF_{10}) + \ldots + PV(CF_{30}) \tag{A9}$$

$$PV = CF_1\,[1/(1+i)^1] + CF_2\,[1/(1+i)^2] + \ldots + CF_{10}\,[1/(1+i)^{10}] + \ldots + CF_{30}\,[1/(1+i)^{30}]$$

$$1{,}000 = 90\,[1/(1+i)^1] + 90\,[1/(1+i)^2] + \ldots + 961\,[1/(1+i)^{10}] + \ldots + 1{,}090\,[1/(1+i)^{30}]$$
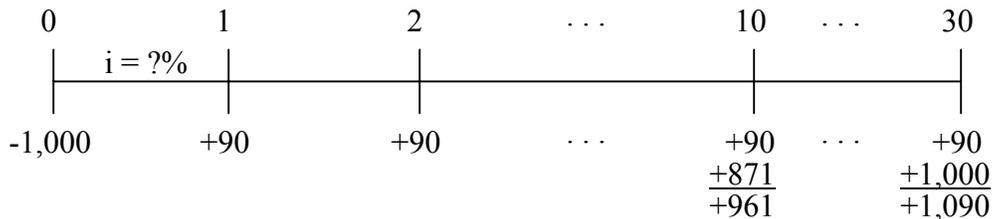
$$i = 12.46\%$$

Exhibit A6. Financing with Bonds and Warrants.

A4. International Financial Markets: Cost of Debt

This section extends the discussion of the cost of capital to include the international financial markets and applies the UA to the CFIIR calculation to find the percentage cost of debt. Consider a corporation which arranges a two-year, $1,000,000 amortized loan which is denominated in French francs, carries a 10% nominal rate, and requires equal semi-annual payments. Assume this scenario: the exchange rate at the time of the loan was 7.75 francs per dollar but immediately dropped to 7.10 FF/U.S.$ before the first payment came due and stayed at that level through the end of the loan period. The corporation must convert U.S. dollars to French francs to make its payments.

The loan in French francs is 7,750,000 (= 1,000,000 x 7.75). The equal semiannual payment for the loan amortization is obtained as follows:

$$PV = PMT\,(PVIFA_{i,N}) \tag{A10}$$

FF 7,750,000 = PMT (PVIFA$_{5\%,4}$)

PMT = FF 2,185,561.20

The semi-annual payments in U.S.$ are:

PMT = FF 2,185,561.20 / 7.10 = U.S.$ 307,825.52 $\hspace{4cm}$ (A11)

The time profiles of cash flows, both in French francs and dollars, are depicted in Exhibit A7. The UA to the CFIIR calculation is used to find the cost of funds, the "i," for the corporation:

PV = PMT (PVIFA$_{i,4}$) $\hspace{6cm}$ (A12)

1,000,000 = 307,825.52 (PVIFA$_{i,4}$)

i = 8.88%

which is the semi-annual rate. The equivalent annual rate, its "i," is:

EAR = $(1 + 0.0888)^2 - 1 = 18.55\%$ $\hspace{5cm}$ (A13)

This is the annual cost of funds to the corporation. In this case, because the French franc has appreciated in value, the U.S. corporation must convert more dollars to buy the necessary French francs to meet the semi-annual payment. Therefore, its interest rate cost, the "i," is greater than was expected. Since the loan was denominated in French francs, the French lender received the expected rate of return, its "i," which is different from the interest rate cost to the U.S. corporation.

0	1	2	3	4	6-Month Period
i = ?%					
FF-7,750,000	2,185,561.20	2,185,561.20	2,185,561.20	2,185,561.20	
U.S.$-1,000,000	307,825.52	307,825.52	307,825.52	307,825.52	

Exhibit A7. International Financial Markets: Debt Financing.

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