Nonlinear Analysis of Concrete Beams Strengthened with Steel Fiber-Reinforced Concrete Layer

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Abstract—The behavior of concrete beams strengthened with a steel fiber-reinforced concrete (SFRC) layer was studied by Nonlinear Finite Element Analysis using ANSYS software. Four beams that were experimentally tested in a previous research were considered. Beam B-1 is made of ordinary reinforced concrete, B-2 is made of SFRC material, B-3 is made of two parts, RC beam with SFRC overlay and B-4 is made of RC beam with SFRC underlay.

Ordinary concrete as well as SFRC were modeled using the multi-linear isotropic hardening constants where they are assumed to have a linear behavior up to 30% of the compressive strength. Afterwards, a multi-linear stress-strain curve was defined. For reinforcing steel, a linear-elastic perfectly-plastic material model was used. Steel fiber reinforced concrete was modeled by the smeared modeling technique.

The results obtained by FEA showed good agreement with those obtained by the experimental program. This research demonstrates the capability of FEA in predicting the behavior of beams strengthened with SFRC layer. It will help researchers in studying beams with different configurations without the need to go through the lengthy experimental testing programs.

Index Terms— None linear FEA, SFRC, RC beams with SFRC overlay.

I INTRODUCTION

The use of steel fiber-added reinforced concrete (SFRC) has become widespread in several structural applications such as tunnel shells, concrete sewer pipes, and slabs of large industrial buildings. Usage of SFRC in load-carrying members of buildings having conventional reinforced concrete (RC) frames is also gaining popularity because of its positive contribution to both the energy absorption capacity and the concrete strength. Recently, SFRC start to make its way into strengthening techniques, such as an additional layer as an underlay or overlay on existing beams, jackets for beams, columns, and other structural members. Members fabricated from SFRC exhibited a remarkable improvement in crack behavior, ductility, compression and tensile strength, and durability.

The effectiveness of using SFRC underlay or overlay techniques depends on the ability of the strengthened beam to act monolithically as one unit while being loaded. Several researches have been conducted to examine the effectiveness of adding SFRC layers on the behaviors of existing RC beams. Most of these researches are based on costly and time consuming experimental programs.

In this study, the validity of numerical analysis will be demonstrated in predicting the behavior of RC beams with SFRC added layers. Four (4) beams which were previously tested in an experimental program are being considered. The beams were strengthened with SFRC overlays, which were mechanically bonded to the original beams. Numerical analysis will be carried out using the well-known FE code ANSYS APDL v13.0.

The importance of this study comes from the fact that if a numerical approach can be validated, it will help researchers in predicting the behavior of different RC/SFRC beams without the need to go through the lengthy and costly experimental programs.

II LITERATURE REVIEW

Steel Fibers drew the attention of many researchers in the last decade, mainly in the strengthening field, due to its ability to enhance the strength, ductility, and durability of RC members. Most of the research was devoted to studying the mechanical properties of SFRC, and its influence on flexural, shear, and torsional behavior of beams. Few studies were found for RC beams with SFRC layers. The following is a brief summary for some related publications.

Tiberti et al [2] conducted an experimental study to investigate the ability of fibers in controlling crack spacing and width. They concluded that the addition of fibers in concrete resulted in narrower and more closely spaced cracks compared to similar members without fibers. They also concluded that added fibers may significantly improve the tension stiffening. It also provided noticeable residual stresses at the crack area due to the bridging effect provided by its enhanced toughness.

Zhang et al [3] conducted three-point bending tests on notched beams of SFRC. The results show that the fracture energy and the peak load increase as the loading rate increases. The gain of the fracture energy and peak load is around 10% compared with its quasi-static values.

Altun et. al. [4] studied the mechanical properties of concrete with different dosage of steel fibers. Experimental tests indicated that beams with SF dosage of 30 kg/m3 exhibited a remarkable increase in strength when compared to RC beams without steel fibers. The same study also showed that increasing the fiber dosage to 60 kg/m3 adds only a small improvement to the beam toughness.

Kim et. al. [5] studied the shear behavior of SFRC members without transverse reinforcement. They proposed a model based on a smeared crack assumption. The model was verified against experimental results and showed a good agreement.

Deluce & Vecchio [6] conducted an experimental study on 4 large-scale SFRC specimens containing conventional reinforcement to study their cracking and tension-stiffening behavior. It was found that the cracking behavior of SFRC was significantly altered by the presence of conventional reinforcement. Crack spacing and crack widths were influenced by the reinforcement ratio and bar diameter of the conventional reinforcing bar, as well as, by the volume fraction and aspect ratio of the steel fiber.

Ziara, M. [7] conducted an experimental test program consisting of nine beams to study the influence of SF overlays on RC beams. He found that beams strengthened by SF overlays exhibited a remarkable increase in load carrying capacity. Mechanically bonded overlays showed better performance when compared with chemically bonded overlay which exhibited inter-laminar shear failure.

From the aforementioned literature review, it can be seen that several experimental researches have been conducted on SFRC. As a result, the behavior and mechanical properties of SFRC are reasonably identified. However, to the best knowledge of the author, no research was found on the numerical analysis of RC beam strengthened with SFRC layers.

III CASE STUDY

To study the behavior of RC beams strengthened with SFRC layers, four beams with different configurations were considered as shown in Figure 1. Beams B1 & B2 are used as control beams. B1 is made entirely of ordinary reinforced concrete (RC) while B2 is made entirely of SFRC material. B3-OL consists of two parts, the bottom part is made of ordinary concrete while the top part (overlay) is made of SFRC. B4-UL is also consists of two parts but the SFRC part is on the bottom (underlay).

The first three beams (B1, B2 and B3-OL) were previously tested in an experimental program conducted by Ziara, [7]. In this study, nonlinear finite element analysis is conducted for the 4 beams. The validity of the numerical analysis is verified by comparing the obtained results with the experimental program results. *Note*: B4-UL was not experimentally tested, but was added to complete the parametric study.

A. Geometric Properties

As shown on Figure-1, all beams have the same geometrical properties, with an overall length of 2000mm and width to depth dimension of 150mm×240 mm. All beams were reinforced with 4 longitudinal bars (2Ø14mm@bottom and 2Ø8mm@top). The stirrups are 8mm@55mm as shown.

B. Material Properties

For the sake of this study, the same mechanical properties for RC and SFRC that were used in the experimental program will be used for the numerical analysis. In SFRC, the steel fibers dosage used is 1.5% of the volume. According to Song and Hwang [8] for this dosage of SFRC, the tensile strength can be taken as 16% of the compressive strength. The material properties used for the beams are as follows:

For ordinary concrete:

Compressive strength	$f_{c'}$ =	25 MPa
Tensile strength	f_t =	2.52 MPa

For SFRC material:

Compressive strength	$f_{c'}$ =	26 MPa
Tensile strength	f_t =	4.16 MPa
Yield strength main steel	f_y =	410 MPa
Yield strength for stirrups	f_y =	280 MPa

C. Loading

All beams were loaded with two symmetrical loads and two simple supports as shown in Figure-1. The loads were gradually increased on the beams until failure.

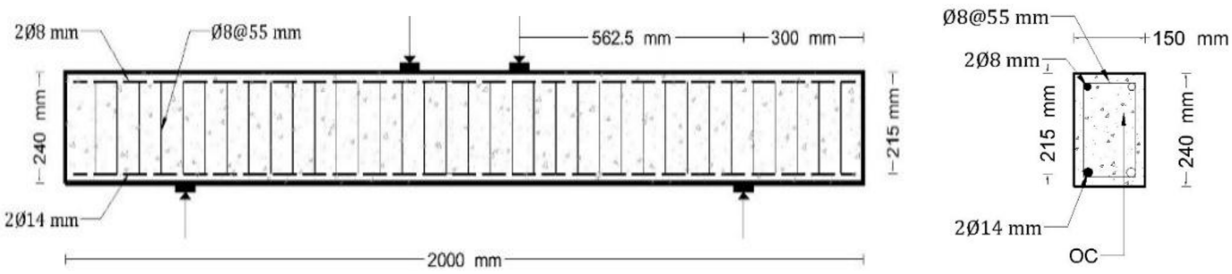

Figure 1-a: *Beam B1-RC (Ordinary Concrete)*

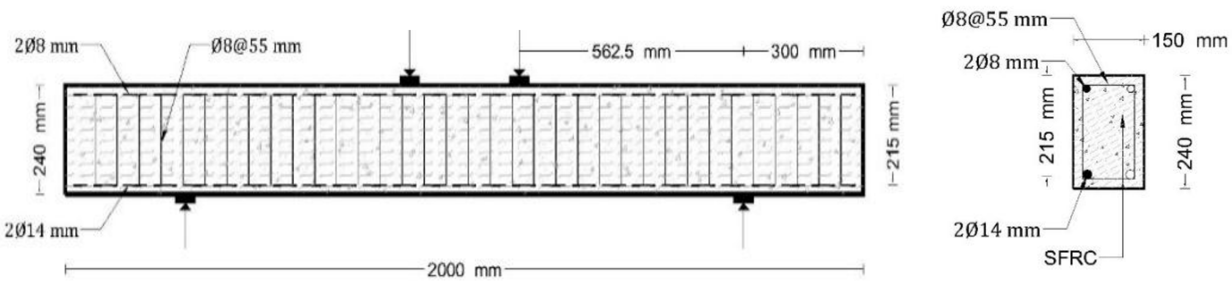

Figure 1-b: Beam B2-SF (SFRC)

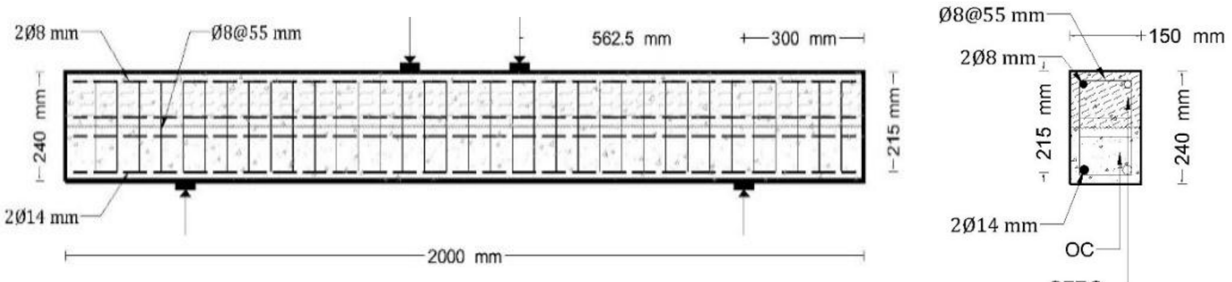

Figure 1-c: *Beam B3-OL (SFRC Overlay)*

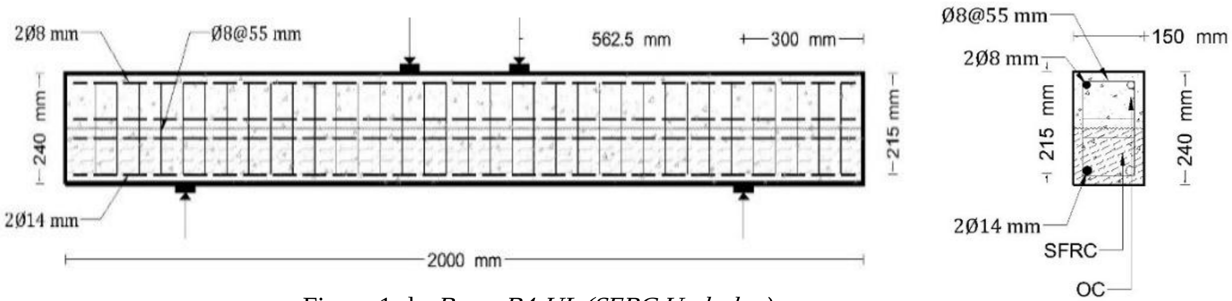

Figure 1-d: *Beam B4-UL (SFRC Underlay)*

Figure 1: Beams Configurations

D. Finite Element Modeling

Nonlinear Finite Element Analysis was conducted on the four beams to predict the behavior of these beams during all loading stages up until failure. The well known FE package ANSYS APDL v13.0 has been used.

The ANSYS program includes a library of elements for different applications. SOLID65 is used for the 3D modeling of concrete with or without reinforcing bars. SOLID65 is capable of cracking in tension and crushing in compression. It can also consider plastic deformation and creep.

For steel rebar, ANSYS presents LINK180 to model reinforcing steel which is simply a pin-joined one dimensional element. The geometry and the coordinate system for the SOLID65 and the LINK180 elements are shown in Figure 2

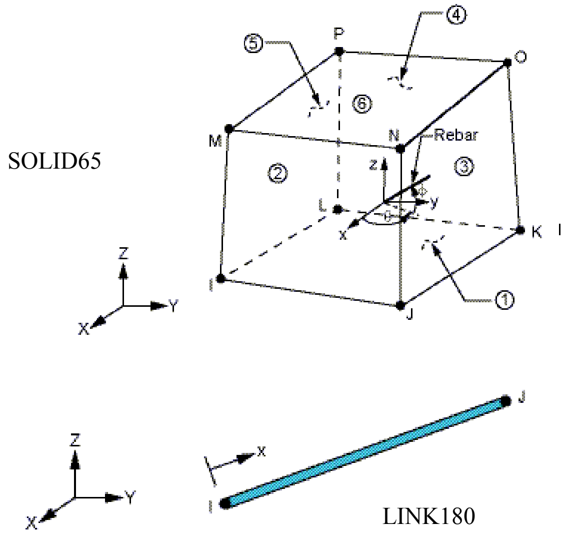

Figure 2: *SOLID65 Geometry*

E. Concrete Material Description

Concrete is a quasi-brittle material and has a highly nonlinear and ductile stress-strain relationship [9]. The nonlinear behavior is attributed to the formation and gradual growth of micro cracks under loading. Fig. 3 shows a typical stress-strain curve for normal weight concrete. In compression, the stress-strain curve for concrete is linearly elastic up to about 30% of the maximum compressive strength. Above this point, the stress increases gradually up to the maximum compressive strength. After it reaches the maximum compressive strength f_{cu}, the curve descends into a softening region, and eventually crushing failure occurs at an ultimate strain ε_{cu}. In tension, the stress-strain curve for concrete is approximately linearly elastic up to the maximum tensile strength. After this point, the concrete cracks and the strength decreases gradually to zero.

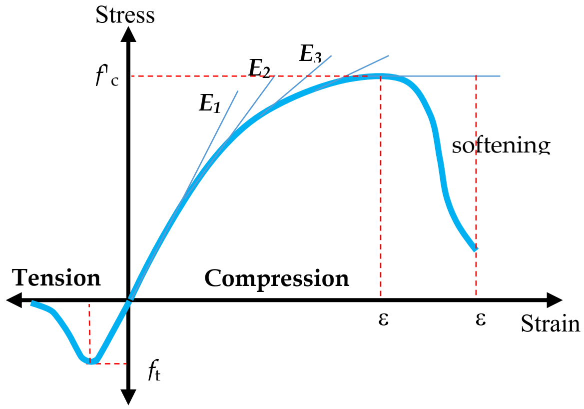

Figure 3: *Typical stress-strain curve for concrete*

In ANSYS, a Multi-linear Isotropic Hardening Constants model is used for concrete. The slope of the first segment of the curve corresponds to the elastic modulus of the material and no segment slope should be larger. No segment can have a slope less than zero. The slope of the stress-strain curve is assumed to be zero beyond the last user-defined stress-strain data point, i.e., the apex of the stress-strain curve.

Linear material properties of normal weight concrete includes modulus of elasticity E_c, and Poisson's ratio υ_c which can be evaluated according to the ACI-318-08 design code as per the following empirical equation [10]:

$$w_c^{1.5} \times 0.043\sqrt{f_c'}$$

For the nonlinear part of the material properties, the popular stress-strain model of Hognestad [11], can be used. This model consists of a second-degree parabola with an apex at a strain of $1.8f_c''/E_c$, where $f_c''=0.9f_c'$, followed by a downward-sloping line terminating at a stress of $0.85f_c'$ and a limiting strain of 0.0038. The stress at any point can be evaluated using the formula [11]:

$$f_c''\left(\frac{2\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2\right)$$

where ε_0 equals $1.8f_c''/E_c$ and ε_c represents strain at different stress values.

In order to get a realistic response of concrete elements and determine the load failure accurately, a failure criterion should be defined. ANSYS uses Willam-Warnke [12] failure criterion, Figure-4, with the following five parameters to define the failure surface.

1. The uniaxial compressive strength, f_c.
2. The uniaxial tensile strength, f_t.
3. The equal biaxial compressive strength, f_{cb}.
4. The high-compression-stress point on the tensile meridian, f_1.
5. The high-compression-stress point on the compressive meridian, f_2.

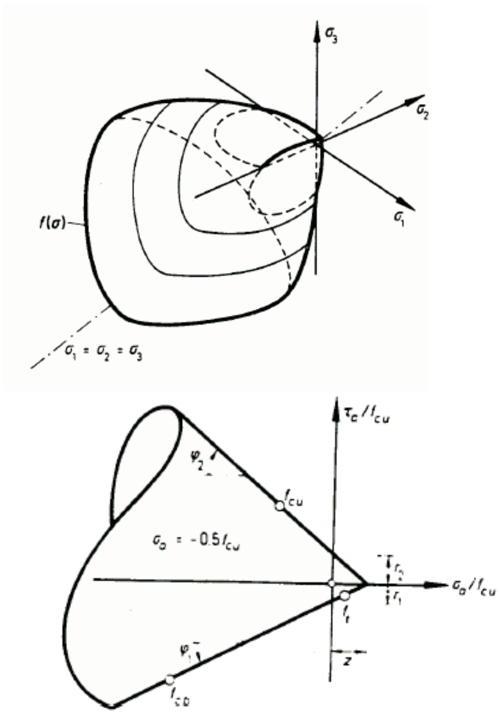

Figure 4: *Willam-Warnke Failure Surface [11]*

However, the failure surface can be specified with a minimum of two constants, f_t and f_c. The other three constants default to Willam and Warnke:

$$f_{cb} = 1.2 \times f_c,\ f_1 = 1.45 \times f_c \text{ and}$$

$$f_2 = 1.725 \times f_c$$

Along with the five parameters, an open-and-close crack retention factor must be defined. This transfer coefficient β_t represents a shear-strength reduction factor for those subsequent loads, which induce sliding (shear) across the crack face. A multiplier to account for the amount of tensile stress relaxation shall be defined as well. The following table summarizes the failure surface, the shear retention, and stress relaxation factors for the four beams:

TABLE 1

Numerical Parameters used in the FE models

Parameter	B1	F1	S4	
			OC	SFRC OL/UL
$\beta_{t\ (OPEN)}$	0.3	0.15	0.3	0.1
$\beta_{t\ (CLOSE)}$	0.8	0.3	0.8	0.2
f_t	2.52	4.16	2.52	3.44
f_c	25.2	26	25.2	-1
T_c	0.6	0.6	0.6	0.6

Typical shear transfer coefficients range from 0.0 to 1.0, with 0.0 representing a smooth crack (complete loss of shear transfer) and 1.0 representing a rough crack (no loss of shear transfer). When the element is cracked or crushed, a small amount of stiffness is added to the element for numerical stability. The stiffness multiplier CSTIF is used across a cracked face or for a crushed element and defaults to 1.0E-6.

F. Steel Material Description

For steel reinforcement, a linear-elastic perfect-plastic material model was adopted. For practical reasons, steel is assumed to exhibit the same stress-strain curve in compression as in tension. Passion's ratio for steel will be set to 0.3, modulus of elasticity will be set to 200 GPa, the yield strength for flexural reinforcement will be set to 420 MPa, while for secondary reinforcement and stirrups it will be set to 280 MPa. The tangent modulus for the flexural, secondary reinforcement and stirrups will be set to 2000 MPa.

G. Model Structure

Establishing an FE model with proper mesh and proper boundary conditions could be a tedious task. Therefore, a special code for automatic mesh generation was developed. Assuming perfect bond, concrete and reinforcement elements were connected at common nodes to assure displacement compatibility. A mesh convergence study indicated that an element with dimensions of 27.5mm×20mm×27.5mm in the x, y, and z directions, respectively, would yield satisfactory results. Concrete elements were densified in locations of contact with loading and supporting plates to consider stress concentration. (Figures 5 & 6)

Considering the symmetry along the mid-span, only one half of the beam were modeled. This required applying additional boundary constrains perpendicular to the axis of symmetry.

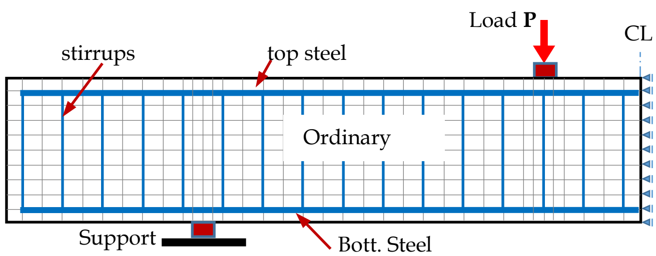

Figure-5: FE mesh for B1/B2 (ordinary RC/SFRC beam)

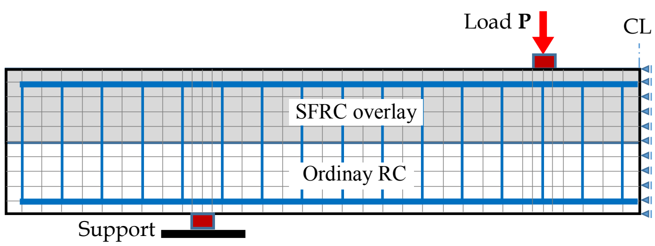

Figure 6: FE mesh for B3 (RC with SFRC overlay)

H. Bonding between the two layers

In the experimental program, the inter laminar shear (bonding) between the ordinary concrete and the SFRC layer was resisted in two different ways, a) by chemical bonding agent, and b) by mechanical dowels. In the experiments, mechanical bonding was shown to be more efficient.

In numerical analysis for beams B3 & B4, mechanical bonding was represented by combining common nodes at the crossing steel (stirrups) locations.

I. Numerical Solution Parameters

Setting numerical solution parameters involves defining the analysis type, specifying load step options and defining convergence criteria. All parameters were set to default, as the analysis will be of a "small displacement static" type. Two convergence criteria were used, i.e. force and displacement with values of 0.005 and 0.05, respectively.

IV. RESULTS

The results obtained from the numerical analysis by ANSYS for the four beams include extensive information that describes the behavior of the beams during all load stages up until failure. Results include first crack load, yield load, failure load, failure deflection, plastic strain, crack pattern and load deflection curve. The following table summarizes the results obtained from the numerical analysis of the four beams:

TABLE 2
Results of Numerical Analysis for the four beams

Description	B1	B2	B3-OL	B4-UL
First Crack Load (KN)	15.65	15.29	15.05	23.52
Yield Load (KN)	103.24	110.39	105.33	109.20
Strain At Failure	0.0092	0.0132	0.0062	0.0345
Failure Load (KN)	113.5	125.0	130.5	141.1
Deflection (mm)	7.16	11.57	7.78	18.80
Failure Mode	Flexural	Flexural	Flexural	Flexural

A. Numerical versus Experimental results

In general, the results of the numerical solution by ANSYS compares very well with the experimental results obtained by Ziara [7] for the same beams in regards to the load carrying capacity, ductility and failure mode. Minor differences in load-deflection curves between numerical and experimental models can be attributed to the shortcomings in numerical material description, constitutive models, and numerical instability in modeling the cracks.

TABLE 3
Comparing numerical to experimental results

Type	B1	B2	B3-OL	B4-UL
Failure Load (KN)				
Numerical	113.48	125.02	130.55	141.12
Experimental[(1)]	113.50	126.80	133.50	NA
Deflection (mm)				
Numerical	7.16	11.57	7.78	18.80
Experimental[(1)]	7.58	11.62	7.40	NA

[(1)] Experimental result obtained by Ziara [7]

The load deflection curves obtained from ANSYS for the four beams were plotted against those of the experimental result as shown in Figure 7, 8 & 9

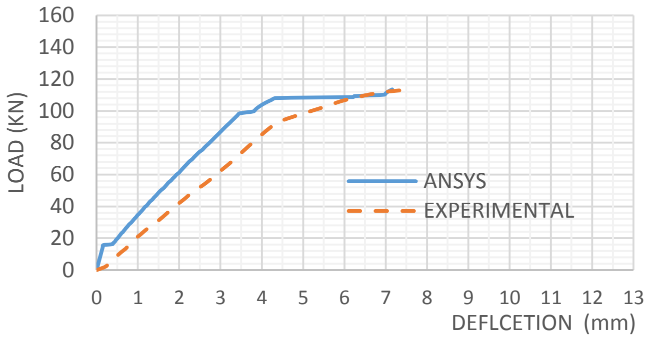

Figure 7: Beam B1-RC

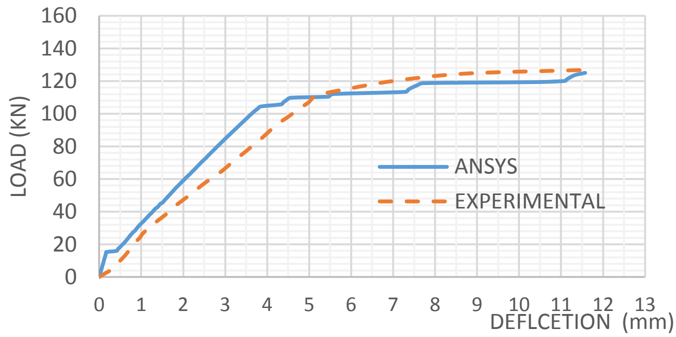

Figure 8: Beam B2-SFRC

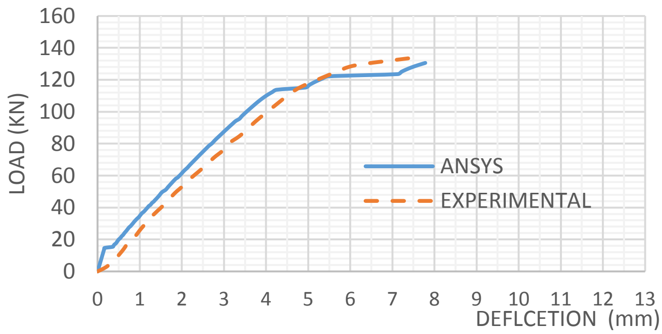

Figure 9: Beam B3-OL

Figures 10& 11 show the crack patterns obtained by ANSYS for the beams B1 and B3.

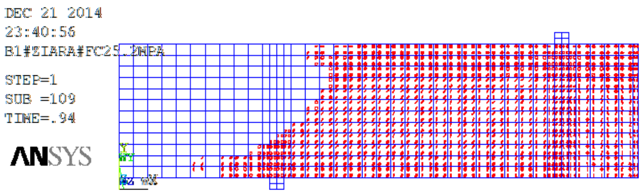

Figure 10: Crack Pattern for B1-RC

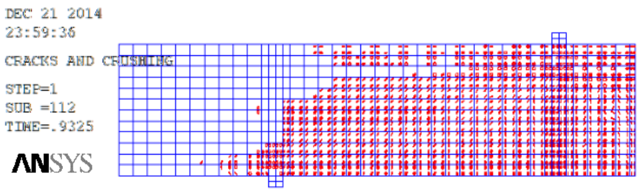

Figure 11: Crack Pattern for B3-OL

B. Discussion of the Results

Referring to the load deflection curves for the four beams, it can be seen that the FEA was able to predict the behavior of the different beams fairly well. It captured the softening phenomena at first crack, major crack proliferation, yield point, and just before complete failure, which is not clear in the experimental results.

Both crack patterns and failure modes of numerical models compared very well to those of the experimental models, this is mainly because failure of the models are controlled by proliferation of cracks into the concrete matrix rather than crushing of concrete.

Due to the presence of steel fiber in **"B2",** this beam shows improved ultimate capacity and ductility as compared to B1. This result is consistent with previous experimental results.

Beam **"B3-OL"** which was strengthened with SFRC overlay containing stirrups in the shear span reached its full flexural capacity, which was equal to 130.55 KN, i.e. 1.15 of beam "B1". The increase of flexural capacity by 15% compared to the control beam can be attributed to the presence of steel fibers and the prevention of inter-laminar shear failure by simulating a welding situation of stirrups in SFRC overlay to the existing stirrups in the ordinary concrete part. However, the ductility of the beam was similar to that of the control beam. Absence of steel fibers from the section under the neutral axis where tensile stresses were formed, led to significant proliferation of cracks, and hence, beam behavior in terms of ductility was similar to the control beam modeled using ordinary concrete properties.

Beam **"B4-UL"** which is a strengthened beam with SFRC underlay containing stirrups in shear span reached its full flexural capacity, which was equal to 141.12 KN, i.e. 1.24 of beam "B1". This beam experienced the highest flexural capacity of all four beams due to the prevention of inter-laminar shear failure by simulating a welding situation of stirrups in the shear span of SFRC underlay to the existing stirrups in ordinary concrete. Moreover, presence of steel fibers in concrete under neutral axis where tensile stresses were developed managed to delay the first crack, prevent sudden cracks from developing, spread under small amounts of loads, and enhance stress redistribution between concrete and steel reinforcement.

Crack width for the aforementioned beams were not detectable. ANSYS is not equipped with discrete crack modeling technology, where crack width can be evaluated based on the separation in the mesh. Smeared crack technology can predict both the crack location and orientation, but not the crack width.

The sudden increase in deflection at the first crack for all of the beams is a reflection of the stress redistribution phenomena, where concrete cannot withstand tensile stresses much longer. In this case, steel reinforcement would resist those tensile stresses formed due to loading of beams. Presence of steel fibers can delay the stress redistribution as in beams "F1" to a certain load, where tensile stresses developed at that load cannot be resisted by steel fibers as well. The same thing goes for the "jumps" in deflection under a small rate of loading, during the entire loading process.

V. Parametric Study

To further demonstrate the validity of the numerical analysis, a parametric study was carried on beam "B3-OL". In this study the influence of SFRC compressive strength and fraction volume on the overall behavior of the beam were examined. The following is a summary of the obtained results.

A. Effect of compressive strength

The influence of compressive strength of the SFRC on the beam load carrying capacity was examined. Three different values of compressive strength were used (31.5 MPa, 41.5 MPa and 51.5 MPa). The obtained results indicated significant improvement in the load carrying capacity of the beam as well as the overall ductility as shown in Figure-12.

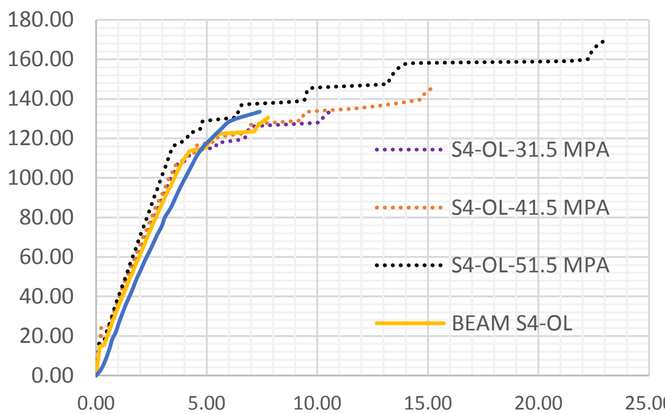

Figure 12: *Effect of Compressive Strength on "S4-OL"*

B. Effect of Fraction volume

Beam "B3-OL" was tested using 0.5%, 1%, and 2% fraction volumes of steel fibers. Results indicated an enhancement in the ductility of the beam at fraction volume of 2.0%. While the load carrying capacity maintained its original levels. (see Figure -13)

Increasing or decreasing the fraction volume did not affect the overall behavior of the beam significantly. This is mainly due to the fact that the SFRC overlay lies above the neutral axis where compressive stresses are formed, while tensile stresses to be resisted by the steel fibers are formed below the neutral axis.

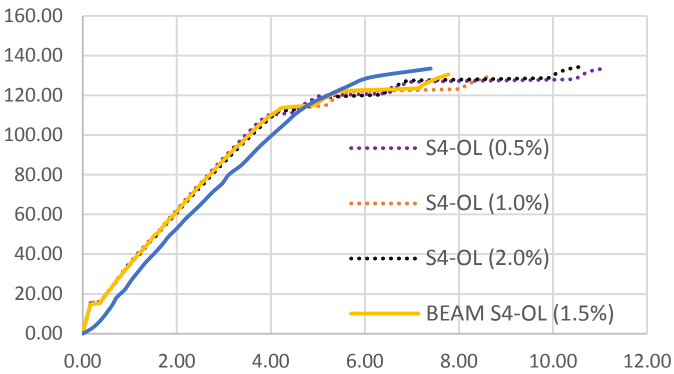

Figure 13: *Effect of Volume Fraction on "S4-OL"*

V. Summary and Conclusion

Experimental studies of reinforced concrete beams could be costly and time consuming. Advancement in computer capabilities and progression in developing sophisticated constitutive models provides a suitable approach that would save time and cost as compared to an actual experimental testing program.

This study is intended to demonstrate the capability of a numerical approach in predicting the behavior of beams strengthened with layers of SFRC.

The use of ANSYS APDL program was demonstrated on four beams, which were previously tested in the lab. The result obtained from the numerical analysis were found to be in good agreement with those obtained from experimental models. The differences between results are within an acceptable range.

The study indicated that the load carrying capacity of RC beams strengthened with SFRC overlays can be improved by about 15%. For beams strengthened with SFRC underlays, the improvement goes far for about 24%. Using SFRC as an underlay shows a remarkable improvement in load carrying capacity and ductility. The ductility obtained by the SFRC underlay seems to control the overall ductility of the beam. Existence of steel fiber in the tension side of the beam helps improve the ductility and assure better stress redistribution.

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