Optimal Portfolio Construction: Application of Sharpe's Single-Index Model on Dhaka Stock Exchange

Imroz Mahmud *)

ABSTRACT

This study aims to find whether Sharpe's single-index model of portfolio construction offers better investment alternatives to the investors of the Dhaka Stock Exchange (DSE). For this purpose, month-ended closing price data of 178 companies listed on the DSE, the prime bourse of Bangladesh, and the month-ended index value of DSEX have been used for the period starting from January 2013 to February 2018. The stocks selected for this study belong to 16 industrial sectors, and purposive sampling technique has been used to select these sectors. Sharpe's model formulates a unique cut-off rate and selects the stocks having an excess return-to-beta ratio above that rate. In this study, 54 stocks qualified to be a part of the optimal portfolio. Hence, the proportion of investment to be made on each of the stock is calculated according to the model. The study reveals that three industries occupy a hefty chunk (65.78%) of the proposed investment portfolio. The constructed portfolio offers a monthly return of 2.1489% and carries 1.9516% risk as measured by standard deviation. The beta of the optimal portfolio is only 0.124003. The constructed portfolio outperforms every individual stock as well as the market index in terms of offering the optimal risk-return combinations. Therefore, this five-and-a-half-decade-old model offers a great opportunity for Bangladeshi investors to optimize return and diversify risk in an efficient manner.

Keywords: Single-Index Model; Sharpe; Optimal Portfolio; Dhaka Stock Exchange

1. INTRODUCTION

1.1 Research Background

The ground-breaking article published by Markowitz in 1952 marked the beginning of modern portfolio theory. His model is concerned with creating an optimal portfolio of assets by risk-averse investors. According to Markowitz (1952), a risk-averse investor should choose efficient portfolios. An efficient portfolio is one that maximizes return for a given level of risk or minimizes risk for a given level of return. Markowitz's pioneering article, named Portfolio Selection, and his subsequent studies have been a source of inspiration for many researchers and scholars. His model was intended to be pragmatic and implementable. However, it is ironic that the volume of work required to construct an optimal portfolio was staggering, and thus the model was rarely used in practice (Elton, Gruber, & Padberg, 1976). To form a portfolio using Markowitz's model we will need $N \times (N-1)/2$ correlation coefficients, where "N" stands for the number of stocks a firm or an investor follows (Elton, Gruber, Brown, & Goetzmann, 2009). Consequently, the implementation of the model is also quite time consuming and expensive. Recognition of these problems has motivated researchers to develop and simplify the portfolio construction process. In quest of a solution, William F. Sharpe came up with a simplified alternative to the Markowitz's model that significantly reduces the data input and computational requirements.

The model developed by Sharpe, known as Sharpe single-index model, is based on the assumption that co-movement between stocks' return is due to movement in return of a broad market index (Sharpe, 1963). Such a return on a broad market index is taken as a valid substitution for the common macroeconomic factor (Bodie, Kane, & Marcus, 2009). According to Sharpe's model, a single value, known as the cut-off rate, measures the desirability of including security in

the optimal portfolio. The portfolio construction process is simplified if the cut-off rate can dictate which securities to include in our optimal portfolio, and which ones to exclude. This simplified model not only allows an investor to pick securities for his/her optimal portfolio but also helps determine how much to invest in each of those securities (Elton, Gruber, & Padberg, 1976). Hence, this five-and-a-half-decade-old model is still considered as one of the simplest and most widely used models for portfolio construction.

Dhaka Stock Exchange (DSE), formerly known as East Pakistan Stock Exchange Association Ltd. (1954-1962) and East Pakistan Stock Exchange Ltd. (1962-1964), began its journey long before the independence of Bangladesh with a paid-up capital of about BDT 4 billion (Dhaka Stock Exchange, 2017). The introduction of DSE has opened the door to many investors and businesses in fulfilling their investment and financing needs. However, DSE evolved slowly and could not properly fulfill these needs due to various political unrest, corruption, military coups and other massive anomalies (Rahaman, Hasan, & Ahsan, 2013). These evils have always haunted the capital market of Bangladesh that resulted in some turmoils in 1990, 1996, 2011 and 2012 for instance. As a result, investors' trust and confidence in DSE have plummeted. Unless we make the necessary changes to this situation, the capital market can't contribute in a desirable way to the economic growth of Bangladesh. These present a great challenge for the rational investors of Bangladesh in picking a good investment, which necessitates the adoption of a systematic approach in the investment decision. A sound and in-depth knowledge of portfolio analysis can help them to diversify the investment risk without unfavorably affecting the return.

The present study has been carried out with a view to evaluating how well the portfolio constructed using Sharpe's single-index model performs in the Bangladeshi capital market. Besides, the study aims to assist both domestic and foreign investors of DSE in simplifying the construction of an optimal portfolio of equity securities. Although there have been extensive studies on this regard in the developed economies, empirical work on simplifying the portfolio construction and evaluation of its performance is comparatively scarce in the developing economies. Specifically, in Bangladesh, there is a significant lacuna of research in this context. Besides, none of the studies carried out in the developing economies consisted of a wide-ranging collection of stocks representing various industrial sectors. As a result, these studies cannot completely reveal the portfolio optimization potential of Sharpe's model.

1.2 Research Problem

a) Does Sharpe's model of portfolio construction offer better risk-return combination to the investors of DSE?
b) Does the portfolio construction model propounded by Sharpe simplify the investment decision for the investors of DSE?

1.3 Research Purpose

The primary objective of this study is to find whether Sharpe's single-index model of portfolio construction offers better investment options to the investors of DSE.

2. THEORETICAL FRAMEWORK

Before Markowitz's monumental work on portfolio selection, investors concentrated on evaluating the risk and return characteristics of individual stocks in constructing their portfolios. The rule of thumb was to pick a stock that offered the best opportunity of earning reward with the least amount of risk (Paudel & Koirala, 2007). Markowitz revolutionized the investment criteria by

suggesting three statistics: mean, standard deviation, and correlation with other stocks for selecting the best portfolio at any given level of risk.

The theoretically sophisticated and conceptually comprehensive model of Markowitz has a severe drawback: the volume of work required to form a portfolio using this model is staggering and beyond the capacity of all but a few specialists. A lot of researchers tried to simplify the portfolio construction process and finally, William F. Sharpe came up with a solution in 1963 that is known as Sharpe's single-index model.

Skarica & Zrinka (2012) stated that the classical Markowitz's model often been criticized because of useless and impractical as they are not structured to apply a mean-variance optimization approach. Elton, Gruber, & Padberg (1976, 1977, 1978) applied a variety of models, such as single-index, multi-index, and constant correlation models, to establish simple criteria for optimal portfolio selection. According to their findings, they have suggested using the single-index model for its simplicity and efficiency. The model was also recommended to apply to a large population of securities as it serves as a simplified alternative to the classical mean-variance approach of portfolio optimization (Haugen, 1993). Paudel & Koirala (2007) conducted a study on the application and comparison of Markowitz and Sharpe's model and advised using either of the models for optimizing portfolio performance. However, the findings of Briec & Kerstens (2009) portrays a contrasting picture. They suggested using Markowitz's model for long-term investments and argued that a single-index model is not suitable for investment portfolios having multi-period durations. To form efficient portfolios using the Markowitz model, clustering of high dimensional stock data was found to be more effectual (Nanda, Mahanty, & Tiwari, 2010). By clustering stocks, the problem of efficient frontier can be solved more efficiently, which results in enhanced diversification.

According to Frankfurter, Phillips, & Seagle (1976), the single-index model is just a simplified variation to the Markowitz's model, and it is based on the assumption that co-movement between stocks' return is due to movement in market return, or return of a broad market index to be specific. As per their study, both models produced identical results under conditions of certainty. However, in conditions of uncertainty, the single-index model produced superior results than Markowitz's model. Nevertheless, when a short span of historical data is used, the single-index model performed worse than Markowitz's model. The findings of Omet (1995) is also consistent with the previous studies. He stated that both models yield similar outcomes, but emphasized using the single-index model, as it is more pragmatic in generating the efficient frontier.

Rani & Bahl (2012) used Sharpe's model for portfolio construction with and without the procedure of short sales on 30 stocks selected from BSE (Bombay Stock Exchange). Both of their portfolios were efficient as they offered the best risk-return combinations possible. But the portfolio prohibiting short sales performed a tad bit better than the one allowing it. Sen & Fattawat (2014) also attempted to construct portfolios from 30 stocks, listed on BSE, using both Markowitz and Sharpe's model. Their study revealed that Sharpe's model outperformed its predecessor in risk reduction. In another study, Singh & Gautam (2014) and Ahuja (2017) formed an optimal portfolio of stocks selected from the banking sector of NSE (National Stock Exchange, Mumbai) and BSE (Bombay Stock Exchange). They also suggested Sharpe's model for its simplicity and efficacy.

From the literature reviewed, we could easily conclude that most of the studies were conducted in the developed economies. Only a few studies were carried out in the developing ones. Specifically, in Bangladesh, there exists a lacuna of research regarding this context. Besides, none of the studies conducted in developing economies were extensive.

2.1 Sharpe's Single-Index Model

William Sharpe (1963) simplified variant of the Markowitz Model, universally known as the Sharpe's single-index model, assumes that the co-movement between stocks' return is due to movement in return of a broad market index, which is DSEX in our case. Hence, the primary equation underlying the single-index model is:

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i \tag{1}$$

Where, R_i= Expected return on security i, α_i =Intercept of the straight line or alpha coefficient (Constant), β_i = Slope of a straight line or beta coefficient, R_m = The rate of return on market index, and ε_i = Error term.

Alpha co-efficient (α_i) and error term (ε_i) are two components of a random variable denoted by a_i. Since the error term (ε_i) has an expected value of zero, therefore the mean return on security can be expressed as:

$$\bar{R}_i = \alpha_i + \beta_i \bar{R}_m \tag{2}$$

To perform the portfolio optimization, the measurement of spread and co-movement of return statistics is also required. The mathematical equations required regarding this respect are:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2 \tag{3}$$

Where, σ_i^2 = Total variance of a security's return, $\beta_i^2 \sigma_m^2$ = Market-related variance, σ_{ei}^2 = Variance of a stock's movement that is not associated with the movement of the market index, i.e. the stock's unsystematic risk.

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \tag{4}$$

Where, σ_{ij} =The covariance of returns between securities i and j, β_i = Beta of security i, β_j = Beta of security j, and σ_m^2 = Variance of market return.

The monthly return of the selected securities is calculated using the following equation.

$$R_{it} = \frac{P_{it}}{P_{it-1}} - 1 \tag{5}$$

Where, R_{it} = Monthly return on stock i at time t, P_{it} = Monthly closing price of the stock i at time t, and P_{it-1} = Monthly closing price of the stock i at time t-1.

In the same manner, the return characteristics of the benchmark market index (DSEX) is calculated using the following equation.

$$R_{mt} = \frac{I_{it}}{I_{it-1}} - 1 \tag{6}$$

Where, R_{mt} = Monthly return on the market index at time t, I_{it} = Monthly closing market index value at time t, and I_{it-1} = Monthly closing market index value at time t-1.

The excess return yielded by risky security is calculated by subtracting the risk-free rate of return, which is 0.4784 percent per month (or 5.74 percent per annum) in our case, from the expected return of that risky security. The sensitivity of security's return to the movement in the market's return, popularly known as systematic risk or beta coefficient, is required for analyzing the risk of a security. The beta coefficient can be calculated as follows:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \tag{7}$$

Where, σ_{im} = Covariance of the stock i return with the market return, and σ_m^2 = Variance of the market return.

Based on the equations depicted above, it is just a matter of a few steps to reach our optimal portfolio. Sharpe's single-index model greatly simplifies the portfolio construction process by taking a single number to quantify the desirability of a stock's inclusion in the optimal portfolio. If one is ready to accept the standard form the single-index model as describing the co-movement between securities, such a number exists. In this instance, the desirability of any stock is directly related to the excess return to the Beta ratio (Elton, Gruber, Brown, & Goetzmann, 2009):

$$\frac{\overline{R}_i - R_f}{\beta_i} \tag{8}$$

Where, $\overline{R}_i$ = Expected return on stock i, R_f = Return on a riskless asset, and β_i = Expected change in the rate of return on stock i associated with a 1% change in the market return. The excess return to Beta ratio measures the additional return on security for each unit of systematic risk or non-diversifiable risk. In a colloquial manner, this ratio expresses the relationship between the potential risk and reward.

Once we have the excess return to the Beta ratio for each of the stocks under consideration, the following step is to rank those stocks in descending order of their respective ratios. Since we are assuming that short selling is prohibited, any stock with a negative excess return to Beta ratio has been excluded from further analyses.

The stocks to be included in the optimal portfolio depend on an unprecedented cutoff rate such that all stocks with higher ratios of $(\overline{R}_i - R_f)/\beta i$ are included and all stocks with lower ratios are excluded. This cut-off point is indicated by C*, and is computed from the characteristics of all the stocks that belong to the optimal portfolio. To determine C*, it is required to estimate its value as if there were different numbers of stocks in the optimal portfolio. If we nominate C_i as a candidate for C*. The value of C_i is calculated when i securities are assumed to belong to the optimal portfolio. For a portfolio of i stocks, C_i is given by:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \frac{(\overline{R}_j - R_f)\beta_j}{\sigma_{ej}^2}}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{ej}^2}} \tag{9}$$

Where, σ_m^2 = The variance in the market index, and σ_{ej}^2 = The variance of a stock's movement that is not associated with the movement of the market index. This is commonly referred to as a stock's unsystematic risk. After estimating the C_i of all the stocks, the highest C_i value is selected as the cut-off point (C*), which is then compared with the excess return to the Beta ratio of each stock. All stocks used in the calculation of C_i have an excess return to Beta above C_i and all stocks not used to calculate C_i have an excess return to Beta below C_i. There will always be one and only one C_i with this property and it is the cut-off point, C*.

After determining which stocks to be included in the optimal portfolio, the investors must find out the optimum percentage of capital to be invested in each of them. The percentage invested in each stock is:

$$X_i = \frac{Z_i}{\sum_{j=1}^{N} Z_j} \tag{10}$$

Where $Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{\overline{R}_i - R_f}{\beta_i} - C^*\right)$ (11)

The former expression indicates the weights on each stock, and they sum up to one. The latter expression determines the relative investment in each stock. The residual variance on each security σ_{ei}^2 plays a vital role in determining the amount to be invested in each security.

After determining the weights on each security, beta and alpha on a portfolio are estimated to find out the portfolio risk and return. Beta on a portfolio, β_p, is a weighted average of the individual β_i on each stock in the portfolio and is denoted by:

$$\beta_p = \sum_{i=1}^{n} X_i \beta_i \tag{12}$$

Similarly, the Alpha on the portfolio, α_p, is calculated as:

$$\alpha_p = \sum_{i=1}^{n} X_i \alpha_i \tag{13}$$

The return on the investor's portfolio can be expressed as:

$$\overline{R}_p = \alpha_p + \beta_p \overline{R}_m \tag{14}$$

Finally the risk of the investor's portfolio, σ_p, as:

$$\sigma_p = \sqrt{(\beta_p^2)\sigma_m^2 + \sum_{i=1}^{N} X_i^2\, \sigma_{ei}^2} \tag{15}$$

All the equations (eq. 1-15) cited above are sourced from the book of Elton, Gruber, Brown, & Goetzmann (2009).

3. RESEARCH METHOD
3.1 Research Design

In the pursuit of constructing an optimal portfolio of equity securities traded on DSE, monthly closing price and benchmark market index (DSEX) data have been used for the period starting from January 2013 to February 2018. The secondary data used in this study are collected from DSE Library.

3.2 Population and Sample

The study was initially aimed at all, 228 to be specific, the "A" category companies offering equity securities that belong to sixteen industrial sectors. The industries covered in this study are listed in Table 1. Purposive sampling technique has been applied to select these industrial sectors. The companies which are regular at holding annual general meetings and have declared a dividend at the rate of ten percent or above in the last calendar year are classified as "A" category companies (Dhaka Stock Exchange, 2018). Due to the unavailability of data during the aboved study period, 50 companies have been excluded from the portfolio construction, which leads to a final sample size of 178 companies. The average monthly cutoff yield of 91-day Treasury Bill has been used as a proxy for the risk-free rate of return (Bangladesh Bank, n.d.).

The 178 sample companies selected following the above-mentioned criteria constitutes 61.59 percent of all the equity securities (289) from the sixteen industrial sectors (refer to Table 1). A number of statistical techniques have been applied to analyze the risk-return characteristics and to construct the optimal portfolio of equity securities using Microsoft Excel 2016. In addition to substantial data coverage, the study covers a period of five years and two months, which reduces the possibility of sampling error and the influence of temporary fluctuations to distort the study outcome. Hence, the sample size can be assumed as adequate to represent the population and make smart investment decisions.

Table 1 Sector-wise Representation of Sample Stocks

Name of the Industry/Sector	Total Number of Companies	Number of Category-A Companies	Sample Companies	Data Coverage (%)
Banks	30	29	29	96.67%
Financial Institutions	20	12	12	60.00%
Engineering	36	29	17	47.22%
Food & Allied	18	9	8	44.44%
Fuel & Power	19	16	13	68.42%
Jute	3	2	2	66.67%
Textile	49	35	18	36.73%
Pharmaceuticals & Chemicals	28	24	16	57.14%
Service & Real Estate	7	6	3	42.86%
Cement	7	6	5	71.43%
IT Sector	8	7	5	62.50%
Tannery Industries	6	5	4	66.67%
Ceramics Sector	5	3	3	60.00%
Insurance	47	42	41	87.23%
Telecommunication	2	2	1	50.00%
Travel & Leisure	4	1	1	25.00%
Total	289	228	178	61.59%

Source: Secondary Data Processed, 2018

3.3 Research Framework
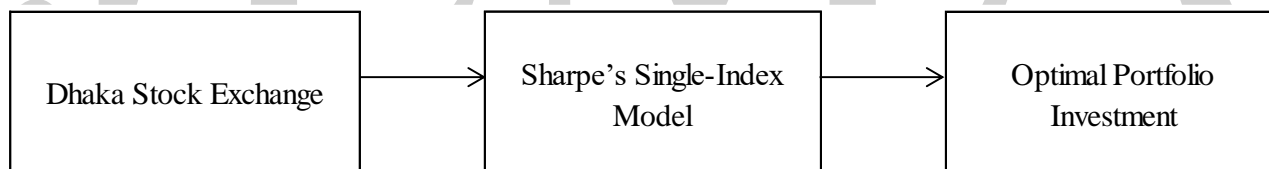
Picture 1 Research Framework

4. RESEARCH RESULT AND ANALYSIS
4.1 Descriptive Analysis

Risk and return have always been the decisive factors in any investment model, and Sharpe's single-index model is of no exception. To apply this model, expected return and risk of each instrument are required. Thus, the month-ended closing price data of the 178 sample stocks as well as the month-ended index value of DSEX, for the period ranging from January 2013 to February 2018, have been used to estimate various risk and return characteristics. The mean monthly return and risk in terms of standard deviation, variance, covariance with market return, and the beta coefficient for each of the selected stocks have been calculated and depicted in Appendix-A.

The risk-return combinations offered by each of the 178 selected stocks have been portrayed in Picture-1. Analysis of the return data on the vertical axis reveals that most of the stocks yielded a monthly return from -0.27 percent to 1.3 percent. However, the average risk carried by the stocks is about 11.5 percent (Picture 1). Although Picture 1 paints an overall picture of the risk and returns

composition offered by the stocks, we cannot pinpoint any precise information from it. An in-depth examination of the data in Appendix-A can be quite useful in this regard.

As revealed by Appendix A, the stock of Northern Jute Manufacturing Co. Ltd. yields the highest monthly return (8.227 percent) followed by Desh Garments Ltd. (5.439 percent) and Renwick Jajneswar & Co (Bd) Ltd. (4.892 percent). In terms of risk, the stock of Legacy Footwear Ltd. has the highest standard deviation (29.845 percent) followed by Grameenphone Ltd. (27.499 percent) and Northern Jute Manufacturing Co. Ltd. (26.969 percent). It is also found that 114 stocks offered a return below the market rate, which is 0.681 percent in our study, and surprisingly none of the stocks in this study had a beta value above the market.
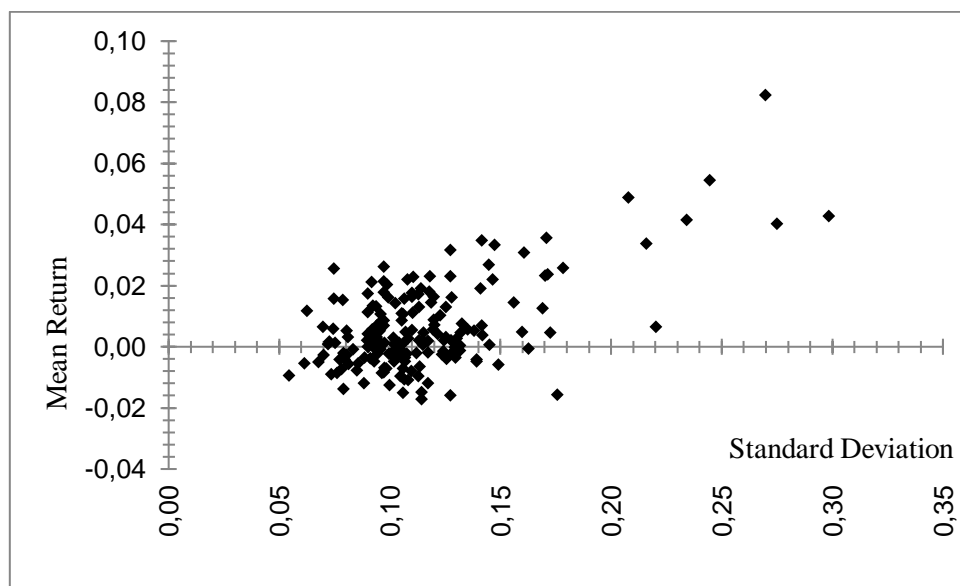
Picture 2 Risk-Return Combination of Each Stock (Secondary Data Processed, 2018)

According to Sharpe's single-index model, the expected return $(\bar{R}_i)$ has two components: a unique or firm-specific part (α_i) and a market-related part $(\beta_i \bar{R}_m)$. Likewise, a security's variance (σ_i^2) has the same two components: a unique risk (σ_{ei}^2) and a market-related risk $(\beta_i^2 \sigma_m^2)$. The stocks having larger beta value also have a greater part of return that is dependent on the market's performance. Breakdowns of risk & return will guide us to glean some idea about how much this model is going to benefit the investors in their quest for optimizing risk and return. Besides, these variables will be used in further calculations related to portfolio construction and performance evaluation. Hence, the decomposition of each firm's return and risk into unique and market-related parts are presented in Appendix-B.

By analyzing the decomposition of return, we can infer that the contribution of a stocks independent component is the dominant factor than its market-related component. Similarly, the breakdown of risk leads us to the identical conclusion: the contribution of a security's unsystematic variance in its total variance is the dominant factor than its systematic variance. The result suggests that the stocks selected for this study offer a significant opportunity for reduction of unsystematic risk, which we will be able to verify when we construct the optimal portfolio and measure its performance.

Sharpe's model provided a simple criterion, called the excess return-to-beta ratio, to quantify the desirability of a stock's inclusion in the optimal portfolio. Therefore, the excess return-to-beta

ratio for each stock has been calculated and the stocks are ranked in descending order according to their respective ratios (Appendix-C). The mean monthly cutoff yield of 91-day Treasury Bill, which is 0.4784 percent, has been used as a proxy for the risk-free rate of return. Of the 178 stocks studied, only 70 stocks have a positive excess return-to-beta ratio. Since short selling is assumed to be prohibited, the rest of the stocks have been excluded from further analysis.

Among the stocks studied, Renwick Jajneswar & Co (Bd) Ltd. secured the top spot with an excess return-to-beta ratio of 6.18333. Legacy Footwear Ltd. and Grameenphone Ltd. were in the second and third spot with the respective ratio of 4.21675 and 1.12589 (Appendix-C). Among the top scorers, only Renwick Jajneswar & Co (Bd) Ltd. secured the third position in terms of highest return generation. These outcomes suggest that the significantly lower beta value of Legacy Footwear Ltd. and Grameenphone Ltd. is the reason why these companies made their way to the top of the chart in terms of the excess return-to-beta ratio.

4.2   Validity and Reliability

This study only used secondary data from DSEX to defined the optimal investment portfolio proposed model, therefore, the validity and reliability test is not needed in this study.

4.3   Research Analysis

According to the single-index model, the selection of stock in the optimal portfolio depends on a unique cut-off rate (C^*) and its comparison with the excess return-to-beta ratio of each stock studied. All stocks used for the estimation of C^* have an excess return-to-beta ratio above the cut-off rate and the stocks excluded from the estimation have an excess return-to-beta ratio below that rate. The detailed calculation of the cut-off rate is portrayed in Table 2.

Table 2 Calculation of The Cut-Off Rate

Rank	Stock No.	$\dfrac{\overline{R}_j - R_f}{\beta_j}$	$\dfrac{(\overline{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\sum\limits_{j=1}^{i} \dfrac{(\overline{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\dfrac{\beta_j^2}{\sigma_{ej}^2}$	$\sum\limits_{j=1}^{i} \dfrac{\beta_j^2}{\sigma_{ej}^2}$	C_i
1	49	6.1833	0.0073	0.0073	0.0012	0.0012	0.00002
2	134	4.2167	0.0039	0.0111	0.0009	0.0021	0.00003
3	130	1.1259	0.0148	0.0260	0.0132	0.0153	0.00006
4	107	1.0012	0.0135	0.0395	0.0135	0.0288	0.00010
5	109	0.8374	0.0276	0.0671	0.0329	0.0617	0.00017
6	135	0.6287	0.0516	0.1187	0.0821	0.1438	0.00029
7	82	0.5761	0.0203	0.1390	0.0353	0.1791	0.00035
8	106	0.4258	0.1135	0.2525	0.2666	0.4457	0.00063
9	44	0.3861	0.1156	0.3682	0.2995	0.7452	0.00091
10	61	0.3066	0.2533	0.6214	0.8262	1.5713	0.00154
11	100	0.2588	0.0416	0.6631	0.1609	1.7322	0.00164
12	114	0.2069	0.1415	0.8046	0.6838	2.4161	0.00199
13	50	0.2031	0.1291	0.9337	0.6356	3.0517	0.00230
14	84	0.1954	0.0619	0.9956	0.3169	3.3686	0.00245
15	108	0.1947	0.1225	1.1181	0.6291	3.9976	0.00275
16	81	0.1537	0.0786	1.1967	0.5114	4.5090	0.00294

Rank	Stock No.	$\dfrac{\overline{R}_j - R_f}{\beta_j}$	$\dfrac{(\overline{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\sum\limits_{j=1}^{i}\dfrac{(\overline{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\dfrac{\beta_j^2}{\sigma_{ej}^2}$	$\sum\limits_{j=1}^{i}\dfrac{\beta_j^2}{\sigma_{ej}^2}$	C_i
17	60	0.1528	0.1357	1.3324	0.8883	5.3973	0.00327
18	128	0.1359	0.0945	1.4269	0.6959	6.0933	0.00349
19	59	0.1270	0.2025	1.6295	1.5950	7.6883	0.00397
20	102	0.1211	0.2675	1.8970	2.2082	9.8965	0.00460
21	125	0.1186	0.0325	1.9295	0.2738	10.1703	0.00468
22	103	0.1176	0.2264	2.1559	1.9256	12.0959	0.00520
23	90	0.1047	0.0410	2.1969	0.3919	12.4878	0.00529
24	113	0.1010	0.1387	2.3356	1.3734	13.8611	0.00561
25	104	0.0992	0.0801	2.4158	0.8079	14.6690	0.00579
26	105	0.0936	0.1938	2.6096	2.0704	16.7394	0.00622
27	64	0.0896	0.0943	2.7038	1.0528	17.7922	0.00643
28	48	0.0868	0.1199	2.8237	1.3806	19.1728	0.00670
29	87	0.0819	0.1334	2.9571	1.6285	20.8013	0.00699
30	25	0.0722	0.3478	3.3050	4.8185	25.6198	0.00772
31	123	0.0647	0.1242	3.4291	1.9183	27.5381	0.00797
32	36	0.0631	0.2517	3.6809	3.9896	31.5277	0.00848
33	67	0.0618	0.3244	4.0052	5.2473	36.7750	0.00912
34	54	0.0613	0.2144	4.2197	3.4989	40.2739	0.00953
35	52	0.0565	0.3183	4.5379	5.6297	45.9035	0.01012
36	43	0.0513	0.1049	4.6428	2.0452	47.9487	0.01031
37	131	0.0511	0.1841	4.8269	3.6000	51.5487	0.01063
38	118	0.0508	0.0415	4.8684	0.8164	52.3652	0.01070
39	132	0.0496	0.4443	5.3127	8.9565	61.3217	0.01145
40	28	0.0426	0.1613	5.4740	3.7847	65.1064	0.01171
41	133	0.0290	0.1108	5.5848	3.8159	68.9222	0.01185
42	116	0.0285	0.0541	5.6388	1.8946	70.8169	0.01191
43	101	0.0280	0.3158	5.9546	11.2757	82.0925	0.01229
44	68	0.0267	0.0759	6.0305	2.8452	84.9377	0.01237
45	120	0.0254	0.1475	6.1780	5.8076	90.7453	0.01253
46	18	0.0240	0.2192	6.3972	9.1414	99.8867	0.01273
47	2	0.0223	0.1510	6.5481	6.7828	106.6695	0.01286
48	110	0.0222	0.0598	6.6079	2.6927	109.3622	0.01291
49	111	0.0219	0.5929	7.2008	27.0920	136.4542	0.01336
50	121	0.0195	0.1831	7.3839	9.4061	145.8602	0.01347
51	119	0.0193	0.0034	7.3873	0.1762	146.0365	0.01347
52	46	0.0172	0.0635	7.4508	3.6943	149.7308	0.01349
53	91	0.0162	0.0283	7.4791	1.7408	151.4716	0.01350

Rank	Stock No.	$\dfrac{\overline{R}_j - R_f}{\beta_j}$	$\dfrac{(\overline{R}_j - R_{f)\beta_j}}{\sigma_{ej}^2}$	$\displaystyle\sum_{j=1}^{i} \dfrac{(\overline{R}_j - R_{f)\beta_j}}{\sigma_{ej}^2}$	$\dfrac{\beta_j^2}{\sigma_{ej}^2}$	$\displaystyle\sum_{j=1}^{i} \dfrac{\beta_j^2}{\sigma_{ej}^2}$	C_i
54	78	0.0145	0.0944	7.5735	6.5132	157.9849	**0.01351**
55	74	0.0108	0.0431	7.6166	3.9735	161.9583	0.01349
56	22	0.0087	0.0587	7.6753	6.7159	168.6742	0.01344
57	39	0.0083	0.0254	7.7008	3.0653	171.7395	0.01341
58	55	0.0080	0.0509	7.7516	6.3729	178.1124	0.01335
59	76	0.0056	0.0483	7.8000	8.7019	186.8143	0.01324
60	16	0.0050	0.0542	7.8542	10.9465	197.7608	0.01308
61	10	0.0048	0.1366	7.9908	28.5501	226.3108	0.01271
62	164	0.0044	0.0032	7.9940	0.7142	227.0250	0.01270
63	122	0.0041	0.0094	8.0034	2.3078	229.3328	0.01267
64	26	0.0034	0.0181	8.0215	5.3407	234.6735	0.01259
65	129	0.0032	0.0142	8.0358	4.3931	239.0666	0.01253
66	117	0.0029	0.0288	8.0646	9.9684	249.0350	0.01238
67	77	0.0020	0.0144	8.0790	7.0901	256.1252	0.01227
68	73	0.0019	0.0140	8.0930	7.2801	263.4053	0.01215
69	89	0.0007	0.0043	8.0973	6.4328	269.8381	0.01204
70	11	0.0002	0.0012	8.0985	6.3358	276.1739	0.01193

Source: Secondary Data Processed, 2018

From Table 2, it is apparent that the highest C_i value is 0.01351 for stock number 78, i.e. MJL Bangladesh Limited; hence, the cutoff rate (C^*) is 0.01351. Sixteen stocks have an excess return-to-beta ratio below the C^*, therefore are excluded from the optimal portfolio. So according to Sharpe's single-index model, the remaining fifty-four stocks, having an excess return-to-beta ratio above the C^*, qualify to be a part of the optimal portfolio.

4.4 Research Discussion
4.4.1 Constructing the Optimal Portfolio

Sharpe's single-index model not only identifies the stocks to be included in the optimal portfolio but also recommends the proportion of fund to be invested in each of them. Thus, the investors can reduce the unsystematic risk and construct a highly diversified portfolio. The following chart (Table 3) depicts the proportion of investment to be made on the fifty-four stocks qualified for the optimal portfolio.

Table 3 Weight of Stocks in the Optimal Portfolio

Rank	Stock No.	Name	Z_i	Weight (X_i)
1	49	Renwick Jajneswar & Co (Bd) Ltd.	1.01938	2.3242%
2	134	Legacy Footwear Ltd.	0.42581	0.9709%
3	130	Grameenphone Ltd.	0.46446	1.0590%
4	107	Kohinoor Chemicals Company (Bangladesh) Ltd.	1.04414	2.3806%
5	109	Libra Infusions Limited	1.03220	2.3534%
6	135	Monno Ceramic Industries Ltd.	1.03242	2.3539%

Rank	Stock No.	Name	Z_i	Weight (X_i)
7	82	Al-Haj Textile Mills Limited	0.62054	1.4148%
8	106	Pharma Aids Ltd.	2.19126	4.9961%
9	44	Eastern Cables Ltd.	1.60392	3.6570%
10	61	British American Tobacco Bangladesh Company Ltd.	3.57568	8.1526%
11	100	Ambee Pharma Ltd.	0.92550	2.1102%
12	114	Marico Bangladesh Ltd.	1.64882	3.7593%
13	50	National tubes Ltd.	0.94147	2.1466%
14	84	Rahim Textile Mills Ltd.	0.59819	1.3639%
15	108	The IBN SINA Pharmaceutical Industry Ltd.	1.21949	2.7805%
16	81	Sonali Aansh Industries Limited	0.88817	2.0250%
17	60	Apex Foods Limited	1.03144	2.3517%
18	128	Daffodil Computers Ltd.	0.86783	1.9787%
19	59	Olympic Industries Limited	1.33037	3.0332%
20	102	GlaxoSmithKline(GSK) Bangladesh Limited	1.74671	3.9825%
21	125	BDCOM Online Ltd.	0.35259	0.8039%
22	103	ACI Limited.	1.30947	2.9856%
23	90	Anlima Yarn Dyeing Ltd.	0.45627	1.0403%
24	113	ACI Formulations Ltd.	0.70168	1.5998%
25	104	Renata Ltd.	0.82110	1.8721%
26	105	Reckitt Benckiser (Bd.)Ltd.	1.18977	2.7127%
27	64	Agricultural Marketing Company Ltd. (Pran)	0.76257	1.7387%
28	48	Quasem Drycells Ltd.	0.61289	1.3974%
29	87	Apex Spinning & Knitting Mills Limited	0.79501	1.8126%
30	25	BRAC Bank Ltd.	1.31509	2.9984%
31	123	Lafarge Surma Cement Ltd.	0.55550	1.2665%
32	36	IPDC Finance Limited	0.87316	1.9908%
33	67	Linde Bangladesh Limited	1.23684	2.8200%
34	54	National Polymer Industries Ltd	0.90478	2.0629%
35	52	Rangpur Foundry Ltd.	1.30733	2.9807%
36	43	Bangladesh Lamps Limited	0.47902	1.0922%
37	131	Apex Tannery Limited	0.59892	1.3655%
38	118	Summit Alliance Port Limited	0.19958	0.4550%
39	132	Bata Shoe Company (Bangladesh) Limited	1.46201	3.3334%
40	28	Trust Bank Ltd.	0.47989	1.0942%
41	133	Apex Footwear Limited	0.28786	0.6563%
42	116	Samorita Hospital Ltd.	0.21311	0.4859%
43	101	Beximco Pharmaceuticals Ltd.	0.53449	1.2187%
44	68	Padma Oil Co.Ltd.	0.18160	0.4141%
45	120	Heidelberg Cement Bangladesh Ltd.	0.30133	0.6870%
46	18	Mutual Trust Bank Ltd.	0.35594	0.8116%
47	2	The City Bank Ltd.	0.20815	0.4746%
48	110	Orion Infusion Ltd.	0.13563	0.3092%
49	111	Square Pharmaceuticals Ltd.	0.72062	1.6430%
50	121	Confidence Cement Ltd.	0.17536	0.3998%

Rank	Stock No.	Name	Z_i	Weight (X_i)
51	119	Unique Hotel & Resorts Limited	0.01107	0.0252%
52	46	Singer Bangladesh Ltd.	0.05939	0.1354%
53	91	H.R. Textile Ltd.	0.02716	0.0619%
54	78	MJL Bangladesh Limited	0.02648	0.0604%
		$\sum Z_i =$	43.85946	100.00%

Source: Secondary Data Processed, 2018

As shown in Table 3, the largest chunk of investment (8.1526%) should be made in British American Tobacco Bangladesh Company Ltd., followed by Pharma Aids Ltd. and GlaxoSmithKline (GSK) Bangladesh Limited with an investment proportion of 4.9961% and 3.9825%, respectively. A sector-wise study on the fifty-four stocks in our optimal portfolio, shown in Picture 2, reveals that most of them belong to three industries: Pharmaceuticals & Chemicals, Engineering, and Food & Allied. These three industrial sectors account for 65.78 percent of the total investment to be made in the optimal portfolio, while the rest of the twelve sectors represent only 34.22 percent of the total investment.
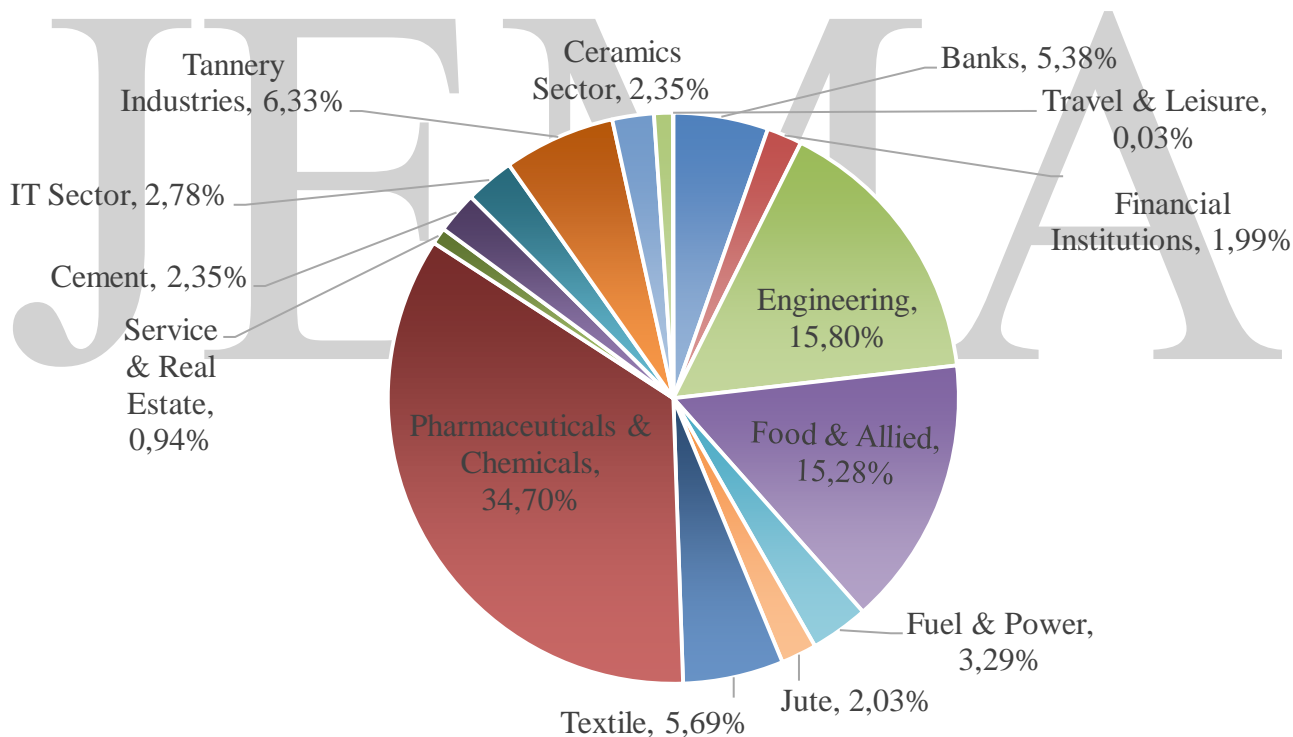
Picture 3 Sector-Wise Proportion of Investment (Secondary Data Processed, 2018)

From the above pie chart (Picture 2), it is evident that Pharmaceuticals & Chemicals sector occupies the highest proportion of fund to be invested in the optimal portfolio, followed by Engineering and Food & Allied industry. Among the sixteen industrial sectors selected for this study, all but Insurance sector have at least one representative stock in the final portfolio. Hence, the final portfolio has stocks from fifteen industrial sectors that react differently to the changes in economic forces.

4.4.2 Performance of Optimal Portfolio

Once we have decided about the stocks along with their respective weights in the optimal portfolio, the final task is to evaluate its overall performance. Therefore, the performance of the constructed portfolio in terms of risk and return is calculated, and the key results are summarized in Table 4. For detailed calculations, refer to Appendix-D.

Table 4 Evaluation of Portfolio Performance

Portfolio Return Calculation		Portfolio Risk Calculation	
Alpha on Portfolio (α_p) =	0.020644	$\sum_{i=1}^{N} X_i^2 \sigma_{ei}^2$ =	0.0003427
Beta on Portfolio (β_p)=	0.124003	$\beta_p^2 \sigma_m^2$=	0.0000382
Return on Market Index $(\overline{R}_m)$=	0.6812%	Portfolio Standard Deviation (σ_p) =	1.9516%
Portfolio Return $(\overline{R}_p)$=	2.1489%	Portfolio Coefficient of Variation (CV) =	90.8184%

Source: Secondary Data Processed, 2018

As apparent from the above table (Table 4), the constructed portfolio offers a generous monthly return of 2.1489 percent. However, the portfolio risk, measured by the standard deviation, is only 1.9516 percent. The portfolio has a CV of 90.8184 percent, a rate that is considerably lower than any individual stock in the optimal portfolio. The portfolio beta, expressing the systematic risk, also leads us to the identical conclusion. A comparison of the portfolio beta with that of the market indicates a substantial absence of systematic risk in the constructed one. The results of this study conform to the earlier researches conducted on portfolio optimization. Since the portfolio formed using Sharpe's single-index model offers the best risk-return combinations possible in the given context, the portfolio can be termed as the optimal portfolio and the stocks consisting this portfolio as the efficient stocks.

5.   RESEARCH CONCLUSION AND LIMITATION
5.1   Conclusion

In addition to simplifying the portfolio construction process, Sharpe's single-index model reduces the number of inputs required to form the optimal portfolio. If we were to form a portfolio using the same number of stocks, Markowitz's model would require 16,109 inputs. Whereas, Sharpe's model required only 536 inputs, which is about 2,905 percent less than that of its predecessor. The findings of the study reveal a huge possibility of risk reduction through diversification while achieving a substantial return for Bangladeshi investors. The constructed portfolio outperforms every individual stock as well as the market index in terms of offering the optimal risk-return combinations. Therefore, this five-and-a-half-decade-old model offers a great opportunity for Bangladeshi investors to optimize return and diversify risk in an efficient manner. Nevertheless, investors should not slack off at this point. They need to evaluate the performance of each stock and make necessary amendments in the optimal portfolio at regular intervals. Otherwise, the risk may go up and return may go down, evaporating the benefits of portfolio optimization.

5.2 Limitation

This study certainly has some limitations. The stocks selected for this study belong to only "A" category companies traded on DSE. Future studies may consider the stocks from other categories and from Chittagong Stock Exchange (CSE), the second bourse of Bangladesh. In addition, the study has excluded fifty "A" category stocks for lack of data. Although the monthly closing price data of five years and two months was adequate to form an optimal portfolio, daily closing price data and longer time horizon may result in better optimization of risk & return. As the study focused on simplifying the Markowitz's model of portfolio optimization using Sharpe's model, a side-by-side comparison of portfolios formed using both techniques can enhance or diminish the credibility of our hypothesis. Lastly, allowing short sales may change the nature of the optimal solution, as stocks having opposite characteristics will be a part of the optimal portfolio.

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*) Imroz Mahmud, Department of Business Administration, University of Asia Pacific, Dhaka, Bangladesh (Email: imroz@outlook.com)

Appendix-A:

Risk and return characteristics of 178 sample stocks and the benchmark market index (DSEX) with

Code A as Stock Number | Code E as Standard Deviation

Code B as Name | Code F as Covariance

Code C as Mean Return | Code G as Beta

Code D as Variance

A	B	C	D	E	F	G
1	AB Bank Limited	-0.00260	0.01528	0.12360	0.00339	0.22182
2	The City Bank Ltd.	0.01113	0.01217	0.11032	0.00347	0.28493
3	IFIC Bank Ltd.	-0.00070	0.02645	0.16264	0.00326	0.12338
4	Islami Bank Bd Ltd	0.00209	0.01626	0.12751	0.00250	0.15359
5	National Bank Ltd.	-0.00479	0.01138	0.10666	0.00262	0.23006
6	Pubali Bank Ltd.	-0.00084	0.00694	0.08332	0.00188	0.27022
7	Rupali Bank Ltd.	0.00042	0.01736	0.13175	0.00322	0.18556
8	United Commercial Bank Ltd.	0.00287	0.01029	0.10145	0.00288	0.28005
9	Uttara Bank Ltd.	0.00001	0.00809	0.08996	0.00265	0.32736
10	Eastern Bank Ltd.	0.00650	0.00485	0.06963	0.00174	0.35953
11	Al-Arafah Islami Bank Ltd.	0.00484	0.01143	0.10692	0.00305	0.26704
12	Prime Bank Ltd.	-0.00472	0.00863	0.09288	0.00168	0.19448
13	Southeast Bank Ltd.	0.00426	0.00812	0.09009	0.00259	0.31930
14	Dhaka Bank Ltd.	-0.00096	0.01042	0.10209	0.00341	0.32680
15	National Credit and Commerce Bank Ltd.	0.00091	0.00520	0.07214	0.00208	0.39944
16	Social Islami Bank Ltd.	0.00599	0.00553	0.07434	0.00134	0.24267
17	Dutch-Bangla Bank Ltd.	0.00311	0.00657	0.08106	0.00215	0.32799
18	Mutual Trust Bank Ltd.	0.01123	0.00808	0.08991	0.00217	0.26881
19	Standard Bank Ltd.	-0.00235	0.00887	0.09419	0.00288	0.32447
20	One Bank Limited	0.00212	0.00805	0.08971	0.00296	0.36771
21	Bank Asia Ltd.	0.00429	0.00826	0.09086	0.00196	0.23776
22	Mercantile Bank Ltd.	0.00689	0.00882	0.09391	0.00213	0.24136
23	Export Import (Exim) Bank of Bangladesh Ltd.	-0.00260	0.00646	0.08035	0.00203	0.31399
24	Jamuna Bank Ltd.	0.00131	0.00565	0.07516	0.00160	0.28406
25	BRAC Bank Ltd.	0.02030	0.00971	0.09852	0.00209	0.21499
26	Shahjalal Islami Bank Ltd.	0.00553	0.00910	0.09539	0.00199	0.21899
27	Premier Bank Ltd.	0.00008	0.00854	0.09241	0.00235	0.27575
28	Trust Bank Ltd.	0.01457	0.01406	0.11857	0.00323	0.22960
29	First Security Islami Bank Ltd.	-0.00085	0.00919	0.09588	0.00268	0.29198
30	IDLC Finance Limited	0.00461	0.01739	0.13186	0.00480	0.27608
31	United Finance Limited	-0.00221	0.01157	0.10756	0.00361	0.31180
32	Uttara Finance and Investments Limited.	-0.00008	0.00815	0.09029	0.00328	0.40203
33	Premier Leasing & Finance Limited.	0.00417	0.01478	0.12157	0.00337	0.22791
34	Islamic Finance & Investment Ltd.	0.00183	0.01323	0.11500	0.00361	0.27282
35	LankaBangla Finance Ltd.	0.00469	0.02970	0.17233	0.00560	0.18862

A	B	C	D	E	F	G
36	IPDC Finance Limited	0.01908	0.01299	0.11399	0.00294	0.22657
37	Union Capital Ltd.	0.00067	0.02101	0.14493	0.00406	0.19313
38	Bangladesh Finance and Investment Co. Ltd.	0.00371	0.02006	0.14165	0.00466	0.23210
39	Phoenix Finance and Investments Ltd.	0.00683	0.02006	0.14162	0.00495	0.24701
40	Bay Leasing & Investment Ltd.	0.00281	0.01693	0.13011	0.00360	0.21247
41	GSP Finance Company (Bangladesh) Limited	0.00478	0.02556	0.15987	0.00384	0.15041
42	Aftab Automobiles Limited	0.00066	0.01322	0.11499	0.00374	0.28280
43	Bangladesh Lamps Limited	0.01305	0.01277	0.11302	0.00206	0.16122
44	Eastern Cables Ltd.	0.03165	0.01617	0.12718	0.00113	0.06957
45	Monno Jute Stafflers Ltd.	0.03474	0.01998	0.14137	-0.00091	-0.04531
46	Singer Bangladesh Ltd.	0.00873	0.01435	0.11977	0.00329	0.22916
47	Atlas Bangladesh Ltd.	0.00279	0.01168	0.10808	0.00238	0.20380
48	Quasem Drycells Ltd.	0.01912	0.01982	0.14077	0.00327	0.16512
49	Renwick Jajneswar & Co (Bd) Ltd.	0.04892	0.04320	0.20785	0.00031	0.00714
50	National tubes Ltd.	0.03079	0.02582	0.16070	0.00331	0.12802
51	Anwar Galvanizing Ltd.	0.03342	0.02169	0.14729	-0.00049	-0.02251
52	Rangpur Foundry Ltd.	0.01526	0.00618	0.07862	0.00115	0.18526
53	S. Alam Cold Rolled Steels Ltd.	-0.00368	0.00830	0.09110	0.00276	0.33294
54	National Polymer Industries Ltd	0.01611	0.00984	0.09920	0.00182	0.18475
55	BSRM Steels Limited	0.00718	0.01438	0.11994	0.00432	0.30040
56	Navana CNG Limited	0.00088	0.01116	0.10566	0.00378	0.33858
57	Deshbandhu Polymer Limited	0.00194	0.01373	0.11719	0.00221	0.16092
58	GPH Ispat Ltd.	0.00326	0.01568	0.12520	0.00425	0.27131
59	Olympic Industries Limited	0.02206	0.01165	0.10793	0.00158	0.13604
60	Apex Foods Limited	0.02311	0.01623	0.12740	0.00195	0.11994
61	British American Tobacco Bangladesh Company Ltd.	0.02554	0.00556	0.07457	0.00038	0.06771
62	Gemini Sea Food Ltd.	0.03379	0.04662	0.21592	-0.00223	-0.04778
63	National Tea Company Limited	-0.00264	0.00489	0.06994	0.00031	0.06378
64	Agricultural Marketing Company Ltd. (Pran)	0.01419	0.01050	0.10245	0.00110	0.10499
65	Fu Wang Food Ltd.	0.00218	0.01547	0.12438	0.00316	0.20425
66	Rangpur Dairy & Food Products Limited	-0.00021	0.01674	0.12937	0.00325	0.19446
67	Linde Bangladesh Limited	0.01745	0.00811	0.09004	0.00166	0.20492
68	Padma Oil Co.Ltd.	0.01029	0.01505	0.12269	0.00310	0.20622
69	Eastern Lubricants Ltd.	0.04159	0.05485	0.23421	-0.00094	-0.01710
70	Summit Power Ltd.	-0.00089	0.01099	0.10482	0.00401	0.36483
71	Dhaka Electric Supply Company Ltd.	-0.00548	0.00734	0.08568	0.00206	0.28067
72	Power Grid Company of Bangladesh Ltd.	0.00283	0.00827	0.09094	0.00220	0.26586
73	Jamuna Oil Com. Ltd.	0.00520	0.00647	0.08042	0.00139	0.21505
74	Meghna Petroleum Ltd.	0.00688	0.00945	0.09721	0.00182	0.19282
75	Titas Gas Trans. & Dist. Co. Ltd.	-0.00547	0.00374	0.06117	0.00154	0.41024
76	Khulna Power Company Ltd.	0.00630	0.00871	0.09334	0.00237	0.27241
77	Baraka Power Limited	0.00537	0.01206	0.10981	0.00349	0.28984
78	MJL Bangladesh Limited	0.00831	0.00921	0.09598	0.00224	0.24299
79	GBB Power Limited	-0.00711	0.00969	0.09845	0.00354	0.36546

A	B	C	D	E	F	G
80	Northern Jute Manufacturing Co. Ltd.	0.08227	0.07274	0.26969	-0.00044	-0.00611
81	Sonali Aansh Industries Limited	0.01719	0.01276	0.11296	0.00103	0.08073
82	Al-Haj Textile Mills Limited	0.02320	0.02898	0.17023	0.00093	0.03196
83	Stylecraft Limited	0.02568	0.03174	0.17815	-0.00033	-0.01040
84	Rahim Textile Mills Ltd.	0.02361	0.02932	0.17122	0.00282	0.09635
85	Saiham Textile Mills Ltd.	-0.00421	0.00634	0.07964	0.00218	0.34376
86	Desh Garments Ltd.	0.05439	0.05976	0.24447	-0.00093	-0.01552
87	Apex Spinning & Knitting Mills Limited	0.01626	0.01211	0.11003	0.00170	0.14013
88	Delta Spinners Ltd.	-0.01595	0.01623	0.12739	0.00230	0.14202
89	Prime Textile Spinning Mills Ltd.	0.00495	0.00908	0.09530	0.00218	0.23979
90	Anlima Yarn Dyeing Ltd.	0.01299	0.01568	0.12523	0.00123	0.07836
91	H.R. Textile Ltd.	0.00762	0.01759	0.13263	0.00307	0.17462
92	Alif Manufacturing Company Ltd.	-0.00579	0.02221	0.14902	0.00299	0.13463
93	Square Textiles Limited	-0.00935	0.00297	0.05449	0.00106	0.35771
94	R.N. Spinning Mills Ltd.	-0.00971	0.01274	0.11287	0.00128	0.10019
95	Malek Spinning Mills Ltd.	0.00250	0.00885	0.09409	0.00184	0.20731
96	Zahintex Industries Limited	-0.00638	0.01283	0.11327	0.00170	0.13270
97	Saiham Cotton Mills Limited	-0.00905	0.00538	0.07334	0.00163	0.30217
98	Generation Next Fashions Limited	-0.01481	0.01303	0.11415	0.00250	0.19177
99	Envoy Textiles Limited	-0.00498	0.00458	0.06767	0.00098	0.21435
100	Ambee Pharma Ltd.	0.01582	0.01131	0.10635	0.00048	0.04265
101	Beximco Pharmaceuticals Ltd.	0.01334	0.00852	0.09230	0.00260	0.30569
102	GlaxoSmithKline(GSK) Bangladesh Limited	0.02127	0.00843	0.09181	0.00115	0.13605
103	ACI Limited.	0.02278	0.01222	0.11054	0.00187	0.15302
104	Renata Ltd.	0.01314	0.00881	0.09385	0.00074	0.08427
105	Reckitt Benckiser (Bd.)Ltd.	0.01783	0.00943	0.09711	0.00131	0.13937
106	Pharma Aids Ltd.	0.02614	0.00944	0.09717	0.00047	0.05015
107	Kohinoor Chemicals Company (Bangladesh) Ltd.	0.01758	0.01209	0.10994	0.00015	0.01278
108	The IBN SINA Pharmaceutical Industry Ltd.	0.02298	0.01390	0.11791	0.00130	0.09345
109	Libra Infusions Limited	0.02678	0.02097	0.14481	0.00055	0.02627
110	Orion Infusion Ltd.	0.00861	0.01110	0.10535	0.00191	0.17229
111	Square Pharmaceuticals Ltd.	0.01167	0.00390	0.06247	0.00123	0.31474
112	Keya Cosmetics Ltd.	-0.01192	0.01373	0.11718	0.00288	0.20939
113	ACI Formulations Ltd.	0.02209	0.02144	0.14641	0.00367	0.17129
114	Marico Bangladesh Ltd.	0.02138	0.00943	0.09709	0.00076	0.08022
115	Active Fine Chemicals Ltd.	-0.00861	0.00926	0.09624	0.00240	0.25872
116	Samorita Hospital Ltd.	0.00859	0.00945	0.09722	0.00126	0.13350
117	Eastern Housing Limited	0.00561	0.00847	0.09202	0.00243	0.28701
118	Summit Alliance Port Limited	0.01253	0.02854	0.16893	0.00435	0.15249
119	Unique Hotel & Resorts Limited	0.00657	0.04854	0.22033	0.00449	0.09248
120	Heidelberg Cement Bangladesh Ltd.	0.01060	0.00916	0.09571	0.00210	0.22901
121	Confidence Cement Ltd.	0.01100	0.01109	0.10533	0.00354	0.31933
122	Meghna Cement Mills Ltd	0.00562	0.01827	0.13515	0.00374	0.20473
123	Lafarge Surma Cement Ltd.	0.01623	0.01639	0.12801	0.00290	0.17688

A	B	C	D	E	F	G
124	M.I. Cement Factory Limited	0.00162	0.00526	0.07252	0.00198	0.37557
125	BDCOM Online Ltd.	0.01447	0.02435	0.15606	0.00199	0.08163
126	Intech Limited	0.00470	0.01326	0.11514	0.00288	0.21731
127	Agni Systems Ltd.	0.00136	0.00948	0.09736	0.00324	0.34223
128	Daffodil Computers Ltd.	0.01811	0.01386	0.11771	0.00136	0.09811
129	aamra technologies limited	0.00559	0.01429	0.11955	0.00356	0.24921
130	Grameenphone Ltd.	0.04033	0.07562	0.27499	0.00239	0.03157
131	Apex Tannery Limited	0.01635	0.01434	0.11976	0.00324	0.22622
132	Bata Shoe Company (Bangladesh) Limited	0.01575	0.00558	0.07470	0.00123	0.22110
133	Apex Footwear Limited	0.01076	0.01119	0.10579	0.00230	0.20568
134	Legacy Footwear Ltd.	0.04283	0.08907	0.29845	0.00080	0.00902
135	Monno Ceramic Industries Ltd.	0.03555	0.02916	0.17078	0.00143	0.04893
136	Fu-Wang Ceramic Industries Ltd.	0.00094	0.01036	0.10179	0.00260	0.25127
137	RAK Ceramics (BD) Ltd.	0.00204	0.01278	0.11303	0.00364	0.28468
138	Bangladesh General Insurance Company Ltd.	-0.00183	0.01372	0.11712	0.00375	0.27374
139	Green Delta Insurance Company Ltd.	0.00062	0.00888	0.09422	0.00200	0.22480
140	United Insurance Company Ltd.	-0.00136	0.01537	0.12399	0.00408	0.26563
141	Peoples Insurance Company Ltd.	-0.00215	0.00622	0.07889	0.00222	0.35636
142	Eastern Insurance Co. Ltd.	-0.00274	0.00632	0.07948	0.00251	0.39755
143	Phoenix Insurance Company Ltd.	-0.00962	0.01097	0.10474	0.00263	0.23969
144	Eastland Insurance Company Limited	-0.01261	0.00995	0.09975	0.00310	0.31146
145	Central Insurance Company Limited	-0.00573	0.00663	0.08143	0.00139	0.20969
146	Karnaphuli Insurance Co. Ltd.	-0.00853	0.00577	0.07593	0.00191	0.33039
147	Rupali Insurance Company Ltd.	-0.01092	0.01173	0.10832	0.00342	0.29158
148	National Life Insurance Co Ltd	-0.00689	0.00952	0.09757	0.00178	0.18738
149	Reliance Insurance Ltd.	-0.00418	0.00596	0.07720	0.00198	0.33274
150	Purabi Gen. Insurance Company Ltd.	-0.01711	0.01308	0.11436	0.00300	0.22962
151	Delta Life Insurance Co. Ltd	-0.01578	0.03089	0.17576	0.00374	0.12120
152	Pragati Insurance Ltd.	-0.00832	0.00944	0.09715	0.00264	0.27977
153	Sandhani Life Insurance Ltd	-0.01503	0.01121	0.10589	0.00278	0.24798
154	Prime Insurance Company Limited	-0.00360	0.01679	0.12958	0.00356	0.21221
155	Pioneer Insurance company Ltd.	-0.01376	0.00623	0.07894	0.00125	0.20022
156	Mercantile Insurance Co. Ltd.	-0.00224	0.00992	0.09959	0.00261	0.26278
157	Agrani Insurance Co Ltd.	-0.00354	0.01143	0.10689	0.00241	0.21098
158	Popular Life Insurance Company Ltd.	-0.00241	0.01622	0.12737	0.00269	0.16608
159	Fareast Islami Life Insurance Co. Ltd.	-0.00355	0.00784	0.08855	0.00287	0.36616
160	Meghna Life Insurance Co. Ltd.	-0.00709	0.01119	0.10580	0.00343	0.30623
161	Nitol Insurance Co. Ltd.	0.00132	0.01083	0.10405	0.00310	0.28661
162	Asia Pacific General Insurance Co. Ltd.	-0.00219	0.01044	0.10216	0.00330	0.31658
163	Sonar Bangla Insurance Ltd.	-0.00769	0.00722	0.08494	0.00213	0.29561
164	Pragati Life Insurance Ltd.	0.00530	0.01903	0.13796	0.00222	0.11649
165	Prime Islami life Insurance Ltd.	-0.00410	0.01936	0.13915	0.00296	0.15289
166	Paramount Insurance Co. Ltd.	-0.00236	0.01021	0.10104	0.00148	0.14503
167	City General Insurance Co. Ltd.	-0.00785	0.01200	0.10953	0.00324	0.26965

A	B	C	D	E	F	G
168	Continental Insurance Ltd.	-0.00123	0.01732	0.13162	0.00338	0.19511
169	Takaful Islami Insurance Ltd.	-0.00403	0.01048	0.10239	0.00129	0.12321
170	Standard Insurance Ltd.	-0.00292	0.01117	0.10570	0.00188	0.16809
171	Northern General Insurance Co. Ltd.	-0.00475	0.01937	0.13918	0.00486	0.25091
172	Republic Insurance Company Ltd.	-0.00391	0.01574	0.12545	0.00329	0.20893
173	Asia Insurance Co. Ltd.	-0.00481	0.01039	0.10191	0.00276	0.26595
174	Rupali Life Insurance Co.Ltd.	-0.01071	0.01135	0.10654	0.00219	0.19293
175	Islami Insurance BD. Ltd.	0.00309	0.00885	0.09409	0.00270	0.30455
176	Provati Insurance Company ltd.	-0.00725	0.00609	0.07802	0.00161	0.26502
177	Dhaka Insurance Co. Ltd.	-0.01203	0.00779	0.08826	0.00272	0.34932
178	Padma Islami Life Insurance Limited	-0.00215	0.01256	0.11208	0.00295	0.23507
179	**Market Index (DSEX)**	**0.00681**	**0.00248**	**0.04985**	**0.00248**	**1.00000**

Appendix-B:

Breakdown of each firm's return and risk into unique and market-related part with

Code A as Stock Number   Code K as σ_{ei}^2

Code H as α_i Code L as $\beta_i^2 \sigma_m^2$

Code I as $\beta_i \overline{R}_m$ Code M as σ_i^2

Code J as $\overline{R}_i$

A	H	I	J	K	L	M
1	-0.00411	0.00151	-0.00260	0.01515	0.00012	0.01528
2	0.00918	0.00194	0.01113	0.01197	0.00020	0.01217
3	-0.00154	0.00084	-0.00070	0.02642	0.00004	0.02645
4	0.00104	0.00105	0.00209	0.01620	0.00006	0.01626
5	-0.00636	0.00157	-0.00479	0.01124	0.00013	0.01138
6	-0.00268	0.00184	-0.00084	0.00676	0.00018	0.00694
7	-0.00084	0.00126	0.00042	0.01727	0.00009	0.01736
8	0.00096	0.00191	0.00287	0.01010	0.00019	0.01029
9	-0.00222	0.00223	0.00001	0.00783	0.00027	0.00809
10	0.00406	0.00245	0.00650	0.00453	0.00032	0.00485
11	0.00302	0.00182	0.00484	0.01126	0.00018	0.01143
12	-0.00605	0.00132	-0.00472	0.00853	0.00009	0.00863
13	0.00209	0.00218	0.00426	0.00786	0.00025	0.00812
14	-0.00319	0.00223	-0.00096	0.01016	0.00027	0.01042
15	-0.00181	0.00272	0.00091	0.00481	0.00040	0.00520
16	0.00433	0.00165	0.00599	0.00538	0.00015	0.00553
17	0.00088	0.00223	0.00311	0.00630	0.00027	0.00657
18	0.00940	0.00183	0.01123	0.00790	0.00018	0.00808
19	-0.00456	0.00221	-0.00235	0.00861	0.00026	0.00887
20	-0.00038	0.00250	0.00212	0.00771	0.00034	0.00805
21	0.00267	0.00162	0.00429	0.00812	0.00014	0.00826
22	0.00525	0.00164	0.00689	0.00867	0.00014	0.00882
23	-0.00474	0.00214	-0.00260	0.00621	0.00024	0.00646
24	-0.00063	0.00194	0.00131	0.00545	0.00020	0.00565
25	0.01884	0.00146	0.02030	0.00959	0.00011	0.00971
26	0.00404	0.00149	0.00553	0.00898	0.00012	0.00910
27	-0.00180	0.00188	0.00008	0.00835	0.00019	0.00854
28	0.01301	0.00156	0.01457	0.01393	0.00013	0.01406
29	-0.00284	0.00199	-0.00085	0.00898	0.00021	0.00919
30	0.00273	0.00188	0.00461	0.01720	0.00019	0.01739
31	-0.00434	0.00212	-0.00221	0.01133	0.00024	0.01157
32	-0.00282	0.00274	-0.00008	0.00775	0.00040	0.00815
33	0.00262	0.00155	0.00417	0.01465	0.00013	0.01478
34	-0.00002	0.00186	0.00183	0.01304	0.00018	0.01323
35	0.00341	0.00128	0.00469	0.02961	0.00009	0.02970

A	H	I	J	K	L	M
36	0.01754	0.00154	0.01908	0.01287	0.00013	0.01299
37	-0.00065	0.00132	0.00067	0.02091	0.00009	0.02101
38	0.00213	0.00158	0.00371	0.01993	0.00013	0.02006
39	0.00515	0.00168	0.00683	0.01990	0.00015	0.02006
40	0.00137	0.00145	0.00281	0.01682	0.00011	0.01693
41	0.00376	0.00102	0.00478	0.02550	0.00006	0.02556
42	-0.00127	0.00193	0.00066	0.01302	0.00020	0.01322
43	0.01195	0.00110	0.01305	0.01271	0.00006	0.01277
44	0.03117	0.00047	0.03165	0.01616	0.00001	0.01617
45	0.03505	-0.00031	0.03474	0.01998	0.00001	0.01998
46	0.00716	0.00156	0.00873	0.01421	0.00013	0.01435
47	0.00140	0.00139	0.00279	0.01158	0.00010	0.01168
48	0.01799	0.00112	0.01912	0.01975	0.00007	0.01982
49	0.04887	0.00005	0.04892	0.04320	0.00000	0.04320
50	0.02992	0.00087	0.03079	0.02578	0.00004	0.02582
51	0.03357	-0.00015	0.03342	0.02169	0.00000	0.02169
52	0.01400	0.00126	0.01526	0.00610	0.00009	0.00618
53	-0.00595	0.00227	-0.00368	0.00802	0.00028	0.00830
54	0.01485	0.00126	0.01611	0.00976	0.00008	0.00984
55	0.00514	0.00205	0.00718	0.01416	0.00022	0.01438
56	-0.00143	0.00231	0.00088	0.01088	0.00028	0.01116
57	0.00085	0.00110	0.00194	0.01367	0.00006	0.01373
58	0.00141	0.00185	0.00326	0.01549	0.00018	0.01568
59	0.02113	0.00093	0.02206	0.01160	0.00005	0.01165
60	0.02229	0.00082	0.02311	0.01620	0.00004	0.01623
61	0.02508	0.00046	0.02554	0.00555	0.00001	0.00556
62	0.03411	-0.00033	0.03379	0.04662	0.00001	0.04662
63	-0.00307	0.00043	-0.00264	0.00488	0.00001	0.00489
64	0.01347	0.00072	0.01419	0.01047	0.00003	0.01050
65	0.00078	0.00139	0.00218	0.01537	0.00010	0.01547
66	-0.00154	0.00132	-0.00021	0.01664	0.00009	0.01674
67	0.01606	0.00140	0.01745	0.00800	0.00010	0.00811
68	0.00888	0.00140	0.01029	0.01495	0.00011	0.01505
69	0.04170	-0.00012	0.04159	0.05485	0.00000	0.05485
70	-0.00337	0.00249	-0.00089	0.01066	0.00033	0.01099
71	-0.00739	0.00191	-0.00548	0.00714	0.00020	0.00734
72	0.00102	0.00181	0.00283	0.00810	0.00018	0.00827
73	0.00373	0.00146	0.00520	0.00635	0.00011	0.00647
74	0.00556	0.00131	0.00688	0.00936	0.00009	0.00945
75	-0.00827	0.00279	-0.00547	0.00332	0.00042	0.00374
76	0.00444	0.00186	0.00630	0.00853	0.00018	0.00871
77	0.00340	0.00197	0.00537	0.01185	0.00021	0.01206
78	0.00665	0.00166	0.00831	0.00907	0.00015	0.00921
79	-0.00960	0.00249	-0.00711	0.00936	0.00033	0.00969
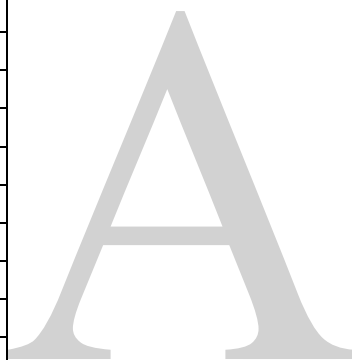
A	H	I	J	K	L	M
80	0.08231	-0.00004	0.08227	0.07274	0.00000	0.07274
81	0.01664	0.00055	0.01719	0.01274	0.00002	0.01276
82	0.02298	0.00022	0.02320	0.02898	0.00000	0.02898
83	0.02575	-0.00007	0.02568	0.03174	0.00000	0.03174
84	0.02295	0.00066	0.02361	0.02929	0.00002	0.02932
85	-0.00656	0.00234	-0.00421	0.00605	0.00029	0.00634
86	0.05449	-0.00011	0.05439	0.05976	0.00000	0.05976
87	0.01531	0.00095	0.01626	0.01206	0.00005	0.01211
88	-0.01692	0.00097	-0.01595	0.01618	0.00005	0.01623
89	0.00331	0.00163	0.00495	0.00894	0.00014	0.00908
90	0.01246	0.00053	0.01299	0.01567	0.00002	0.01568
91	0.00643	0.00119	0.00762	0.01752	0.00008	0.01759
92	-0.00670	0.00092	-0.00579	0.02216	0.00005	0.02221
93	-0.01179	0.00244	-0.00935	0.00265	0.00032	0.00297
94	-0.01039	0.00068	-0.00971	0.01272	0.00002	0.01274
95	0.00108	0.00141	0.00250	0.00875	0.00011	0.00885
96	-0.00728	0.00090	-0.00638	0.01279	0.00004	0.01283
97	-0.01111	0.00206	-0.00905	0.00515	0.00023	0.00538
98	-0.01612	0.00131	-0.01481	0.01294	0.00009	0.01303
99	-0.00644	0.00146	-0.00498	0.00446	0.00011	0.00458
100	0.01553	0.00029	0.01582	0.01131	0.00000	0.01131
101	0.01126	0.00208	0.01334	0.00829	0.00023	0.00852
102	0.02034	0.00093	0.02127	0.00838	0.00005	0.00843
103	0.02173	0.00104	0.02278	0.01216	0.00006	0.01222
104	0.01257	0.00057	0.01314	0.00879	0.00002	0.00881
105	0.01688	0.00095	0.01783	0.00938	0.00005	0.00943
106	0.02580	0.00034	0.02614	0.00944	0.00001	0.00944
107	0.01749	0.00009	0.01758	0.01209	0.00000	0.01209
108	0.02234	0.00064	0.02298	0.01388	0.00002	0.01390
109	0.02660	0.00018	0.02678	0.02097	0.00000	0.02097
110	0.00743	0.00117	0.00861	0.01102	0.00007	0.01110
111	0.00953	0.00214	0.01167	0.00366	0.00025	0.00390
112	-0.01335	0.00143	-0.01192	0.01362	0.00011	0.01373
113	0.02092	0.00117	0.02209	0.02136	0.00007	0.02144
114	0.02084	0.00055	0.02138	0.00941	0.00002	0.00943
115	-0.01037	0.00176	-0.00861	0.00910	0.00017	0.00926
116	0.00768	0.00091	0.00859	0.00941	0.00004	0.00945
117	0.00366	0.00196	0.00561	0.00826	0.00020	0.00847
118	0.01149	0.00104	0.01253	0.02848	0.00006	0.02854
119	0.00594	0.00063	0.00657	0.04852	0.00002	0.04854
120	0.00904	0.00156	0.01060	0.00903	0.00013	0.00916
121	0.00883	0.00218	0.01100	0.01084	0.00025	0.01109
122	0.00422	0.00139	0.00562	0.01816	0.00010	0.01827
123	0.01503	0.00120	0.01623	0.01631	0.00008	0.01639

A	H	I	J	K	L	M
124	-0.00094	0.00256	0.00162	0.00491	0.00035	0.00526
125	0.01391	0.00056	0.01447	0.02434	0.00002	0.02435
126	0.00322	0.00148	0.00470	0.01314	0.00012	0.01326
127	-0.00098	0.00233	0.00136	0.00919	0.00029	0.00948
128	0.01745	0.00067	0.01811	0.01383	0.00002	0.01386
129	0.00389	0.00170	0.00559	0.01414	0.00015	0.01429
130	0.04012	0.00022	0.04033	0.07562	0.00000	0.07562
131	0.01481	0.00154	0.01635	0.01421	0.00013	0.01434
132	0.01425	0.00151	0.01575	0.00546	0.00012	0.00558
133	0.00935	0.00140	0.01076	0.01109	0.00011	0.01119
134	0.04277	0.00006	0.04283	0.08907	0.00000	0.08907
135	0.03522	0.00033	0.03555	0.02916	0.00001	0.02916
136	-0.00077	0.00171	0.00094	0.01020	0.00016	0.01036
137	0.00010	0.00194	0.00204	0.01258	0.00020	0.01278
138	-0.00369	0.00186	-0.00183	0.01353	0.00019	0.01372
139	-0.00091	0.00153	0.00062	0.00875	0.00013	0.00888
140	-0.00317	0.00181	-0.00136	0.01520	0.00018	0.01537
141	-0.00458	0.00243	-0.00215	0.00591	0.00032	0.00622
142	-0.00545	0.00271	-0.00274	0.00592	0.00039	0.00632
143	-0.01126	0.00163	-0.00962	0.01083	0.00014	0.01097
144	-0.01473	0.00212	-0.01261	0.00971	0.00024	0.00995
145	-0.00716	0.00143	-0.00573	0.00652	0.00011	0.00663
146	-0.01078	0.00225	-0.00853	0.00549	0.00027	0.00577
147	-0.01291	0.00199	-0.01092	0.01152	0.00021	0.01173
148	-0.00817	0.00128	-0.00689	0.00943	0.00009	0.00952
149	-0.00645	0.00227	-0.00418	0.00569	0.00028	0.00596
150	-0.01867	0.00156	-0.01711	0.01295	0.00013	0.01308
151	-0.01661	0.00083	-0.01578	0.03086	0.00004	0.03089
152	-0.01023	0.00191	-0.00832	0.00924	0.00019	0.00944
153	-0.01672	0.00169	-0.01503	0.01106	0.00015	0.01121
154	-0.00504	0.00145	-0.00360	0.01668	0.00011	0.01679
155	-0.01512	0.00136	-0.01376	0.00613	0.00010	0.00623
156	-0.00403	0.00179	-0.00224	0.00975	0.00017	0.00992
157	-0.00497	0.00144	-0.00354	0.01131	0.00011	0.01143
158	-0.00354	0.00113	-0.00241	0.01616	0.00007	0.01622
159	-0.00605	0.00249	-0.00355	0.00751	0.00033	0.00784
160	-0.00918	0.00209	-0.00709	0.01096	0.00023	0.01119
161	-0.00063	0.00195	0.00132	0.01062	0.00020	0.01083
162	-0.00435	0.00216	-0.00219	0.01019	0.00025	0.01044
163	-0.00970	0.00201	-0.00769	0.00700	0.00022	0.00722
164	0.00451	0.00079	0.00530	0.01900	0.00003	0.01903
165	-0.00514	0.00104	-0.00410	0.01930	0.00006	0.01936
166	-0.00335	0.00099	-0.00236	0.01016	0.00005	0.01021
167	-0.00969	0.00184	-0.00785	0.01182	0.00018	0.01200

A	H	I	J	K	L	M
168	-0.00256	0.00133	-0.00123	0.01723	0.00009	0.01732
169	-0.00487	0.00084	-0.00403	0.01045	0.00004	0.01048
170	-0.00406	0.00115	-0.00292	0.01110	0.00007	0.01117
171	-0.00646	0.00171	-0.00475	0.01921	0.00016	0.01937
172	-0.00534	0.00142	-0.00391	0.01563	0.00011	0.01574
173	-0.00662	0.00181	-0.00481	0.01021	0.00018	0.01039
174	-0.01202	0.00131	-0.01071	0.01126	0.00009	0.01135
175	0.00102	0.00207	0.00309	0.00862	0.00023	0.00885
176	-0.00906	0.00181	-0.00725	0.00591	0.00017	0.00609
177	-0.01440	0.00238	-0.01203	0.00749	0.00030	0.00779
178	-0.00375	0.00160	-0.00215	0.01242	0.00014	0.01256

JEMA

Appendix-C:

Ranking of stocks according to their excess return-to-beta ratio with

Code N as Rank Code Q as $(\overline{R}_i - R_f)$

Code O as Company Name Code R as β_i

Code P as $(\overline{R}_i)$ Code S as $\frac{\overline{R}_i - R_f}{\beta_i}$

N	O	P	Q	R	S
1	Renwick Jajneswar & Co (Bd) Ltd.	0.04892	0.04413	0.00714	6.18333
2	Legacy Footwear Ltd.	0.04283	0.03805	0.00902	4.21675
3	Grameenphone Ltd.	0.04033	0.03555	0.03157	1.12589
4	Kohinoor Chemicals Company (Bangladesh) Ltd.	0.01758	0.01279	0.01278	1.00118
5	Libra Infusions Limited	0.02678	0.02200	0.02627	0.83736
6	Monno Ceramic Industries Ltd.	0.03555	0.03077	0.04893	0.62871
7	Al-Haj Textile Mills Limited	0.02320	0.01841	0.03196	0.57611
8	Pharma Aids Ltd.	0.02614	0.02135	0.05015	0.42578
9	Eastern Cables Ltd.	0.03165	0.02686	0.06957	0.38610
10	British American Tobacco Bangladesh Company Limited	0.02554	0.02076	0.06771	0.30655
11	Ambee Pharma Ltd.	0.01582	0.01104	0.04265	0.25884
12	Marico Bangladesh Ltd.	0.02138	0.01660	0.08022	0.20693
13	National tubes Ltd.	0.03079	0.02600	0.12802	0.20314
14	Rahim Textile Mills Ltd.	0.02361	0.01883	0.09635	0.19539
15	The IBN SINA Pharmaceutical Industry Ltd.	0.02298	0.01819	0.09345	0.19466
16	Sonali Aansh Industries Limited	0.01719	0.01241	0.08073	0.15372
17	Apex Foods Limited	0.02311	0.01833	0.11994	0.15279
18	Daffodil Computers Ltd.	0.01811	0.01333	0.09811	0.13586
19	Olympic Industries Limited	0.02206	0.01727	0.13604	0.12698
20	GlaxoSmithKline(GSK) Bangladesh Limited	0.02127	0.01648	0.13605	0.12113
21	BDCOM Online Ltd.	0.01447	0.00968	0.08163	0.11864
22	ACI Limited.	0.02278	0.01799	0.15302	0.11757
23	Anlima Yarn Dyeing Ltd.	0.01299	0.00821	0.07836	0.10474
24	ACI Formulations Ltd.	0.02209	0.01730	0.17129	0.10103
25	Renata Ltd.	0.01314	0.00836	0.08427	0.09916
26	Reckitt Benckiser (Bd.)Ltd.	0.01783	0.01305	0.13937	0.09361
27	Agricultural Marketing Company Ltd. (Pran)	0.01419	0.00940	0.10499	0.08956
28	Quasem Drycells Ltd.	0.01912	0.01433	0.16512	0.08681
29	Apex Spinning & Knitting Mills Limited	0.01626	0.01148	0.14013	0.08192
30	BRAC Bank Ltd.	0.02030	0.01552	0.21499	0.07219
31	Lafarge Surma Cement Ltd.	0.01623	0.01145	0.17688	0.06473
32	IPDC Finance Limited	0.01908	0.01430	0.22657	0.06310
33	Linde Bangladesh Limited	0.01745	0.01267	0.20492	0.06182
34	National Polymer Industries Ltd	0.01611	0.01132	0.18475	0.06129
35	Rangpur Foundry Ltd.	0.01526	0.01047	0.18526	0.05653
36	Bangladesh Lamps Limited	0.01305	0.00827	0.16122	0.05127

N	O	P	Q	R	S
37	Apex Tannery Limited	0.01635	0.01157	0.22622	0.05115
38	Summit Alliance Port Limited	0.01253	0.00774	0.15249	0.05079
39	Bata Shoe Company (Bangladesh) Limited	0.01575	0.01097	0.22110	0.04960
40	Trust Bank Ltd.	0.01457	0.00979	0.22960	0.04263
41	Apex Footwear Limited	0.01076	0.00597	0.20568	0.02903
42	Samorita Hospital Ltd.	0.00859	0.00381	0.13350	0.02853
43	Beximco Pharmaceuticals Ltd.	0.01334	0.00856	0.30569	0.02800
44	Padma Oil Co.Ltd.	0.01029	0.00550	0.20622	0.02668
45	Heidelberg Cement Bangladesh Ltd.	0.01060	0.00582	0.22901	0.02540
46	Mutual Trust Bank Ltd.	0.01123	0.00645	0.26881	0.02398
47	The City Bank Ltd.	0.01113	0.00634	0.28493	0.02226
48	Orion Infusion Ltd.	0.00861	0.00382	0.17229	0.02219
49	Square Pharmaceuticals Ltd.	0.01167	0.00689	0.31474	0.02188
50	Confidence Cement Ltd.	0.01100	0.00622	0.31933	0.01947
51	Unique Hotel & Resorts Limited	0.00657	0.00179	0.09248	0.01932
52	Singer Bangladesh Ltd.	0.00873	0.00394	0.22916	0.01720
53	H.R. Textile Ltd.	0.00762	0.00284	0.17462	0.01624
54	MJL Bangladesh Limited	0.00831	0.00352	0.24299	0.01450
55	Meghna Petroleum Ltd.	0.00688	0.00209	0.19282	0.01085
56	Mercantile Bank Ltd.	0.00689	0.00211	0.24136	0.00874
57	Phoenix Finance and Investments Ltd.	0.00683	0.00205	0.24701	0.00830
58	BSRM Steels Limited	0.00718	0.00240	0.30040	0.00798
59	Khulna Power Company Ltd.	0.00630	0.00151	0.27241	0.00555
60	Social Islami Bank Ltd.	0.00599	0.00120	0.24267	0.00495
61	Eastern Bank Ltd.	0.00650	0.00172	0.35953	0.00479
62	Pragati Life Insurance Ltd.	0.00530	0.00052	0.11649	0.00443
63	Meghna Cement Mills Ltd	0.00562	0.00084	0.20473	0.00408
64	Shahjalal Islami Bank Ltd.	0.00553	0.00074	0.21899	0.00339
65	aamra technologies limited	0.00559	0.00081	0.24921	0.00324
66	Eastern Housing Limited	0.00561	0.00083	0.28701	0.00289
67	Baraka Power Limited	0.00537	0.00059	0.28984	0.00203
68	Jamuna Oil Com. Ltd.	0.00520	0.00041	0.21505	0.00192
69	Prime Textile Spinning Mills Ltd.	0.00495	0.00016	0.23979	0.00067
70	Al-Arafah Islami Bank Ltd.	0.00484	0.00005	0.26704	0.00019
71	GSP Finance Company (Bangladesh) Limited	0.00478	0.00000	0.15041	-0.00002
72	Intech Limited	0.00470	-0.00008	0.21731	-0.00038
73	LankaBangla Finance Ltd.	0.00469	-0.00009	0.18862	-0.00050
74	IDLC Finance Limited	0.00461	-0.00018	0.27608	-0.00064
75	Southeast Bank Ltd.	0.00426	-0.00052	0.31930	-0.00164
76	Bank Asia Ltd.	0.00429	-0.00050	0.23776	-0.00209
77	Premier Leasing & Finance Limited.	0.00417	-0.00061	0.22791	-0.00268
78	Bangladesh Finance and Investment Co. Ltd.	0.00371	-0.00107	0.23210	-0.00461
79	Dutch-Bangla Bank Ltd.	0.00311	-0.00167	0.32799	-0.00511
80	Islami Insurance BD. Ltd.	0.00309	-0.00169	0.30455	-0.00556

N	O	P	Q	R	S
81	GPH Ispat Ltd.	0.00326	-0.00152	0.27131	-0.00562
82	United Commercial Bank Ltd.	0.00287	-0.00191	0.28005	-0.00684
83	One Bank Limited	0.00212	-0.00266	0.36771	-0.00724
84	Power Grid Company of Bangladesh Ltd.	0.00283	-0.00195	0.26586	-0.00734
85	M.I. Cement Factory Limited	0.00162	-0.00316	0.37557	-0.00842
86	Bay Leasing & Investment Ltd.	0.00281	-0.00197	0.21247	-0.00928
87	RAK Ceramics (BD) Ltd.	0.00204	-0.00274	0.28468	-0.00964
88	National Credit and Commerce Bank Ltd.	0.00091	-0.00387	0.39944	-0.00969
89	Atlas Bangladesh Ltd.	0.00279	-0.00199	0.20380	-0.00979
90	Agni Systems Ltd.	0.00136	-0.00343	0.34223	-0.01002
91	Islamic Finance & Investment Ltd.	0.00183	-0.00295	0.27282	-0.01081
92	Malek Spinning Mills Ltd.	0.00250	-0.00229	0.20731	-0.01104
93	Navana CNG Limited	0.00088	-0.00390	0.33858	-0.01153
94	Nitol Insurance Co. Ltd.	0.00132	-0.00346	0.28661	-0.01208
95	Uttara Finance and Investments Limited.	-0.00008	-0.00487	0.40203	-0.01211
96	Jamuna Bank Ltd.	0.00131	-0.00348	0.28406	-0.01225
97	Fu Wang Food Ltd.	0.00218	-0.00261	0.20425	-0.01277
98	Uttara Bank Ltd.	0.00001	-0.00477	0.32736	-0.01459
99	Aftab Automobiles Limited	0.00066	-0.00413	0.28280	-0.01459
100	Fu-Wang Ceramic Industries Ltd.	0.00094	-0.00385	0.25127	-0.01531
101	Summit Power Ltd.	-0.00089	-0.00567	0.36483	-0.01554
102	Premier Bank Ltd.	0.00008	-0.00470	0.27575	-0.01705
103	Islami Bank Bd Ltd	0.00209	-0.00270	0.15359	-0.01757
104	Dhaka Bank Ltd.	-0.00096	-0.00575	0.32680	-0.01759
105	Deshbandhu Polymer Limited	0.00194	-0.00284	0.16092	-0.01766
106	Green Delta Insurance Company Ltd.	0.00062	-0.00416	0.22480	-0.01851
107	Eastern Insurance Co. Ltd.	-0.00274	-0.00753	0.39755	-0.01893
108	First Security Islami Bank Ltd.	-0.00085	-0.00564	0.29198	-0.01931
109	Peoples Insurance Company Ltd.	-0.00215	-0.00694	0.35636	-0.01947
110	Pubali Bank Ltd.	-0.00084	-0.00562	0.27022	-0.02080
111	Union Capital Ltd.	0.00067	-0.00412	0.19313	-0.02133
112	Standard Bank Ltd.	-0.00235	-0.00713	0.32447	-0.02199
113	Asia Pacific General Insurance Co. Ltd.	-0.00219	-0.00698	0.31658	-0.02204
114	United Finance Limited	-0.00221	-0.00700	0.31180	-0.02244
115	Fareast Islami Life Insurance Co. Ltd.	-0.00355	-0.00834	0.36616	-0.02277
116	United Insurance Company Ltd.	-0.00136	-0.00615	0.26563	-0.02314
117	Rupali Bank Ltd.	0.00042	-0.00436	0.18556	-0.02352
118	Export Import (Exim) Bank of Bangladesh Ltd.	-0.00260	-0.00739	0.31399	-0.02352
119	Bangladesh General Insurance Company Ltd.	-0.00183	-0.00661	0.27374	-0.02416
120	Titas Gas Trans. & Dist. Co. Ltd.	-0.00547	-0.01026	0.41024	-0.02500
121	S. Alam Cold Rolled Steels Ltd.	-0.00368	-0.00846	0.33294	-0.02542
122	Rangpur Dairy & Food Products Limited	-0.00021	-0.00500	0.19446	-0.02571
123	Saiham Textile Mills Ltd.	-0.00421	-0.00900	0.34376	-0.02618
124	Mercantile Insurance Co. Ltd.	-0.00224	-0.00702	0.26278	-0.02673

N	O	P	Q	R	S
125	Reliance Insurance Ltd.	-0.00418	-0.00897	0.33274	-0.02695
126	Padma Islami Life Insurance Limited	-0.00215	-0.00693	0.23507	-0.02950
127	Continental Insurance Ltd.	-0.00123	-0.00602	0.19511	-0.03084
128	GBB Power Limited	-0.00711	-0.01189	0.36546	-0.03254
129	AB Bank Limited	-0.00260	-0.00738	0.22182	-0.03327
130	Asia Insurance Co. Ltd.	-0.00481	-0.00960	0.26595	-0.03609
131	Dhaka Electric Supply Company Ltd.	-0.00548	-0.01026	0.28067	-0.03656
132	Northern General Insurance Co. Ltd.	-0.00475	-0.00953	0.25091	-0.03798
133	Meghna Life Insurance Co. Ltd.	-0.00709	-0.01188	0.30623	-0.03879
134	Agrani Insurance Co Ltd.	-0.00354	-0.00832	0.21098	-0.03943
135	Prime Insurance Company Limited	-0.00360	-0.00838	0.21221	-0.03950
136	Square Textiles Limited	-0.00935	-0.01414	0.35771	-0.03952
137	Karnaphuli Insurance Co. Ltd.	-0.00853	-0.01331	0.33039	-0.04030
138	National Bank Ltd.	-0.00479	-0.00958	0.23006	-0.04162
139	Republic Insurance Company Ltd.	-0.00391	-0.00870	0.20893	-0.04163
140	Sonar Bangla Insurance Ltd.	-0.00769	-0.01248	0.29561	-0.04220
141	Popular Life Insurance Company Ltd.	-0.00241	-0.00719	0.16608	-0.04330
142	IFIC Bank Ltd.	-0.00070	-0.00548	0.12338	-0.04443
143	Provati Insurance Company ltd.	-0.00725	-0.01204	0.26502	-0.04542
144	Envoy Textiles Limited	-0.00498	-0.00976	0.21435	-0.04555
145	Saiham Cotton Mills Limited	-0.00905	-0.01383	0.30217	-0.04578
146	Standard Insurance Ltd.	-0.00292	-0.00770	0.16809	-0.04583
147	Pragati Insurance Ltd.	-0.00832	-0.01310	0.27977	-0.04684
148	City General Insurance Co. Ltd.	-0.00785	-0.01263	0.26965	-0.04685
149	Dhaka Insurance Co. Ltd.	-0.01203	-0.01681	0.34932	-0.04812
150	Prime Bank Ltd.	-0.00472	-0.00951	0.19448	-0.04888
151	Paramount Insurance Co. Ltd.	-0.00236	-0.00715	0.14503	-0.04929
152	Central Insurance Company Limited	-0.00573	-0.01052	0.20969	-0.05015
153	Active Fine Chemicals Ltd.	-0.00861	-0.01339	0.25872	-0.05176
154	Rupali Insurance Company Ltd.	-0.01092	-0.01571	0.29158	-0.05386
155	Eastland Insurance Company Limited	-0.01261	-0.01739	0.31146	-0.05583
156	Prime Islami life Insurance Ltd.	-0.00410	-0.00889	0.15289	-0.05812
157	Phoenix Insurance Company Ltd.	-0.00962	-0.01441	0.23969	-0.06011
158	National Life Insurance Co Ltd	-0.00689	-0.01168	0.18738	-0.06231
159	Takaful Islami Insurance Ltd.	-0.00403	-0.00882	0.12321	-0.07156
160	Alif Manufacturing Company Ltd.	-0.00579	-0.01057	0.13463	-0.07851
161	Keya Cosmetics Ltd.	-0.01192	-0.01670	0.20939	-0.07978
162	Sandhani Life Insurance Ltd	-0.01503	-0.01981	0.24798	-0.07989
163	Rupali Life Insurance Co.Ltd.	-0.01071	-0.01549	0.19293	-0.08031
164	Zahintex Industries Limited	-0.00638	-0.01116	0.13270	-0.08413
165	Pioneer Insurance company Ltd.	-0.01376	-0.01854	0.20022	-0.09261
166	Purabi Gen. Insurance Company Ltd.	-0.01711	-0.02189	0.22962	-0.09535
167	Generation Next Fashions Limited	-0.01481	-0.01960	0.19177	-0.10218
168	National Tea Company Limited	-0.00264	-0.00742	0.06378	-0.11640

N	O	P	Q	R	S
169	R.N. Spinning Mills Ltd.	-0.00971	-0.01449	0.10019	-0.14467
170	Delta Spinners Ltd.	-0.01595	-0.02073	0.14202	-0.14599
171	Delta Life Insurance Co. Ltd	-0.01578	-0.02056	0.12120	-0.16967
172	Gemini Sea Food Ltd.	0.03379	0.02900	-0.04778	-0.60706
173	Monno Jute Stafflers Ltd.	0.03474	0.02995	-0.04531	-0.66110
174	Anwar Galvanizing Ltd.	0.03342	0.02863	-0.02251	-1.27203
175	Stylecraft Limited	0.02568	0.02090	-0.01040	-2.00855
176	Eastern Lubricants Ltd.	0.04159	0.03680	-0.01710	-2.15210
177	Desh Garments Ltd.	0.05439	0.04960	-0.01552	-3.19625
178	Northern Jute Manufacturing Co. Ltd.	0.08227	0.07749	-0.00611	-12.69015

Appendix-D:

Evaluation of Portfolio Performance with

Code A as Stock Number

Code N as Rank

N	A	β_i	X_i	$X_i\beta_i$	α_i	$X_i\alpha_i$	σ_{ei}^2	$X_i^2\sigma_{ei}^2$
1	49	0.00714	0.02324	0.00017	0.04887	0.001136	0.04320	0.000023336
2	134	0.00902	0.00971	0.00009	0.04277	0.000415	0.08907	0.000008395
3	130	0.03157	0.01059	0.00033	0.04012	0.000425	0.07562	0.000008480
4	107	0.01278	0.02381	0.00030	0.01749	0.000416	0.01209	0.000006850
5	109	0.02627	0.02353	0.00062	0.02660	0.000626	0.02097	0.000011613
6	135	0.04893	0.02354	0.00115	0.03522	0.000829	0.02916	0.000016157
7	82	0.03196	0.01415	0.00045	0.02298	0.000325	0.02898	0.000005800
8	106	0.05015	0.04996	0.00251	0.02580	0.001289	0.00944	0.000023554
9	44	0.06957	0.03657	0.00254	0.03117	0.001140	0.01616	0.000021614
10	61	0.06771	0.08153	0.00552	0.02508	0.002045	0.00555	0.000036880
11	100	0.04265	0.02110	0.00090	0.01553	0.000328	0.01131	0.000005034
12	114	0.08022	0.03759	0.00302	0.02084	0.000783	0.00941	0.000013300
13	50	0.12802	0.02147	0.00275	0.02992	0.000642	0.02578	0.000011881
14	84	0.09635	0.01364	0.00131	0.02295	0.000313	0.02929	0.000005449
15	108	0.09345	0.02780	0.00260	0.02234	0.000621	0.01388	0.000010731
16	81	0.08073	0.02025	0.00163	0.01664	0.000337	0.01274	0.000005226
17	60	0.11994	0.02352	0.00282	0.02229	0.000524	0.01620	0.000008957
18	128	0.09811	0.01979	0.00194	0.01745	0.000345	0.01383	0.000005415
19	59	0.13604	0.03033	0.00413	0.02113	0.000641	0.01160	0.000010675
20	102	0.13605	0.03983	0.00542	0.02034	0.000810	0.00838	0.000013296
21	125	0.08163	0.00804	0.00066	0.01391	0.000112	0.02434	0.000001573
22	103	0.15302	0.02986	0.00457	0.02173	0.000649	0.01216	0.000010840
23	90	0.07836	0.01040	0.00082	0.01246	0.000130	0.01567	0.000001696
24	113	0.17129	0.01600	0.00274	0.02092	0.000335	0.02136	0.000005468
25	104	0.08427	0.01872	0.00158	0.01257	0.000235	0.00879	0.000003081
26	105	0.13937	0.02713	0.00378	0.01688	0.000458	0.00938	0.000006904
27	64	0.10499	0.01739	0.00183	0.01347	0.000234	0.01047	0.000003165
28	48	0.16512	0.01397	0.00231	0.01799	0.000251	0.01975	0.000003856
29	87	0.14013	0.01813	0.00254	0.01531	0.000278	0.01206	0.000003962
30	25	0.21499	0.02998	0.00645	0.01884	0.000565	0.00959	0.000008624
31	123	0.17688	0.01267	0.00224	0.01503	0.000190	0.01631	0.000002616
32	36	0.22657	0.01991	0.00451	0.01754	0.000349	0.01287	0.000005099
33	67	0.20492	0.02820	0.00578	0.01606	0.000453	0.00800	0.000006364
34	54	0.18475	0.02063	0.00381	0.01485	0.000306	0.00976	0.000004151
35	52	0.18526	0.02981	0.00552	0.01400	0.000417	0.00610	0.000005417
36	43	0.16122	0.01092	0.00176	0.01195	0.000131	0.01271	0.000001516
37	131	0.22622	0.01366	0.00309	0.01481	0.000202	0.01421	0.000002651

N	A	β_i	X_i	$X_i\beta_i$	α_i	$X_i\alpha_i$	σ_{ei}^2	$X_i^2\sigma_{ei}^2$
38	118	0.15249	0.00455	0.00069	0.01149	0.000052	0.02848	0.000000590
39	132	0.22110	0.03333	0.00737	0.01425	0.000475	0.00546	0.000006065
40	28	0.22960	0.01094	0.00251	0.01301	0.000142	0.01393	0.000001668
41	133	0.20568	0.00656	0.00135	0.00935	0.000061	0.01109	0.000000478
42	116	0.13350	0.00486	0.00065	0.00768	0.000037	0.00941	0.000000222
43	101	0.30569	0.01219	0.00373	0.01126	0.000137	0.00829	0.000001231
44	68	0.20622	0.00414	0.00085	0.00888	0.000037	0.01495	0.000000256
45	120	0.22901	0.00687	0.00157	0.00904	0.000062	0.00903	0.000000426
46	18	0.26881	0.00812	0.00218	0.00940	0.000076	0.00790	0.000000521
47	2	0.28493	0.00475	0.00135	0.00918	0.000044	0.01197	0.000000270
48	110	0.17229	0.00309	0.00053	0.00743	0.000023	0.01102	0.000000105
49	111	0.31474	0.01643	0.00517	0.00953	0.000157	0.00366	0.000000987
50	121	0.31933	0.00400	0.00128	0.00883	0.000035	0.01084	0.000000173
51	119	0.09248	0.00025	0.00002	0.00594	0.000001	0.04852	0.000000003
52	46	0.22916	0.00135	0.00031	0.00716	0.000010	0.01421	0.000000026
53	91	0.17462	0.00062	0.00011	0.00643	0.000004	0.01752	0.000000007
54	78	0.24299	0.00060	0.00015	0.00665	0.000004	0.00907	0.000000003
		$\beta_p =$	0.124003		$\alpha_p =$	0.020644	$\sum_{i=1}^{N} X_i^2\sigma_{ei}^2 =$	0.000342655