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Modeling decisions from experience: How models with a set of parameters for aggregate choices explain individual choices

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One of the paradigms (called "sampling paradigm") in judgment and decision-making involves decision-makers sample information before making a final consequential choice. In the sampling paradigm, certain computational models have been proposed where a set of single or distribution parameters is calibrated to the choice proportions of a group of participants (aggregate and hierarchical models). However, currently little is known on how aggregate and hierarchical models would account for choices made by individual participants in the sampling paradigm. In this paper, we test the ability of aggregate and hierarchical models to explain choices made by individual participants. Several models, Ensemble, Cumulative Prospect Theory (CPT), Best Estimation and Simulation Techniques (BEAST), Natural-Mean Heuristic (NMH), and Instance-Based Learning (IBL), had their parameters calibrated to individual choices in a large dataset involving the sampling paradigm. Later, these models were generalized to two large datasets in the sampling paradigm. Results revealed that the aggregate models (like CPT and IBL) accounted for individual choices better than hierarchical models (like Ensemble and BEAST) upon generalization to problems that were like those encountered during calibration. Furthermore, the CPT model, which relies on differential valuing of gains and losses, respectively, performed better than other models during calibration and generalization on datasets with similar set of problems. The IBL model, relying on recency and frequency of sampled information, and the NMH model, relying on frequency of sampled information, performed better than other models during generalization to a challenging dataset. Sequential analyses of results from different models showed how these models accounted for transitions from the last sample to final choice in human data. We highlight the implications of using aggregate and hierarchical models in explaining individual choices from experience.

Keywords: Aggregate choice, individual choice, sampling paradigm, decisions from experience, computational models, likelihood

With the advent of Internet, online shopping for products has gained popularity (Stevens, 2016). For making satisfying online purchases, a consumer could first sample information about different products and then make a choice for the preferred item (Horrace et al., 2009). However, the act of making choices based upon sampled information is not limited to choosing between different products; rather, it is a very common exercise involving different facets of our daily lives (e.g., choosing food items, life partners, and careers). In fact, information search before a choice constitutes an integral part of Decisions from Experience (DFE) research, where the focus is on explaining human decisions based upon one's experience with sampled information (Hertwig & Erev, 2009).

To study people's information search and consequential choice behaviors in the laboratory, researchers have proposed the "sampling paradigm" (Hertwig & Erev, 2009). In the sampling paradigm, people are presented with two or more options to choose between. These options are represented as blank buttons on a computer screen. People are first asked to sample as many outcomes as they wish from different button options (information search). Once people are satisfied with their sampling of options, they decide from which option to make a single consequential choice for actual awards.

Several computational cognitive models have been proposed in the sampling paradigm, where these models help explain how people search for information and make consequential choices (Erev et al., 2010; Gonzalez & Dutt, 2011). Some of these models have a set of parameter values calibrated to each individual participant (called "individual models"; Busemeyer & Diederich, 2010; Kudryavtsev & Pavlodsky, 2012; Frey, Mata, & Hertwig, 2015). The parameter calibration exercise in these models results in a set of parameter values per individual participant, where the number of parameter sets from a model equal the number of participants in data. For example, Kudryavtsev and Pavlodsky (2012) tested three variations of two models, Prospect Theory (PT) (Kahneman & Tversky, 1979) and Expectancy-Valence (EVL) (Busemeyer & Stout, 2002) by calibrating model parameters to each participant's choice. As another example, Shteingart, Neima and Loewenstein (2013) modeled many repeated choices of individual participants in the

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Technion Prediction Tournament (TPT) dataset, considering a specific reinforcement-learning algorithm. These authors showed that there was a substantial effect of the first experience on choice behavior and this behavior could be accounted by the reinforcement learning model if the outcome of first experience rested the values of the experienced actions. Similarly, Frey, Mata, and Hertwig (2015) presented a modeling analysis at the individual level showing that a simple delta-learning rule model with parameters calibrated to younger and older adults separately best described the learning processes for both these age groups.

Furthermore, certain computational models have been proposed where model parameters are calibrated to the choice proportions of a group of participants (called “aggregate models”; Busemeyer & Diederich, 2010; Estes & Maddox, 2005). Here, a single set of values of model parameters is calibrated to the average decision computed across several participants (Busemeyer & Diederich, 2010; Erev, Ert, Roth, et al., 2010; Gonzalez & Dutt, 2011; 2012; Lejarraga, Dutt, & Gonzalez, 2012). The calibration exercise results in only one set of values of parameters from a model and these parameters explain the averaged decision computed across all participants. For example, Gonzalez and Dutt (2011) calibrated one set of values for three parameters in an Instance-Based Learning (IBL) model to the risky proportions averaged over all participants and problems in different DFE datasets. Similarly, Erev et al. (2010) compared several models, each with a single set of values for parameters, in their ability to capture average risk-taking in the TPT datasets.

There is still a third approach to model calibration where model parameters follow certain distributions (possessing density functions) that are defined across the choice proportions of a group of participants (called “hierarchical models”; Lee, 2008; Rouder & Lu, 2005). For example, in the Choice Prediction Competition (Erev, Ert, Plonsky, et. al., 2015), the Best Estimation and Simulation Techniques (BEAST) model was hierarchical and it contained a set of distribution parameters that were calibrated to the choice proportions across many participants.

Although literature has focused on calibrating parameters of individual, hierarchical, and aggregate models (Estes & Maddox, 2005; Gonzalez & Dutt, 2011; Rouder & Lu, 2005); however, little is currently known on how aggregate or hierarchical models and their set of single or distribution parameter values, respectively, account for decisions of individual participants. In this paper, we address this question by considering both aggregate and hierarchical models with a set of single or distribution parameter values and evaluate how these models explain individual choices. We perform our evaluation by calibrating and generalizing a set of parameter values in aggregate or hierarchical models to choices made by individual participants in large publically available datasets in the sampling paradigm. For example, the aggregate IBL model consists of a set of two parameters, d and σ where these two parameters possess single values and explain the average risk-taking in DFE datasets (Dutt & Gonzalez, 2012; Gonzalez & Dutt, 2011; 2012). In this paper, however, we recalibrate the d and σ parameters in the IBL model by assigning them a value each to predict individual choices in DFE datasets.

The aggregate models that possess a single set of parameter values and predict aggregated choices, i.e., choices that are averaged over several participants, may or may not explain individual choices well. One reason for this expectation is that if several individuals learn linearly at different points in time, then the average learning curve is likely to be curvilinear (Gallistel et al., 2004). Thus, even if models with a single set of parameter values explain a group’s aggregate curvilinear learning, it is possible that such models may not explain individual linear behavior. Another reason why these models may not explain individual behavior is due to the degree of heterogeneity present in individual choices (Busemeyer & Diederich, 2010): A single set of parameter values may not be sufficient to explain many individual choices. However, hierarchical models possess a set of distribution parameters. If these models account for aggregate choices, then they are also likely to account for individual choices. That is because the parameter values are resampled in a hierarchical model from their density functions for each individual participant and this resampling may allow these models to account for individual choices.

In addition, there seems to be a tradeoff between aggregate models (like IBL; Dutt & Gonzalez, 2012) that possess cognitive mechanisms (like recency, frequency, and blending of outcomes) and a single set of parameter values that are fixed across individuals; and, hierarchical models (like BEAST; Erev, Ert, Plonsky, et al., 2015) that possess mathematical functions to account for individual biases with a set of parameters that vary across individuals according to distributions. On one hand, one expects that aggregate models with cognitive mechanisms and a set of single parameters would account for individual choices; however, one may also expect that hierarchical models with mathematical functions and a set of distribution parameters would also account for individual choices.

In this paper, we test these expectations by taking both aggregate and hierarchical models where these models’ parameters are calibrated to individual choices. Furthermore, using the sampling paradigm, we also evaluate the sequential decisions of participants from their last sample to final choice as accounted by different aggregate and hierarchical models. This sequential analysis helps us showcase the ability of aggregate models in accounting for individual differences in decisions with a set of single or distribution parameters.

To calibrate aggregate and hierarchical model parameters to individual choices, we use the estimation dataset from TPT (Erev, Ert, Roth, et al., 2010), the largest publically available DFE dataset. We compare calibrated aggregate and hierarchical models by generalizing them to two different DFE datasets in the sampling paradigm. Furthermore, we investigate an aggregate or hierarchical model's ability in capturing individual differences in data with a set of single or distribution parameter values. In what follows, we first motivate our model choices, different datasets used, and the working of different models. Furthermore, we discuss the method used for calibrating a set of single or distribution parameters in models to choices made by individual participants. Finally, we present the results of model evaluations both during calibration and generalization and close the paper by discussing the implications of our results for predicting individual choices from experience.

Models in the sampling paradigm

Two classes of models have been proposed in the sampling paradigm (Hertwig, 2012): associative-learning models (e.g., Instance-Based Learning) and cognitive heuristics (e.g., Natural Mean Heuristic). Among the associative-learning class, human choice is conceptualized as a learning process (for example, see Busemeyer & Myung, 1992; Bush & Mosteller, 1955). Learning is captured by changing the propensity to select a gamble based on the experienced outcomes. Good experiences boost the propensity of choosing the gamble associated with them, and bad experiences diminish it (e.g., Barron & Erev, 2003; Denrell, 2007; Erev & Barron, 2005; March, 1996). Some of the models in the associative class include the Instance-Based Learning (IBL) model (Dutt & Gonzalez, 2012; Gonzalez and Dutt, 2011; 2012, Lejarraga, Dutt, & Gonzalez, 2012), Value Updating model (Hertwig et al., 2004), and Fractional Adjustment model (March, 1996). The IBL model (Dutt & Gonzalez, 2012; Gonzalez & Dutt, 2011; 2012, Lejarraga, Dutt, & Gonzalez, 2012) consists of experiences (called instances) stored in memory. Each instance's activation is a function of the frequency and recency of the corresponding outcomes observed during sampling in different options, where the activation function is borrowed from the Adaptive Control of Thought - Rational (ACT-R) cognitive framework (Anderson & Lebiere, 1998). Activations are used to calculate the blended value for each option and the model makes a final choice for the option with the highest blended value. Gonzalez and Dutt (2011; 2012) showed that an aggregate IBL model with three parameters performed efficiently in accounting for choices aggregated over many participants across two DFE paradigms. In fact, this IBL model was overall the best model in explaining aggregate choices with fewest parameters.

The second class of models are referred to as cognitive heuristics and this class aims to describe both the process and outcome of choice as heuristic rules (Brandstätter et al., 2006; Hertwig, 2012). A popular cognitive heuristic that focuses on the expected-value of outcomes obtained during sampling is the Natural-Mean Heuristic (NMH) (Hertwig & Pleskac, 2010; Hertwig, 2012). As per Hertwig (2012), the NMH model has the following interesting properties: (a) it is well tailored to sequentially encountered outcomes; and, (b) it arrives at its choice prediction by the expected-value of options based upon sampled outcomes. Two other heuristics proposed in the cognitive-heuristic class include the Maximax Heuristic (Hau et al., 2008) and the Lexicographic Heuristic (Luce & Raifa, 1957). In Maximax heuristic, the option with best possible outcome, no matter how likely it is, is chosen. A lexicographic heuristic generally consists of three building blocks (Gigerenzer & Goldstein, 1996): Search rule: Look up attributes in order of validity. Stopping rule: Stop search after the first attribute discriminates between alternatives. Decision rule: Choose the alternative that this attribute favors. Hau et al. (2008) and Brandstätter et al. (2006) have shown that both these heuristics seem to underperform compared to the NMH model. Furthermore, a very commonly used baseline heuristic is the Primed-Sampler (PS) model (Erev, Glozman & Hertwig, 2008). The PS model depends upon the recency of sampled information and it looks few samples back on each option during sampling before making a final choice (Gonzalez & Dutt, 2011). A variant of the PS model is the PS model with variability (Erev, Ert, Roth, et al., 2010). In this model variant, the look-back sample size k is varied between participants and problems. The PS model with variability is a special case of the NMH model (as the NMH model looks back the entire sample size while deriving a choice).

Furthermore, Hau et al. (2008) have shown that a Cumulative Prospect Theory (CPT) model (Tversky & Kahneman, 1992), which is a popular mathematical model (sometimes referred to as a "measurement model" or an "as-if" model), seems to perform about the same as the NMH model to account for aggregated choices. In the CPT model, a weighing function and a value function is associated with each probability and outcome, respectively. The model chooses the option that has the highest prospect value, where the prospect value is determined by multiplying the value with its corresponding weight. Furthermore, a linear combination heuristic model (Ensemble) was submitted to TPT (Erev, Ert, Roth, et al., 2010). The Ensemble model contains four heuristic rules, PS, CPT, Priority Heuristic (PH), and NMH, and it was shown to be the best model in the sampling paradigm. Most recently, Erev, Ert, Plonsky, et al. (2015) proposed the BEAST model, which consisted of several heuristic rules like expected value and mental simulations with a set of distribution parameters. The BEAST model performed well to capture 14-different aggregate phenomena in the 2015 Choice Prediction Competition. The 14-different aggregate phenomena refer to anomalies such as Ellsberg paradox, Allais paradox, Reflection effect and others described by Erev, Ert, Plonsky, et al. (2015).

Across the associative-learning models, mathematical models, and cognitive heuristics, there are aggregate models that possess a single set of parameter values and predict aggregated choices, i.e., choices that are averaged over several participants (Busemeyer & Wang, 2000; Dutt & Gonzalez, 2012; 2015; Gonzalez & Dutt, 2011; 2012; Lejarraga, Dutt, & Gonzalez, 2012). Also, there exist hierarchical models that possess a set of distribution parameters to predict aggregated choices, i.e., choices that are averaged over several participants (Erev, Ert, Plonsky, et al., 2015; Lee, 2008; Rouder & Lu, 2005).

The IBL, NMH, and CPT models are aggregate models (possessing a set of single parameter values); whereas, the BEAST and Ensemble models are hierarchical models and they possess a set of distribution parameter values. Within the aggregate and hierarchical models, some of the models (like IBL) possess cognitive processes like recency, frequency, or blending; whereas, other models (like CPT and Ensemble) possess mathematical functions that account for biases in people's decisions. If possessing a set of distribution parameter values helps models to account for individual choices, then we expect hierarchical models like BEAST and Ensemble to perform well in explaining individual choices. In contrast, if possessing cognitive mechanisms helps models account for individual choices, then we expect models like IBL to perform well in explaining individual decisions. In contrast, if mathematical functions can accurately account for biases in individual decisions, then we expect that models like CPT and Ensemble to perform well in explaining individual choices. We test these expectations in this paper by calibrating different models to human data in large datasets involving the sampling paradigm.

Model selection

Among all associative-learning models, the IBL model (Dutt & Gonzalez, 2012; Lejarraga, Dutt, & Gonzalez, 2012) has been shown as the best performing aggregate model in the sampling paradigm (Gonzalez & Dutt, 2011; 2012). Gonzalez and Dutt (2011) showed that the IBL model accounts for aggregate final choices with a small error. Thus, we choose the IBL model as one of the models for our evaluation. For this purpose, we first test the original IBL model (called IBL (LDG) model; Lejarraga, Dutt, & Gonzalez, 2012) in explaining individual choices with a set of parameter values. Next, we recalibrated a set of parameter values of this model to individual choices (called IBL (TPT) model) in the TPT dataset.

Popular maximax and lexicographic heuristics (Hau et al., 2008; Luce & Raifa, 1957) have underperformed compared to the NMH model (Brandstätter et al., 2006; Hau et al., 2008). The NMH model has been reported in literature as explaining aggregate final choices in the sampling paradigm (Hau et al., 2008; Hertwig, 2012). Thus, we chose the NMH model as another aggregate model for evaluating individual choices.

Furthermore, Hau et al. (2008) have also shown that different variants of the CPT model (Tversky & Kahneman, 1992) perform about the same as the NMH model to account for aggregate choices. Due to these reasons, we consider three variants of CPT model for our evaluation. The first, CPT (TK) model, is based upon parameters defined by Tversky and Kahneman (1992). The second, CPT (Hau) model, is based upon recalibrated parameters from Hau et al. (2008) to derive aggregated final choices. The third, CPT (TPT) model, has its parameters recalibrated to individual choices in the TPT dataset.

Erev et al. (2010) have shown the hierarchical Ensemble model, consisting of the PS, CPT, PH, and NMH models, to perform best in TPT's E-sampling condition.[1] Given that the Ensemble model contains a collection of several popular heuristic models, we consider two variants of this model for our evaluation: Ensemble (TPT) model, which used parameters proposed by Erev et al. (2010); and, Ensemble (Individual), where we recalibrated a set of parameter values of this model to individual choices in the TPT dataset.

In addition to the above models, we also considered the hierarchical BEAST model, which has recently been shown to account for 14-different phenomena in aggregate choices (Erev, Ert, Plonsky, et. al., 2015). We considered two variants of the BEAST model: BEAST (CPC) model, which was based on the same set of distribution parameters as reported by Erev, Ert, Plonsky, et. al. (2015); and, BEAST (TPT), which consisted of a set of distribution parameters calibrated to individual choices in the TPT dataset.

The Technion Prediction Tournament datasets

The Technion Prediction Tournament (TPT) (Erev et al., 2010) was a competition in which several participants were subjected to an experimental setup, the E-sampling condition. In this condition, participants sampled the two blank button options in a problem before making a final consequential choice for one of the options. During sampling, participants were free to click both button options one-by-one and observe the resulting outcome. Participants were asked to press the "choice-stage" key when they felt that they had sampled enough (but not before sampling at least once from each option). The outcome of each sample was determined by the structure of the relevant problem. One option corresponded to a choice where each sample provided a medium (M) outcome. The other option corresponded to a choice where each sample provided a High (H) payoff with some probability (pH) or a low (L) payoff with the complementary probability (1 - pH). At the choice stage, participants were asked to select once between the two options. Their choice yielded a random draw of one outcome from the selected option and this outcome was considered at the end of the experiment to determine the final payoff. Competing models submitted to TPT were evaluated

[1] The CPT model within this Ensemble model estimates the weighting function using approximations.

following the generalization criterion method (Busemeyer &Wang, 2000). As per the generalization criterion method, models were calibrated to aggregate human choices in 60 problems (the estimation set) and later tested in a new set of 60 problems (the competition set) with the set of parameters obtained in the estimation set. The M, H, pH, and L in a problem were generated randomly, and a selection algorithm was used so that the 60 problems in each set differed in its M, H, pH, and L from other problems. For more details about the TPT, please refer to Erev, Ert, Roth, et al. (2010).

In all the models described here, we have considered an individual human or model participant playing a problem in a dataset as an individual observation. Also, all model parameters have been calibrated by using the estimation dataset from TPT that consisted of 60 problems and 1,170 observations.[2] In the experiment involving the TPT's estimation dataset, forty participants were randomly assigned to two different sub-groups, where each sub-group contained 20 participants who were presented with a representative sample of 30 problems. Next, calibrated models were generalized on 60 problems from TPT's competition set (composed of 1,200 observations) and the Six-Problems (SP) dataset (Hertwig et al., 2004; composed of 150 observations). In the experiment involving the TPT's competition dataset, forty new participants were randomly assigned to two different subgroups, where each sub-group contained 20 participants who were presented with a representative sample of 30 problems. In the experiment involving the Six-Problems (SP) dataset, fifty participants were equally divided into two groups, where one group played the first three problems and the other group played the remaining three problems.

Working of Models

In this section, we detail the working of aggregate or hierarchical models with a set of point or distribution parameters values calibrated to individual choices. In every model, the final choice for each individual observation is estimated by using the following softmax function (Bishop, 2006; Daw, 2011; Sutton & Barto,1998):

$$Prob(OptionX) = \frac{e^{S_{Mean}X}}{e^{S_{Mean}X} + e^{S_{Mean}Y}} \qquad (1)$$

where, $S_{Mean}X$ and $S_{Mean}Y$ are the sample means or expectations of the two options X and Y for a model participant in a problem; and, *Prob(Option X)* is the probability of choosing *Option X* by a model participant. If *Option X* was chosen by a human participant in a problem, then the *Prob(Option X)* is used to calculate the log-likelihood from a model given its parameters. The log-likelihood function L is defined as:

$$L = \sum_{i=1}^{N} \ln\left(Prob(OptionX_i)\right) \qquad (2)$$

Where, i refers to the ith observation (a combination of a participant playing a problem) and N is the total number of observations in human data.[3] The refers to the natural log and the log-likelihood is negative as *Prob(Option X)* is a proportion. The log-likelihoods measure the goodness-of-fit for individual choices from a model and greater log-likelihoods values imply better fits from a model (Busemeyer & Diederich, 2010). As suggested by Busemeyer and Diederich (2010), in this paper, to calibrate aggregate or hierarchical model parameters, we minimize L . That is because our goal is to derive the likelihood of a model making the same choice as made by a human participant. We detail more about this calibration process in a future section. Next, we detail the working of models that we considered for evaluating individual choices.

Ensemble Model

The Ensemble model (Erev et al., 2010) assumes that each choice is made based on one of four equally likely rules and the predicted choice rate is a simple average across the predictions of four different rules. The first rule is similar to the Primed-Sampler model with variability (Erev, Glozman, & Hertwig, 2008). Decision-makers are assumed to sample each option m times, and select the option with the highest sample mean. The value of m is uniformly drawn from the set 1, 2, 3,. . . , 9. The second rule is identical to the first, but m is drawn from the distribution of sample sizes observed in the estimation set, with samples larger than 20 treated as 20. The third rule in the Ensemble model is a stochastic variant of CPT (Tversky & Kahneman, 1992), where the weighting function is approximated based upon certain parameters (the model does not use the sampling data to determine the weighting function). The final rule is a stochastic version of the lexicographic priority heuristic (Brandstätter et al., 2006; Rieskamp, 2008). The probabilities with which search orders for final rule were p_{order1} and p_{order2}. The first order begins by comparing minimum outcomes (i.e., minimum gain or minimum loss depending on the domain of gambles), then their associated probabilities, and finally the maximum outcomes. The second order begins with probabilities of the minimum outcomes, then proceeds to check minimum outcomes, and ends with the maximum outcomes (the probabilities with which both search orders are implemented were determined from the estimation set). The Ensemble model

[2]The data of one observation was missing in the original estimation dataset downloaded from the website.

[3]N = 1,170 observations in TPT's estimation set.

computes expectations for choosing options from its constituting models. These expectations are averaged to give a net expectation. Given a human participant's choice, the net expectation (averaged across all rules) is used to calculate the log-likelihood (using equation 2). In one version of the Ensemble model (called Ensemble (Herzog)), we used the original parameters proposed in Erev et al. (2010) to evaluate the model against individual choices. However, in a second version of the same model (called Ensemble (TPT)), we calibrated a set of Ensemble model's distribution parameters to individual choices using the log-likelihood function. The Ensemble (TPT) model had a set of 11 parameters. These parameters were assigned single values when they were recalibrated to individual choices. Among the 11 parameters, α, β,γ, δ, λ, μ belonged to the stochastic variant of CPT; while, parameters T_o, T_p, σ, p_{order1}, and p_{order2} were part of the priority heuristic. The σ was a free distribution parameter that defined the variance of a normal distribution. If the subjective difference involving the first comparison in each search order exceeds a threshold t, then the more attractive option is selected based on this comparison; otherwise, the next comparison is executed. The values of thresholds are other free distribution parameters. The estimated values are T_o for the minimum- and maximum- based comparisons, and T_p for the probability-based comparison (both T_o and T_p enabled define the mean of the normal distribution). The α, β, γ, δ, λ, μ, σ and T_o parameters were varied between 0 and 1.5, σ and T_o were varied between 0 and 1 while probabilistic parameters p_{oder1}, p_{order2} and T_p were varied between 0 and 1.0. These ranges ensured that the optimization could capture the optimal parameter values with high confidence. During model calibration, the initial parameter population was set to parameters from Erev et al. (2010).

Natural Mean Heuristic (NMH) Model

The NMH model (Hertwig & Pleskac, 2010) involves the following steps:

Step 1. Calculate the natural mean of observed outcomes for each option by summing, separately for each option, all n experienced outcomes and then dividing by n.

Step 2. Apply equation 1, where the sample mean for an option is replaced by its natural mean. In the NMH model, there are no free parameters. Like the Ensemble model, we evaluate the log-likelihood value from the NMH model (using equation 2).

Instance-Based Learning (IBL) Model

The IBL model (Dutt & Gonzalez, 2012; Gonzalez & Dutt, 2011; 2012; Lejarraga, Dutt, & Gonzalez, 2012) is based upon the ACT-R cognitive framework (Anderson & Lebiere, 1998). In this model, every occurrence of an outcome of an option is stored in the form of an instance in memory. An instance is made up of the following structure: SDU, here S is the current situation (many blank option buttons on a computer screen), D is the decision made in the current situation (choice for one of the option buttons), and U is the goodness (utility) of the made decision (the outcome obtained upon making a choice for an option). When a decision choice needs to be made, instances belonging to each option are retrieved from memory and blended together. The blended value of an option j (e.g., a gamble that pays \$5 with 0.9 probability or \$0 with probability 0.1) at any trial t is defined as:

$$V_{j,t} = \sum_{i=1}^{n} p_{i,j,t} x_{i,j,t} \qquad (3)$$

where $x_{i,j,t}$ is the value of the U (outcome) part of an instance (e.g., either \$5 or \$0, in the previous example) i on option j in trial t. The $p_{i,j,t}$ is the probability of retrieval of instance i on option j from memory in trial t. Because $x_{i,j,t}$ is value of the U part of an instance i on option j in trial t, the number of terms in the summation changes when new outcomes are observed within an option j (and new instances corresponding to observed outcomes are created in memory). Thus, n=1 if j is an option with one possible outcome. If j is an option with two possible outcomes, then n=1 when one of the outcomes has been observed on an option (i.e., one instance is created in memory) and n=2 when both outcomes have been observed (i.e., two instances are created in memory).

In any trial t, the probability of retrieval of an instance i on option j is a function of the activation of that instance relative to the activation of all instances (1, 2, … n) created within the option j, given by:

$$p_{i,j,t} = \frac{e^{(A_{i,j,t})/\tau}}{\sum_{i=1}^{n} e^{(A_{i,j,t})/\tau}} \qquad (4)$$

where τ, is random noise defined as $\sigma * \sqrt{2}$ and σ is a free noise parameter. Noise in Equation (4) captures the imprecision of recalling past experiences from memory. Activation of an instance is a function of the frequency and recency of observed outcomes that occur on choosing options during sampling. The activation of an instance i corresponding to an observed outcome on an option j in a given trial t is a function of the frequency of the outcome's past occurrences and the recency of the outcome's past occurrences (as done in ACT-R). In each trial t, activation $A_{i,j,t}$ of an instance i on option j is given by:

$$A_{i,j,t} = \sigma * \ln\left(\frac{1-\gamma_{i,j,t}}{\gamma_{i,j,t}}\right) + \ln \sum_{t_p \varepsilon \{1..,t-1\}} (t - t_p)^{-d} \qquad (5)$$

where d is a free decay parameter; $\gamma_{i,j,t}$ is a random draw from a uniform distribution bounded between 0 and 1 for instance i on option j in trial t; and T_p is each of the previous trials in which the outcome corresponding to instance i was observed in the binary-choice task. The IBL model has two free parameters that need to be calibrated: d and σ. The d parameter controls the reliance on recent or distant sampled information. Thus, when d is large (> 1.0), then the model gives more weight to recently observed outcomes in computing instance activations compared to when d is small (< 1.0). The σ parameter helps to account for the sample-to-sample variability in an instance's activation. Thus, blended value of each option is a function of activation of instances corresponding to outcomes observed on the option. In this model, we feed the sampling of individual human participants to generate instance activations and blended values. Every time a choice is made and outcome is observed, the instance associated with it is activated and thereafter blended values are computed for options faced by an individual participant. At final choice, the likelihood is computed from the blended values that replace the option means in Equation 1. In one version of the model, IBL (LDG), we used single value of parameters suggested by Lejarraga, Dutt, and Gonzalez (2012) to test the model against individual choices. However, in a second version of the model, IBL (TPT), we calibrated a set of d and σ parameters in the IBL model to individual choices. For this calibration, we determine the model's log-likelihood value for making the same choice as made by each human participant. During optimization, both d and σ parameters were varied between 0 and 20. These ranges ensured that the optimization could capture the optimal parameter values with high confidence. During parameter calibration, the initial parameter population was set to parameters from Lejarraga, Dutt, and Gonzalez (2012).

Cumulative Prospect Theory (CPT) Model

The CPT model (Hau et al., 2008; Tversky & Kahneman, 1992) assumes that people first form subjective beliefs of the probability of events, and then enter these beliefs into cumulative prospect theory's weighting function (Fox & Tversky, 1998; Tversky & Fox, 1995). Similarly, people associate a value (utility) corresponding to outcomes observed in options. The CPT consists of the following four steps:

Step 1. Assess the sample probability, p_j, of the nonzero outcome in given option j.

Step 2. Calculate the expected gain (loss) of option j, E_j

$$E_j = w(p_j)v(x_j) \qquad (6)$$

where w represents a weighting function for the probability experienced in the option j, and v represents a value function for the experienced outcome x_j in the option j. According to Tversky and Kahneman (1992), the weighting function w is defined as:

$$w(p_j) = \begin{cases} \dfrac{p_j^{\gamma}}{\left(p_j^{\gamma} + (1-p_j)^{\gamma}\right)^{1/\gamma}} & \text{,if } x \geq 0 \\ \dfrac{p_j^{\delta}}{\left(p_j^{\delta} + (1-p_j)^{\delta}\right)^{1/\delta}} & \text{,if } x < 0 \end{cases} \qquad (7)$$

The γ and δ are adjustable parameters that fit the shape of the function for gains and losses, respectively. The weighting function w has an S-shape that underweights small probabilities and overweighs larger ones (Hertwig, 2012). The x represents the outcome associated with the probability pj. The value function v is defined as:

$$v(x_j) = \begin{cases} x_j^{\alpha} & \text{,if } x_j \geq 0 \\ -\lambda\left(|x_j|^{\beta}\right) & \text{,if } x_j < 0 \end{cases} \qquad (8)$$

Here, α and β are adjustable parameters that fit the curvature for gain and loss domains, respectively. Finally, the λ parameter ($\lambda > 1$) scales loss aversion. The x_j represents the outcome associated with the option j.

Step 3. Assess the prospect value of the option by multiplying the weight with the value obtained.

Step 4. Given a human participant's choice, calculate the log-likelihood value of model making this choice using Equation 1 and Equation 2. The prospect value replaces the sample mean in Equation 1.

As seen above, the CPT model has 5 parameters, α, β, γ, δ, and λ; and, we investigated three versions of CPT model. In the first model, CPT (TK), we tested the set of parameter values estimated by Tversky and Kahneman (1992) against individual choices. In the second model, CPT (Hau), we tested the set of parameter values estimated by Hau et al. (2008) against individual choices. In the third model, CPT (TPT), we recalibrated a set of parameter values in the CPT model to individual choices. All five parameters were varied between 0 to 5. These ranges ensured that the optimization could capture the optimal parameter values with high confidence. During calibration, the initial parameter population was set to parameters from Hau et al. (2008).

Best Estimate and Simulation Techniques (BEAST) Model

The BEAST model captures the joint effect of and the interaction between 14-choice phenomena at aggregate level discussed in the 2015 Choice Prediction Competition (Erev, Ert, Plonsky, et. al., 2015). The first assumption in this model is to compute the expected values of options (since people try to maximize payoffs). The second assumption uses mental simulations that were found to lead to good outcomes in similar situations in the past (Marchiori, Di Guida, & Erev, 2015; Plonsky, Teodorescu, & Erev, 2015). Each simulation uses four different techniques, unbiased, uniform, contingent pessimism, and sign. The unbiased technique implies random and unbiased draws, either from an option's described distributions or from an option's observed history of outcomes. The other three techniques are "biased" and imply overgeneralizations. They can be described as mental draws from distributions that differ from the objective problem distributions. The three biased techniques are each used with equal probability. The simulation technique uniform yields each of the possible outcomes with equal probability. This technique enables the model to capture underweighting of rare events and the splitting effect.[4] The simulation-technique contingent pessimism is like the priority heuristic (Brandstätter et al., 2006); it depends on the sign of the best possible payoff and the ratio of the minimum payoffs. This technique helps the model capture loss aversion and the certainty effect. The simulation technique sign implies high sensitivity to the payoff sign. It is identical to the technique unbiased, with one important exception: positive drawn values are replaced by R, and negative outcomes are replaced by -R, where R is the payoff range (the difference between the best and worst possible payoffs in the current problem). This model has six distribution parameters, σ, κ, β, γ, ϕ, and θ, where each of these parameters defines the upper bound of uniform distributions with a 0.0 lower bound (κ defined the upper bound of a discrete uniform distribution with a 0.0 lower bound). Four of these parameters (σ, κ, β, and γ) are needed to capture decisions under risk without feedback. The parameter ϕ captures attitude toward ambiguity, and θ abstracts the reaction to feedback. In this model, the expectation for one of the options, option A, equals $BEV_{\mathrm{A}}(r) + ST_{\mathrm{A}}(r) + e(r)$ and that for the other option, option B, equals $BEV_{\mathrm{B}}(r)$ + $ST_{\mathrm{B}}(r)$. Here, $BEV_{\mathrm{A}}(r)$ and $BEV_{\mathrm{B}}(r)$ are the best estimates of the expected values of both options A and B after r samples; $ST_{\mathrm{A}}(r)$ and $ST_{\mathrm{B}}(r)$ are the expectations based on mental simulations techniques after r samples, and $e(r)$ is an error term after r samples ($e(r)$ is drawn from a normal distribution with a mean 0 and standard deviation σ). Given a human participant's choice, the expectations on different options are used to determine the log-likelihood in the model (using Equation 1 and Equation 2). In one of the BEAST versions, BEAST (CPC), we used the set of parameter values reported by Erev, Ert, Plonsky, et. al. (2015) against individual choices. However, in another version, BEAST (TPT), we recalibrated the set of distribution parameter values to individual choices. All six parameters were varied between 0 to 20. These ranges ensured that the optimization could capture the optimal parameter values with high confidence. During recalibration, the initial population of parameters was taken from Erev, Ert, Plonsky, et. al. (2015).

Method

Dependent variables

In this paper, we account for final choices made by individual participants in different problems. For this purpose, given a choice made by a human participant in a problem, we calculate the log-likelihood of a model participant making the same choice in the same problem. In all models, if the probability of making a human participant's choice is greater than 0.5, then it is assumed that the model choice coincides with the human choice. Using this 0.5 rule, we compare whether both model and human participants select the maximizing option in a problem. The maximizing option is the one that has the highest expected value among both options (expected value is calculated by using the objective probability distribution of outcomes in options). If both human participants and model participants select the maximizing option or the non-maximizing option in a problem, then the model can explain the human participant's choice. Using this method, in the TPT's estimation set, the final choices made by model observations are compared to 1,170 human observations, i.e., the total number of human observations available. The comparison between human choices and model choices is used to compute the incorrect proportion for each model, which is the main criteria for capturing individual behavior by a model. The incorrect proportion is simply a proportion of human choices that were different from model predictions. It is defined as:

$$\text{Incorrect Proportion} = (M_H N_M + N_H M_M)/ (M_H N_M + N_H M_M + N_H N_M + M_H M_M) \quad (9)$$

where, $M_H N_M$ is the number of observations where the human participant makes a maximizing choice but the model predicts a non-maximizing choice. $N_H M_M$ is the number of observations where the human participant makes a non-maximizing choice but the model predicts a maximizing choice. Similarly, the $M_H M_M$ and $N_H N_M$ are the number of observations, where the human participant observation makes the same choice (maximizing or non-maximizing) as predicted by the model. The smaller the value of the incorrect proportion, the more accurate is the model in accounting for individual human choices. Once model parameters were calibrated to individual choices using the log-likelihood function, the incorrect proportions were computed from different models and compared.

[4] According to Birnbaum (2008) and Tversky and Kahneman (1986) splitting an attractive outcome into two distinct outcomes can increase the attractiveness of a prospect even when it reduces its expected value. This phenomenon is referred to as the splitting effect.

Parameter calibration

Given the choice made by a human participant, we use Equation 1 and Equation 2 to compute the log-likelihood from a model of making the same choice as made by a human participant. Classically, Equation 1 has used an inverse temperature parameter β which scales the sample means (Busemeyer & Diederich, 2010). In this paper, we assume $\beta = 1$ across all models as we did not want to introduce an additional free parameter beyond those already present in models. That is because, the β parameter's recalibration to individual choices could benefit models differently. As $\beta = 1$ across all models, the β parameter does not favor some models over others.

The NMH model did not require parameter calibration as this model did not possess any parameters. The set of parameters of Ensemble, BEAST, CPT, and IBL models were recalibrated using a Genetic Algorithm (GA) program. The GA is a probabilistic (stochastic) trial-and-error method of optimization that is different from other deterministic methods like steepest gradient descent. Due to GA's trial-and-error nature and its dependence on processes like reproduction, crossover, and mutation, the algorithm provides good chances of avoiding local optima in the parameter search space (Jakobsen, 2010; Gonzalez & Dutt, 2011; Houck, Joines & Kay, 1995). In addition, prior research involving models have used the GA procedure for model calibration (Gonzalez & Dutt, 2011; 2012; Lejarraga, Dutt, & Gonzalez, 2012). In our model calibrations, the GA repeatedly modified a population of parameter tuples to find the tuple that minimized the negative of model's log-likelihood function (Equation 2) across all human participants. In each generation, the GA selected parameter tuples randomly from a population to become parents and used these parents to select children for the next generation. For each parameter tuple in a generation, each model was run five times across 1,170 participants to minimize the negative of model's average log-likelihood function over five runs.[5] Over successive generations, the population evolved toward an optimal solution. The population-size was set to 20 randomly-selected parameter tuples per generation (each tuple contained a certain value for each of the model's parameters). The mutation and crossover fractions were both set at 0.5 after a grid search for the best combination. The best combination for mutation and crossover fractions was found by calibrating the IBL (LDG) model to aggregate choices using its known parameters (d = 5.0; σ = 1.5). We systematically varied the mutation and crossover fractions in steps of 0.1 in the interval [0, 1] for finding their best combination. The optimal values of mutation and crossover fraction (= 0.5) were those where the optimization converged the IBL (LDG) parameters to their optimal values in the least number of generations. These optimal values of mutation and crossover fraction found were used for calibration of model parameters to individual choices. The GA procedure was implemented in Matlab® toolbox (Houck, Joines & Kay, 1995; Mathworks, 2012), where the stopping criteria in optimization of model parameters involved the following constraints: stall generations = 200, function tolerance = $1\text{x}10^{-8}$, and when the average relative change in the fitness function value over 200 stall generations was less than function tolerance ($1\text{x}10^{-8}$).

Results

Calibration in TPT's estimation set

Table 1 shows parameter calibration results from different models in TPT's estimation dataset. The table lists different models, calibrated parameter values, combinations obtained due to comparison of human and model final choices, log-likelihoods, and incorrect proportions.

Calibrated parameters

The best model in terms of log-likelihood values was CPT (TPT). Five parameters were calibrated in the CPT (TPT) model and the calibrated model possessed the log-likelihood of -634.7, which was significantly larger than that for the CPT (TK) model (-662.8) and CPT (Hau) model (-643.9). The calibrated parameter values were: α = 1.008; β =0.96; γ =2.00; δ =0.92; λ =1.03. The free parameters for the value function indicated slightly less magnitude of disutility for losses compared to the utility for gains. The value function for the CPT (TPT) model was aligned with risk-neutral behavior for both gains and losses, which was different from the behavior in the CPT (Hau) model and in the CPT (TK) model. Furthermore, the weighting function of the CPT (TPT) model showed underweighting of small probabilities for positive outcomes and about equal weighting of small probabilities for negative outcomes. Furthermore, the weighting function for the CPT (Hau) and CPT (TK) models overweight small probabilities for both positive and negative outcomes. Please see Appendix D for shapes of value and weighting functions for different CPT models.

The Ensemble (TPT) model was second best model where the model exhibited a log-likelihood of -691.0. The model's calibrated parameters were α=0.75, β = 1.46, γ = 1.42, δ = 1.03, λ = 1.13, μ= 0.37, T_0=0.001, p_{order1}= 0.38, σ = 0.020, T_p= 0.18 and p_{order2} = 0.62. The first six parameters from the model depicted underweighting of rare events and loss aversion with losses perceived as more damaging compared to gains. The later five parameters from priority heuristic showed smaller variance in distribution of σ parameter compared to the last round. Also, results indicated underweighting of small probabilities, overweighting of

[5]The number of runs were set to five after analyzing the run-to-run variability in models with stochasticity (e.g., IBL and BEAST). Five runs were chosen as there was little change in the standard deviation by increasing the number of runs beyond five.

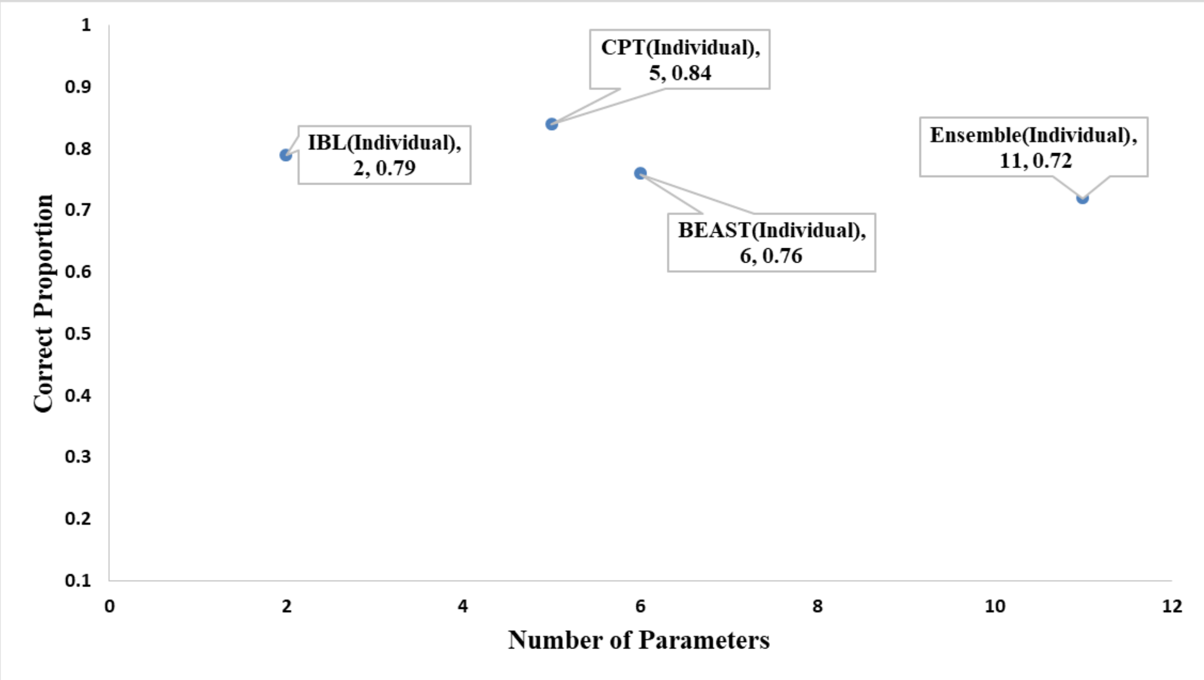

Figure 1. The correct proportions against number of parameters from different models calibrated in the TPT's estimation set.

large probabilities, and diminishing sensitivity to gains and losses.

The next best model was the IBL (TPT) model, where this model exhibited a considerably larger log-likelihood value of -929.0 compared to the IBL (LDG) model. The IBL (TPT) model's calibrated parameters were: $d = 5.39$ and $\sigma = 0.04$. These parameters indicated reliance on recency of sampled information, which provides a plausible account of recency's role in human participant's sampling and subsequent choice. The recency reliance for individual choices is also in agreement with documented reliance on recency in aggregate choices (Dutt & Gonzalez, 2012; Gonzalez & Dutt, 2011; 2012; Hertwig et al., 2004; Lejarraga, Dutt, & Gonzalez, 2011). In fact, the d parameter value was higher for the model calibrated to individual choices compared to the model calibrated to the aggregate choices. Furthermore, the participant-to-participant variability (captured by the σ) was smaller in the IBL (TPT) compared to the IBL (LDG) model. This observation showed less variability among individual participants in their choices.

For the BEAST (TPT) and NMH model, the log-likelihood values (-1129.0, -1386.5) were much smaller compared to those for the individual versions of CPT, Ensemble, and IBL models. Please see Table 1 for log-likelihood values of different models.

Incorrect proportion

In the calibration datasets, the CPT (TPT) model possessed the best incorrect proportion of 0.15. In the CPT (TPT) model, the desirable $N_H N_M$ and $M_H M_M$ combinations were 39% and 45%, respectively. In contrast, the erroneous $N_H M_M$ and $M_H N_M$ combinations were 10% and 6%, respectively. The CPT (Hau) model showed an error proportion of 0.16. The model showed 39% of $N_H N_M$ combinations and 45% of $M_H M_M$ combinations. The erroneous combinations included, 9% for $N_H M_M$ and 7% for $M_H N_M$. The incorrect proportion for the CPT (TK) model was 0.18. The proportions of desirable $N_H N_M$ and $M_H M_M$ combinations were 41% and 42%, respectively. In addition, the erroneous $N_H M_M$ and $M_H N_M$ combinations were 8% and 10%, respectively.

The next best model was the IBL (TPT) model, where the model exhibited incorrect proportion of 0.21. The IBL (TPT) model showed 39% and 40% of desirable $N_H N_M$ and $M_H M_M$ combinations. The erroneous $N_H M_M$ and $M_H N_M$ combinations were 9% and 12%, respectively.

Beyond the IBL (TPT) model, the BEAST (TPT) model did well with an incorrect proportion of 0.24. The four combination proportions for BEAST (TPT) model were: 36% ($N_H N_M$), 41% ($M_H M_M$), 13% ($N_H M_M$) and 10% ($M_H N_M$).

Next, to gauge the benefit of explaining individual choices with different model parameters, we plotted correct proportions from calibrated models against their number of free parameters (see Figure 1). Models closer to the origin are the ones that explain individual choices with least number of free parameters. The magnitude distance of IBL (TPT) and CPT (TPT) models from origin (= 2 and 5 units, respectively) was much less than that for the BEAST (TPT) and Ensemble (TPT) models (= 6 and 11 units, respectively). Thus, based upon the distance metric, IBL and CPT models explained individual choices with fewer number of free parameters. Thus, it seems that cognitive mechanisms like recency, frequency, and blending as well as mathematical functions that underweight rare outcomes and value gains and losses differently are appropriate to account for individual choices.

Table 1. Calibration results from models in TPT's estimation dataset.

	Percentage of 1170 Observations									
Combinations from Human and Model Data H/M	**Ensemble (Herzog)**	**Ensemble (TPT)**	**NMH**	**IBL (LDG)**	**IBL (TPT)**	**CPT (TK)**	**CPT (Hau)**	**CPT (TPT)**	**BEAST (CPC)**	**BEAST (TPT)**
	α=1.19,	α=0.75,								
	β=1.35,	β=1.46,								
	γ=1.42,	γ=1.42,							σ=7.00,	σ=0.24,
	δ=1.54,	δ=1.03,				α=0.88,	α=0.94,	α=1.008,	κ=3.00,	κ=1.99,
	λ=1.19,	λ=1.13,				β=0.88,	β=0.86,	β=0.96,	β=2.6,	β=0.06,
	μ=0.41,	μ=0.37,		*d*=5.00,	*d*=5.39,	γ=0.61,	γ=0.99,	γ=2.00,	γ=0.50,	γ=1.16,
	T_0=0.0001,	T_0=0.001,	-	σ=1.50	σ=0.04	δ=0.69,	δ=0.93,	δ=0.92,	φ=0.07,	φ=0.03,
	p_{order1}=0.38,	p_{order1}=0.38,				λ=1.00	λ=1.00	λ=1.03	θ=1.00,	θ=1.17,
	σ=0.037,	σ=0.02,								
	T_p=0.11,	T_p=0.18,								
	p_{order2}=0.62	p_{order2}=0.62								
$N_H N_M^1$	31	32	29	26	39	41	39	39	33	36
$M_H M_M$	40	40	33	32	40	42	45	45	37	41
$N_H M_M$	17	18	19	23	09	08	09	10	15	13
$M_H N_M$	12	11	19	20	12	10	07	06	15	10
Incorrect proportion	0.29	0.28	0.37	0.43	0.21	0.18	0.16	0.15	0.31	0.24
Log Likelihood	-696.2	-691.0	-1386.5	-3158.0	-929.0	-662.8	-643.9	-643.7	-1971.0	-1129.0

Note. [1] N_H and M_H refers to non-maximizing and maximizing human choices, respectively. N_M and M_M refers to non-maximizing and maximizing model choices, respectively.

Generalization to different datasets

Up to now, different models predicted choices of individual participants in TPT's estimation set using a single set of parameter values. These models, however, possess different number of free parameters. Due to differences in model parameters it becomes difficult to compare model performance during parameter calibration. One method that allows us to compare models and account for parameter differences is generalization (Busemeyer & Diederich, 2010; Busemeyer & Wang, 2000; Dutt & Gonzalez, 2012). In generalization, models with calibrated parameters are run in new problems (Busemeyer & Wang, 2000). Ideally, new problems encountered during generalization should be different from those encountered during calibration; otherwise, generalization may favor models that show superior performance during calibration. In what follows, we first generalize calibrated models to problems in TPT's competition set. Generalization of this kind was also followed for models submitted to TPT (Erev et al., 2010). However, problems in TPT's competition set were derived using the same algorithm as in TPT's estimation set (Erev et al., 2010). Thus, it is likely that the nature of problems across competition and estimation sets were similar and that TPT's competition set provided a weaker generalization dataset with respect to TPT's estimation set. To overcome this limitation, we also generalized calibrated models to the Six-Choice (SC) problems dataset (Hertwig et al., 2004), where the SC problems were different in structure and nature compared to the TPT problems. We will first report on the generalization results for the TPT's competition set, then those for the SC dataset.

Generalization to competition set. TPT's competition set was like the estimation set with two exceptions: problems in competition set were different from those in the estimation set and different subjects participated in the competition set compared to the estimation set (Erev et al., 2010). The 60 problems in the competition set were selected using the same algorithm as used for the estimation set. To explain individual choices, all models were run in the competition set using the parameters obtained in the estimation set.

Table 2 shows generalization results from different models in the competition set. In all models, parameters were set to values reported in Table 1. Overall, the incorrect proportions obtained from models in the competition set were like those obtained in the estimation set. Calibrated models performed better compared to their uncalibrated counterparts that borrowed parameter values for aggregate choices from literature. The incorrect proportion was the lowest for the CPT (TPT) model, where IBL (TPT) and Ensemble (TPT) models took the second and third places, respectively. Also, all three models performed significantly better than the BEAST and NMH models. These results highlight the role of the certain mechanisms in explaining individual choices: recency, frequency, and blending of encountered information during sampling, the underweighting of rare events, and the differential valuation of gains and losses.

Sequential analysis. To gauge models in accounting for individual differences, we evaluated the proportion of sequential decisions in models from last sample to final choice. Here, human and model choices were analyzed sequentially. Thus, we evaluated decisions made by human participants during their last sample and consequential choice and then compared these sequential decisions to those from models. Table 3 presents the proportion of model participants and likelihoods showing a transition that was similar to or different from human participants in TPT's competition dataset. Based upon the last sample and consequential choice among human participants, the following four transition possibilities existed: $N \to N$, $N \to M$, $M \to N$, and $M \to M$, where the first letter (before the arrow) corresponds to the choice made by a human participant during her last sample and the second letter (after the arrow) corresponds to the final choice made by the same participant after sampling. For each last sample and final choice transition by a human participant, there are two transition possibilities for the model: first, like the human participant; and, second, different from the human participant. If the model is suggestive of individual choice, then the model should show a transition between last sample and final choice like human participants for more than 50 percent (i.e., a majority) of its participants. We evaluated sequential decisions in the top four models: CPT (TPT), IBL (TPT), Ensemble (TPT) and NMH model. As shown in Table 3, across all transitions, $N \to N$, $N \to M$, $M \to N$, and $M \to M$, the CPT (TPT) model performed better compared to all other models. Thus, the CPT (TPT) model made stronger correct predictions for human transitions from last sample to final choice compared to the Ensemble (TPT), NMH, and IBL models. The IBL (TPT) model performed superior to the Ensemble (TPT) model on two kinds of transitions: NN and MN. Overall, these results show that underweighting of experienced probabilities, loss-aversion due to negative outcomes, and recency and frequency processes seem to account for sequential individual choices in data.

Six Choice (SC) dataset. In the section above, we generalized models to TPT's competition set. However, the problems in the competition set were similar to those in the estimation set as the problem-generation algorithm remained the same between the two sets. Due to this observation, the competition set provides a weaker generalization dataset. In order to overcome this limitation, we also generalized calibrated models to the Six Choice (SC) dataset (Hertwig et al., 2004; Appendix C), where the structure of options across problems in SC dataset was different from that in TPT's estimation and competition sets. In the SC dataset, all six problems presented options that differed with respect to expected value;

Table 2. Generalization results from models in TPT's competition dataset.

	Percentage of 1200 Observations									
Combinations from Human Data and Model H/M	**Ensemble (Herzog)**	**Ensemble (TPT)**	**NMH**	**IBL (LDG)**	**IBL (TPT)**	**CPT (TK)**	**CPT (Hau)**	**CPT (TPT)**	**BEAST (CPC)**	**BEAST (TPT)**
$N_H N_M$	20	20	25	22	29	32	33	33	21	24
$M_H M_M$	46	46	39	40	43	53	49	50	36	39
$N_H M_M$	21	20	15	19	12	09	08	09	19	17
$M_H N_M$	14	13	21	20	17	09	10	07	24	20
Incorrect proportion	0.34	0.33	0.36	0.39	0.28	0.17	0.18	0.16	0.42	0.37

Note. [1] N_H and M_H refers to non-maximizing and maximizing human choices, respectively. N_M and M_M refers to non-maximizing and maximizing model choices, respectively.

Table 3. Proportion of model participants following a transition that is similar to or different from human participants in the Competition dataset.

Human Transition (Last Sample → Final Choice)	Model Transition (Last Sample → Final Choice)	CPT (TPT) (%)	IBL (TPT) (%)	Ensemble (TPT) (%)	NMH (%)
N→N	N→N	79	73	54	62
	N→M	21	27	46	38
N→M	N→M	80	70	77	64
	N→N	20	30	23	36
M→N	M→N	77	67	51	62
	M→M	23	34	49	38
M→M	M→M	87	74	78	66
	M→N	13	26	22	34

Note. N and M refers to non-maximizing and maximizing choices, respectively.

Table 4. Generalization results from models in the SC problems dataset.

	Percentage of 150 Observations									
Combinations from Human Data and Model H/M	**Ensemble (Herzog)**	**Ensemble (TPT)**	**NMH**	**IBL (LDG)**	**IBL (TPT)**	**CPT (TK)**	**CPT (Hau)**	**CPT (TPT)**	**BEAST (CPC)**	**BEAST (TPT)**
$N_H N_M$	45	46	55	41	51	37	37	39	33	34
$M_H M_M$	20	19	26	23	32	25	31	27	31	31
$N_H M_M$	14	13	03	18	07	22	21	20	25	25
$M_H N_M$	21	22	15	19	09	17	11	10	10	11
Incorrect proportion	0.35	0.35	0.19	0.37	0.16	0.39	0.34	0.33	0.36	0.35

Note. [1] N_H and M_H refers to non-maximizing and maximizing human choices, respectively. N_M and M_M refers to non-maximizing and maximizing model choices, respectively.

Table 5. Proportion of model participants following a transition that is similar to or different from human participants in the SC problems dataset.

Human Transition (Last Sample → Final Choice)	Model Transition (Last Sample → Final Choice)	NMH (%)	IBL (TPT) (%)	Ensemble (TPT) (%)	CPT TPT (%)
N→N	N→N	96	81	79	66
	N→M	4	19	21	34
N→M	N→M	54	75	36	64
	N→N	46	25	64	36
M→N	M→N	91	89	71	66
	M→M	9	11	29	34
M→M	M→M	71	74	59	68
	M→N	29	26	41	32

Note. N and M refers to non-maximizing and maximizing choices, respectively.

four of them offered positive prospects and two offered negative prospects. All problems in the SC dataset were run in the sampling paradigm format: free sampling of options followed by a final choice for one of the options for real. During sampling, participants could sample options in whatever order they desired, and however often they wished. They were encouraged to sample until they felt confident enough to decide from which option to draw a real payoff. Like the TPT dataset, each problem consisted of choosing between two options. However, unlike the TPT dataset, problems in the SC dataset could have both options risky: Both options could independently contain high and low outcomes with predefined probability distributions. Problems in SC dataset belonged to both positive and negative domains. In positive domain, the associated non-zero outcomes were positive; whereas, in the negative domain, the associated non-zero outcomes were negative. Overall, the TPT dataset and SC dataset differed on the number of outcomes possible on options and the presence of the mixed domain in TPT and its absence in the SC problems.

Table 4 shows the generalization results from running different models in the SC dataset (model parameters were calibrated in the estimation set). As shown in Table 4, the IBL (TPT) model was the best performing model with an incorrect proportion of 0.16. The NMH model was the second-best model with an incorrect proportion of 0.19. The CPT (TPT) model was the third-best model with an incorrect proportion of 0.33. Other hierarchical models like Ensemble and BEAST did not perform as well in the SC dataset and possessed higher incorrect proportions. Furthermore, models with recalibrated parameters performed better compared to models with parameters for aggregate choices borrowed from literature. These results show that when a more challenging generalization is performed, models like IBL and CPT that are based upon activations and recency and frequency mechanisms as well as assumptions of underweighting of rare outcomes and different valuation of gains and losses perform better compared to other model that rely on heuristics rules and biased sampling techniques.

Sequential Analyses. To evaluate models at explaining individual differences, we analyzed the top four models in the SC dataset. Table 5 shows the transition from the last sample to final choice for human and model participants in the SC problems dataset. As seen in Table 5, both IBL and NMH models were suggestive of human-like transitions for all four combinations based upon the 50% rule. The IBL (TPT) model performed better compared to the NMH model in NM and MM transitions and poorer compared to the NMH in NN and MN transitions. Overall, these results show the role of recency and frequency processes during sampling in individual choices.

Discussion

Till recently, researchers had evaluated how aggregate or hierarchical models with a set of parameter values explained aggregate choices made from experience (Dutt & Gonzalez, 2012; Gonzalez & Dutt, 2011; 2012; Lee, 2008; Lejarraga, Dutt, & Gonzalez, 2012; Rouder & Lu, 2005). Also, researcher had evaluated how models with a single set of parameter values calibrated to each participant explained individual choices (individual models; Kudryavtsev & Pavlodsky, 2012; Frey, Mata, & Hertwig, 2015). However, little was known on how aggregate or hierarchical models with a set of single or distribution parameter values would perform when they are made to account for individual choices. In this paper, we contributed to this investigation by calibrating aggregate or hierarchical models with a set of single or distribution parameter values to individual choices across three different datasets. Aggregate and hierarchicalmodels were calibrated in the Technion Prediction Tournament (TPT)'s estimation set using the log-likelihood function and later generalized to TPT's competition dataset (Erev, Ert, Roth, et al., 2010) and the Six-Choice (SC) problems dataset (Hertwig et al., 2004). We followed the traditional approach of model comparison via generalization as proposed by Busemeyer and Wang (2000).

Overall, our results revealed that both aggregate and hierarchical models performed above chance (= 50%) when their parameters were calibrated to individual choices. Even parameter values calibrated to aggregate choices (borrowed from literature) performed above chance in these models. The CPT model performed well overall in the calibration and generalization datasets from TPT. Models such as Ensemble and CPT possess rules like weighting and value functions that abstract the sampling process experienced by human participants. From our results, these constructs help such models in cases where the generalization environment is similar to the calibration environment (as in TPT); however, not when the generalization environment is different from the calibration environment (as in SC dataset).

However, upon performing a generalization to the SC problems dataset, the IBL model, relying on recency, frequency, and blending mechanisms, showed superior performance compared to other models employing mathematical functions (Ensemble or CPT) or biased sampling techniques (BEAST). Also, the NMH model, which incorporates frequency and magnitude of experienced outcomes, also performed well to account for individual decisions. One likely reason for this observation is the presence of cognitive constructs like expectations, instances, activations, and blended values in the IBL model and the averaging mechanism in the NMH model. These mechanisms help these models account for individual experiences gained during sampling of options. For example, the IBL model is motivated from the ACT-R theory of cognition (Anderson & Libere, 1998). The IBL model's reliance on recency and frequency of experiences during sampling (exhibited through activations and blended values) helps this model to make human-like choices. Similarly, the natural means in the NMH model are computed based upon experienced outcomes during the sampling process. These natural means represent expectations of choosing different options and enables this model to account for individual choices.

Next, we found that the IBL model performed consistently well in both calibration and generalization datasets standing among the top two models even though it possessed only two parameters. One likely reason for this observation could be that the IBL model uses the blending mechanism, where for every option, the values of all the observed outcomes are weighted by their activation strengths. Blending of experiences considers both the activation of outcomes in memory as well as their magnitude. Perhaps, the IBL model's blending mechanism makes the model blend outcomes correctly for both maximizing and non-maximizing choices. Other factors affecting performance of IBL model are its two parameters d and σ. The calibrated value of d parameter was higher for individual choices compared to its calibrated value for aggregate choices (the latter being done by Lejarraga, Dutt, and Gonzalez, 2012). The increased d value shows that individual choices rely excessively on recency of outcomes. Furthermore, the σ parameter helped the IBL model account for sample-to-sample variability in instance activations. Here, when the model parameters were calibrated to individual choices, the σ parameter's value was much smaller and closer to its ACT-R default compared to when the same model was calibrated by Lejarraga, Dutt, and Gonzalez (2012) to aggregate choices. The smaller value of σ parameter closer to its ACT-R default showcases lesser variation in outcome activations among individual choices.

This research work builds upon literature in judgment and decision making in several ways. First, the BEAST and Ensemble models were hierarchical, where these models possessed distribution parameters to account for individual choices. The parameters in these models assumed different values from distribution for different participants in the dataset. Thus, these distribution parameters should have helped these models to account for individual choices due to parameter heterogeneity. However, in our results, the BEAST and Ensemble models did not account for individual choices as well as those models (like IBL or CPT) that possessed single parameters. This finding likely shows that it is more important for a model to possess the right cognitive or mathematical mechanisms compared to possessing heterogeneity among its parameters for different participants.

Second, we performed generalizations to large datasets that were similar or dissimilar to the calibration dataset. An insight from this generalization exercise it that the true picture emerges when the generalization dataset is different in its structure from the calibration dataset. The SC dataset possessed problems where the problem structure was different from that of the TPT datasets (both options could be risky in SC dataset). Thus, it is recommended that generalizations should be performed to datasets that possess structural differences from the calibration datasets.

Third, we used individual-level techniques like likelihoods and incorrect proportions, where these techniques enabled us to evaluate aggregate and hierarchical models at the individual participant level. In summary, the likelihood approach is powerful and it enables us to calibrate models at the individual level. However, beyond calibration, one needs to test models based upon dependent measures that account for model error at the individual level. This need is especially true for generalizations, where the calibration measures like likelihood cannot be used as parameters have already been fixed to their calibrated values.

In this paper, our focus was on investigating how aggregate and hierarchical models with a set of single or distribution parameters performed when their parameters were calibrated to individual choices rather than aggregate choices. As part of our future research, we plan to also perform individual modeling: calibrate a set of model parameters to each individual decision such that we get a set of parameters for each participant in the dataset. This evaluation will enable us to test the tradeoffs between aggregate modeling, hierarchical modeling, and individual modeling when these models are evaluated for explaining individual decisions (as in this paper). Individual modeling may help us to account for individual differences well; however, these models also run the risk of overfitting individual decisions due to too many parameter values (one for each individual participant). To provide a robust comparison of this tradeoff, as part of our future research, we plan to generalize individual models across both similar and dissimilar datasets within the same paradigm or across datasets in different paradigms.

Furthermore, as part of our future research, we plan to extend our investigation to decision tasks where decision-makers make decisions across multiple options rather than make a binary choice. An example of this task is the Iowa-Gambling Task (Bechara, Damasio, Damasio, & Anderson, 1994), where the problem consists of making a choice between four options. In this paper, we took problem environments that were static in terms of outcomes and probabilities. Thus, outcomes and probabilities in a problem did not change during sampling. In future, it would be worthwhile to extend the evaluation of models in explain individual choices in dynamic environments, where outcomes and probabilities change during information search. Some of these ideas form the immediate next steps that we would like to undertake as part of our research.

Conclusion

This paper helped to bridge the gap in literature on how aggregate and hierarchical models with a set of parameter values (either single or distribution) would perform when they are made to account for individual choices. We contributed to this investigation by calibrating different models with a set of parameter values to individual choices across three different datasets. Models with constructs that abstract the sampling process performed well when generalized to problems that were similar to the calibration problems. However, generalization to other problems that were structurally different from the calibration problems revealed that model mechanisms like differential valuing of gains and losses, recency, frequency, blending and, underweighting of rare outcomes were important to account for individual choices. Also, models using distribution parameters with heuristic rules and biased techniques did not perform well in accounting for individual choices when these models were generalized to different problems.

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References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, *19*(6), 716–723. doi:10.1109/tac.1974.1100705

Anderson, J. R., & Lebiere, C. (1998). *The atomic components of thought*. Hillsdale, NJ: Erlbaum.

Barron, G., & Erev, I. (2003). Small feedback-based decisions and their limited correspondence to description-based decisions. *Journal of Behavioral Decision Making*, *16*(3), 215–233. doi:10.1002/bdm.443

Bechara, A., Damasio, A. R., Damasio, H., & Anderson, S. W. (1994). Insensitivity to future consequences following damage to human prefrontal cortex. *Cognition*, *50*(1–3), 7–15. doi:10.1016/0010-0277(94)90018-3

Birnbaum, M. H. (2008). New paradoxes of risky decision making. *Psychological review*, *115*(2), 463. doi:10.1037/0033-295X.115.2.463

Bishop, C. M. (2006). *Pattern recognition and machine learning*. New York, NY: Springer.

Busemeyer, J. R., & Diederich, A. (2010). *Cognitive modeling*. Thousand Oaks, CA: Sage.

Busemeyer J. R., Myung, I. J. (1992). An adaptive approach to human decision making: Learning theory, decision theory, and human performance. *Journal of Experimental Psychology: General*, *121*(2), 177–194. doi:10.1037/0096-3445.121.2.177

Busemeyer, J. R., & Stout, J. C. (2002). A contribution of cognitive decision models to clinical assessment: decomposing performance on the Bechara gambling task. *Psychological Assessment*, *14*(3), 253–262.doi:10.1037//1040-3590.14.3.253

Busemeyer, J. R., & Wang, Y. (2000). Model comparisons and model selections based on the generalization criterion methodology. *Journal of Mathematical Psychology*, *44*(1), 171–189. doi:10.1006/jmps.1999.1282

Bush, R. R., & Mosteller, F. (1955). *Stochastic models for learning*. Oxford, England: Wiley & Sons.

Brandstätter, E., Gigerenzer, G., & Hertwig, R. (2006). The priority heuristic: making choices without trade-offs. *Psychological Review*, *113*(2), 409–432. doi:10.1037/0033-295X.113.2.409

Daw, N. D., Gershman, S. J., Seymour, B., Dayan, P., & Dolan, R. J. (2011). Model-based influences on humans' choices and striatal prediction errors. *Neuron*, *69*(6), 1204–1215. doi:10.1016/j.neuron.2011.02.027

Denrell, J. (2007). Adaptive learning and risk taking. *Psychological Review*, *114*(1), 177–187. doi:10.1037/0033-295X.114.1.177

Dutt, V., & Gonzalez, C. (2012). The role of inertia in modeling decisions from experience with instance-based learning. *Frontiers in Psychology*, *3*(177). doi:10.3389/fpsyg.2012.001777

Dutt, V. & Gonzalez, C. (2015). Accounting for outcome and process measures and the effects of model calibration. *Journal of Dynamic Decision Making*, *1*(2),1–10. doi:10.11588/jddm.2015.1.17663

Erev, I., & Barron, G. (2005). On adaptation, maximization, and reinforcement learning among cognitive strategies. *Psychological Review*, *112*(4), 912–31. doi:10.1037/0033-295X.112.4.912

Erev, I., Ert, E., Plonsky, O., Cohen, D., & Cohen, O. (2015). *From anomalies to forecasts: A choice prediction competition for decisions under risk and ambiguity*. Mimeo, 1–56.

Erev, I., Ert, E., Roth, A. E., Haruvy, E., Herzog, S. M., & Hau, R. (2010). A choice prediction competition: Choices from experience and from description. *Journal of Behavioral Decision Making*, *23*(1), 15–47. doi:10.1002/bdm.683

Erev, I., Glozman, I., & Hertwig, R. (2008). What impacts the impact of rare events. *Journal of Risk and Uncertainty*, *36*(2), 153–177. doi:10.1007/s11166-008-9035-z

Estes, W. K., & Todd Maddox, W. (2005). Risks of drawing inferences about cognitive processes from model fits to individual versus average performance. *Psychonomic Bulletin & Review*, *12*(3), 403–408.doi:10.3758/bf03193784

Fox, C. R., & Tversky, A. (1998). A belief-based account of decision under uncertainty. *Management Science*, *44*(7), 879–895.doi:10.1287/mnsc.44.7.879

Frey, R., Mata, R., & Hertwig, R. (2015). The role of cognitive abilities in decisions from experience: Age differences emerge as a function of choice set size. *Cognition*, *142*, 60–80. doi:10.1016/j.cognition.2015.05.004

Gallistel, C. R., Fairhurst, S., & Balsam, P. (2004). The learning curve: implications of a quantitative analysis. *Proceedings of the National Academy of Sciences of the United States of America*, *101*(36), 13124–13131. doi:10.1073/pnas.0404965101

Gigerenzer, G., & Goldstein, D. G. (1996). Reasoning the fast and frugal way: models of bounded rationality. *Psychological Review*, *103*(4), 650–669.doi:10.1037//0033-295x.103.4.650

Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, *18*(2), 141–153. doi:10.1016/0304-4068(89)90018-9

Gonzalez, C., & Dutt, V. (2011). Instance-Based Learning: Integrating Sampling and Repeated Decisions From Experience. *Psychological Review*, *118*(4), 523–551. doi:10.1037/a0024558

Gonzalez, C., & Dutt, V. (2012).Refuting data aggregation arguments and how the instance-based learning model stands criticism: A reply to Hills and Hertwig. *Psychological Review*, *119*(4), 893–898.doi:10.1037/a0029445

Hau, R., Pleskac, T. J., Kiefer, J., & Hertwig, R. (2008). The description-experience gap in risky choice: The role of sample size and experienced probabilities. *Journal of Behavioral Decision Making*, *21*(5), 493–518. doi:10.3758/s13423-015-0924-2

Hertwig, R. (2012). The psychology and rationality of decisions from experience. *Synthese*, *187*(1), 269–292. doi:10.1007/s11229-011-0024-4

Hertwig, R., Barron, G., Weber, E. U., & Erev, I. (2004). Decisions from experience and the effect of rare events in risky choice. *Psychological Science*, *15*(8), 534–539. doi:10.1111/j.0956-7976.2004.00715.x

Hertwig, R., & Erev, I. (2009). The description-experience gap in risky choice. *Trends in Cognitive Sciences*, *13*(12), 517–523. doi:10.1016/j.tics.2009.09.004

Hertwig, R., & Pleskac, T. J. (2010). Decisions from experience: Why small samples? *Cognition*, *115*(2), 225–237. doi:10.1016/j.cognition.2009.12.009

Horrace, R. H., William, C., and Jeffrey, M. P. (2009), *Variety: Consumer choice and optimal diversity. Food Marketing Policy, Center Research Report*, 115.

Houck, C. R., Joines, J., & Kay, M. G. (1995). *A genetic algorithm for function optimization: a Matlab implementation*. North Carolina State University, Technical Report NCSU-IE TR 95-09.

Jakobsen, T. (2010). Genetic algorithms. Retrieved from http://subsimple.com/genealgo.asp

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 263–291.doi:10.2307/1914185

Kudryavtsev, A., & Pavlodsky, J. (2012). Description-based and experience-based decisions: individual analysis. *Judgment and Decision Making*, *7*(3), 316–331.

Lebiere, C. (1999). Blending: An ACT-R mechanism for Aggregate retrievals. *Paper presented at the 6th Annual ACT-R Workshop at George Mason University*. Fairfax County, VA.

Lee, M. D. (2008). Three case studies in the Bayesian analysis of cognitive models. *Psychonomic Bulletin & Review*, *15*(1), 1–15. doi:10.3758/PBR.15.1.1

Lejarraga, T. & Dutt, V. & Gonzalez, C. (2012). Instance-Based Learning: A general model of repeated binary choice. *Journal of Behavioral Decision Making*, *25*(2),143–153. doi:10.1002/bdm.722

Luce, R. D., & Raiffa, H. (1957). *Games and decisions: Introduction and critical surveys*. New York: Wiley.

March, J. G. (1996). Learning to be risk averse. *Psychological Review*, *103*(2), 309–319. doi:10.1037/0033-295X.103.2.309

Marchiori, D., Di Guida, S., & Erev, I. (2015). Noisy retrieval models of over-and under sensitivity to rare events. *Decision*, *2*(2), 82–106. doi:10.1037/dec0000023

Mathworks (2012). MATLAB and Statistics Toolbox Release 2012b [Computer software]. Natick, Massachusetts, United States: The MathWorks, Inc.

Plonsky, O., Teodorescu, K., & Erev, I. (2015). Reliance on small samples, the wavy recency effect, and similarity-based learning. *Psychological Review*, *122*(4), 621–647. doi:10.1037/a0039413

Rieskamp, J. (2008). The probabilistic nature of preferential choice. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *34*(6), 1446–1465. doi:10.1037/a0013646

Rouder, J. N., & Lu, J. (2005). An introduction to Bayesian hierarchical models with an application in the theory of signal detection. *Psychonomic Bulletin & Review*, *12*(4), 573–604. doi:10.3758/BF03196750

Shteingart, H., Neiman, T., & Loewenstein, Y. (2013). The role of first impression in operant learning. *Journal of Experimental Psychology: General*, *142*(2), 476–488. doi:10.1037/a0029550

Stevens, L. (2016, June 8). Survey shows rapid growth in online shopping. *The Wall Street Journal*. Retrieved from https://www.wsj.com/articles/survey-shows-rapid-growth-in-online-shopping-1465358582.

Sutton, R. S., & Barto, A. G. (1998). *Reinforcement learning: An introduction* (Vol. 1, No. 1). Cambridge: MIT Press.

Tversky, A., & Kahneman, D. (1986). Rational choice and the framing of decisions. *Journal of business*, *59*(S4), S251–S278. doi:10.1086/296365

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*(4), 297–323.doi:10.1007/bf00122574

Tversky, A., & Fox, C. R. (1995). Weighing risk and uncertainty. *Psychological Review*, *102*(2), 269–283. doi:10.1037/0033-295X.102.2.269

Appendix

Appendix A: Estimation Set (TPT)

Problem	Set	High	P(High)	Low	Medium
1	Est	-0.3	0.96	-2.1	-0.3
2	Est	-0.9	0.95	-4.2	-1.0
3	Est	-6.3	0.3	-15.2	-12.2
4	Est	-10	0.2	-29.2	-25.6
5	Est	-1.7	0.9	-3.9	-1.9
6	Est	-6.3	0.99	-15.7	-6.4
7	Est	-5.6	0.7	-20.2	-11.7
8	Est	-0.7	0.1	-6.5	-6.0
9	Est	-5.7	0.95	-16.3	-6.1
10	Est	-1.5	0.92	-6.4	-1.8
11	Est	-1.2	0.02	-12.3	-12.1
12	Est	-5.4	0.94	-16.8	-6.4
13	Est	-2.0	0.05	-10.4	-9.4
14	Est	-8.8	0.6	-19.5	-15.5
15	Est	-8.9	0.08	-26.3	-25.4
16	Est	-7.1	0.07	-19.6	-18.7
17	Est	-9.7	0.1	-24.7	-23.8
18	Est	-4.0	0.2	-9.3	-8.1
19	Est	-6.5	0.9	-17.5	-8.4
20	Est	-4.3	0.6	-16.1	-4.5
21	Est	2.0	0.1	-5.7	-4.6
22	Est	9.6	0.91	-6.4	8.7
23	Est	7.3	0.8	-3.6	5.6
24	Est	9.2	0.05	-9.5	-7.5
25	Est	7.4	0.02	-6.6	-6.4
26	Est	6.4	0.05	-5.3	-4.9
27	Est	1.6	0.93	-8.3	1.2
28	Est	5.9	0.8	-0.8	4.6
29	Est	7.9	0.92	-2.3	7.0
30	Est	3.0	0.91	-7.7	1.4
31	Est	6.7	0.95	-1.8	6.4
32	Est	6.7	0.93	-5.0	5.6
33	Est	7.3	0.96	-8.5	6.8
34	Est	1.3	0.05	-4.3	-4.1
35	Est	3.0	0.93	-7.2	2.2
36	Est	5.0	0.08	-9.1	-7.9
37	Est	2.1	0.8	-8.4	1.3
38	Est	6.7	0.07	-6.2	-5.1
39	Est	7.4	0.3	-8.2	-6.9
40	Est	6.0	0.98	-1.3	5.9
41	Est	18.8	0.8	7.6	15.5
42	Est	17.9	0.92	7.2	17.1
43	Est	22.9	0.06	9.6	9.2
44	Est	10.0	0.96	1.7	9.9
45	Est	2.8	0.8	1.0	2.2
46	Est	17.1	0.1	6.9	8.0
47	Est	24.3	0.04	9.7	10.6
48	Est	18.2	0.98	6.9	18.1
49	Est	13.4	0.5	3.8	9.9
50	Est	5.8	0.04	2.7	2.8
51	Est	13.1	0.94	3.8	12.8
52	Est	3.5	0.09	0.1	0.5
53	Est	25.7	0.1	8.1	11.5
54	Est	16.5	0.01	6.9	7.0
55	Est	11.4	0.97	1.9	11.0
56	Est	26.5	0.94	8.3	25.2
57	Est	11.5	0.6	3.7	7.9
58	Est	20.8	0.99	8.9	20.7
59	Est	10.1	0.3	4.2	6.0
60	Est	8.0	0.92	0.8	7.7

Appendix B: Competition Set (TPT)

Problem	Set	High	P(High)	Low	Medium
1	Comp	-8.7	0.06	-22.8	-21.4
2	Comp	-2.2	0.09	-9.6	-8.7
3	Comp	-2.0	0.1	-11.2	-9.5
4	Comp	-1.4	0.02	-9.1	-9.0
5	Comp	-0.9	0.07	-4.8	-4.7
6	Comp	-4.7	0.91	-18.1	-6.8
7	Comp	-9.7	0.06	-24.8	-24.2
8	Comp	-5.7	0.96	-20.6	-6.4
9	Comp	-5.6	0.1	-19.4	-18.1
10	Comp	-2.5	0.6	-5.5	-3.6
11	Comp	-5.8	0.97	-16.4	-6.6
12	Comp	-7.2	0.05	-16.1	-15.6
13	Comp	-1.8	0.93	-6.7	-2.0
14	Comp	-6.4	0.2	-22.4	-18.0
15	Comp	-3.3	0.97	-10.5	-3.2
16	Comp	-9.5	0.1	-24.5	-23.5
17	Comp	-2.2	0.92	-11.5	-3.4
18	Comp	-1.4	0.93	-4.7	-1.7
19	Comp	-8.6	0.1	-26.5	-26.3
20	Comp	-6.9	0.06	-20.5	-20.3
21	Comp	1.8	0.6	-4.1	1.7
22	Comp	9.0	0.97	-6.7	9.1
23	Comp	5.5	0.06	-3.4	-2.6
24	Comp	1.0	0.93	-7.1	0.6
25	Comp	3.0	0.2	-1.3	-0.1
26	Comp	8.9	0.1	-1.4	-0.9
27	Comp	9.4	0.95	-6.3	8.5
28	Comp	3.3	0.91	-3.5	2.7
29	Comp	5.0	0.4	-6.9	-3.8
30	Comp	2.1	0.06	-9.4	-8.4
31	Comp	0.9	0.2	-5.0	-5.3
32	Comp	9.9	0.05	-8.7	-7.6
33	Comp	7.7	0.02	-3.1	-3.0
34	Comp	2.5	0.96	-2.0	2.3
35	Comp	9.2	0.91	-0.7	8.2
36	Comp	2.9	0.98	-9.4	2.9
37	Comp	2.9	0.05	-6.5	-5.7
38	Comp	7.8	0.99	-9.3	7.6
39	Comp	6.5	0.8	-4.8	6.2
40	Comp	5.0	0.9	-3.8	4.1
41	Comp	20.1	0.95	6.5	19.6
42	Comp	5.2	0.5	1.4	5.1
43	Comp	12.0	0.5	2.4	9.0
44	Comp	20.7	0.9	9.1	19.8
45	Comp	8.4	0.07	1.2	1.6
46	Comp	22.6	0.4	7.2	12.4
47	Comp	23.4	0.93	7.6	22.1
48	Comp	17.2	0.09	5.0	5.9
49	Comp	18.9	0.9	6.7	17.7
50	Comp	12.8	0.04	4.7	4.9
51	Comp	19.1	0.03	4.8	5.2
52	Comp	12.3	0.91	1.3	12.1
53	Comp	6.8	0.9	3.0	6.7
54	Comp	22.6	0.3	9.2	11.0
55	Comp	6.4	0.09	0.5	1.5
56	Comp	15.3	0.06	5.9	7.1
57	Comp	5.3	0.9	1.5	4.7
58	Comp	21.9	0.5	8.1	12.6
59	Comp	27.5	0.7	9.2	21.9
60	Comp	4.4	0.2	0.7	1.1

Appendix C: SC Problems Set

Problem	Set	High	P(High)	Low	Medium
1	SC Problems	4	0.8	0	3
2	SC Problems	4	0.2	0	3
3	SC Problems	-3	1	0	-32
4	SC Problems	-3	1	0	-4
5	SC Problems	32	0.1	0	3
6	SC Problems	32	0.025	0	3

Appendix D: CPT Models' Value and Weighting Functions