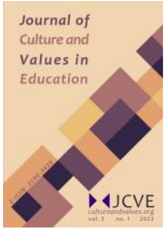

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Twinned Teachers’ Mathematical Discourse Using Problem-Solving

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ABSTRACT
Discussing and debating mathematical ideas through integrating natural and mathematical language is vital for conceptual understanding and ultimately for learner performance. In schools with low performance, it is likely that teaching mathematics follows an approach largely involving rote learning. In rural Limpopo where schools are isolated, professional development takes the form of twinning, where a well-functioning school twins with a school in need. In this study two teachers twinned to teach algebraic word problems to Grade 11 to improve learner performance through a problem-solving approach. A central aspect of the study was attention to discourse informed by the commognitive framework, where both natural language (learners home language and the language of instruction) and language of mathematics spoken by the teachers and encouraged in the learners, was the focus. This quasi-experimental design was implemented to examine the effect of the twinned teachers discourse in a class where English, the language of instruction, was a second language. In the study a pre-test and post-test were administered to the experimental group of 34 learners, and to a comparison group of 40 learners. The study aimed at testing the hypothesis that the twinned mathematics teachers’ discourse using a problem-solving approach informed by the commognitive framework has a significant effect on learner performance in algebraic word problems. In summary, using the commognitive framework in implementing problem-solving approach with the twinned teachers’ mathematical discourse had significant effect in improving learner performance of algebraic word problems in the target group.

KEYWORDS
Algebraic word problems; twinning; learner performance; mathematical discourse; problem solving

INTRODUCTION

This paper investigated the effectiveness of the twinned teachers' mathematical discourse in teaching Grade 11 algebraic word problems by using a problem-solving approach with English Second Language learners in a rural secondary school in Limpopo province, South Africa. English in South Africa is used as a language of learning and teaching (LoLT) from Grade 4 to tertiary education (Department of Basic Education, 2010): it mediates comprehensibility and comprehension of mathematical language (Siyepu & Ralarala, 2014). Siyepu and Ralarala further note that English is a pre-requisite for learner understanding of mathematics. Robertson and Graven (2019) add that many former colonies of Great Britain, including South Africa, selected English rather than their own indigenous languages as the official language for teaching and learning, and for government communications. In this context the LoLT plays a pivotal part in solving mathematical problems.

Mathematical problem-solving (PS) was given prominence by Polya (1945) and has since gained popularity in the late 1980s and 1990s internationally (Hiebert et al. (1996) and in South Africa with the work of, Murray, Olivier and Human (1998). Studies have been conducted in mathematical problem-solving, and concluded, for example, that PS can be applied as an approach to teaching for deeper understanding of mathematical concepts (Gurat, 2018). In the Martins and Martinho's (2021) study they found that PS encouraged learner collaboration in the exchange of ideas, justifying those ideas and understanding peer ideas in the process of developing critical thinking. PS can also develop learner abilities to select appropriate strategies to solve mathematical problems (Hoon et al., 2013). Learner self-confidence and creativity can be developed in mathematical PS and learners can become independent thinkers (Özreçberoğlu & Çağanağa, 2018). PS was found to have significant effect when used by the twinned teachers teaching Grade 11 financial mathematics (Makgakga, 2022).

Twinning is an engagement of two or more schools sharing knowledge, skills, expertise, and resources. Lock (2011) describes twinning as a cluster of schools with teachers working together to promote the schools, providing peer support to teachers by presenting lessons at the other schools. Lock (2011) and Collaborne and West-Burnham (2008) postulate that twinning can prevent the isolation of schools in rural areas and encourage working together among teachers. Communities are likely to succeed when they embark and participate in twinning; they become interdependent and heterogeneous. In this study, two teachers are twinned from two schools, one well performing, and the other less well-performing in teaching Grade 11 algebraic word problems using a problem-solving approach during classroom mathematical discourse to improve performance.

The mathematical discourse of the twinned teachers teaching Grade 11 algebraic word problem using PS is the focus of this paper. Wisniewsk (2006) postulates that discourse has its genesis in the Latin "discursus" which denotes "speech, conversation" (Cohen, Manion & Morrison, 2018). A discourse is a way of thinking, a cultural or institutional condition which is legitimated by communities, often those with power. Crystal (1992) defines discourse as an unremitting expanse of language larger than a sentence, often found in a logical unit such as an argument, a joke, a narrative, or sermon. Drawing from all of the above, we can define mathematical discourse as language used in the mathematical classroom which abides by the mathematical canon and engages both natural language and mathematical language to communicate and debate concepts and procedures.

This approach consists of firstly understanding the problem, then secondly analysing the word problems by translating words into mathematical expressions or equations. Word problems are important for mathematics learning as they provide learners with understanding of the nature of story problems (Morin, Watson et al., 2017; Vula et al., 2017; Zhu, 2015), and they highlight lack of reading comprehension skills (Kurshumlia & Vula, 2019). However, studies have shown word-problems (WP) to be problematic for most mathematics learners (Emanuel et al, 2021; Verschaffel et al., 2020). Verschaffel et al. report that the challenges learners face in solving word problems in mathematics emanate from the problem of comprehension which involves transformation of words into equations or expressions. WP represent a series of actions by the learners when solving problems (Zhu, 2015); it requires a high-level of thinking, more than just procedural knowledge or memorizing procedures to solve problems (Bhagwonparsadh & Pule, 2023; Kurshumlia & Vula, 2019).

In solving WP, learners need to recite, comprehend, and use mathematical comprehension (Fatmanissa et al., 2020). Fatmanissa et al. (ibid) hold the view that the inability of learners to comprehend vocabulary, to transform everyday vocabulary into mathematical forms, or to understand symbols when solving WP, is fundamentally a linguistic difficulty. This paper explores the effect of the mathematical discourse of the twinned teachers when teaching Grade 11 algebraic word problems using a problem-solving approach to improve learner performance. One school performed well, and the other school performed poorly in mathematics. The primary intent of this article is to explore how the twinned teachers used the PS approach in their mathematical discourses to improve learner performance. This study sought to test the following hypotheses:

H_0: The twinned teachers' mathematical discourse has no significant difference in effect between the experimental group and comparison group on learner performance when teaching Grade 11 WPs using a PS approach.

H_1: The twinned teachers' mathematical discourse has a significant difference in effect between the experimental group and comparison group on learner performance when teaching Grade 11 WPs using PS approach.

Theoretical Lens

The teachers mathematical discourse requires representations of mathematical concepts and ideas that involve communication. Communication can be interpersonal for individuals and can be either verbal or non-verbal using cognitive processes (Sfard, 2015). This study espoused Sfard's (2007) commognitive framework which offers conceptual tools for capturing and exploring social participation and mathematical discourse during teaching. Commognition is the amalgamation of cognitive process and communication in the discourse of mathematics (Siyepu & Ralarala, 2014). Mbhiza (2021) views this form of communication as an illustration occurring at an interpersonal (communication in a social space) level and intrapersonal (cognitive) level in teaching mathematics. Sfard (2008) regards communication and the cognitive process as dissimilar expressions of the same phenomenon. Sfard (2006, p.157) views this theory as a unit of analysis in a discursive activity, referring to the "collective doing, patterned," in the activity. This article focuses on the twinned teachers' mathematical discourse when teaching algebraic word problems using a problem-solving approach to improve learner performance.

Sfard (2008) provides four distinct traits of discourse in mathematical: texts and how they are used (1), visual mediators (2). narratives that are endorsed (3), and routines (4). visual mediator.

1. Words and their uses entail the unique vocabulary, specific words, are used by the learners and teachers in the discourse of mathematics teaching. Siyepu and Ralarala (2014) argue that mathematics language shares words similar with ordinary English but provides dissimilar meaning in the mathematics situation. The mathematics register can be improved by means of extending the mathematical vocabulary. Cuevas (1984) defines a mathematics register as the meaning that aligns to the natural language utilised in mathematics teaching and learning. Siyepu and Ralarala (2014) add that the mathematics register includes ways of presenting arguments and styles of meaning within the mathematics situation.
2. In this study, keywords need to be identified to differentiate the meanings as use in natural language, ordinary English, and the mathematics situation.
3. Visual mediators are the visual objects identified as objects of their discourse and coordinates the communications (Siyepu & Ralarala, 2014). Mbhiza (2021) views visual mediators as objects within mathematics that are operated upon in discursive practice. Algebraic visual mediators could be tables used to make explicit patterns to assist learners to make sense of those problems in preparation for answering the given questions.
4. Endorsed narratives refer to the facts and ideas presented in mathematics knowledge (Siyepu & Ralarala, 2014), relations between processes that are performed, and the mathematical objects that are rejected or endorsed within mathematics communities (Mbhiza, 2021). Examples of endorsed narratives are mathematical definitions and equations that could be used by teachers during mathematical discourse teaching algebraic WPs using PS approach.
5. Making sense of algebraic WPs and identifying keywords to analyse the problems during mathematical discourse are key issues within this study to communicate ideas at the interpersonal and intrapersonal levels.
6. Lastly, routines are repetitive actions (Mbhiza, 2021), or procedures following repeated steps to determine the solutions of mathematical problems. Routines are fundamentally

helpful in mathematics discourse as learner actions in the new situations depend on their past experiences or prior knowledge (Tabach & Nachlieli, 2011). Mbhiza (2011) argues that few concepts and ideas about mathematics routines are needed in classroom practices, which could be applicable in the teaching and learning of algebraic WPs. Ritual and exploration routines are distinguished in Sfard and Prusak's (2005) work. Ritual routine is a practice whose aim is collective, and which supports creating and maintaining connections with others (Berger, 2013, p.3). while exploration routines focus on the goal that produces new descriptions (Sfard, 2015, p.131), which could be related to routines that emphasise the analysis of repetitive examples of mathematical discourse.

This study concentrates on narratives, word use and routines with learners determining the unknown values of the WPs, translating words into mathematical equations, characterizing the teacher mathematical discourse when teaching WPs using a PS approach. Visual mediators could be used whenever teacher lessons are analysed.

Sfard has made the following claims for teaching mathematics:

1. Focus on the object of learning: What significant effect/no significant effect could occur because of mathematical discourses in teaching algebraic WPs using PS approach to improve learner performance.
2. Focus on the process: How do teachers and learners collaborate in the mathematical discourse towards bringing significant difference or effects using the PS approach to improve learner performance?
3. Focus on the outcome: Has the PS approach had significant effect or difference in the outcomes following the twinned teachers mathematical discourse teaching WPs?

Siyepu and Ralarala (2014) postulate that the shift from an acquisitionist to a participationist approach emerged as a concern of the educationists and researchers when observing learners learning mathematical concepts. Learners need to participate in their own learning to make sense of mathematical concepts to improve their performance.

RELATED LITERATURE

Literature sources in this paper focus on mathematical discourse, difficulties in word problem solving, and the problem-solving approach.

Mathematical discourse

Language culture in the mathematics classroom is outlined by Adler (2001) in her study on language issues in South African schools. Firstly, urban, and suburban, schools have a strong English language culture, with other languages also in use. Secondly, township or rural schools have less English language background than other indigenous languages. Then thirdly, there are other foreign language conditions where learners mainly speak and hear English at school with learners sharing same language other than English at home or in the playground. This last scenario appears to have a direct bearing on this study because all the learners who participated in this study hail from one of the rural districts in Limpopo province, where there has been little research into the teaching strategies in mathematics within these farm and rural schools (Mbhiza, 2019). This study can therefore expand and diversify the scope of research in mathematical discourse in rural schools, to make sense of their classroom discourse and investigate how this discourse facilitates or constrains learner comprehension of mathematics (Mbhiza, 2021).

Kersaint (2015) argues that for a meaningful mathematical discourse, teachers need to have a plan, then to initiate and orchestrate discourse in ways that can encourage learning of the subject. In planning a meaningful discourse, five teacher practices that may be considered in their planning, that is selecting, sequencing, anticipating, connecting and monitoring (Smith & Stein, 2011) are briefly outlined. Firstly, teachers need to anticipate approaches learners use when solving mathematical problems in which they could be guided by the questions *(how might learners interpret the problem? what approaches they might employ? what are the errors and misconceptions they commit?)*. Secondly, the teacher can pay attention to learner mathematical thinking and solution strategies individually or in groups to know which ideas to emphasise during whole class discussion. Thirdly, selecting learners who can do presentations after group discussion could be informed in advance to prepare, organize, and gather their thoughts. Fourthly, sequencing the order of learner presentations, i.e. starting with the group with the incorrect approach and outcome highlighting common misconceptions, and then moving to the group with the correct approach and outcome. Lastly, the teacher needs to understand how learners make connections between their solutions by comparing with the solutions given by other learners. The following questions may be used to guide how learners make connections (*how are these two ideas similar? how are they different? how does the second idea build on the new idea?*). These practices can build on each other for Grade 11 mathematics teachers to orchestrate mathematical discourse in meaningful ways when teaching WPs using PS approach to improve learner performance.

Difficulties in mathematical word problem solving

Informative studies have been conducted on difficulties when solving mathematical word problems (for example, Emanuel et al., 2021; Fatmanisa et al., 2020; Kurshumlia & Vula, 2019; Mingke & Alegre, 2019; Seifi, Haghverdi, & Azizmohamadi, 2012). Seifi et al. (2012) used an interview guide with 52 teachers in their study to detect learner difficulties when solving word problems. Their study revealed the challenges in solving word problems as not understanding texts, encountering unfamiliar contexts and using inappropriate strategies. Emanuel et al. (2021) showed that learners have difficulties when translating words into mathematical expressions or equations, that is identifying known and unknown variables, and have difficulties with transformation. Mingke and Alegre (2019) investigated learner difficulties in solving word problems using a survey questionnaire with 100 learners. These scholars concur that learner difficulties comprise firstly, translations of mathematical word problems into mathematical symbols, and secondly, there is a resistant attitude towards WP solving.

Some studies revealed language issues as a challenge in solving mathematical word problems (for example, Fatmanisa et al., 2020; Rahman et al., 2015; Mulyadi et al., 2015). These scholars concur that learner inability to comprehend vocabulary, translate texts to mathematical equations and comprehend symbols resides fundamentally in linguistic difficulties. The linguistic problem in understanding word problems impedes learners making meaning of given texts and translating these into mathematical equations (Fatmanissa et al., 2020). Multiple representations such as comprising everyday language, using symbols, and visual representations were found as one of the difficulties in solving WPs (O'Halloran, 2010, 2015).

Problem-solving as a strategy

Problem-solving is referred to as key in mathematics teaching and this skill contributes towards learners participating in modern society (Gravemeijer et al., 2017). Lester and Cai (2016) postulate that PS as a teaching approach permits learners to apply, integrate and connect isolated pieces of information which then contribute to a deeper conceptual understanding of mathematical models. PS is largely influenced by the work of George Polya (1945) in which he outlined four steps: understanding a problem, analysing a problem, executing a plan, and reflecting upon the suggested solutions. Degrande et al. (2016) point out that learners' experience difficulties in PS, especially in WPs whereby they cannot identify the underlying model but can only provide superficial characteristics of the problem.

Klang et al. (2021) suggest that small group discussion can enhance learner PS abilities. Small group discussions focused on problem solving, provide a space in which learners are enabled to explain their solutions, to think using language for reasoning, and to gain understanding of peer perceptions around the given question (Fujita et al., 2019). Gurat (2018) introduced mathematical PS strategies among pre-service teachers which appeared to be influential in academic performance. Gurat (ibid) classifies PS strategies as metacognitive, cognitive, and other strategies. Thus, cognitive approaches are identified as elaboration, rehearsal, and organization, while metacognition strategies are classified as self-regulation and critical thinking. Some strategies such as orientation or prediction, monitoring, planning, and evaluation, are considered in the category of other strategies.

RESEARCH METHODS

This article espoused a quantitative research approach within a non-equivalent pre-test-intervention-post-test design because it is not practical to perform random assignment in this research design (McMillan & Schumacher, 2014). Quantitative research tests hypotheses and theories using quantitative data to confirm or disconfirm (Johnson & Christensen, 2014). The primary purpose for this research approach is to test the hypotheses to see if the results confirm the null hypothesis or alternative hypothesis. A non-equivalent pre-test and post-test design consists of a treatment group and a comparison group in which the two tests were administered in the two groups without random assignment (Creswell & Creswell, 2018). The experimental group in this study refers to the school that under-performed in mathematics, and the comparison group is the school that performed better when compared to its counterpart school. This study has used two schools that performed differently to allow the teacher from the comparison group to share expertise and knowledge to improve practice and performance in the experimental group. The treatment condition was administered in the experimental group immediately after administering the pre-test to the two groups, the study then implemented an intervention or exposed the experimental school to treatment and after a certain period of time the researcher administered a posttest in the two-study group.

The same instrument was given for the pretest and posttest, at the beginning and the end of the study. McMillan and Schumacher (2014) identified in this design type the threats to validity for the improvement of the results, for example, the history, maturation, testing, instrumentation, and regression artefacts which can influence the post-test results. The timeframe provided may have been enough for learners to not remember the questions and not to remember the responses of the post-test as noted by Creswell & Creswell (2018). History is an event that co-occurs with the treatment, and that can be experienced by the participants outside of that treatment but that could affect the post-test results in the two schools. In this study the experimental school received only this treatment and the comparison group received only the standard teaching, therefore any improvement in performance could be due to the treatment. Maturation implies that it is the change in age and experience rather than the treatment which affects the outcome. In this case both the comparison and the experimental group increased in age.

Research participants

The research participants comprised 78 Grade 11 learners (experimental group = 38 and comparison group = 40) who wrote the two tests, and two teachers who implemented the experimental treatment in one of the secondary schools in Limpopo province. The Grade 11 learners needed to be prepared for subsequent grade 12 year, in a school that under performed in mathematics over the past three years. The two participating teachers from each of the two schools were purposively selected, as they participated in the twinning project teaching algebra. The two teachers needed to possess skills and knowledge of teaching secondary school mathematics in rural areas.

Permission to visit schools was sought from the College of Education Ethics Committee chairperson under the auspices of the Departmental Engaged Scholarship Research Project in mathematics education. The researcher also sought permission from the two mathematics learners and teachers in the experimental group. This study aimed at elucidated the purpose of the study to the participants and participation was declared voluntary. Table 1 shows the biographical information of the teachers who took part in the study. Pseudo names were used for schools and teachers to protect their identities and ensure their anonymity.

Table 1

Teacher Information

Pseudo name	Gender	Name of school	Qualifications	Teaching experience	Institution trained
Malesela	Male	Maletswai Secondary School (comparison group)	Honours in Mathematics Education	20	University of Limpopo
Raesibe	Female	Reratile Secondary School (experimental group)	Bachelor of Education major in Mathematics Education	6 years	University of Limpopo

Data Collection

Data was collected between March and April 2022 before the mid-year examination was administered. The collected data was managed using Microsoft Excel to prepare for analysis. The data set was mined from the pre-test administered to 78 learners in both study groups where experimental group has learners [N = 38, females = 19 and males = 15] and the comparison group has learners [N = 40]. Learner answer scripts were marked and analysed quantitatively a day after administering the pre-test. After analysing the pre-test results of the two study groups, the researcher implemented the treatment (explained the next section) in the experimental group, with twinned teachers using PS approach in their mathematical discourse to teach Grade 11 WPs with the aim of improving performance. After the intervention, the researcher then administered the post-test which was then marked and analysed quantitatively to measure the effects of the experimental treatment. The rationale for administering the pre-test and the post-test was to confirm the hypotheses test, whether the use of PS approach has significant effects or not when Grade 11 teachers adapted the mathematical discourse through a problem-solving approach to teach WPs, on learner academic achievement in the two groups.

Table 2

Pre-test and Post-test Instrument

Items	Questions
1	Form equations to describe the following scenarios (use x and/or y as the unknown number)
1.1	The sum of two numbers is 18, and the product of these two numbers 56. (2)
1.2	The product of two successive even integers is 452. (2)
1.3	A certain number is doubled and increased by twenty. The answer is ten less than 3 times the number. (2)
2	Determine the unknown number(s): (use x and/or y as the unknown number)
2.1	The sum of three successive natural numbers is 18. What are the numbers? (4)
2.2	The product of two consecutive integers is 72. Find the numbers. (6)
2.3	Nolo's father is six times as old as Nolo. The product of their ages is 150 years. What are their respective ages? (5)
2.4	A small rectangular vegetable garden is enlarged by increasing the length by 3m and width by 1m. The area of the new garden is three times larger than that of the original garden. Determine the original dimensions of the garden if its area was $6m^2$. (7)
2.5	A man travels 180km from his farm to town in a loaded truck. On the return journey he is able to travel 30km/h faster having unloaded his produce in town. It takes him one hour less on the return trip. At what speed did he travel to town? (7)

Twinning process

The two Grade 11 mathematics teachers whose schools in the same vicinity performed differently in Grade 12 mathematics for the past four years embarked on a twinning project. One school performed well, and the other school performed poorly in mathematics. The primary aim for this project was to focus on Grade 11 WPs as it was found to be one of the topics that learners perform well in, but it was also revealed by their teacher in the low-performing school that most of the learners even in other grades do not attempt the WP questions. The two teachers met and planned how twinning could be implemented in the low-performing school to see how they can assist learners to understand WPs.

Prior to implementing the treatment in the experimental group, the researcher organised a meeting with the twinned teachers to plan the intervention. The researcher had discussed with the twinned teachers about the goals and activities that would roll out during the treatment and ensured that they understood their responsibilities and roles within the project. The responsibilities and roles of the twinned teachers were outlined as firstly, understanding what twinning is, secondly, to share skills and resources for the change in practice to improve performance, and thirdly to respect each other's thoughts and present lessons using PS approach. The role of the teacher from the comparison school was to share skills and expertise with the teacher in the experimental school. The researcher's role was to facilitate the two-mentioned meetings. Two one-and-half hour meetings were conducted to discuss how PS approach could implemented in the experimental school prior to the treatment. The twinned mathematics teachers discussed lesson plans to agree with the types of activities that would be given to the learners.

The twinning project took six consecutive weeks; lessons were conducted twice a week for one and half hours after school to avoid disrupting the smooth running of the school. During the treatment, small groups of four to five learners discussed activities on WPs with the guidance of the teacher, and the learners used the language of their choice when discussing WP in their lessons. Prior to the intervention stage, two baseline lessons were observed in the respective schools to establish the teaching strategies used to teach WPs. The teacher from the comparison group was found to be exposed to PS approach and used it to teach WPs during baseline lesson observations. The intention was also to avoid replication of approaches which could not serve the purpose in the experimental school. Table 3 shows the dates and activities planned over six weeks by the twinned teachers.

Table 3 describes the activities that the twinned teachers planned together prior the implementation of twinning using PS approach teaching WPs in their mathematical discourse. In their planning they outlined the topics on WPs to be presented and the skills that the learners in the experimental school could acquire through mathematical discourses using PS approach. The teacher from the experimental school showed to have learned skills and knowledge of how to use PS approach to WPs as was the one who facilitated lesson 4, 5 and 6 with the help of the teacher from the comparison school.

Table 3

Dates and Activities planned during Experimental Treatment

Date	Topic	skills	Facilitator(s)
03/05 and 05/05/22	Solving problems with equations (transition from algebraic expressions to algebraic equations: time, month and age)	How to translate word problems to form equations involving time, months and age	Malesela/ Raesibe
10/05 and 12/05/22	Algebraic language (time, months, and age)	How to interpret and solve word problem involving time, months and age	Malesela/ Raesibe
17/05 and 19/05/22	Solving problems with equations (transition from algebraic expressions to algebraic equations: speed, distance, time)	How to translate word problems to form equations involving speed, distance and time	Malesela/ Raesibe
24/05 and 26/05/22	Algebraic language (speed, distance, time)	How to interpret algebraic language to solve problems involving speed, distance and time	Malesela/ Raesibe
31/05 and 02/06/22	Algebraic language and solving word problems equations (rate and problem solving with integers)	How to interpret algebraic language to solve word problems involving rate and problem solving with integers	Malesela/ Raesibe
07/06 and 09/06	Solving problems with equations (general word problems)	How to interpret word problems to find the solution	Malesela/ Raesibe

Data Analysis

For statistical data capturing and management, the researcher used Microsoft Excel (Microsoft, 2013), and used Wilcoxon Rank-Sum test for statistical data analysis using Strata 13 software. As data was not normally distributed, a non-parametric test was used. The interpretation of statistical data was performed at a 95% confidence limit.

Results and interpretation

Seventy-eight Grade 11 mathematics learners participated in the pre-test and post-test focused in solving WPs. Two teachers were twinned to introduce the PS approach to teaching WPs. Commognition as a combination of cognitive process and communication was espoused in the discourse of mathematics with the twinned teachers. Though this is a quantitative study, the researcher presented the background finding of the baseline lesson observations conducted before the treatment could be implemented. Prior to the treatment being implemented in the baseline lesson observations, learners in the experimental school appeared to be more passive as compared to the comparison group during lesson observations. Furthermore, the teacher in the experimental school appeared not have used the PS approach to teach learners and used the traditional teaching method. The teacher in the comparison group has complemented his traditional way of teaching with PS approach to teach WPs in the baseline lesson observations and most of the learners participated in his lessons. Further, the teacher from the comparison group used small group discussion of five to seven learners to discuss activities. Imagine a line along a white wall. This line represents normal mathematical functioning according to the DBE. Now your experimental group functions far below the line. The comparison group functions on a par with that line. Why, because this class has progressive teaching. Now in this study the researcher wants to see whether twinning would enable your experimental group to improve to the level of the comparison group.

All the variables are not normally distributed (*all p-values* < 0.05). The non-parametric test is warranted due to abnormality distribution of learners' marks. Wilcoxon rank-sum (Mann-Whitney) test was used to test the significant difference between the experimental and comparison groups. The two study groups were compared by using rank-sum test. The statistical interpretation is performed at 95% confidence limit. Table 4.1 presents the two sample Wilcoxon rank-sum test testing the statistically significant difference between the two groups.

Table 4.1

Pre-test Results

Group	rank sum	obs	expected
Comparison	2266	40	1806
Experimental	1389	38	1849
combined	3655	78	3655

z = -4.052

Prob > |z| = 0.0001

Table 4.1 shows that the experimental and comparison groups yielded significantly differently in the pre-test, comparison group (rank-sum score = 2266), the experimental group (rank-sum score = 1389) yielded significantly differently with the p-value (Prob > |z| = 0.0001) which was less than 0.05 at the 95% interval. The results reveal that the comparison group has recorded rank-sum score greater and the experimental group in the pre-test which assessed learner WP solving abilities and skills. Most of the experimental group learners were shown to have lacked understanding of translating WPs into mathematical expressions and equations as Fatmanisa et al. (2020) have found. The teaching approach used in the experimental group used in the baseline lesson observations may have contributed to the poor learner performance in solving WPs. In addition, Emanuel et al. (2019) and Mingke and Alegre (2019) note that most learners have challenges in translating word statements into mathematical expressions and equations. By contrast, learner performance in the comparison group may have been affected the traditional teaching approach and PS approach used by their teacher to teach WPs. This supports what Lester and Cai (2016) postulated that PS approach permits learners to apply, integrate and connect isolated pieces of information which then contribute to a deeper conceptual knowledge of concepts in mathematics. Table 4.2 presents post-test results after the intervention.

Table 4.2

Post-test Results

Group	rank sum	obs	expected
Comparison	2476.5	38	2085
Experimental	1727.5	40	2018
Combined	4103	78	4103

z = -1.972

Prob > |z| = 0.0486

Table 4.2 depicts that the comparison group still recorded higher rank-sum score as compared to the experimental group even after the intervention when solving WPs using a problem-solving approach. The results show statistically significant difference between the two study groups with the comparison group (*rank-sum score = 2476.5*) performing better than the experimental group (*rank-sum score = 1727.5*) in the post-test with the p-value (*Prob > |z| = 0.0486*). Learners who received treatment showed to have improved in solving WPs using PS during mathematical discourse. Mathematical discourse showed to have yielded positive results in learners' learning WPs using PS. This concurs with Kersaint's (2015) study that a meaningful mathematical discourse, teachers need to have a plan, then initiate and orchestrate discourse in ways that can encourage learning of the subject. The twinned teachers in planning their discourse, focused on how learners interpreted the WPs, methods used to solve problems, and errors learners committed when solving WPs. Learners appeared to have been able to identify keywords to analyse and create visual objects to interpret WPs to execute the solutions to the problems. For learners to effectively solve WPs, teachers need to anticipate the approaches learners use such as learners' interpretation to the problem, strategies they use and errors, and misconceptions they have (Smith & Stein, 2011).

DISCUSSION

This article shed light on Grade 11 mathematics teacher discourse in teaching WPs using a PS approach. There was a significant difference between the experimental and comparison schools. The comparison school (rank-sum score = 2266) performed significantly better than the experimental group (rank-sum score = 1389) when solving Grade 11 WPs with the p-value (*Prob > |z| = 0.0001*) which less than 0.05 at 95% confidence level. The performance in the experimental could have been affected by the issue of understanding vocabulary as learners in this study are English Second Language speakers, which could have led to being unable to translate words into mathematical expressions and equations. Fatmanissa et al. (2020) and Mulyadi et al. (2015) showed that the inability of learners in understanding vocabulary can affect them in translating word statements into mathematical expressions and equations. Emanuel et al. (2021) and Mingke and Alegre (2019) support that learners have challenges in translating word statements into mathematical expressions and equations, identifying the known and unknown variables. In other words, learners could have lacked an understanding of identifying keywords to interpret WPs in order to translate word statements into mathematical expressions and equations. Morin et al. (2017) and Vula et al. (2017) postulate that learners need to understand the story problems for them solve WPs. Learners' comprehension of story problems in mathematics would enable them to translate those problems, as these problems need higher order thinking of learners than just procedural knowledge using memorised procedures (Verschaffel et al. 2020; Kurshumlia and Vula, 2019), in which learners may forget those procedures to solve WPs.

Similarly, the two groups reported statistically significant difference after the intervention in the post-test. The comparison group recorded higher scores (rank-sum score = 2476.5) than the experimental school (rank-sum score = 1727.5) after the intervention with the p-value (*Prob > |z| = 0.0486*) less than 0.05 at 95% confident limit. The assumption was that the experimental group would perform significantly better as compared to the comparison group in the post-test. However, the experimental school improved significantly in the post-test (rank-sum score = 1727,5) than the pre-test (1389). The small group discussions in the mathematics discourse have enhanced learners' abilities in identifying words used and visual mediators used during mathematics discourse in WPs using PS (Klang et al., 2021; Sfard, 2008). Martins and Martinho, (2021) add that PS during mathematics discourse encourages learner collaboration and exchange of ideas, justifying those ideas and understanding peer ideas to develop critical thinking, and to translate word statements into mathematical expressions and equations. Mecer and Sams (2006) note that PS solving in the discourse of mathematics assists learners in explaining their solutions, thinking using language for reasoning, and deeper conceptual understanding, therefore can influence academic achievement (Gurat, 2018). The post-test results after the intervention revealed that learners were able to understand the WPs, devising a plan to solve WPs, executed the WPs and reflected upon the suggested solutions (Polya, 1945).

Though this study investigated the effectiveness of the implementation of PS in mathematical discourses in WPs, the study did not focus on how PS mathematical can facilitate and/ or constrain learner understanding of WPs. Kersaint (2015) highlighted that teachers' plans, initiations, and orchestration of meaningful discourse in mathematics can have an effect in learning and teaching. Smith and Stein (2011) suggested five practices in planning mathematical discourse: monitoring, anticipating, sequencing, selecting, and connecting as the ones that can be used in implementing effective mathematical discourse in WPs. It was also found that learners experience challenges in understanding how to translate words into equations or expressions. Fatmanissa et al. (2020) revealed that the issue of language plays a role in learners' difficulties in solving WP, and translating daily vocabulary into mathematical form. Teachers also identified other challenges as understanding texts, unfamiliar contexts and using inappropriate strategies (Seifi et al., 2012). In addition, Degrande et al. (2016) showed that leaners in solving WPs can experience difficulties in identifying the underlying models but can only provide superficial characteristics of the problem. Smith and Stein (2011) identified the teaching practices that could be used in planning a meaningful mathematical discourse, which includes, monitoring, anticipating, selecting, sequencing and connecting.

Since the comparison group has performed significantly better in the post-test than in the experimental group and did not participate in the intervention, it provides a challenge as what contributed to the better performance of the group in the two tests. Özreçberoğlu and Çağanağa (2018) highlighted that learners who have abilities in solving WPs appear to have self-confidence, creative and are independent thinkers in mathematics PS. Makgakga (2022) found that learners with mathematical PS abilities perform significantly better than those who lack those PS abilities in WPs. In addition, Hoon et al. (2013) postulated that learners with mathematical PS abilities can select appropriate strategies to solve problems in mathematics.

CONCLUSION

The results of this study indicated the effect of mathematics discourse in teaching WPs using a PS approach. However, the comparison group showed significantly better performance in the pre-test and post-test in which the study cannot confirm the various factors that the study can account for. The communication and cognition of commognitive theory supports that mathematics discourse when teaching WPs using PS approach, was implemented. The experimental group was equipped to a point where they could be given the post-test after the treatment, in which the mathematics discourse in the teaching of WPs using PS approach was implemented. In other words, the PS approach in mathematical discourse teaching WPs accounts for the two test results.

In dealing with the learners who struggle with mathematical WPs, one approach like this intervention cannot be sufficient to improve their performance. Although the mathematical discourse, informed by commognitive theory, in teaching WPs was used as an intervention in this study, teaching cannot hold other factors constant to obtain the desired results. Teachers may use other approaches, models, and strategies to account for an improvement in the performance of learners. Therefore, the education stakeholders have a responsibility to explore more strategies to empower mathematics teachers, among others, providing them continuous support and training for effective mathematics teaching and learner support. Understanding a meaningful mathematics discourse teaching WPs using a PS approach may aid other mathematics teachers to know how to offer learner support and guidance to improve performance in WPs. Further study may be conducted teachers and learners' perceptions/ experiences on mathematics discourse when teaching WPs using PS approach.

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