THE MEDIATING ROLE OF METACOGNITIVE SELF-REGULATION SKILLS IN THE RELATIONSHIP BETWEEN PROBLEM-POSING SKILLS AND MATHEMATICS ACHIEVEMENT OF PRIMARY PRE-SERVICE TEACHERS

Research article

Özlem Özçakır Sümen 🆔

Ondokuz Mayıs University

ozlem.ozcakir@omu.edu.tr

Biodata:

Özlem Özçakır Sümen is an Assistant Professor in the Primary Education Department of Faculty of Education in Ondokuz Mayıs University. Her research interests are primary school mathematics education, STEM education, pre-service teacher training.

THE MEDIATING ROLE OF METACOGNITIVE SELF-REGULATION SKILLS IN THE RELATIONSHIP BETWEEN PROBLEM-POSING SKILLS AND MATHEMATICS ACHIEVEMENT OF PRIMARY PRE-SERVICE TEACHERS

Özlem Özçakır Sümen

ozlem.ozcakir@omu.edu.tr

Abstract

Metacognitive self-regulation is the ability to organize an individual's mental activities according to his/her goals, and it has been found to affect students' mathematics achievement. However, its relationship with problem-solving and posing skills is still not clarified. This study aims to examine the mediating role of metacognitive self-regulation skills in the relationship between primary pre-service teachers' problem-posing skills and mathematics achievement. Participants consist of 165 primary pre-service teachers studying at different grade levels in the Primary School Teaching Department. The data were collected with the metacognitive self-regulation scale and the problem-posing test consisting of semi-structured problem-posing questions. The proposed hypothesis regarding the role of metacognitive self-regulation in the relationship between problem-posing and mathematics achievement in the study was tested by structural equation modeling and confirmed by bootstrap analysis. Analysis results revealed that problem-posing and metacognitive self-regulation significantly predicted mathematics achievement, but metacognitive self-regulation was not a significant mediator between problem-posing and mathematics achievement.

Keywords: mathematics achievement, metacognitive self-regulation, problem-posing

1. Introduction

It was late understood in mathematics education that it is crucial and valuable for students to be able to solve problems as well as to pose new problems based on what is given. Today, problem-posing have become routine activities in mathematics classes along with problem-solving. Metacognition is the active and conscious control of an individual's cognitive activities (Brown, 1987; Flavell, 1979). Relationships of metacognition with academic performance (Mega et al., 2014; Zohar & Peled, 2008), problem-solving (Borkowski et al., 1989; Davidson et al., 1994; Mayer, 1998), and problem-posing (Ding & Shen, 2008; Ghasempour & Baker, 2012; Ghasempour et al., 2013; Karnain et al., 2014) are some of the topics studied in the mathematics education. The place and importance of metacognitive self-regulation, which includes the regulation of cognitive activities in line with the individual' goals, is a current research topic in this field. In this study, the mediating role of metacognitive self-regulation in the relationship between problem-posing and mathematics achievement was examined. Although the mediating role of metacognitive skills in the relationship between problem-solving and mathematics achievement has been examined in the literature (Hassan & Rahman, 2017), there is no study examining the relationship with problem-posing. Therefore, the study will fill the gap in this area.

1.1. Theoretical Framework

Problem-posing is a process in which concrete situations are interpreted and expressed as mathematical problems that ensure understanding of mathematical concepts and achieving learning goals (Bonotto, 2006). Problem-posing is creating new questions from a problem situation (National Council of Teachers of Mathematics [NCTM], 2000) and is directly linked to problem-solving (Cai, 1997; Silver & Cai, 1996). Problem-posing activities improve students' problem-solving skills, enable them to understand mathematics, and improve their attitudes towards mathematics (Grundmeier, 2003; Silver, 1994; Silver & Cai, 1996). Moreover, it provides them flexible thinking skills, enhanced and enriched basic mathematical concepts (English, 1997). In problem-posing activities, students pose problems using their own life experiences; this provides them with opportunities to reveal the problems they enjoy solving, thus creating a more complex and more motivating learning environment (Lowrie, 2002). Problem-solving instruction, which is given with a problem-posing approach, enables students to understand the problem better and show higher-level qualitative reasoning skills (Cankoy & Darbaz, 2010). Besides, children with a developed sense of number can pose problems better than those with a limited sense of number because they understand the problem's structure better (English, 1997). The teacher's guidance is also vital in problem-posing activities. The teacher should provide opportunities for students to discover and create their math problems (Kilpatrick, 1987). In this way, good guidance to the mathematics learning process makes it easier for the child to acquire problem-solving and problem-posing skills (Chang, 2007). While children can pose one-and two-stage problems in the early stages of problem-posing activities, it has been proven that they can pose increasingly complex, open-ended, and new problems with the guidance of the teacher (Lowrie, 2002).

According to Gonzales (1994), problem-posing is the fifth step of Polya's problem-solving steps. Problem-posing can be accomplished by posing a new problem or reconstructing a given problem (Cai, 2003; Silver, 1994). Problem-posing activities are divided into three as free, semi-structured, and structured problem-posing. Free problem-posing is activities in which students pose problems using a situation from daily life, without any limitation. In semi-structured problem-posing, students explore the problem's structure in an open-ended situation and complete the problem using their mathematical knowledge. Structured problem-posing, on the other hand, is the re-establishment of a problem solved before by changing its conditions or questions (Stoyanova & Ellerton, 1996 as cited in Bonotto 2013).

In the studies, different criteria were developed to evaluate the problem-posing activities. For example, Silver and Cai (1996) suggested three steps to evaluate posed problems. These are whether the established problem expresses a mathematical question, its solubility and complexity. There are also different classifications: fluency and flexibility (Van Harpen & Sriraman, 2013); relevance, complexity, and diversity (Chen et al., 2015); The problem text, the problem's compatibility with mathematical principles, the type (structure) of the problem and the solubility of the problem (Şengül & Katrancı, 2015) are the evaluation criteria developed in different studies on this subject.

1.1.1. Metacognitive self-regulation

Metacognition is the mental or emotional interventions that affect the cognitive activities and are carried out consciously (Flavell, 1979). Metacognition is the knowledge of an individual about his / her cognitive system, and it is the ability to control his/her own cognitive system (Brown, 1987). Metacognition is effective in all processes of control and regulation of thinking and learning processes, effective learning, critical thinking, and problem-solving (Hartman, 1998). It enables the individual to select, evaluate and review cognitive tasks, goals, and strategies in line with their abilities and interests (Flavell, 1979). It

includes planning, monitoring, and evaluation skills. Planning is determining strategies in line with goals, and objectives and organizing resources. Monitoring is awareness of one's performance and evaluation is the judgment by evaluating the performance of a person according to some criteria (Schraw & Moshman, 1995). Metacognitive self-regulation, on the other hand, has many superior features such as setting effective goals for students, using self-confidence and metacognitive strategies more effectively to achieve these goals, awareness of their cognitive features, knowing how to learn and using different learning strategies effectively, following and evaluating their learning processes (Pintrich, 2000; Risemberg & Zimmerman, 1992; Schraw & Moshman, 1995). Metacognition also has essential effects on self-regulated learning (Puustinen & Pulkkinen, 2001) and academic performance (Mega et al., 2014; Zohar & Peled, 2008).

Metacognitive knowledge in mathematics expresses the students' ideas about mathematical processes, techniques, and the nature of mathematics (Özsoy, 2011). Positive effects of metacognition on mathematics performance have been found (Desoete et al., 2001; Efklides & Vlachopoulos, 2012; Özsoy, 2011; Schneider & Artelt, 2010). Desoete et al. (2001) determined that metacognitive knowledge and skills constitute 37% of mathematical problem-solving performance. Kramarski and Revach (2009) investigated the effects of self-regulated learning education given to mathematics teachers. The results of the observation analysis showed that teachers who received self-regulated learning education performed more teaching practices that encourage students' understanding and support their regulation of learning. Rozen and Kramarski (2014) developed self-regulated learning activities, including metacognitive regulation and motivational-emotional regulation and conducted two experimental intervention studies. As a result, the metacognitive self-regulation group showed the best performance in metacognitive self-regulation compared to the other groups. Tian et al. (2018) found that metacognitive knowledge, self-efficacy, and intrinsic motivation significantly predicted math performance.

Studies are also carried out on metacognitive skills and problem-posing. For example, studies have been conducted to develop a theoretical framework that includes developing problem-posing activities for metacognitive awareness (Ghasempour & Baker, 2012) and presenting examples of different studies to improve students' metacognitive skills (Ghasempour et al., 2013). It has been found that the problem-posing approach improves the metacognitive awareness (Akben, 2020), and research-based teaching supported by metacognitive strategies improves students' problem-solving and posing skills (Divrik et al., 2020). Ding and Shen (2008) examined the relationships between metacognition level, achievement, and mathematical problem-posing skills. As a result of the study, it was found that middle school students' metacognitive knowledge level was high, metacognitive experience level was low, metacognitive monitoring skill was relatively weak, and mathematical problem-posing level was found to below. Also, it was revealed that there are significant differences in mathematical problem-posing skills of students with upper, middle, and low metacognitive levels. Karnain et al. (2014) examined the metacognitive skills of 21 middle school students during problem-posing activities. It was determined that the students used planning and monitoring equally among the metacognitive skills consisting of planning, monitoring, and evaluation types, and those who combined these metacognitive skills showed higher monitoring levels. However, no study investigating the mediating role of metacognitive skills in the relationship between problem-posing and mathematics achievement has not been found in the literature. Therefore, this research will contribute to the field.

1.2. Purpose of The Research

This study aims to examine the mediating role of metacognitive self-regulation skills in the relationship between problem-posing skills and mathematics achievement. The hypothesis proposed in the study is as follows;

Hypothesis: Metacognitive self-regulation skills have a mediating role in the relationship between primary pre-service teachers' problem-posing skills and mathematics achievement.

2. Method

The method of the study is relational scanning type. Relational scanning aims to determine the presence and degree of change between two or more variables (Karasar, 2005). In the study, it was examined that the relationships between the mathematics achievement, problem-posing skills and metacognitive self-regulation skills of primary pre-service teachers and whether problem posing skills and metacognitive self-regulation skills have significant effects on mathematics achievement by using the relational screening method.

2.1. Participants

This study was conducted in the Faculty of Education of a university in the fall semester of the 2020-2021 academic year. Within the research scope, the data collection tools were applied to all pre-service teachers attended the Basic Mathematics and Mathematics Teaching I courses using random sampling method. 193 questionnaires were filled online by the pre-service primary teachers on a voluntary basis. However, when the data were examined, a total of 28 data, which were empty and incomplete, filled in twice and constituted extreme values, were detected and deleted. The data of the remaining 165 pre-service teachers were evaluated. 127 of the pre-service teachers are female (77 %), 38 are male (23 %). 67 of them are in first grade (40,6 %), 12 are in second grade (7,3 %), 70 are in third grade (42,4 %), 16 are in fourth grade (9,7 %).

2.2. Data Collection Tools

The data were collected with the metacognitive self-regulation scale and the problem-posing test consisting of semi-structured problem-posing questions. Pre-service teachers' final grades of the Basic Mathematics course were considered as the mathematics achievements of them. Since the pre-service teachers' Basic Mathematics final grades include the achievements in many mathematics subjects they learned during the term, it was thought to be a more comprehensive measure of mathematics achievement. Therefore, it was used in the research considering that the final grades express a more general success level.

2.2.1. Metacognitive self-regulation scale

The original scale was developed by Howard et al. (2000) to measure the metacognitive awareness and metacognitive self-regulation skills of students aged 12-18 in the process of mathematical and scientific problem-solving. For this purpose, the researchers combined the two scales previously developed for metacognition, and problem-solving. They conducted the validity and reliability studies and developed a new scale consisting of 35 items in five-point Likert type. The scale was adapted to Turkish by Çelik (2017), and a five-factor structure was obtained. These factors are knowledge of cognition, objectivity, problem representation, subtask monitoring, and evaluation. As a result of the confirmatory factor analysis performed in this study, the fit indices of the scale were found to be $\chi 2(501, N=165)=666,812$; $p=0,000$; CFI=0,916; TLI=0,900; IFI=0,920; RMSEA=0,045; SRMR=0,0755. In the adaptation study, the Cronbach Alpha coefficient of the scale was calculated as 0,91. The Cronbach Alpha reliability of the scale in this study was also found as 0,91.

2.2.2. Problem-posing test

The problem-posing test consists of five semi-structured problem-posing questions that measure the pre-service primary teachers' ability to pose word problems in natural numbers. The problem-posing test aimed to measure pre-service teachers' ability to pose routine problems at fourth-grade level. Therefore, in the test, solutions including mathematical operations in natural numbers were given, and the pre-service teachers were asked to pose problems that require these solutions. The solutions given in the problem posing test were asked according to the order in the mathematics program (Ministry of National Education [MoNE], 2018):

1.   Posing two-step and then three-step problems that require addition and subtraction in three and four-digit numbers,

2.   Posing two-step problems involving addition and then subtraction that require multiplication of two-digit numbers,

3.   Posing a two-step problem that requires division in three-digit numbers

The solutions that are included in the problem-posing test are as follows:

1. $350 + (1000 - 475) = ?$

2. $(6500 + 2750) - (1350 + 2370) = ?$

3. $(18 \times 3) + 75 = ?$

4. $2100 - (65 \times 20) = ?$

5. $(560 : 80) + 5 = ?$

The problem-posing test was presented to an expert from Mathematics Teaching Department to evaluate the suitability of the solutions to fourth grade level and mathematics program. After the problem-posing test was arranged in line with expert opinions, it was used in the study.

2.3. Collection of Data

The data of the study were collected online due to distance education. The pre-service teachers were asked to complete the metacognitive self-regulation scale and problem-posing test on a voluntary basis. While sharing the scales, the purpose of the study was explained; they were asked to individually pose problems for the procedures in the problem-posing test and leave blank questions that they could not pose a problem. Ethical approval of the study was obtained with the decision of the Ondokuz Mayıs Ethics Committee dated 26.02.2021 and numbered 2021/186.

2.4. Data Analysis

In the problem-posing test, pre-service teachers' posed problems were analyzed using the rubric developed within the research scope. The rubric was developed by examining the relevant studies, and criteria were determined to evaluate the semi-structured problem-posing activities. The posed problems were evaluated according to the criteria of mathematical accuracy (posing the problem correctly and being mathematically correct), suitability to the given solution (including all the operations given in the solution), and comprehensibility (being correct in terms of language and expression). The posed problems were evaluated separately by the researcher and an expert working in mathematics education. The differences were determined by comparing the results. Then, by exchanging views, results were concluded. So the scoring was finalized. The rubric used in the evaluation of the posed problems is presented as ANNEX.

The data was examined primarily in terms of missing and extreme values. After 28 data were discarded, the data were analyzed in terms of normal distribution, and it was observed that it was normally distributed (for Shapiro Wilk and Kolmogorov-Smirnov tests; p>0,05). Then, descriptive statistics of the data and correlations between variables were calculated. In the next step, measurement model was created and tested with Confirmatory Factor Analysis (CFA). In the measurement model, the problem-posing was the latent variable, and the scores of five questions in the problem-posing test were assigned as the observed variables of problem-posing. Besides, observed variables of the metacognitive self-regulation scale were created by item parceling. With item parceling, latent variables are represented with reliable and valid indicators, so the reliability of the data increases and the model fits better (Bandalos, 2002; Little et al., 2002, Little et al., 2013). A balancing approach was used in item parcels. Exploratory factor analysis was performed according to the balancing approach, and the items were ordered in descending order according to factor loadings and distributed to parcels. Each time, the distribution order was applied in reverse order to ensure a balanced distribution of the items across parcels (Güler & Çetin, 2019; Little et al., 2002). In this way, five parcels were created, and the observed variables of metacognitive self-regulation were formed by naming parcel1, parcel2, parcel3, parcel4, and parcel 5. The path diagram for the measurement model is presented in Figure 1.

After the analysis of the measurement model, a hypothetical model consisting of two latent (metacognitive self-regulation and problem-posing) and 11 observed variables (math achievement - MA, prb1, prb2, prb3, prb4, prb5, parcel1, parcel2, parcel3, parcel4, and parcel 5) was established and tested with the Structural Equation Model (SEM). SEM enables the creation of multiple data sets based on the research sample with bootstrap analysis and the estimation of parameters and fit indices through these data sets. Bootstrap analysis results generate averages of model fit indices and parameters based on all data sets (Tam et al., 2019). This process produces valid results even in asymmetrically distributed data, and small samples (Briggs, 2006; Hoyle & Kenny, 1999; Ichikawa & Konishi, 1997; Preacher & Hayes, 2004). Multivariate extreme values were examined with the Mahalanobis coefficient, and it was found that they were not included in the data set. Mardia's multivariate normality test showed that the data were distributed normally (critical ratio=5,115). Besides, bootstrap analysis was used for the significance of direct and indirect effects. The significance of direct and indirect effects and confidence intervals were examined by increasing the sample size to 5000. SPSS 17.0 and AMOS programs were used for analysis.

3. Results

Descriptive statistics of the variables examined in the study and Pearson Correlation coefficients between them are presented in Table 1.

Table 1. *Descriptive statistics and Pearson Correlation coefficients*

		M	SD	Range	Mathematics Achievement	Metacognitive Self-Regulation
1	Mathematics Achievement	3,37	0,71	4,00		
2	Metacognitive Self-Regulation	128,95	14,91	74,00	,179*	
3	Problem-posing	25,53	3,28	24,00	,664**	-,013

*p<0,05   **p<0,01

In Table 1, it is seen that there is a significant low level relationship (r=0,179; p<0,05) between mathematics achievement and metacognitive self-regulation, and a significant

medium level (r=0,664; p<0,01) relationship between mathematics achievement and problem-posing. The relationship between metacognitive self-regulation and problem-posing is not significant (p>0,05).

To examine the factor structure of the hypothetical model proposed in the study, a measurement model was created and analyzed with CFA. The fit indices obtained as a result of the CFA are at an excellent and acceptable level, χ2 (34, N=165)=62,346; p=0,002; CFI=0,968; NFI=0,932; TLI=0,957; RMSEA=0,071; SRMR=0,0645 (Hu & Bentler, 1999; Özdamar, 2017; Tabachnick & Fidell, 2001). The measurement model is presented in Figure 1.
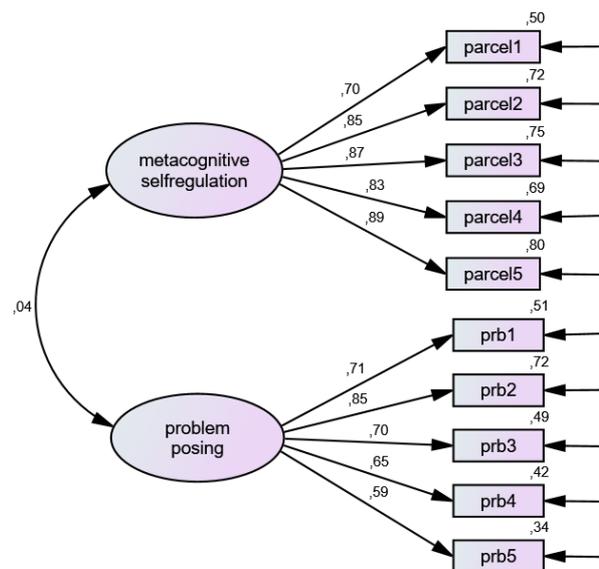

Figure 1. The measurement model

Then, the hypothetical model, which includes the mediating role of metacognitive self-regulation in the relationship between mathematics achievement and problem-posing was tested. Analysis results showed that the hypothetical model's fit indexes were good and excellent; χ2 (42, N=165)=74,147; p=0,002; CFI=0,967; NFI=0,928; TLI=0,957; RMSEA=0,068; SRMR=0,0597 (Hu & Bentler, 1999; Özdamar, 2017; Tabachnick & Fidell, 2001). The hypothetical model is seen in Figure 2.
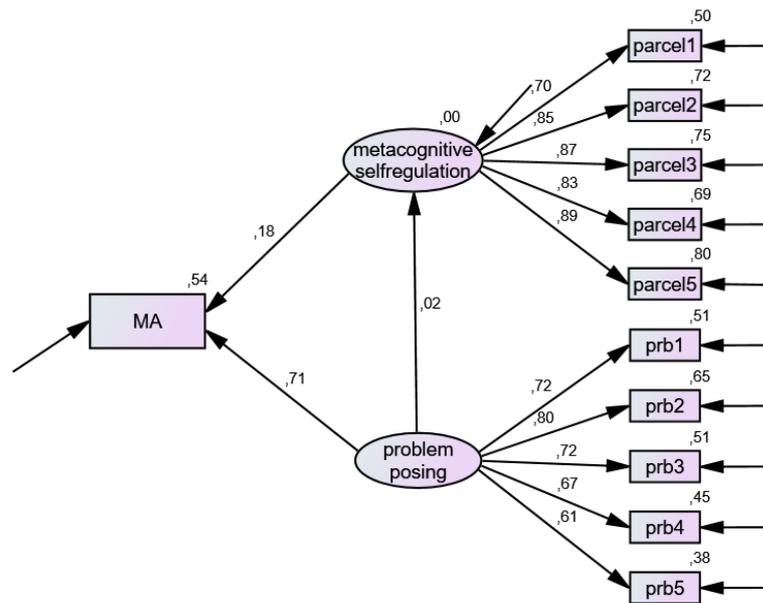
Figure 2. Hypothetical model proposed in the study

Analysis results showed that the path coefficients from the latent variables in the model to the indicators were significant (metacognitive self-regulation: range=0,70 – 0,89; problem-posing: range=0,61 – 0,80). As a result of SEM analysis, it was revealed that both problem-posing (β=0,71; p<0,001) and metacognitive self-regulation (β=0,18; p<0,001) had significant direct effects on mathematics achievement. However, the effect of problem-posing on metacognitive self-regulation was not significant (β=0,02; p>0,05). Therefore, there is no mediating effect of metacognitive self-regulation in the relationship between problem-posing and mathematics achievement.

The significance of the direct effects in the model was evaluated by bootstrap analysis. The results were analyzed according to whether the lower and upper limits of the confidence intervals of direct effect estimates contain zero; if it does not contain zero, that direct effect is interpreted as meaningful (Shrout & Bolger, 2002). The coefficients and confidence intervals of the direct effects resulting from the Bootstrap analysis are presented in Table 2. As it can be seen in the table, the direct effects of problem-posing (bootstrap coefficient=0,71; 95% CI=[0,58-0,82]) and metacognitive self-regulation (bootstrap coefficient=0,18; 95% CI=[0,04-0,30]) on mathematics achievement is significant. However, the direct effect of problem-posing on metacognitive self-regulation is not significant (bootstrap coefficient=0,02; 95% CI=[-0,21-0,29]). These results revealed that the mediating role of metacognitive self-regulation in the relationship between problem-posing and mathematics achievement was not significant. Therefore, the hypothesis that "metacognitive self-regulation skills have a mediating role in the relationship between primary pre-service teachers' problem-posing skills and mathematics achievement " was rejected.

Table 2. *Bootstrap analysis results of the model*

		Coefficients	% 95 Confidence Interval	
			Lower	Upper
Direct	Problem posing – MA	0,71	0,58	0,82
	Metacognitive self-regulation – MA	0,18	0,04	0,30
	Problem posing – Metacognitive self-regulation	0,02	-0,21	0,29
Indirect	Problem posing – Metacognitive self-regulation – MA	0,004	-0,04	0,05

4. Discussion and Conclusion

Because of the crucial effects of metacognition and problem-posing on mathematics performance, both are essential skills that should be focused on in mathematics education (Desoete et al., 2001; Kilpatrick, 1987). In this study, the mediating role of metacognitive self-regulation in the relationship between problem-posing and mathematics achievement was examined, and data analysis results revealed that metacognitive self-regulation did not have a mediating role in this relationship. In the literature, there is no study examining the mediating effect of metacognitive self-regulation in the relationship between problem-posing skills and mathematics achievement, but its mediating effect on problem-solving skills has been investigated. Hassan and Rahman (2017) examined the mediating role of metacognitive awareness in the relationship between middle school students' problem-solving skills and their mathematical achievement. As a result of the research, it was found that metacognitive awareness had a mediating role on this relationship, and students' problem-solving skills affected their mathematics achievement through metacognitive awareness. The results of the research are not consistent with this study. This result may be due to the difference in sub-dimensions of metacognitive skills (metacognitive self-regulation and awareness) in the studies, or it may be because problem-posing is a different skill than problem-solving. While problem-solving mainly includes the generalization thinking skill, problem-posing includes the generative thinking skill (Cai & Hwang, 2002). It is understood that the power of metacognition and its sub-dimensions to predict these thinking skills, which dominate the problem-solving and posing processes, is different, but more research is needed on this subject.

In the study, it was concluded that both problem-posing and metacognitive self-regulation significantly predicted mathematics achievement. Similarly, Ding and Shen (2008) found significant relationships between metacognition, achievement, and mathematical problem-posing skills. Mirzaei et al. (2012) found that metacognition predicts mathematics achievement. Besides, studies show that education supported by metacognitive strategies improves students 'academic performance (Divrik et al., 2020; Zohar & Peled, 2008), and problem-posing approach improves students' problem-solving and metacognitive awareness (Akben, 2020). Moreover, it has been found that metacognitive experiences have a mediating effect between metacognitive knowledge and problem-solving performance (Aşık & Ertkin, 2019). Studies on this subject point to the multifaceted relationships between problem-solving, posing, mathematics achievement and metacognitive skills. It is understood that the achievements of students in these fields mutually affect each other. In this context, it is essential for teachers to include their students in problem-posing activities through metacognitive approaches (Ghasempour et al., 2013).

As a result of the research, the significant effects of problem-posing and metacognitive self-regulation on mathematics achievement were revealed. Therefore, educating students about metacognitive skills will increase their mathematics achievement. Also, including more problem-posing activities in mathematics classes is vital for students' mathematics performance. It was also found that metacognitive self-regulation does not mediate the relationship between problem-posing and mathematics achievement, but more research is needed in this field. Studies investigating these relationships in different samples and education levels will reveal the network of relationships between these variables and guide educators and researchers.

References

Akben, N. (2020). Effects of the problem-posing approach on students' problem solving skills and metacognitive awareness in science education. *Research in Science Education, 50*(3), 1143-1165. https://doi.org/10.1007/s11165-018-9726-7

Aşık, G., & Erktin, E. (2019). Metacognitive experiences: Mediating the relationship between metacognitive knowledge and problem solving. *Education and Science, 44*(197), 85-103. http://dx.doi.org/10.15390/EB.2019.7199

Bandalos, D. L. (2002). The effects of item parceling on goodness-of-fit and parameter estimate bias in structural equation modeling. *Structural Equation Modeling, 9,* 78-102. https://doi.org/10.1207/S15328007SEM0901_5

Bonotto, C. (2006). Extending students' understanding of decimal numbers via realistic mathematical modeling and problem-posing. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics* (2, pp. 193–200). Charles University.

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics, 83*(1), 37-55. https://doi.org/10.1007/s10649-012-9441-7

Borkowski, J. G., Estrada, M. T., Milstead, M., & Hale, C. A. (1989). General problem-solving skills: Relations between metacognition and strategic processing. *Learning Disability Quarterly, 12*(1), 57-70. https://doi.org/10.2307/1510252

Briggs, N. (2006). *Estimation of the standard error and confidence interval of the indirect effect in multiple mediator models.* [Unpublished doctoral dissertation]. Ohio State University.

Brown, A. L. (1987). Metacognition, executive control, self-regulation, and other mysterious mechanisms. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 65-116). Erlbaum.

Cai, J. (1997). An investigation of US and Chinese students' mathematical problem posing and problem solving. *Mathematics Education Research Journal, 10*(1), 37-50. https://doi.org/10.1007/BF03217121

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: an exploratory study. *International journal of mathematical education in science and technology, 34*(5), 719-737. https://doi.org/10.1080/00207390310001595401

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *The Journal of mathematical behavior, 21*(4), 401-421. https://doi.org/10.1016/S0732-3123(02)00142-6

Cankoy, O., & Darbaz, S. (2010). Effect of a problem posing based problem solving instruction on understanding problem. *H. U. Journal of Education, 38*(38), 11-24.

Chang, N. (2007). Responsibilities of a teacher in a harmonic cycle of problem solving and problem posing. *Early Childhood Education Journal, 34*(4), 265-271. https://doi.org/10.1007/s10643-006-0117-8

Chen, L., Van Dooren, W., & Verschaffel, L. (2015). Enhancing the development of Chinese fifth-graders' problem-posing and problem-solving abilities, beliefs, and attitudes: A

design experiment. F. M. Singer, N. F. Ellerton & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp.309-329), Springer.

Çelik, E. (2017). Metacognitive self-regulation scale: the reliability and validity study of the Turkish form. *Studies in Psychology*, *37*(1), 53-71.

Davidson, J. E., Deuser, R., & Sternberg, R. J. (1994). The role of metacognition in problem solving. In R. Metcalfe & A. P. Shimamura (Eds.), *Metacognition: Knowing about knowing* (pp. 207-226). The MIT Press.

Desoete, A., Roeyers, H., and Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *J. Learn. Disabil. 34*, 435–449. https://doi.org/10.1177/00222194040370010601

Ding, Y. Y., & Shen, Z. J. (2008). A study on relationship of metacognition level, achievement and mathematical problem-posing ability. *Journal of Anhui Radio & TV University, 1*.

Divrik, R., Pilten, P., & Tas, A. M. (2020). Effect of inquiry-based learning method supported by metacognitive strategies on fourth-grade students' problem-solving and problem-posing skills: A mixed methods research. *International Electronic Journal of Elementary Education, 13(*2), 287-308.

Efklides, A., & Vlachopoulos, S. P. (2012). Measurement of metacognitive knowledge of self, task, and strategies in mathematics. *European Journal of Psychological Assessment, 28*(3), 227-239. https://doi.org/10.1027/1015-5759/a000145

English, L. (1997) Promoting a problem-posing classroom. *Teaching Children Mathematics, 3,* 172-179. https://doi.org/10.5951/TCM.4.3.0172

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive–developmental inquiry. *American psychologist, 34*(10), 906. https://doi.org/10.1037/0003-066X.34.10.906

Ghasempour, Z., Bakar, N., & Jahanshahloo, G. R. (2013). Innovation in teaching and learning through problem posing tasks and metacognitive strategies. *International journal of pedagogical innovations, 1*(1), 53-62. http://dx.doi.org/10.12785/IJPI/010108

Ghasempour, Z., & Baker, M. N. (2012). Pedagogical perspective on problem posing and metacognitive. In S. N. S. Hassan & N. Bakar (Eds.), *International Conference on Active Learning* (pp. 53-58), Universiti Teknikal Malaysia Melaka.

Gonzales, N. A. (1994). Problem posing: A neglected component in mathematics courses for prospective elementary and middle school teachers. *School Science and Mathematics, 94*(2), 78-84. https://doi.org/10.1111/j.1949-8594.1994.tb12295.x

Grundmeier, T. A. (2003). *The effects of providing mathematical problem posing experiences for K-8 pre-service teachers: Investigating teachers' beliefs' and characteristics of posed problems* [Unpublished doctoral dissertation]. University of New Hampshire.

Güler, M., & Çetin, F. (2020). Item parceling in organizational behavior studies: method and application. *Ömer Halisdemir University Academic Review of Economics and Administrative Sciences, 13*(1), 61-71. https://doi.org/10.25287/ohuiibf.660689

Hartman, H. J. (1998). Metacognition in teaching and learning: An introduction. *Instructional Science − International Journal of Learning and Cognition, 26*,1−3.

Hassan, N. M., & Rahman, S. (2017). Problem solving skills, metacognitive awareness, and mathematics achievement: A mediation model. *The New Educational Review, 49*(3), 201-212. doi: 10.15804/tner.2017.49.3.16

Howard, B., McGee, S., Shia, R., & Hong, N. (2000). Metacognitive self- regulation and problem solving: Expanding the theory base through factor analysis. Paper presented at *The Annual Meeting of the American Educational Research Association.* New Orleans. [ERIC Ed 470973].

Hoyle, R. H. & Kenny, D. A. (1999). Sample size, reliability, and tests of statistical mediation. In R. H. Hoyle (Ed.), *Statistical strategies for small sample research* (pp. 195–222). Sage.

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling, 6*(1), 1-55. doi:10.1080/10705519909540118

Ichikawa, M., & Konishi, S. (1997). Bootstrap tests for the goodness of fit in factor analysis. *Behaviormetrika, 24*(1), 27-38. https://doi.org/10.2333/bhmk.24.27

Karasar, N. (2005). *Bilimsel araştırma yöntemi [Scientific research method]*. Nobel Yayın Dağıtım.

Karnain, T., Bakar, M. N., Siamakani, S. Y. M., Mohammadikia, H., & Candra, M. (2014). Exploring the metacognitive skills of secondary school students' use during problem posing. *Sains Humanika, 67*(1), 27-32. https://doi.org/10.11113/sh.v67n1.121

Kilpatrick, J. (1987). Where do good problems come from?. In A. H. Schoenfeld, (Ed), *Cognitive science and mathematics education*, (pp. 123-148). Lawrence Erlbaum Associates, Inc., Publishers.

Kramarski, B., & Revach, T. (2009). The challenge of self-regulated learning in mathematics teachers' professional training. *Educational Studies in Mathematics, 72*(3), 379-399. https://doi.org/10.1007/s10649-009-9204-2

Little, T. D., Cunningham, W. A., Shahar, G., & Widaman, K. F. (2002). To parcel or not to parcel: Exploring the question, weighing the merits. *Structural Equation Modeling, 9*, 151-173. https://doi.org/10.1207/S15328007SEM0902_1

Little, T. D., Rhemtulla, M., Gibson, K., & Schoemann, A. (2013). Why the items versus parcels controversy needn't be one. *Psychological Methods. 18*(3), 285-300. https://doi.org/10.1037/a0033266

Lowrie, T. (2002). Designing a framework for problem posing: young children generating open-ended tasks. *Contemporary Issues in Early Childhood, 3*(3), 354-364. https://doi.org/10.2304/ciec.2002.3.3.4

Mayer, R. E. (1998). Cognitive, metacognitive, and motivational aspects of problem solving. *Instructional science, 26*(1), 49-63. https://doi.org/10.1023/A:1003088013286

Mega, C., Ronconi, L., & De Beni, R. (2014). What makes a good student? How emotions, self-regulated learning, and motivation contribute to academic achievement. *J. Educ. Psychol. 106*, 121–131. https://doi.org/10.1037/a0033546

Ministry of National Education [MoNE], (2018). *Mathematics lesson curriculum (primary and secondary school 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th grades)*. http://mufredat.meb.gov.tr/Dosyalar/201813017165445-

MATEMAT%C4%B0K%20%C3%96%C4%9ERET%C4%B0M%20PROGRAMI%20 2018v.pdf

Mirzaei, F., Phang, F. A., Sulaiman, S., Kashefi, H., & Ismail, Z. (2012). Mastery goals, performance goals, students' beliefs and academic success: Metacognition as a mediator. *Procedia-Social and Behavioral Sciences, 46,* 3603-3608. https://doi.org/10.1016/j.sbspro.2012.06.113

National Council of Teachers of Mathematics [NCTM], (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Özdamar, K. (2017). *Ölçek ve test geliştirme yapısal eşitlik modellemesi [Scale and test development structural equation modeling]*. Nisan Kitabevi.

Özsoy, G. (2011). An investigation of the relationship between metacognition and mathematics achievement. *Asia Pac. Educ. Rev. 12,* 227–235. https://doi.org/10.1007/s12564-010-9129-6

Pintrich, P. R. (2000). The role of goal orientation in self-regulated learning. In M. Boekarts, P. R. Pintrich, & M. Zeidner (Ed.), *Handbook of self-regulation* (pp. 451-495). Academic Press.

Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in simple mediation models. *Behavior Research Methods, Instruments & Computers, 36*, 717–731. https://doi.org/10.3758/BF03206553

Puustinen, M., & Pulkkinen, L. (2001). Models of self-regulated learning: a review. *Scand. J. Educ. Res. 45*, 269–286. https://doi.org/10.1080/00313830120074206

Risemberg, R., & Zimmerman, B. J. (1992). Self-regulated learning in gifted students. *Roeper review, 15*(2), 98-101. https://doi.org/10.1080/02783199209553476

Rozen, T., M., & Kramarski, B. (2014). Metacognition, motivation and emotions: Contribution of self-regulated learning to solving mathematical problems. *Global Education Review, 1*(4), 76-95.

Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. *ZDM, 42,* 149–161. https://doi.org/10.1007/s11858-010-0240-2

Schraw, G., & Moshman, D. (1995). Metacognitive theories. *Educational Psychology Review, 7*(4), 351-371.

Shrout, P.E., & Bolger, N. (2002). Mediation in experimental and nonexperimental studies: New procedures and recommendations. *Psychological Methods, 7*(4), 422–445. https://doi.org/10.1037/1082-989X.7.4.422

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics, 14*(1), 19-28.

Silver E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school. *Journal for Research in Mathematics Education, 27*(5), 521-539. https://doi.org/10.5951/jresematheduc.27.5.0521

Şengül S., & Katrancı Y. (2015). The analysis of the problems posed by prospective mathematics teachers about 'ratio and proportion' subject. *Procedia - Social and Behavioral Sciences, 174*, 1364 – 1370. https://doi.org/10.1016/j.sbspro.2015.01.760

Tabachnick, B. G., & Fidell, L. S. (2001). *Using multivariate statistics (4th ed.).* Allyn & Bacon.

Tam, Y. P., Wong, T. T. Y., & Chan, W. W. L. (2019). The relation between spatial skills and mathematical abilities: The mediating role of mental number line representation. *Contemporary Educational Psychology, 56,* 14-24. https://doi.org/10.1016/j.cedpsych.2018.10.007

Tian, Y., Fang, Y., & Li, J. (2018). The effect of metacognitive knowledge on mathematics performance in self-regulated learning framework—multiple mediation of self-efficacy and motivation. *Frontiers in psychology, 9,* 2518. https://doi.org/10.3389/fpsyg.2018.02518

Van Harpen, X. Y., & Sriraman, B. (2013). Creativity and mathematical problem posing: An analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics, 82*(2), 201-221. https://doi.org/10.1007/s10649-012-9419-5

Zohar, A., & Peled, B. (2008). The effects of explicit teaching of metastrategic knowledge on low- and high-achieving students. *Learn. Instr. 18,* 337–353. https://doi.org/10.1016/j.learninstruc.2007.07.001

ANNEX. The rubric used to evaluate the posed problems

	Mathematical Accuracy			Suitability to The Given Solution			Comprehensibility			Total
	Not a Problem 0 score	There are shortcomings 1 score	Correct problem 2 scores	Not posed for the given solution 0 score	Includes some of what is given 1 score	Including all the given operations 2 scores	Insufficient 0 score	partially sufficient 1 score	Sufficient 2 scores	
Problem 1										
Problem 2										
Problem 3										
Problem 4										
Problem 5										