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ERRORS AND MISCONCEPTIONS OF EIGHTH-GRADE STUDENTS REGARDING OPERATIONS WITH ALGEBRAIC EXPRESSIONS[1]

Research article

(Corresponding author)
Funda Aydın-Güç https://orcid.org/0000-0002-3922-017X
Giresun University
fundaydin05@gmail.com & funda.guc@giresun.edu.tr

Derya Aygün https://orcid.org/0000-0003-3372-0920
Ministry of National Education
deryaaygun88@gmail.com

Funda Aydın-Güç, is an Assistant Professor of Mathematics Education at the Department of Mathematics and Science Education of Faculty of Education in Giresun University. Her research interests include problem solving, mathematical modeling, mathematics teacher education and mathematical misconceptions.

Derya Aygün, is a Mathematics Teacher and an expert in Mathematics Education in Giresun, Turkey. She is interested in mathematical language formation, concept cartoons and mathematical misconceptions.

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Funda Aydın-Güç

funda.guc@giresun.edu.tr

Derya Aygün

deryaaygun88@gmail.com

Abstract

The study aimed to determine the mistakes of students in operations with algebraic expressions, and their misconceptions that may lead to errors. The study adopted case study method, one of the qualitative research models. The participants were composed of 48 (24 boys, 24 girls) randomly selected among 8th grade students in three different classes from three different schools that were selected via convenience sampling method. To determine students' errors and misconceptions, the "Misconceptions Diagnostic Test for Operations with Algebraic Expressions" was developed by the researchers, considering the curriculum, relevant literature, and researchers' teaching experience. The diagnostic test included 10 open-ended questions. In addition, clinical interviews were conducted with all students on their wrong answers. Results indicated that the most common misconception was that the minus at the beginning of the algebraic expression had no meaning and that half of the students had this misconception. It was also observed that the number of students with the following misconceptions were close to each other: "The operation on one side of the equation should also be applied to the other side so that the equality is not broken", "we should take into account the order of operation priority in integers while writing the sentences as algebraic expressions," and "everything before the parenthesis is distributed to the parentheses." The reasons for the emergence of these misconceptions were found to be epistemological barriers and over-generalization of information in arithmetic.

Keywords: Algebraic expressions; algebraic operations; student errors; misconceptions

1. Introduction

Algebra is the language used to explain views and thoughts in any discipline (Sutherland and Rojano, 1993). Stacey and MacGregor (2001), in contrast, have defined it as a system used to describe the relationships between numbers and general rules. While individuals are analyzing many situations in daily life and explaining relationships, they unconsciously use algebra and algebraic thinking (Davidenko, 1997). For these reasons, algebraic thinking skill is among the important skills addressed in mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2000). When the primary mathematics curriculum published by the Ministry of National Education in Turkey [MNE] (2018) is examined, it has been observed that the algebra learning began in the sixth grade with the meaning of algebraic expressions and finding rules in number patterns, and students are expected to algebraically express the rule of the pattern to find numbers that are not given in number patterns. In the seventh grade, students are expected to be able to add and subtract algebraic expressions, multiply a natural number and an algebraic expression. They are also expected to solve a first-order unknown equation and related problems by understanding the concept of equality. In the eighth grade, students are asked to show algebraic expressions in different ways, multiply algebraic expressions, grasp the linear relationship, and make sense of inequalities. It is seen that the achievements of learning algebra end with the examination of the linear relationship and equation solutions. When the skills expected from students are grouped, it is seen that the following three basic skills are focused on: "Making sense of the concept of a variable, making operations with algebraic expressions, and working on equations and inequalities by making sense of the concept of equality." It is stated in the literature that algebraic thinking consists of three key topics: (1) the use of patterns that lead to generalizations (especially with operations), (2) the explanation of the change, and (3) the concept of function (Van de Walle, Karp, & Bay-Williams, 2019). It is emphasized that for students to be successful in algebraic thinking, they must have a deep understanding of the number system, operations, and properties associated with operations (Seeley & Schielack, 2007). Kaput (1999) highlights five different forms of algebraic reasoning:

1. Generalizing from arithmetic and patterns
2. Meaningful use of symbols
3. Study of structures in the number system
4. Studying functions and patterns
5. Using the mathematical modeling process by combining four items

As seen, the curriculum and literature emphasize that one of the important parts of algebraic thinking is the ability to make operations with algebraic expressions. In this study, the ability to make operations with algebraic expressions is discussed. Knowing what kind of understanding students have regarding algebraic expressions and operations, which are the basic components of algebraic thinking skills, is important in terms of revealing their learning environment needs.

1.1 Errors and Common Misconceptions Regarding Algebraic Expressions and Operations

The literature indicates that students who operate with algebraic expressions make many mistakes. For example, Hall (2002) states that some students trying to solve the equation $4x = 1$ perceive the crossing operation in the expression $4x$ as $4 + x$ and consider the constant number as the number that should be subtracted from the other side of the equation. Here, it is stated that the student matched the multiplication operation with another operation, addition. Matz (1982) asserts that the reason for this error is that students generalize their experience that the fraction $3\frac{3}{4}$ can be expressed as $3+\frac{3}{4}$. The student with this idea uses $x = 1-4$ to solve the equation. This *error* is called *the other inverse error*. It has been revealed by other studies (Akyüz & Hangül, 2014; Gomes & Jaques 2020; Kieran, 1992) that students make *the other inverse processing* errors. It is known that some students think that the solution sets of the equation $x + 37 = 150$ and the equation $x = 37 + 150$ are the same (Kieran, 1992). Here, the student thinks that he can change the addition process as he wishes. The change feature of the addition operation works incorrectly in the equation solution; this error is called a *switching addends error* of the additive. Researchers such as Hall (2002), Akyüz and Hangül (2014), and Larino (2018) state that there are students who have this error. The students also think that the solution set of the equation $x + 37 = 150$ and the solution set for the equation $x + 37-10 = 150 + 10$ are the same (Kieran, 1992). Here, students tend to add the number they subtract from one side to the other in order not to break the equality. In other words, they are mishandling the crossing method. This error is expressed as a *redistribution error*. Grouws (1992) also revealed that there are students with this error. Kieran (1992) states that some students solve the equation x / 2 + 3 = 5 as x + 3 = 10. Here, the student primarily deals with the division process, which is a randomly chosen process. This error stems from the student's blind application of the “change side-change sign” rule by making random choices among terms (Kiernan, 1992). This error is called a *reversal error*; this error has been demonstrated in other studies (Cohen & Kanim 2005; Hall, 2002; Rosnick & Clement, 1980; Tooher & Johnson, 2020; Yasseen, Yew, & Meng, 2020).

Some students show the solution approaches given in Figure 1 while solving the 3x−4 = x + 9 and -3x + 6 = 2x +16 equations (Vlassis, 2001).

$$
\begin{array}{c}
3x-4=x+9 \\
3x\ -\boxed{4-4}=x+9-4 \\
3x=x+5
\end{array}
\qquad\qquad
\begin{array}{c}
-3x+6=2x+16 \\
-2x \qquad\quad -2x \\
-\boxed{3x-2x}+6=\boxed{2x-2x}+16 \\
1x+6=16
\end{array}
$$

Figure 1. Solutions for the error of negligence of minus

Here, students ignore the minus at the beginning of the terms and solve the equation. This error is called the *neglect of minus* error. As seen, not being able to make sense of the negative sign prevents the success of solving equations (Booth & Koedinger, 2008). Different researchers also revealed that students made this mistake (Das, 2020; Draper & Lott, 2020; Gomes & Jaques 2020; Herscovics & Linchevski 1994; Vlassis, 2001). Conversely, students often use a reverse operation strategy when solving systems of univariate equations (Umanah, 2020); this causes some errors. Kieran (1984) affirms that while some students solved the x / 4 + 22 = 182 equations, they first reached the equation x / 4 = 160 by subtracting 22 from both sides of the equation, then converting the equation to x = 160/4 to reach the result. Kiernan (1984) states that students have a lot of experience in finding the inverse of addition; however, their experience of finding the inverse of subtraction, multiplication, and division is limited. He asserts that the reason for this error is the lack of experience in finding the opposite of operations other than addition. This error is called a *limited implementation of the inverse operation*. It was also revealed by other studies (Akyüz & Hangül, 2014; Oktaç, 2010) that students made the mistake of *applying reverse procedure in a limited way*.

The correct use of parentheses is extremely important for teaching algebra (Das, 2020). Tavsan (2020) revealed that students made the mistake of ignoring parentheses while interpreting algebraic expressions and writing them as sentences. It is known that some students do not consider the parenthesis while writing the verbal algebraic expression, and they write the algebraic expression corresponding to the sentence “Add n by 5, then multiply by 3” as “n + 5 · 3 = n + 15” (Şimşek & Soylu, 2018). The reason for this misconception is thought to be that students operate from left to right, without considering the priority of operation, while writing algebraic expressions (Şimşek & Soylu, 2018). This error is called an *equation error without considering the brackets*. It is known that some students consider it correct to write 6 (x + 3) as 6x + 3 (Perso, 1992). This mistake is considered as errors caused by arithmetic operations and is called failure to consider parentheses when performing operations. Here, students do not take into account the parenthesis and do not pay attention to the order of the operation. It was also revealed by other researchers (Akkaya & Durmuş, 2006; Erbaş, Çetinkaya, & Ersoy, 2009; Gürel & Okur 2017; Keşan & Akbulut, 2019) that students have this error.

As seen, different errors have been revealed in studies on algebraic expressions and operations. This is an indication that different students may have insights that can make different mistakes. Furthermore, researchers found that during their teaching experiences, the students made different mistakes than those stated in the literature. It is thought that systematically presenting these errors and determining whether they are caused by misconceptions or not will contribute to the field. In this context, the aim of this study is to determine the mistakes made by primary school eighth-grade students toward operations with algebraic expressions and misconceptions that lead to errors. The research questions of the study were formulated as in the following:

1. What causes eighth-grade students to make errors and misconceptions when performing operations with algebraic expressions?
2. What are the insights that lead students to make errors?

2. Method

The study adopted the case study method as a type of qualitative research because it aimed to summarize an existing situation via descriptive methodology due to its nature. A case study is used to identify and see the details that make up a situation, to develop possible explanations about a situation and to evaluate a situation (Gall, Gall & Borg, 2007). In the study, the misconceptions that the participant students had about algebraic expressions were determined, and no intervention was made to the existing situations.

2.1. Participants

The participants consisted of 48 students who were selected randomly and on voluntary bases in three classes from three different schools that were selected via convenience sampling method. Table 1 below shows the distribution of the participants' gender.

Table 1. *Distribution of participants' gender*

	Girls	Boys	Total
A	6	5	11
B	8	6	14
C	10	13	23
Total	24	24	48

When Table 1 is examined, it is seen that the distribution of the participants according to their gender was equal; this allows errors to be handled independently of the gender variable. Schools A and B were village schools. Students in the surrounding villages were also educated in these schools. Since there were no other schools for students to choose from, it can be said that the classes were heterogeneous according to their success level. While there was a very successful student in a class, there were also students with very low academic success. The success status of the schools in the Placement Exam held throughout the country was below average. School C was a school in a district center that also provided education through the bussing system. Since there were many schools in the district center, students were able to choose different schools. However, this did not change the class's heterogeneity. In the classes of this school, there are both very successful students and those with very low levels of success. The success level of the school in the Placement Exam held throughout the country was average.

The reason why these schools were chosen among the others in the village and district center was to include students with different levels of achievement in the study. Classes in schools in the city center were generally close to homogeneous classes formed by students from the same environment. This would damage the possibility of maximum sampling diversity, requiring that the obtained results should belong to a certain level of success.

The study was conducted in the spring semester of the 2017–2018 school year. It was ensured that all the classes to be included in the study had learned about operations with algebraic expressions. In other words, the participants had the readiness of the algebraic expressions and procedures required by the diagnostic test offered in the curriculum.

2.2. Data Collection Tool

One of the data collection tools of the research was the "Misconceptions Diagnostic Test Regarding Operations with Algebraic Expressions." The test development process was as follows:

1. The objectives of the curriculum (MoNE, 2018) were examined. The expected achievements in the curriculum about the operations with algebraic expressions in the learning area of algebra were as given in Table 2 below.

Table 2. *Achievements regarding operations with algebraic expressions (MoNE, 2018)*

Sixth grade	M.6.2.1.1. Writes an algebraic statement corresponding to a verbally given situation and a verbal case corresponding to a given algebraic statement.
Seventh grade	M.7.2.1.1. Makes addition and subtraction with algebraic expressions. Appropriate models are used in addition and subtraction with algebraic expressions.
	M.7.2.1.2. Multiplies a natural number by an algebraic expression.
Eighth grade	M.8.2.1.2. Makes the multiplication process of algebraic expressions.

2. In accordance with the expectations of the curriculum, errors and misconceptions in the literature about algebraic expressions were determined and the diagnostic tests related to the subject were examined. In line with these mistakes and misconceptions, questions were developed and added to the test.

3. Questions about faulty student insights that researchers noticed during their teaching experience and were not included in the literature were developed and added to the test. Detailed information is given in the relevant section.

4. To ensure the validity and reliability of the test, the opinions of three mathematics teachers with a master's degree in mathematics education and a specialist mathematics educator who has studies on misconceptions were consulted. Experts and mathematics teachers were asked to examine the questions in the test and to provide their opinions on issues such as comprehensibility, simplicity, whether they were aimed at revealing the desired misconceptions, and whether the items were necessary.

5. Essential arrangements have been made by taking into account the feedback on the comprehensibility of the questions. Experts did not give negative feedback on other aspects of the questions.

6. The "Misconceptions Diagnostic Test Regarding Operations with Algebraic Expressions" consisting of open-ended questions that can reveal misconceptions about algebraic expressions has been developed.

7. The pilot implementation of the test was conducted with 17 students who were not included in the main study. The answers given by the students were analyzed and it was examined whether the questions were understood or not and whether the errors and mistakes were revealed. The questions that caused misunderstanding and gave the impression that the goal was abandoned or that no student could answer because it was difficult for them were excluded from the test. Since the removed questions did not affect content validity, no new questions were added.

The final test consists of 10 open-ended questions aiming to identify errors and misconceptions about operations with algebraic expressions. The students' mistakes/misconceptions, questions about mistakes/misconceptions, and expected incorrect answers in the test are given in Table 3.

Table 3. *Student errors, examples of related acquisition, and test items*

Error/Misconception	**Question**	**Expected Incorrect Answers**
Other inverse error	If 6x = 12, find the value of x. (The number of questions has been changed by Hall (2002) to reveal the error.)	12−6 = 6 x = 6
Switching addends error	Find the value of x if x + 37 = 150. (Kieran, 1992)	x = 150 + 37 = 190
Redistribution error	x + 37−10 = 150 + 10 y + 37 = 150 Compare the x and y in their equations. (Kieran, 1992)	The solution set of these two equations is the same. So, x = y.
Reversal error	Find the solution set of the equation if x / 2 + 4 = 5. (Akyüz & Hangül, 2014)	x + 4 = 5.2 x + 4 = 10 x = 10−4 x = 6
Neglect of minus	In the equation -3x + 6 = 2x + 16, write the new equation obtained if -2x is added to both sides of the equation. (The number of the question given by Vlassis (2001) to reveal the relevant error has been changed.)	-3x−2x + 6 = 2x−2x + 16 x + 6 = 16 x = 10
	In the equation -3x + 6 = 2x + 16, write the new equation obtained by subtracting -2x from both sides of the equation. (The number of the question given by Vlassis (2001) to reveal the relevant error has been changed.)	-3x−2x = 2x−2x + 16 x = 16
Limited application of reverse operation	Find the solution set of the equation x / 4 + 22 = 182. (Oktaç, 2010)	x / 4 = 160 x = 160: 4 x = 40
An equation without considering the parentheses	Write the sentence “5 times more plus 2 the number of students in a class” as an algebraic expression. (The number of the question given by Şimşek and Soylu (2018) to reveal the relevant error has been changed.)	5x + 2
	Write the sentence “Half of the two more students in a class” as an algebraic expression.	x / 2 + 2
Ignoring parentheses when operating	Perform the operations stated in 7 + 5 (2x + 3).	14x + 21 + 10x + 15 24x + 36

When the 10-question “Diagnostic test of misconceptions about operations with algebraic expressions” is examined, it is seen that all questions are about the errors and misconceptions that exist in the literature. One researcher noted in his teaching experience that some students made mistakes similar to the error of *constructing equations without taking into account* the *braces* in the literature; however, this error was applied differently in mathematical sentences containing division. In this context, a question including splitting was added to the diagnostic test. Similarly, another researcher found that the students applied the error of *ignoring the parenthesis while performing the* procedure specified in the literature to all the operations in front of the brackets. In this context, a question with various operations before the brackets was added to the diagnostic test.

In the test, there is a question for each error or misconception. There are two questions about the error of "neglecting minus." The reason for this is to be able to determine errors for removing a variable with a coefficient of −2 from an algebraic expression in one question and adding a variable with a coefficient of −2 to the algebraic expression in the other. Since this type of error can occur in two different situations, a question has been added for each situation. Similarly, two questions were added about the error of "ignoring the parenthesis while making an operation." There is only one case in other errors or mistakes. Since it would be determined by clinical interviews with students whether the mistakes are caused by errors or misconceptions, asking one question for each error and misconception was not seen as an obstacle to reveal systematicity.

2.3. Implementation Process

The diagnostic test was administered to the participants by their teachers, and the students were given the time they needed. It was stated that the answers would not be evaluated as grades. Student answers were evaluated as correct, incorrect, and empty. Wrong answers were examined by two researchers, and unrelated answers were eliminated. Thus, student responses that may have been mistakes and misconceptions were determined. To determine whether the students' mistakes were due to misconception or not, a clinical interview was conducted with all the students participating in the study to solve the questions they made wrong.

During the clinical interviews, the students were asked to solve the questions they got wrong and to express their thoughts in detail during the solution process. No guidance was given to the students in the clinical interview. To enable them to express their thoughts clearly, the following questions were directed: "What is the reason for giving this answer?" "Why do you think this solution is correct?", and "How would the result change when the situation (the special case given in the question) change?" This process was recorded by the researcher conducting the interview.

2.4. Data Analysis

The records of clinical interviews were analyzed by two researchers, and the students' mistakes and misconceptions were tried to be revealed. In cases where a decision was not reached, the answer was re-negotiated with the student and the reason for the wrong answer was discussed and confirmed. Thus, analysis reliability was tried to be provided. In the following section, an example is given regarding the process carried out to serve as an example for the analysis.

The students were asked to write "5 times more of 2 plus the number of students in a class" as an algebraic expression. The answer given by S1 to the eighth question is as in Figure 2.

$$(x+2)5$$

Figure 2. S1's answer

As seen, the student determined the unknown that shows the number of students in the class as x, took two more than the number of students, and prioritized this process by using parentheses. He was then able to write the algebraic expression correctly by multiplying it by five. This answer of the student was coded as "correct." S38's answer to the same question is given in Figure 3.

$$(2.x).5$$
$$2x.5$$
$$x=10$$

Figure 3. S38's answer

As seen, the student tends to multiply everything without considering what is required in the sentence. Moreover, he tries to find the unknown instead of writing the algebraic expression. However, his actions are random. In this context, the student's answer was coded as irrelevant. S36's answer to the same question is illustrated in Figure 4.

$$x+2(.5)$$

Figure 4. S36's answer

As seen, the student thought the unknown representing the number of students in the class as x and took 2 more as requested in the question. However, instead of enclosing this expression in parentheses, he encloses the operation that indicates the fivefold in parentheses; this is the wrong approach. In addition, this answer points to the error of *constructing an equation without considering the brackets* in the literature. In this context, this answer was coded as incorrect, and a clinical interview was conducted with the student to determine the underlying understanding of the error. An example of the functioning of the interview can be examined in the process of S22. The answer given by S22 to the same question is shown in Figure 5.

$$x.5+2$$

Figure 5. S22's answer

As seen, the student thought of the unknown representing the number of students in the class as x. Instead of taking 2 more as requested in the question, he/she first took fivefold and then two more; this is also a wrong approach. Additionally, this answer points to a different error than the error of *equation without considering the brackets* in the literature. In this context, this answer was coded as incorrect, and a clinical interview was conducted with the student to determine the underlying understanding of the error. The interview between the researcher and S22 is as follows:

A: Why did you give this answer?

S22: Since we need to comply with the priority of the process, I first took the fivefold and then added 2.

A: How did you decide which action to come first?

S22: No matter what order in integers, we do the multiplication first and then addition.

When the student's answer and explanations regarding his answer are examined, it is seen that the student has generalized the operation priority rules in integers to the process of writing algebraic expressions of sentences. It is perceived that the student has the understanding that *we must take into account the order of operation priority in integers while writing the sentences as algebraic expressions*, and hence he has misspelled the given algebraic expressions. Therefore, this was coded as having misconception of *we should take into account the order of operation priority in integers while writing* an *algebraic expression*.

3. Results

The answers given by the students to the questions in the diagnostic test were examined and the answers for each question were analyzed as correct, incorrect, and empty. The distribution of student answers for each question is given in Table 4.

Table 4. *Distribution of student responses*

Misconception	Question	True	False	Null
Other inverse error	1	45	2	1
Switching addends errors	2	40	3	5
Redistribution error	3	30	11	7
Reversal error	4	20	14	14
Neglect of minus	5	34	11	3
	6	16	24	8
Limited application of reverse operation	7	19	17	12
Equation without considering the parentheses	8	30	13	5
	9	30	14	4
Ignoring braces when making an operation	10	20	20	8

When Table 4 is examined, it is seen that there are students who provide wrong answers to every question prepared for the errors in the literature. Since only one question is asked for each error or misconception, it is not known whether these errors stem from a systematic understanding. Moreover, it is not understood in detail whether the students gave incorrect answers for reasons other than the error or misconception that they tried to reveal. For this reason, those who gave wrong answers for each question were determined, and clinical interviews were conducted with these students based on their wrong answers, which made it possible to determine whether the mistakes made by the students were due to error or a systematic understanding. As a result of the clinical interviews conducted with the students, it was seen that some errors in the literature did not occur, and the questions prepared to reveal the errors in the literature allowed to reveal different misconceptions. In this context, the misconceptions revealed by sample student answers are given in the following section.

Different types of questions were asked in Questions 1, 2, 3, 4, and 7 of the diagnostic test to reveal the students' different errors and misconceptions. The common feature of these questions is that the solutions of the given equations are expected. When the answers are examined, it is seen that some students answer incorrectly for different reasons other than the errors and misconceptions that are expected to be revealed in these questions. For example, in Question 7, the solution set of the equation x / 4 + 22 = 182 was asked to be found. Although this question is designed for the error of *applying* the *reverse operation in a limited way* existed in the literature, it was observed that the students who gave the wrong answer had a different error than this one. As an example, S41's answer can be examined. The answer given by S41 is in Figure 6.

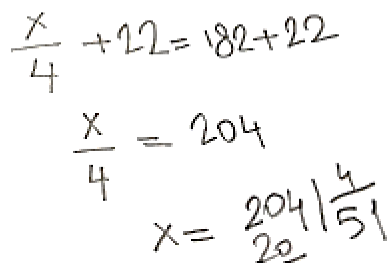

Figure 6. S41's answer to Question 7

The interview between the researcher and S41 is as follows;

A: Why did you give this answer?

S41: Because there are operations, I did them. It is divided by 4 on the left side and summed by 22. I did the same to the right.

A: What did you do this for?

S41: Because the teacher said, "What is done to the left must be done to the right, so that equality is not violated." I did the same on the left to the right.

When S41's solution and his explanations on the solution are examined, it is seen that the student thinks that he should do the operations on the left to the right in order to achieve the balance in equality. As seen, the student applies the operations that already exist in the equation, not for the operations added later to the equation. It is observed that the reason for his wrong answer to the question is the misconception that "the *operation on one side of the equation should be applied to the other side so that the equality is not broken*."

In Question 5, students were asked to add −2x to both sides of the given equation. The question was: Write the new equation obtained if −2x is added to both sides of the equation −3x + 6 = 2x + 16. When the answers are examined, some students expressed −3x + (−2x) + 6 as 5x + 6, and the equation 2x + (−2x) +16 as 4x + 16. For example, the answer given by S23 is presented in Figure 7.

Figure 7. S23's answer to Question 5

To clearly understand the reason for this mistake, an interview was conducted with S23 upon his answer. The interview between the researcher and S23 is as follows:

A: Why did you give this answer?

S23: I added 2x on both sides.

A: Can you tell us how did you do this?

S23: I gathered the unknowns. I wrote that when 3x and 2x are added, it is 5x. I wrote that when 2x and 2x are added, it is 4x.

A: What can you say about the signs at the beginning of the phrases?

S23: No need to pay attention to them, that's right.

As seen, the student does not care about the minus sign at the beginning of algebraic expressions and thinks that it has no meaning. The answer given by the student can be explained as in Figure 8.

$$-3x+6=2x+16$$

$$-\boxed{3x+(-2x)}+6=\boxed{2x+(-2x)}+16$$

$$5x+6=4x+16$$

Figure 8. Detailed solution of S23's understanding

In Question 6, which represents the other dimension of the same error, students were asked to subtract −2x from both sides of the given equation. The question was: Write the new equation obtained if −2x is subtracted from both sides of the equation −3x + 6 = 2x + 16. When the answers are examined, some students expressed −3x − (−2x) + 6 as x + 6 and the expression 2x − (−2x) +16 as +16. As an example, let's handle the answer of S23 again. The answer of S23 is illustrated in Figure 9.

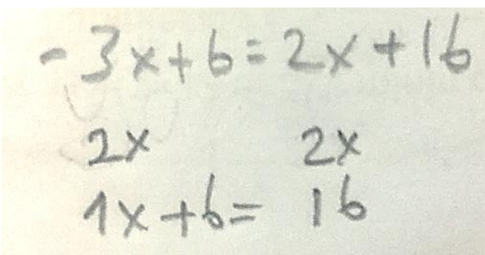

Figure 9. S23's answer to Question 6

An interview was conducted with S23 upon his answer to Question 6. The interview between the researcher and S23 is as follows:

A: Why did you give this answer?

S23: I subtract 2x from both sides.

A: Can you tell us how did you do this?

S23: 3x−2x so I subtracted 1x. When it is 2x−2x it becomes zero. We don't need to type zero, I just typed 16.

A: What can you say about the signs at the beginning of the phrases?

S23: As with the other question (pointing to Question 5), there is no need to pay attention to them.

As can be seen, the student does not care about the minus sign at the beginning of algebraic expressions and thinks that it has no meaning. The answer given by the student is explained in Figure 10.

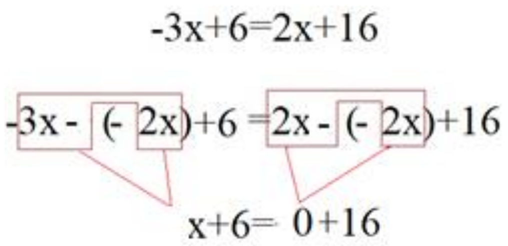

Figure 10. Detailed solution of S23's understanding

When examining both answers and the explanations made by the student on the answers, it is seen that he made the mistake of neglecting the minus and that this error is caused by the misconception that "the minus at the beginning of algebraic expression has no meaning."

When the wrong answers given by the students to Questions 8 and 9, which were put on the test to determine whether there were students who had the error of equation without considering the parenthesis, it was seen that the students showed similar approaches to the expected error. Question 8 was: Write "5 times more of 2 plus the number of students in a class" as an algebraic expression." As an example, the answer given by S22 to the question is in Figure 11.

Figure 11. S22's answer to Question 8

The interview between the researcher and S22 is as follows;

A: Why did you give this answer?

S22: Since we need to comply with the priority of the process, I first took the 5 times and then added 2.

A: How did you decide which action should come first?

S22: No matter what order in integers, we do the multiplication first, then addition.

S22’s answer to Question 9 for similar understanding is given in Figure 12. The question was: Write the sentence “2 plus the number of students in a class” as an algebraic statement.

**Sayı* means *number* in Turkish.

Figure 12. S22’s answer to Question 9

The interview between the researcher and S22 about Question 9 is as follows:

A: Why did you give this answer?

S22: Since we have to comply with the priority of the process, we must first find the half of the number and then add 2.

A: How did you decide which action should come first?

S22: Same as this one (showing his answer to the eighth question). In whole numbers, multiplication and division are done first, then addition and subtraction.

Examining the students’ answers and their explanations for these answers, it is seen that the students generalized the operation priority rules in integers to the process of writing algebraic expressions of sentences. It is observed that the students have the understanding that *we should consider the order of operation priority in integers while writing sentences as algebraic expressions* and therefore they misspell the algebraic expressions of the given the sentences.

In Question 10 that was included in the test to reveal the error of ignoring the parentheses during the operation, students distributed all the numbers in front of the parenthesis into the parentheses. The question was: “Do the operations in the expression $7 + 5 \cdot (2x + 3)$.” As an example, S39’s answer to Question 10 is given in Figure 13.

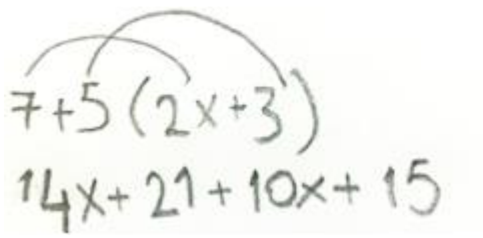

Figure 13. S39’s answer to Question 10

The interview between the researcher and S39 is as follows;

A: Why did you give this answer?

S39: I distributed the numbers in front of the parenthesis into the parentheses.

A: How did you determine the number that you should distribute into the parentheses?

S39: We distribute everything in front of the parentheses into the parentheses.

As seen, the student distributes all the operations in front of the parenthesis into the parenthesis in the expression $(7 + 5)(2x + 3)$, as well as in the expression $7 + 5(2x + 3)$. In other words, the student is over-generalizing. When the answers and explanations of the students are examined, it is seen that the wrong answer *is* due to the understanding *that everything before the parenthesis is distributed into the parentheses*.

The number of students with misconceptions and incorrect answers revealed as a result of clinical interviews conducted on incorrect answers given by students are provided in Table 5.

Table 5. *Misconceptions of students and frequencies*

Misconceptions	f	%
The minus at the beginning of the algebraic expression has no meaning.	24	50
The operation on one side of the equation should be applied to the other side so that equality is not broken.	18	38
When writing sentences as algebraic expressions, we must consider the order of precedence of integers.	14	29
Everything before the parentheses is distributed into the parentheses.	12	25

When Table 5 is examined, it is seen that students have four different misconceptions about algebraic expressions and operations. It is noteworthy that the most common misconception is *that the minus at the beginning of the algebraic expression has no meaning* and that half of the students have this misconception. The number of students who have the misconceptions *The operation on one side of the equation should also be applied to the other side so that the equality is not broken, we should take into account the order of operation priority in integers while writing the sentences as algebraic expressions,* and *everything before the parenthesis is distributed into the parentheses* are observed to be very close to each other.

4. Discussion, Conclusion, and Suggestions

As a result of the study, it was noted that the students had various misconceptions about operations with algebraic expressions. It was seen that the most common misconception was that "*minus at the beginning of algebraic expression has no meaning*." Half of the students participating in the study have this misconception. This result coincides with *the* error of *neglecting* the *minus* stated by Das (2020), Draper and Lott, (2020), Gomes and Jaques (2020), Herscovics and Linchevski (1994), and Vlassis (2001). Contrarily, in the present study, clinical interviews were conducted with students to demonstrate the understanding of the student who made this mistake. It is thought that the reason why students mostly have this misconception is the epistemological difficulties in learning the concept of "*negative*." There are also studies in the literature showing that students have difficulty operating with negative numbers (Booth & Koedinger, 2008; Vlassis, 2004). In this context, the epistemological obstacles to the learning of the concept of negative can be overcome by having classroom discussions about what negative numbers mean and how to remove them from a plurality.

When the answers given by the students to the questions regarding the solution of the given equations are examined, it can be observed that they often have misconceptions *that the operation on one side of the equation should be applied to the other side so that the equality is not broken*. This finding is similar to *the switching addends errors* of the provided by Kieran (1992), Hall (2002), Akyüz and Hangül (2014), Larino (2018) and the errors of *limited implementation of the reverse process* given by Kieran (1992), Oktaç (2010), Akyüz and Hangül (2014). Kieran (1992) states that the students who make the *switching addends* *errors* think that the solution sets of the equation x + 37 = 150 and the equation x = 37 + 150 are the same and that the addition can be replaced as they wish. Students who make this mistake use the change feature of the addition operation in the solution of the equation. Conversely, the students who make the mistake of *applying the inverse operation in a limited* way subtract 22 from both sides of the equation first while solving the equation x / 4 + 22 = 182 and reach the equation x / 4 = 160 correctly, eventually reaching the equation x = 160/4 (Kieran, 1992). In contrast, the current study found out that students systematically apply the same process to the other side of equality not for some of the processes in the question but for all of them. Similarly, Perso (1992) also revealed the results that students tend to apply the operation to the left of the equality to the right. As a result of the clinical interviews with the students, the students explained their mistakes as: "The operation to the left of the equation should be done to the right so that the equality is not broken. For example, there is an addition on the left, so we must add the same number to the right." It seems that students generalize their experience that operations added to the equation should be applied to both sides in order to not disrupt the equality in such a way that they also apply to operations that already exist in the equation. It is known that students have difficulty in adding and subtracting the same term from both sides while working on equality and inequality solutions (Cortes and Pfaff, 2000; Pomerantsev and Korosteleva, 2003; Şandır, Ubuz & Argün, 2007). The difficulties experienced during these procedures may have caused students to overgeneralize. In this context, classroom practices are recommended for students to conceptualize the concept of balance in equality.

As a result of the study, it is seen that some students have the misconception that *we should consider the order of operation priority in integers while writing sentences as algebraic expressions*; this misconception that has emerged is different from the error "equation is not taken into account without considering the parenthesis," which was put forward by Akkaya and Durmuş (2006); Erbaş, Çetinkaya, and Ersoy (2009); Gürel and Okur (2017); Keşan and Akbulut (2019); and Şimşek and Soylu (2018). The misconception presented in the current study does not stem from not considering parentheses but from writing a mathematical sentence algebraically expressing the operation priority of the operations in the sentence at the beginning. In other words, students generalize the priority of operations in arithmetic to algebraic expressions. In this context, while studying operations with algebraic expressions with the students, awareness of the validity of the rules can be provided by bringing together the cases where the operation priority rules in arithmetic are both valid and not valid.

As a result of the clinical interviews, it was concluded that the students had *the* misconception that *everything in front of the parenthesis is distributed into the parentheses*. It is known from the literature that some students think it correct to write 6 (x + 3) as 6x + 3 (Perso, 1992). The misconception obtained in the present study is different from the error presented by Perso (1992) who reveals that students ignore the parenthesis, while in the present study, students distribute all the operations before the parentheses into the parentheses rather than ignoring them. This situation indicates that students overgeneralize the dispersion feature and demonstrate that the misconception is transferred to operations with algebraic expressions due to the experiences in arithmetic operations. It is known that the lack of knowledge about arithmetic operations leads students to errors in algebraic expressions (Arnawa & Nita, 2019; Sarımanoğlu, 2019). In this context, it can be said that the characteristics and rules experienced in arithmetic lay the groundwork for misconceptions in algebra and that it is necessary to provide students working in arithmetic to prevent this. Teachers may be advised to choose examples that will draw attention to this misconception while providing their students with the experience of distribution in parentheses in arithmetic.

In addition, such operations can be matched with real-life situations, and the operations made might be turned into being meaningful.

When the literature is examined, it is seen that students make some mistakes while performing operations with algebraic expressions. In this study, it was aimed to reveal the student understanding underlying these mistakes. In this context, while trying to identify misconceptions, instead of increasing the number of identical questions to reveal the systematicity of student concepts, clinical interviews were conducted with all students who gave wrong answers. Clinical interviews have shown that although written responses point to existing errors in the literature, insights underlying the errors may differ. This reveals, once again, the role of clinical interviews in revealing the deep understanding underlying student insights. In contrast, it has been observed that questions designed for different purposes allow to reveal different misconceptions; this shows that further studies are needed to determine students' misconceptions. It reveals that different students in different sociocultural settings may have different misconceptions. Knowing the different misconceptions that students may have provides information about which conceptions teachers can encounter with students. Furthermore, the test developed in the study can be used by teachers to measure their students' pre-understanding. Therefore, it permits to include activities that will not allow for the misconceptions put forward in the design of learning environments or that will eliminate the existing misconceptions. It is thought that the misconceptions revealed as a result of the study will be very useful in this context.

Although the results obtained from this study reveal students' understanding of operations with algebraic expressions, there are some limitations that should be taken into account. The most important limitation of the study is that the clinical interviews were conducted only on wrong answers. This situation prevented the disclosure of misconceptions that directed students to the correct answer. Conversely, although it is not intended to generalize the results of the study, it should be taken into consideration that different results can be obtained from a larger group of participants. Another important limitation is that the results are restricted to the "operations with algebraic expressions" questions in the data collection tool. Different results can be obtained in different dimensions of algebraic expressions.

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