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Experimental and numerical evaluation of the mechanical behavior of diagonally reinforced plates subjected to the effect of residual thermal stresses

Evaluación experimental y numérica del comportamiento mecánico de placas con refuerzos diagonales sometidas al efecto de tensiones térmicas residuales

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ABSTRACT

This paper presents an experimental and numerical study of the effect of residual thermal stresses on the mechanical behavior of diagonally reinforced plates. The study focuses on the analysis of carbon/epoxy square plates onto which diagonal reinforcements were glued. These reinforcements were glued using two different methodologies: the first method was to glue the reinforcements at operating temperature (22 °C), while in the second methodology reinforcements were cured in an autoclave at 177 °C. Mechanical behavior assessment was based on the stiffness, free vibration, and buckling tests. For the study of the stiffness of the plates, an optical technique to determine the transversal displacement caused by the presence of a static load was employed. For the vibration tests, the natural frequencies associated with the first four modes of free vibration were determined by using a dynamic signal analyzer. For the linear and non-linear buckling, compression tests on a universal testing machine were performed, determining the displacements produced during the test using the digital image processing method. All experimental results were compared with results obtained from numerical approximations made with commercial software. The results show the effect of residual thermal stresses caused during the manufacturing process on the mechanical performance of diagonally reinforced plates.

Keywords: Thermal residual stresses, stiffness, free vibration, buckling, diagonally reinforced plates.

RESUMEN

Este artículo presenta el estudio experimental y numérico del efecto de las tensiones térmicas en el comportamiento mecánico de placas reforzadas.

El estudio se centra en el análisis de placas cuadradas de carbono/epoxi en las cuales fueron pegados refuerzos diagonales. Estos refuerzos fueron pegados siguiendo dos metodologías diferentes: la primera consistió en pegar los refuerzos a temperatura de servicio (22 °C), mientras que en la segunda, los refuerzos fueron curados en autoclave a temperatura de 177 °C. La evaluación del comportamiento mecánico se basó en la realización de ensayos de rigidez, vibración libre y pandeo. Para el estudio de la rigidez de las placas, se utilizó una técnica óptica que permite determinar de manera precisa los desplazamientos fuera del plano cuando una fuerza estática transversal es aplicada. Para los ensayos de vibración fueron determinadas las frecuencias naturales asociadas a los primeros cuatro modos de vibración libre mediante el uso de un analizador de señales dinámicos. Para los ensayos de pandeo, fueron realizados ensayos de compresión en una máquina universal de ensayos, determinando los desplazamientos producidos durante el ensayo por medio de un procedimiento de procesamiento digital de imágenes. Todos los resultados obtenidos experimentalmente fueron comparados con resultados logrados a partir de aproximaciones numéricas realizadas con software comercial. Los resultados muestran el efecto de las tensiones térmicas residuales que se originan durante la manufactura de las placas en el desempeño mecánico de placas reforzadas diagonalmente.

Palabras clave: Tensiones térmicas residuales, rigidez, vibración libre, placas con refuerzos diagonales.

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Introduction

The efficient design of composite structures requires proper study of the factors that directly or indirectly influence their performance and can even cause changes in the

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mechanical properties of the material, at both the micro- and the macro-mechanical level (Parveliet *et al.*, 2006). The strongly anisotropic character and the inhomogeneity of polymer laminates are factors that contribute to the appearance of residual stresses during the curing process of the material (Hosseini-Toudeshky & Mohammadi, 2009). The study of these stresses is all the more relevant because they can lead to negative effects not only on the strength of the material but also on its dimensional stability, and the material may experience overall premature failure, delamination, and fracture mechanics (Kim *et al.*, 2006). It is for this reason that recent research has focused on developing methods to modify the curing processes, seeking thereby to reduce residual thermal stresses in order to help improve the mechanical performance of the laminate (Kim *et al.*, 2012). Recently, reports show that differences in the thermal expansion coefficients of the constituent materials of composite laminates can generate large residual stresses during the curing process, reducing the fatigue resistance of the material. Kim *et al*. (2013) propose a monitoring system based on methods such as the dielectrometric method and the use of sensors based on Bragg networks, which allows monitoring and controlling cure cycles, thus achieving a reduction in the residual thermal stresses in reinforced carbon fiber composites.

It has been found that the use of materials with a high modulus of elasticity as filler for reinforcing polymeric matrices can improve the characteristics of composite laminates. In this regard, Shokrieh *et al*. (2014) evaluated the mechanical properties of polymeric composites into which carbon nanotubes were incorporated as additives. The results show a significant reduction in the magnitude of the stresses generated during the cure of the laminate, and thus they represent an appreciable improvement in mechanical performance.

Advanced studies indicate that the use of carbon nanotubes as reinforcing material can reduce the differences between the thermal expansion coefficients of the constituent materials of a polymeric composite. Godara *et al*. (2009) used carbon nanotubes to reinforce a composite formed by epoxy resin and carbon fibers, plotting profiles of the variation of viscosity as a function of temperature for different types of nanotubes. In this study, an important reduction of the value of the coefficients of thermal expansion and an increase in the fracture toughness of the material was reported.

Due to the difference between the thermal expansion coefficients of the fiber and the matrix, residual thermal stresses may be induced in the composite (Quek, 2004). Almeida and Hansen (1999) suggest that it is possible to take advantage of these stresses that arise due to of the curing of composite laminates in order to improve their mechanical properties, and they proved analytically that residual thermal stresses can be used to enhance important properties such as the first natural frequency of free vibration and the critical load of linear buckling for laminates with different types of reinforcements (Almeida & Hansen, 2002). One way to experimentally evaluate the residual thermal stresses in the mechanical performance of epoxy resin laminates reinforced with carbon fibers was introduced by Sánchez *et al*. (2016). This study evaluates the effect of residual thermal stresses on the buckling of plates reinforced around their perimeter, showing experimental and numerical values of the critical load of linear buckling mode shapes and features for the first three linear buckling modes. These results serve as a complement to a preliminary study presented by Sánchez and Almeida (2013), which constituted a first approach to the experimental characterization of composites reinforced with different types of reinforcements, reporting values for a statistical analysis of the flexibility test, values for the first frequency of free vibration, and preliminary results for the critical load of buckling for plates with reinforcements with different forms. These theoretical and experimental studies provided the motivation for this paper, which experimentally and numerically evaluates the effect of residual thermal stresses on the stiffness, vibration, and buckling of plates with diagonal reinforcements.

Finite element method (FEM) has been used in the analysis of laminated composite plates (Ochoa & Reddy, 1992). Zhang and Yang (2009) used the finite element method to analyze the mechanical behavior of laminates, specifically in the numerical determination of free vibration frequencies, the influence of geometrical inertia, buckling, post-buckling and, the failure modes. Numerical examples with different typologies of square plates have been recently presented. For instance, Tinh Quoc *et al*. (2011) analyzed several factors affecting the natural frequency of free vibration of the model as fiber orientations, lay-up number and stiffness ratios. Sadamoto *et al*. (2017) used the Galerkin mesh-free flat shell formulation for the buckling behavior of stiffness plates.

In carrying out this investigation, twelve plates with diagonal reinforcements were analyzed. The plates were manufactured according to the pre-impregnation method described by Besednjak (2005), using a base fabric consisting of four laminas with orientation [(0,90)] S, on which the reinforcements were placed. To cause the presence of thermal residual stresses, two groups of plates were produced, varying the methodology used in the bonding process and the curing of the diagonal reinforcements. For each of the study groups, the transversal displacement caused by the action of a static load, the natural frequencies for the first four modes of free vibration, and the behavior of the material under conditions of linear buckling were determined. To develop the experimental procedure for the stiffness and the buckling tests, digital processing of images obtained with optical equipment proposed by Fantin (1999) was applied. This experimental methodology allows accurate results that can be compared with results obtained from numerical models developed with the use of commercial finite-element software.

Experimental Procedure

Characteristics of the diagonally reinforced plates

For the manufacture of the plates, square laminates of carbon/epoxy of 354 × 354 mm, formed by a symmetrical laminate pre-preg with orientation [0/90]S, were used. On each side of the plates, diagonal reinforcements 35,4 mm wide and 354 mm long were glued. The laminate used as reinforcement consisted of four laminas with variable orientation according to their location. To form the plates for the experimental and numerical development, it was assumed that during the manufacturing process no internal forces occur in the plates. For this reason, the residual thermal stresses originate during the cooling to operating temperature, a stage in which an effect of residual tensile stress on the base laminate and residual compression stress occur on the laminate serving as diagonal reinforcement.

To evaluate the effect of the residual thermal stresses induced in the plates, two groups were analyzed. For the first group, six plates were fabricated in which the base fabric and the laminate used as reinforcement were manufactured separately in order to later attach them at the chosen operating temperature. This form of manufacture of the plates does not generate the presence of thermal stresses in the material, and was denoted as "DB". For the second group, six plates were developed in which both the reinforcement and the base fabric were processed in an autoclave at 177 °C simultaneously for subsequent cooling to operating temperature. With the methodology of the second group, residual stresses of thermal origin were induced in the material. For this second group, a "DC" nomenclature was adopted. The design and the location of the reinforcements are shown in Figure 1. The orientation of the laminas in both, the diagonal reinforcements and in the base laminate, are shown in Table 1, where CF refers to the base tissue and CT refers to the laminate used as lateral reinforcement.

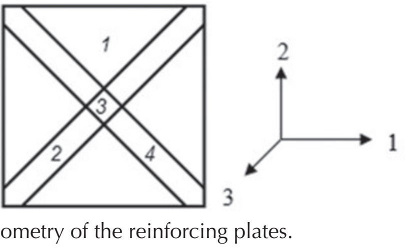

Figure 1. Geometry of the reinforcing plates.
Source: Authors

Table 1. Sequence of lamination

Region of laminate	Orientation
1	$[(0,90)^{CF}]_S$
2	$[45_4^{CT}/(0,90)^{CF}]_S$
3	$[(45,-45)_2^{CT}/(0,90)^{CF}]_S$
4	$[-45_4^{CT}/(0,90)^{CF}]_S$

Source: Authors

For the fabrication of the laminate, a basic fabric from the company "Hexcel Composites" with nomenclature "T7G190-12"-F584-21" was used. This denomination indicates the use of fibers of type "T7G190" immersed in a "F584-21" resin, which has a glass transition temperature between 121 °C and 171 °C. The mechanical properties of the materials from which the plates were manufactured are shown in Table 2.

Table 2. Mechanical properties of constituent's materials

Mechanical Properties	Tape	Fabric
Longitudinal modulus of elasticity, MPa	130100	66600
Poisson's coefficient	0,30	0,05
In plane shear modulus, MPa	5800	4600
Transversal shear modulus, MPa	3360	3360
Longitudinal thermal expansion coefficient, $0C^{-1}$	$-0,9 \times 10^{-6}$	$1,79 \times 10^{-6}$
Transversal thermal expansion coefficient, $0C^{-1}$	27×10^{-6}	$1,79 \times 10^{-6}$
Ply thickness, mm.	0,17	0,35
Density, g/cm^3	1,56	1,56

Source: Authors

Procedure for the stiffness tests

For the stiffness tests, an optical technique for the determination of the three-dimensional coordinates of the surface of the plate by means of digital image processing was used. This technique involves using a projector that reflects a striped pattern on the surface of the plate. Two digital cameras capture the projected pattern simultaneously. The captured images are divided into vertical and horizontal phase maps and are processed by software, from which the three-dimensional coordinates are obtained at any point located on the surface of the element.

For the stiffness tests, a metal frame in which the plate is simply supported at its corners was designed (see Figure 2). This test consists of applying a force of 10 N, for which two points for the application of the load were selected (see Figure 3). To obtain the displacement, the methodology described above was used (see Figure 4). For better contrast in the images, all plates were painted white. With the help of TPLA40012 software, four vertices were defined for each plate, considering the largest region that can be captured simultaneously by the two cameras (330 mm × 330 mm). Once the region of interest is selected, the software allows reducing noise, which can produce errors in the results. This methodology allows storing a number of surface maps for the same plate for different load conditions. Subtraction of two images obtained sequentially supplies the value of the transverse displacement caused by the force applied.

Procedure for free vibration tests

The vibration tests focused on the determination of the natural frequencies of free vibration. The tests were performed by exciting the plates through an impulsive force and measuring their response. To generate the excitation of the plates, a model PCB 086C03 hammer with a load cell adapted at its tip was used. The response of the specimens was measured using an accelerometer attached to the plates. The signals received by the accelerometer were sent to a dynamic signal conditioner model PCB 480E09 and were amplified and sent to a dynamic signal analyzer model HP 35665. This analyzer process allowed calculating the natural frequencies of free vibration for all the studied elements. This method of data collection is based on signal processing and the use of an algorithm based on fast Fourier transform (FFT).

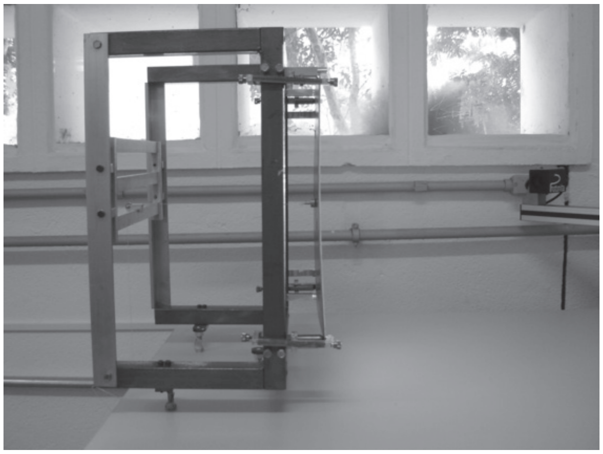

Figure 2. Supports used in stiffness test.
Source: Authors

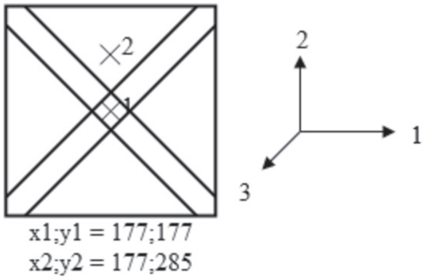

Figure 3. Points of application of the load in stiffness tests.
Source: Authors

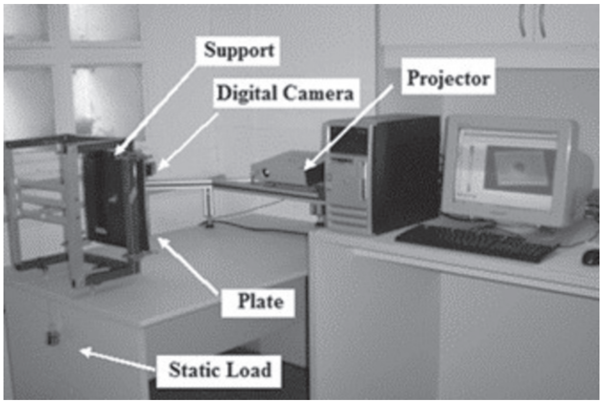

Figure 4. Experimental setup for stiffness tests.
Source: Authors

For test purposes, the plate was simply supported at the four corners. To avoid difficulties in capturing the free vibration modes, both the position of the accelerometer and the excitation point of the plate were selected such that no possible nodal lines coincided with the plate. Seeking greater accuracy in capturing the vibration modes and a decrease in noise, twelve consecutive repetitions were performed for each test. The experimental setup is shown in Figure 5.

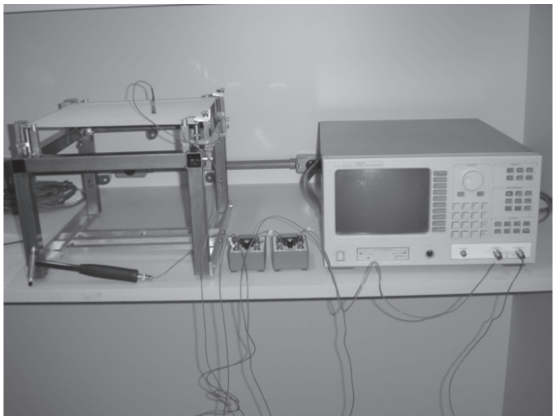

Figure 5. Experimental setup for vibration tests.
Source: Authors

Procedure for the buckling tests

For the buckling tests, a universal mechanical testing machine with a maximum capacity of 30 kN was used. To simulate the conditions of support of the plates, they were placed in a metal frame that was designed for this purpose (see Figure 6). A piece of aluminum was designed to couple the load cell to the metal frame in which the plates were supported.

As in the stiffness tests, the out-of-plane displacement was determined using an optical procedure. The experimental methodology selected involved the implementation of displacements applied sequentially at constant intervals of 0,2 mm at a speed of 1 mm/min. This methodology facilitated the capture of multiple images for each measurement interval.

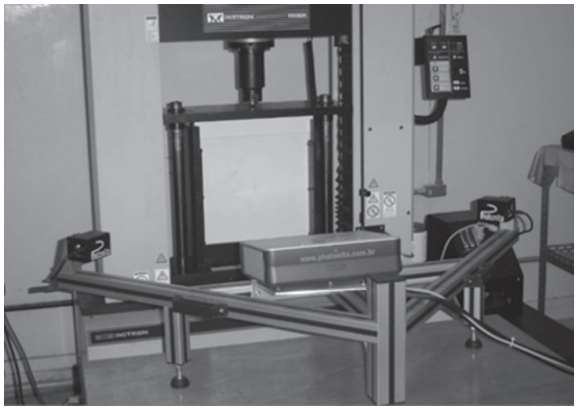

Figure 6. Experimental setup for buckling tests.
Source: Authors

Characteristics of numerical models

To verify the influence of the thermal residual stresses on the mechanical behavior of the plates, numerical models were developed according to the two established case studies. The numerical approximation was performed using the commercial program ABAQUS®. This software allows defining the orientation angle of the plies forming the plate and performs the assembly to create the laminate. For this composite, a single mesh can be defined because the plate is very thin and all the plies that compose it are of the same material.

A 3D model was used for simulating the behavior of diagonally reinforced plates. This type of model allowed carrying out the analysis of large displacements and the occurrence of out-of-plane strain in orthotropic laminates that have been subjected to a uniform temperature field without increasing processing time. For the elaboration of the finite element mesh, an element of S8R type, thin shell, standard, square, with 6 degrees of freedom per node was selected. The selection of the mesh was based on convergence tests, in which experimental results were verified during the stiffness, vibration, and buckling tests using structured meshes of 4000, 8000, 12000, and 16000 elements. Based on these results, and taking into account the processing time, it was decided to use a mesh of 4000 elements.

In this study, a plane-stress shell element having linear elastic behavior and an orthotropic constitutive law was adopted for modeling the diagonally reinforced plates. Mechanical properties such as the longitudinal modulus of elasticity, transverse modulus of elasticity, Poisson's coefficient, transverse shear modulus, longitudinal and transverse thermal expansion coefficients are required for implementing the numerical model. In ABAQUS®, linear elasticity in an orthotropic composite can be defined in terms of these engineering constants. The orthotropic plane-stress failure criterion was used as an indicator of failure of composite materials. These criterions can be used in conjunction with a linear elastic material model.

A very important factor in the development of the numerical model was the consideration of the initial geometric imperfections that the material possessed. The commercial program used in this study provides the ability to directly specify the initial imperfections in the geometry of the element studied, for which a first model of the plate with the help of CATIA® commercial software was developed, using as reference points the coordinates of the vertices of the plate obtained from the processing of the images supplied by digital image processing. The generated model was imported into ABAQUS® to simulate its mechanical behavior under the real conditions of the study.

Assuming that the mechanical properties of the constituent materials are not affected by temperature changes, and in order to induce the presence of residual thermal stresses in the numerical model that simulates the behavior of plates with residual thermal stress ("DC"), the analysis was divided into two stages:

Step 1: Determination of thermal residual stresses, for which the plate is not restricted at its vertices when it is subjected to a uniform temperature field.

Step 2: Verification of the mechanical behavior, considering the residual thermal stresses determined in step one.

The numerical verification of the stiffness of the plates was analyzed using a linear elastic analysis of a standard type, as defined in the commercial software used, varying the point of application of the load and its magnitude, in which the plate is considered to be simply supported at its four corners, as is shown in Figure 7.

For modeling the behavior of the plates when subjected to free vibration, the element was considered to be subjected to linear perturbation, from which the eigenvalues were determined using the Lanczos method pre-established in the software used. To calculate the residual thermal stress, a linear elastic analysis was used. The boundary conditions were the same as those used in the stiffness tests (see Figure 7).

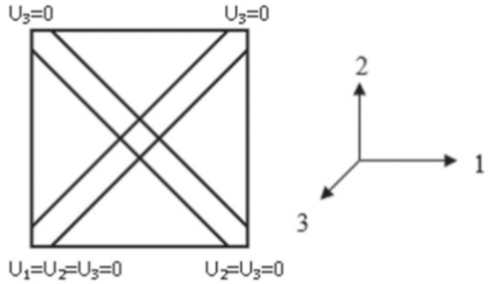

Figure 7. Boundary conditions used in the models of stiffness and free vibration.
Source: Authors

For the buckling analysis, an option of linear perturbation available in the software used was used. This type of analysis enables obtaining the response of the structure to linear problems, considering only the elastic properties of the material. The determination of the critical load of buckling is performed by the option "Predicting Buckling Eigenvalues" available in commercial software. As for the degrees of freedom, six degrees corresponding to displacements and rotations in all three axes of the composite were activated. For verification of the mechanical behavior for values that exceed the critical load of linear buckling, the modulus "Explicit Dynamic Analysis", whose considerations are predefined in the software, was used. For numerical simulation, the boundary conditions shown in Figure 8 were considered, where U represents the restrictions in the displacements and UR represents the restrictions in the rotations, while the subscripts x, y, and z are related to the reference system adopted.

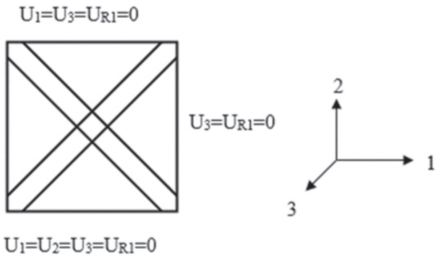

Figure 8. Boundary conditions used in the buckling model.
Source: Authors

To simulate the compressive load, the application of a unit force line around the top edge of the plate was adopted**.** This makes it possible to obtain the magnitude of the critical Euler load for the first three modes of linear buckling, multiplying the eigenvalue by the width of the plate.

Results and Discussion

The influence of residual thermal stresses on the stiffness of diagonal reinforcing plates was evaluated using 5 N and 10 N loads at the points indicated in Figure 3. The transverse displacements were obtained through digital image processing. An image of the resulting deformation in the plates is shown in Figure 9. It is noteworthy that on applying a load at point 1, located in the center of the plate, it can be seen that the presence of thermal residual stresses does not substantially affect the stiffness of the plate. This response is associated with the fact that this point is located directly on the reinforcement located in the center of the plate, this being an area of high stiffness that is caused by the design of the laminate itself. By applying the force at point 2, which is displaced 108 mm from the center of the plate relative to the y axis, it can be seen that the presence of residual thermal stress causes a decrease in the transverse displacement on the order of 10 % for plates to which a load of 5 N was applied and on the order of 24 % for loads of 10 N. These results demonstrate that the presence of thermal residual stresses causes an increase in the stiffness of the reinforcing plates. The results of the maximum and minimum displacements obtained in the stiffness test compared with numerical models are shown in Tables 3 and 4.

Figure 9. Result of the deformations that occur during stiffness test.
Source: Authors

Table 3. Transversal displacements when forces are applied in the center of the plates with diagonal reinforcement

Plate	Transversal displacement (mm)					
	Load (P): 5 N			Load (P): 10 N		
	w_{min}	w_{max}	Model	w_{min}	w_{max}	Model
DB	0,90	1,10	1,12	2,06	2,30	2,26
DC	0,92	1,10	1,12	2,05	2,30	2,08

Source: Authors

Table 4. Transversal displacements when static forces are applied to 108 mm from the center of the diagonal reinforcement plates

Plate	Transversal displacement (mm)					
	Load (P): 5 N			Load (P): 10 N		
	w_{min}	w_{max}	Model	w_{min}	w_{max}	Model
DB	1,01	1,39	1,68	2,55	3,15	2,49
DC	0,97	1,17	0,92	2,07	2,28	1,94

Source: Authors

An important factor in the analysis of the transversal displacement is the perception of effects of nonlinearity in the experimental and numerical results. These effects becomes evident if you notice that upon doubling the value of the applied load, the displacements increase to a lesser extent than does the increased load. This nonlinear behavior is more visible for plates that do not have thermal residual stresses, and it can be associated with the boundary conditions adopted.

To analyze the effect of residual thermal stresses on the natural frequencies of the diagonally reinforced plates, graphics of the variation of the frequency as a function of the amplitude were created through the results obtained with the dynamic signal analyzer. The results obtained for a representative "DB" type plate and a representative "DC" type plate are shown in Figure 10. Frequency peaks that characterize the first four modes of free vibration are listed for each type of plate.

From the frequency peaks shown in Figure10, it is possible to note that the presence of residual thermal stresses has a different effect on each of the first four modes of free vibration. The results show an increase of about 60 % for the first mode of free vibration in plates with residual thermal stress, while for the second, third, and fourth modes of free vibration, this increase resulted in about 20 %, 15 %, and 30 %, respectively.

To analyze possible variations in modal configurations, numerical models were developed, varying the temperature to which the plate is subjected. In the first stage of the numerical model, the plate was subjected to uniform temperature fields of 22 °C, 50 °C, 100 °C, and 177 °C, obtaining in each case the characteristic modal shape for the first four modes of free vibration. The results are shown in Figure 11, and they show significant variations in the modal form that characterized the first three modes of free vibration, principally for temperatures above 100 °C. As can be seen, the induction of residual thermal stresses affects not only the magnitude of the frequency but also the characteristic modal shape. However, it is important to note that depending on the applied temperature, each vibration mode can be affected differently. The results obtained for such plates show that the use of a high cure temperature in the manufacturing process of diagonally reinforced plates modifies both the value of the first frequency of free vibration and its characteristic mode shape. However, this behavior cannot be standardized for other modes of vibration, because as can be verified, it is possible that an increase will occur in the value of the frequency of free vibration without this leading to modification of the characteristic modal shape of the excited mode. The experimental and numerical results are consistent with the results presented by Almeida and Hansen (1999), and Sánchez and Almeida (2013).

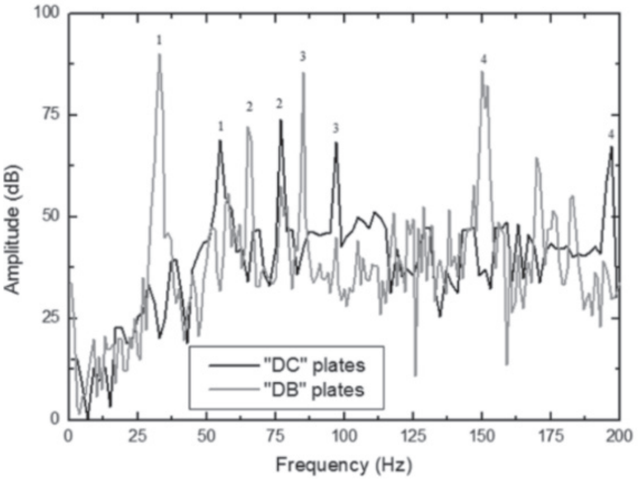

Figure 10. Frequency response obtained experimentally.
Source: Authors

To analyze the effect of thermal residual stresses on the linear buckling of diagonally reinforced plates, the Euler critical load was determined for plates of type "DB" and type "DC". The results can be compared with the results obtained from the numerical model approach and are shown in Table 5.

The characteristic modal shape of the first three modes of linear buckling was determined numerically by varying the temperature between 22 °C and 177 °C, which generates a temperature gradient of -155 °C. The results are shown in Figure 12. The results show that induction of thermal residual stresses causes not only a significant increase in the critical load linear buckling of the reinforced plates but also variations in the modal form that characterizes the first three buckling modes.

To evaluate the mechanical behavior of the plates at values exceeding the critical linear buckling load, profiles were performed, varying the transverse displacement along an imaginary horizontal axis passing through the center of the plate at a compression force of 2000 N. The average displacements obtained experimentally and numerically are shown in Figure 13 for the two types of plates studied. From these results, it is possible to observe that induction of residual thermal stresses causes an increase in the stiffness of the plates, which can be confirmed by the decrease in the transverse displacements that occurs in the plates that have residual thermal stresses for the same values of acting load.

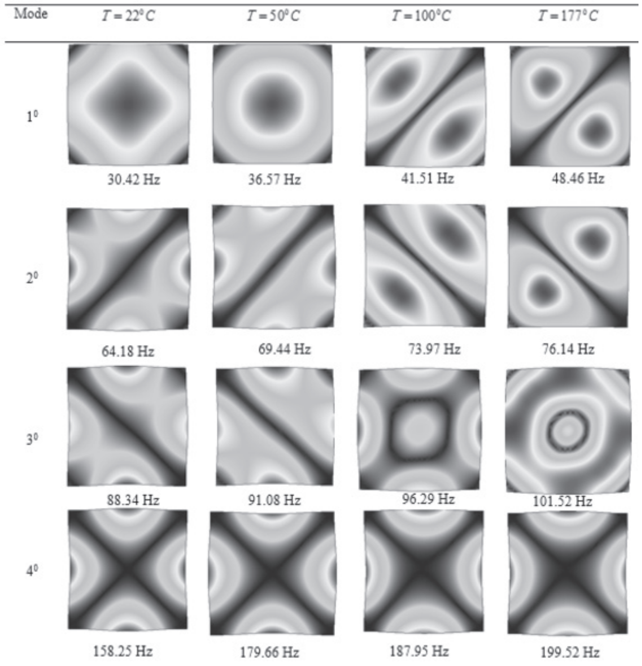

Figure 11. Modal characteristics forms obtained in vibration test. T is the temperature.
Source: Authors

Table 5. Critical load of linear buckling for diagonally reinforced plates. Comparison with results of the numerical model

Type of Plate	P (N) (Experimental)	P (N) (Model)
DB	320	310
DC	1100	905

Source: Authors

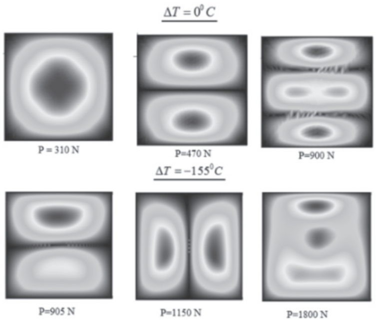

Figure 12. Modal forms for the first three modes of linear buckling.
Source: Authors

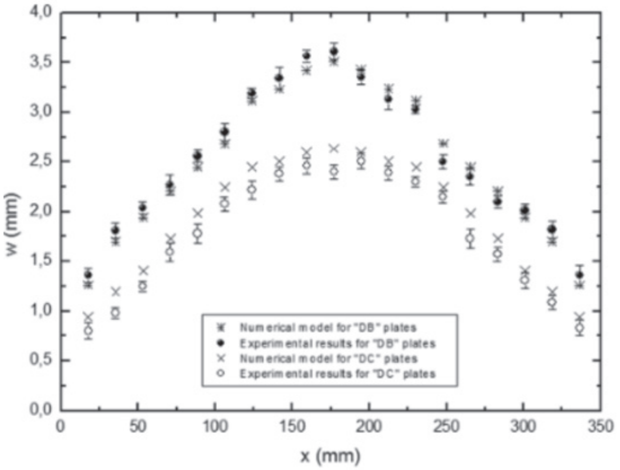

Figure 13. Variation of transverse displacement along the horizontal axis (x) for a load of 2000 N.
Source: Authors

Conclusions

This paper presents an experimental and numerical procedure to evaluate the effect of residual thermal stresses generated during the manufacturing process on the mechanical behavior of reinforced laminates, specifically analyzing their influence on stiffness, free vibration, and buckling behavior. These results show that upon simultaneously processing both the reinforcements and the base fabric in an autoclave raised to a temperature of 177 °C and then cooling them to operating temperature, a thermal gradient that globally affects the rigidity of the material is generated. The appearance of this gradient reduces the transversal displacement that can occur when a static load test is applied, increasing the natural frequencies of free vibration and increasing the linear buckling critical load.

From the assays of free vibration and linear buckling it can be seen that the presence of residual thermal stresses affects not only the magnitude of free vibration frequencies and the critical load of linear buckling but also the characteristic modal shapes. The results obtained from the analysis of the behavior of the plates when they receive an external excitation that causes a state of free vibration show that applying high temperatures during the curing phase can significantly increase the value of the first natural frequency of free vibration. However, this behavior cannot be generalized, because as was proven, each vibration mode can be affected differently.

On analyzing the critical load values obtained in the linear buckling test, they are shown to be consistent with those reported by other authors in previous research, showing that it is feasible to induce the presence of thermal residual stress in order to improve the mechanical performance of the laminate composites.

The results obtained for values of load above the critical load of linear buckling show that the use of high temperatures in the manufacturing process causes an increase in rigidity when the structure is in a state of unstable equilibrium. Similar results have recently been reported in the technical literature and allow one to conclude that it is possible to use this information in future studies that focus on the search for a suitable temperature for processing these materials in order to help improve their mechanical performance.

The use of digital image processing, a suitable experimental methodology, resulted in the development of assays of stiffness and buckling. The absence of contact between the surface of the plates and the optical system adopted avoided errors in determining displacement and enabled the rapid determination of the transversal displacement for a large number of defined points within the region of interest on the surface of the plates. Moreover, the capture of numerous phase maps (horizontal and vertical) simultaneously and further processing reduced sources of error associated with the initial geometric imperfections of the plates.

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