Modulating electrocardiographic signals with chaotic algorithms

Modulación de señales electrocardiográficas mediante algoritmos caóticos

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ABSTRACT

Chaos theory is becoming increasingly applied to areas like communications, telemedicine and processing signals and images. Algorithms generating chaotic signals can be used for generating, encrypting and encoding carriers. This work proposes a transmission method allowing electrocardiogram (ECG) signals obtained from a patient to be combined with algorithms generating chaotic signals, based on the Lorenz equation system. Acceptable results were obtained regarding noise and other traditional modulation methods.

Keywords: algorithm, chaotic signal, ECG signal.

RESUMEN

Las aplicaciones de la teoría del caos en ramas como las comunicaciones, la telemedicina y el procesamiento de señales e imágenes, son cada vez más frecuentes. Los algoritmos que generan señales caóticas pueden ser utilizados para la generación de portadoras, la encriptación y la codificación, entre otras aplicaciones. En este trabajo se plantea un método de transmisión que permite combinar señales electrocardiográficas (ECG) obtenidas de un paciente, con algoritmos que generan señales caóticas utilizando como base el sistema de ecuaciones de Lorenz. En este proceso se obtienen resultados aceptables frente al ruido y otros métodos de modulaciones tradicionales.

Palabras clave: algoritmos, ecuaciones caóticas, señal de electrocardiograma.

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Introduction

Chaos theory is concerned with the qualitative study of unstable aperiodic behavior in nonlinear, deterministic dynamic systems. Aperiodic behavior is observed when a variable reflects regular repetition of values describing system state.

Non-linearity is fundamental, especially when manifest in chaotic algorithms and equation systems (Barbará, Martinez, 2008).

This paper discusses the concept of chaos as a system of equations resulting from a deterministic process occurring in nonlinear and feedback systems.

This study was aimed at modulating and demodulating electrocardiographic signals using Lorenz model equations. The models used improve randomness without altering the original chaotic state; results are compared against noise and traditional modulation methods.

Model chaotic equations

The model for generating chaotic signals discussed in this research was related to the Lorenz model (Rodríguez, Alvarez, 2010) (González, Larrondo, 2006). Lorenz was interested in hydrodynamic equation solutions' predictability; the system was obtained by equations 1.1, 1.2 and 1.3:

$$\frac{dx}{dt} = \sigma(y - x) \tag{1.1}$$

$$\frac{dy}{dt} = rx - y - xz \tag{1.2}$$

$$\frac{dz}{dt} = xy - bz \tag{1.3}$$

The above system was a continuous system expressed in terms of derivatives of x, y and z, needing to be discretised for further processing an electrocardiographic signal. This was performed by first order Euler equations yielding a system as shown in 1.4, 1.5 and 1.6 (Blanchet, 2006), (Murali, 1993).

$$\tilde{X}_{n+1} = \tilde{X}_n + \left[-\sigma\left(\tilde{X}_n - \tilde{Y}_n\right)\right] \tag{1.4}$$

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$$\tilde{Y}_{n+1} = \tilde{Y}_n + k[-\tilde{X}_n\tilde{Z}_n + r\tilde{X}_n - \tilde{Y}_n] \tag{1.5}$$

$$\tilde{Z}_{n+1} = \tilde{Z}_n + k[\tilde{X}_n\tilde{Y}_n - b\tilde{Z}_n] \tag{1.6}$$

where k was a scalar parameter of time. The system was expressed in discrete form (i.e. a 3D map). The following transformations were applied to reduce complexity and elevation scale shown in equations 1.7, 1.8 and 1.9:

$$X_n = (\tilde{X}_n + B)S \tag{1.7}$$

$$Y_n = (\tilde{Y}_n + B)S \tag{1.8}$$

$$Z_n = (\tilde{Z}_n + B)S \tag{1.9}$$

where B and S were elevation and scaling parameters, respectively. Equations 1.10, 1.11 and 1.12 show the results:

$$X_{n+1} = X_n + k\sigma(Y_n - X_n) \tag{1.10}$$

$$\begin{aligned} Y_{n+1} = (1-k)Y_n + (kB + kr)X_n + kBZ_n \\ - {}^{k}\!/_{SX_nZ_n} + BS(k - kr - kB) \end{aligned} \tag{1.11}$$

$$\begin{aligned} Z_{n+1} = (1-kb)Z_n - kB(X_n + Y_n) + {}^{k}\!/_{SX_nY_n} \\ + BS(kB + kb) \end{aligned} \tag{1.12}$$

This study has adopted the following system simulation values:

$$k = \frac{1}{64};\ \sigma = 8; r = 24; b = 2; B = 40; S = 512$$

Applying these values to equations 1.10, 1.11, 1.12 resulted in 1.13, 1.14, and 1.15:

$$X_{n+1} = X_n + \frac{Y_n}{8} - \frac{X_n}{8} \tag{1.13}$$

$$\begin{aligned} Y_{n+1} = Y_n - \frac{Y_n}{64} + X_n + \frac{Z_n}{2} + \frac{Z_n}{8} - \left(\frac{X_n}{256}\right)\left(\frac{Z_n}{128}\right) \\ - 20160 \end{aligned} \tag{1.14}$$

$$\begin{aligned} Z_{n+1} = Z_n - \frac{Z_n}{32} + X_n - \frac{X_n + Y_n}{2} + \frac{X_n + Y_n}{8} \\ + \left(\frac{X_n}{256}\right)\left(\frac{Y_n}{128}\right) + 13440 \end{aligned} \tag{1.15}$$

The system of equations 1.13, 1.14 and 1.15 is known as 3D mapping where the modulating key comprises the variable's least significant Xn 8 bit (González and Larrondo, 2006; Sobhy and Sheata, 1997).

Transformations and elevation and arithmetic scales with natural numbers were used to simplify the computational implementation of this system of equations.

This map was structurally stable. Graphical analysis of equation 1.12, 1.13 and 1.14 system may be checked to ensure that it actually reaches chaos. This was simulated in Matlab mathematical assistant (V-7.10.0); initial condition values used were Xn=19400, Yn=21315, and Zn=32032. These values were selected as they belonged to the phase space in which the algorithm had chaotic behaviour (Figures 1 and 2).

The design and simulation of a electrocardiographic signal (ECG) modular system obtained from a real patient using chaotic variables Xn+1 and Zn+1 as carriers is described. These signals were generated using the system of equations 1.12, 1.13 and 1.14 shown above.

The block diagram in Figure 3 describes modulation by the algorithm for the disruption of a chaotic signal.

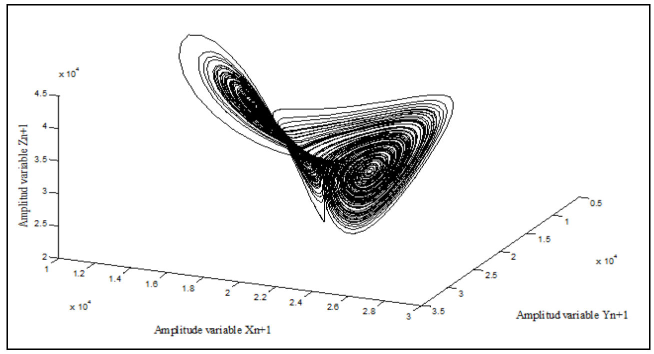

Figure 1. Variable Xn+1 chaotic system of equations corresponding to 1.13, 1.14 and 1.15

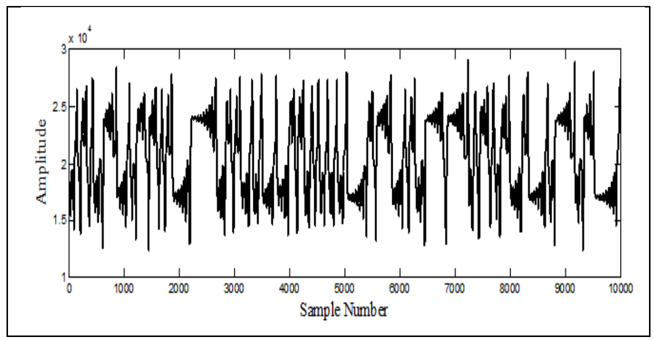

Figure 2. Fractal for the system of equations 1.13, 1.14, and 1.15.

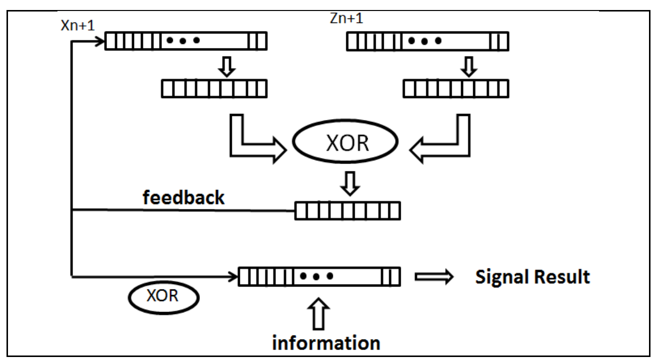

Figure 3. Schematic of modulation and chaotic disturbance generator

Variables Xn+1 and Zn+1 in this algorithm were taken with an initial 16 bit length; each 8 bit was taken from the bottom by a logical AND operation. An XOR followed and was stored in an 8 bit register. Such bits were then fed back again to the position where they were taken in variable Xn+1 to form a new 16 bit word (Figure 3). This feedback was an excitation to the system of equations forming the basis for the disturbance performed in each iteration of this algorithm. The chaotic signal had new behaviour from this moment on and it could be said that the Lorenz system had then been disturbed (González, Larrondo, 2006).

Figure 4 shows the chaotic behaviour of variable originally Xn+1 and then after being disturbed.

The variables' pattern differed as time elapsed; the correlation coefficients matrix obtained with Matlab for the Xn+1 and Xn+1 signal disturbed below shows that both signals had very low correlation (Yu, 2001; Murali, Lakshmanan, 1993).

$$\begin{matrix} 1.0000 & 0.0559 \\ 0.0559 & 1.0000 \end{matrix}$$

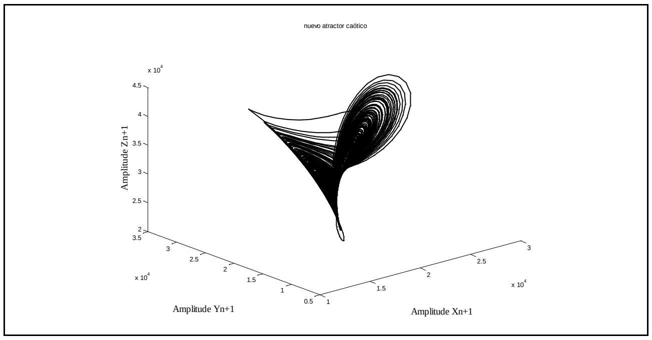

Figure 4. Signal Xn+1 in the upper and Xn+1 disturbed by the variable Zn+1

Although new signal Xn+1 pattern became modified, this continued to be chaotic and retained these systems' properties (González, 2006; Lawrence, 2000).

Figure 5 shows the Lorenz fractal generated from new modified signal Xn+1.

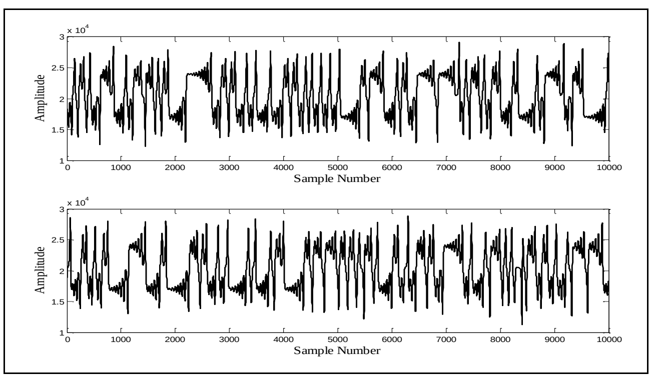

Figure 5. New Lorenz fractal generated from disturbed variable Xn+1 and other elements of the system of equations 1.13, 1.14 and 1.15

Modulation occurred when performing XOR between modified chaotic variable Xn+1 as described above. The ECG signal to be conveyed provided useful data or information in this case (Figure 6).

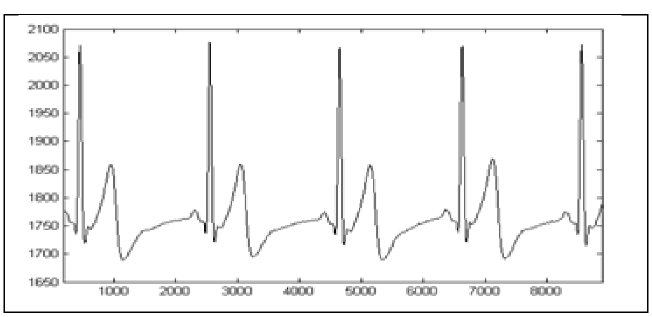

Figure 6. Fragment of the ECG signal used in the modulation

Figure 7 shows applied modulation between chaotic and ECG signal.

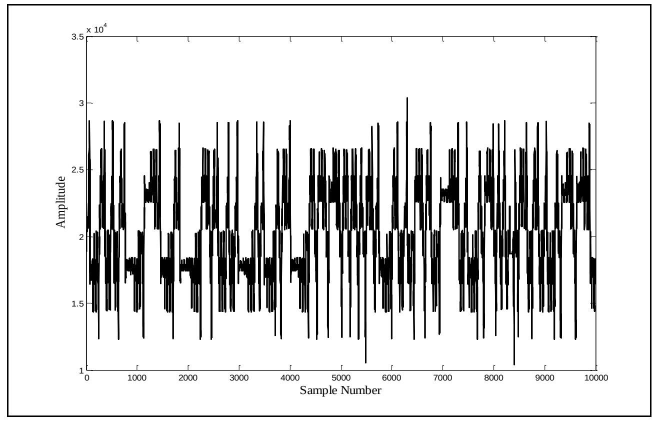

Figure 7. Modulation between the variable chaotic and ECG signal

Demodulation algorithm

The demodulation algorithm was similar to that shown in Figure 3, but in this case, the useful signal for ECG had to perform the XOR logic operation. The chaotic variable disturbed by the modulated signal was received, again obtaining a useful signal. It is worth noting that a chaotic system of equations used to transmit in the modulator must be generated exactly in the unit; this must be perfectly synchronised with the initial conditions mentioned above. Figure 8 shows the block diagram for this process.

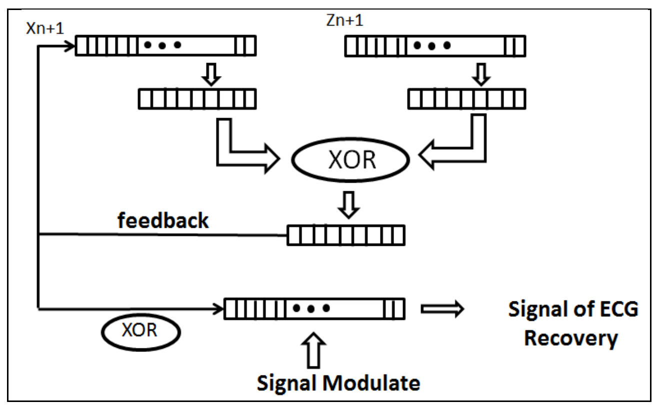

Figure 8. Demodulation algorithms for recovering the modulated ECG signal

Results and Discussion

This model's behaviour was analysed against noise and compared to traditional modulation method.

The simulations were performed in Matlab (Blanchet and Charbit, 2006) and the noise chosen for this process was additive white Gaussian noise (AWGN) as this is a mathematical model of noise resembling overall communications' channel features.

Recovery was simulated ECG signal for various signal to noise ratio (SNR) values. The signal to be modulated was contaminated with AWGN in a first case, maintaining SNR = 10 dB. The ECG signal recovered with the detection scheme shown in Figure 8 is illustrated in Figure 9.

Values were calculated from the matrix of coefficient correlation between original signal used in modulation and that recovered in the above conditions, resulting in:

$$\begin{matrix} 1.0000 & 0.7648 \\ 0.7648 & 1.0000 \end{matrix}$$

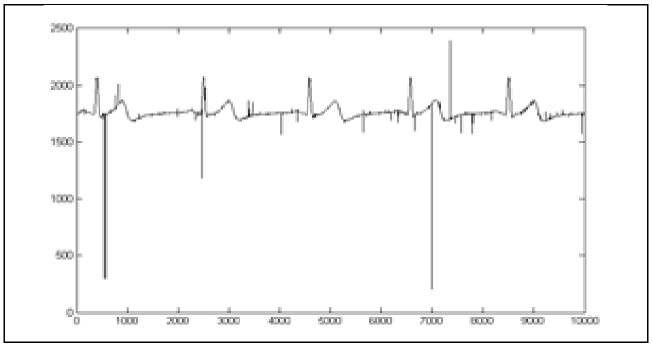

Figure 9. ECG signal demodulated by the proposed algorithm (SNR = 10 dB)

Recovery was also simulated for 15 dB and 18 dB SNR values (Figures 10 and 11, respectively.

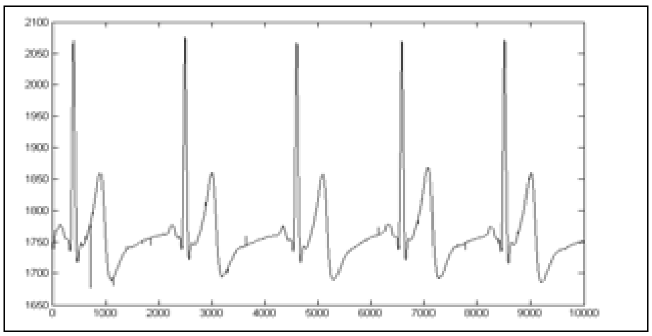

Figure 10. ECG signal demodulated by the proposed algorithm (SNR = 15 dB)

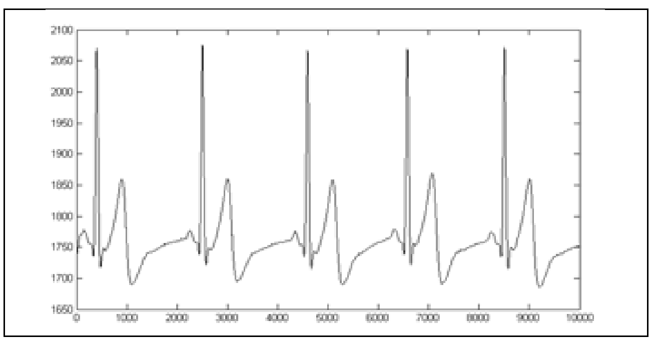

Figure 11. ECG signal demodulated by the proposed algorithm (SNR = 18 dB)

The matrix of correlation coefficients with the original signal for the demodulated signal having SNR = 18dB resulted in:

$$\begin{matrix} 1.0000 & 1.0000 \\ 1.0000 & 1.0000 \end{matrix}$$

Comparisons were also made with traditional modulation methods, such as frequency shift keying (FSK) digital modulation frequency. The ECG signal was modulated in this case by using 256 levels so that a symbol consisted of 8 bits. Initial separation between carrier frequencies was 185 Hz. This signal was contaminated with AWGN noise (SNR = 18 dB). A non-coherent FSK receiver was used for demodulation (Sklar, 2002), which is easy to implement and does not require synchronisation. The result was obtained by demodulating the signal (as shown in Figure 12).

The modulation described above involved a parameter regarding

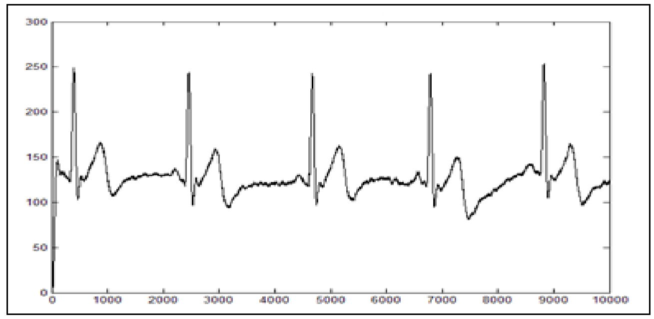

Figure 12. ECG signal demodulated with 256FSK, and 185 Hz separation between frequencies

separation between desired modular frequency tones; this parameter had an impact on signal demodulation. As it became smaller, separation between tones became reduced so that it became more difficult to identify amongst tones close proximity, and worsened detection. When frequency tones were spaced at 30 Hz, maintaining SNR = 18 dB, then it could be seen that the recovered ECG signal had noise levels (Figure 13).

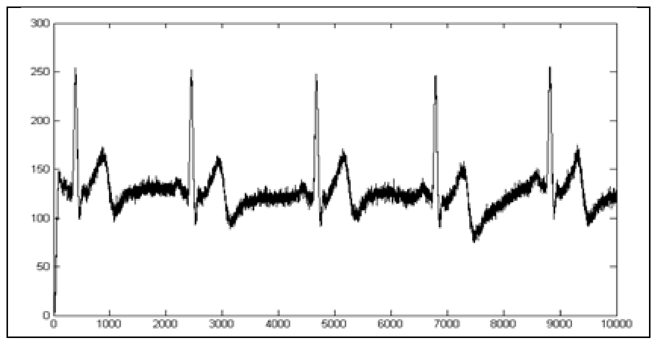

Figure 13. ECG signal demodulated with 256FSK, and 30 Hz separation between frequencies

Another modulation method proposed for comparison was phase shift keying (PSK) linear phase digital modulation using 256 levels.

Then the resulting modulation was contaminated by additive white Gaussian noise (18 dB SNR).

The ECG signal recovered for this SNR value had the shape shown in Figure 14. The effects of noise can be seen, introducing distortion into the retrieved information. Correlation coefficient matrix values with the original signal resulted in:

$$\begin{matrix} 1.0000 & 0.9566 \\ 0.9566 & 1.0000 \end{matrix}$$

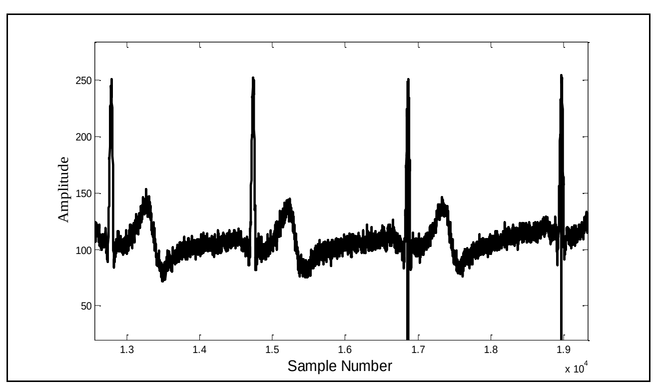

Figure 14 ECG signal demodulated with 256PSK.

The SNR signal was recovered with better fidelity in the proposed algorithm for the chaotic system Lorenz equations, taking into account the above conditions (Figure 15). The resulting correlation coefficient matrix was:

$$\begin{matrix} 1.0000 & 1.0000 \\ 1.0000 & 1.0000 \end{matrix}$$

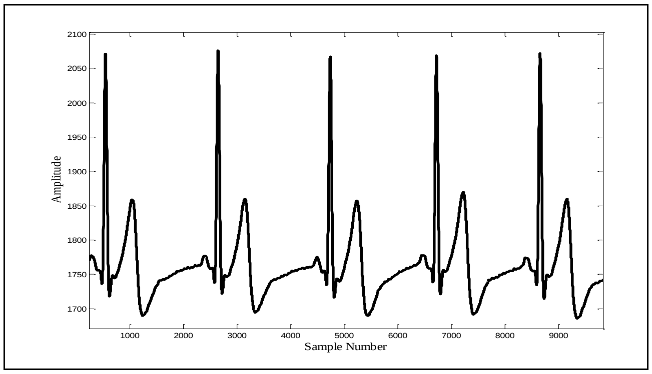

Figure 15. ECG signal demodulated by the proposed chaotic algorithm (SNR = 18 dB)

Better results were obtained for the proposed chaotic modulation algorithm for the same SNR value.

Conclusions

This work has shown an algorithm allowing ECG signal modulation using a chaotic carrier signal. Simulations were also performed for the recovery of an ECG signal modulated with different SNR values. Acceptable results were obtained; this modulation method had acceptable characteristics against noise and also provided advantages over traditional modulation methods.

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