Hydrodynamic evaluation of a hydraulic clarifier through hydraulic behaviour indicators and simplified flow models

Evaluación hidrodinámica de un clarificador hidráulico mediante indicadores de comportamiento hidráulico y modelos simplificados de flujo

Paola **Patiño**[1], Camilo **Cruz**[2], Patricia **Torres**[3], Santiago **Laín**[4]

RESUMEN

En los clarificadores de las plantas de potabilización ocurren fenómenos hidrodinámicos asociados a factores de tipo físico, operativo y ambiental que pueden afectar la calidad del agua clarificada. En este estudio se evaluó el comportamiento hidrodinámico de un clarificador hidráulico con recirculación de lodos mediante ensayos de trazadores de tipo continuo, a partir de los cuales se determinaron diferentes indicadores de comportamiento hidráulico y modelos simplificados de flujo. El clarificador presentó flujo dual con predominio de mezcla completa durante las horas en las que el agua afluente reportó temperaturas mayores a las del interior del reactor, ocasionando la formación de corrientes de densidad térmicas que promovieron la mezcla en el reactor y aumentaron la turbiedad en el efluente; adicionalmente, se observó que los indicadores hidráulicos y el modelo de Wolf-Resnick mostraron mayor sensibilidad a la influencia de la temperatura sobre la hidrodinámica del reactor.

Palabras clave: Clarificador hidráulico, evaluación hidrodinámica, modelos de flujo, ensayo de trazadores.

ABSTRACT

Hydrodynamic phenomena take place within water treatment plants associated with physical, operational and environmental factors which can affect the water quality. This study evaluated a hydraulic clarifier's hydrodynamic pattern using sludge recirculation through continuous tracer test leading to determining hydraulic behaviour indicators and simplified flow models. The clarifier had dual flow with a predominantly complete mixture during the hours in which higher temperatures were reported for affluent water compared to those reported inside the reactor, causing the formation of density currents promoting mixing in the reactor and increased turbidity in the effluent. The hydraulic indicators and the Wolf-Resnick model had higher sensitivity to the influence of temperature on reactor hydrodynamics.

Keywords: hydraulic clarifier, hydrodynamic evaluation, flow model, tracer study.

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Introduction

Solid-liquid separation in water treatment plant (WTP) clarifiers plays a fundamental role along with coagulation and flocculation, since most high-turbidity removal occurs here. This is related to the presence of some pathogens such as *Giardia* and *Cryptosporidium*; their elimination in disinfection is more limited. The EPA (1998) advocates that clarified water turbidity must be less than 2 UNT in WTP for ensuring water treatment quality.

According to MWH (2005), exact prediction of clarifier efficiency through mathematical and experimental methods is a challenge even for the best design engineers. Factors such as density current, temperature gradient, wind effect, energy dissipation at the entrance, outflow and motion equipment may affect hydrodynamics and therefore a clarifier's performance.

Tools such as hydraulic performance indicators and simplified flow models based on tracer studies have led to a reactor's hydrodynamic evaluation from field data. According to the dosage method, tracer studies may be instantaneous addition type (using C_o concentration at the reactor inlet for a very short period) or continuous addition (continuously injecting a C_o concentration of tracer at a constant rate during a period of not less than three times the hydraulic retention time (HRT) and then abruptly interrupting the dosage) (AWWA, 2011; Ministry of Economic Development, 2000).

Tracer study data is used directly (residence distribution time (RDT) curves) or together with indicators and/or simplified flow models to represent and predict flow behaviour within a reactor (MWH, 2005). RDT knowledge is fundamental for reactor design since it allows knowing system kinetics for planning design options to maintain desired flow pattern and compare the influence of

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[1] Sanitary Engineer, Specialist, Universidad del Valle, Colombia. Investigation Assistant, Grupo de Investigación Estudio y Control de la Contaminación Ambiental - Universidad del Valle, Colombia. E-mail:paopatin@univalle.edu.co

[2] Sanitary Engineer, Universidad del Valle, Colombia. MSc. Universidad São Paulo, Brasil. Associated Professor, Facultad de Ingeniería. Grupo de Investigación Estudio y Control de la Contaminación Ambiental - Universidad del Valle, Colombia. E-mail:camilo.cruz@correounivalle.edu.co

[3] Sanitary Engineer, Universidad del Valle, Colombia. MSc. y PhD., Universidad São Paulo, Brasil. Titular Professor, Facultad de Ingeniería. Grupo de Investigación Estudio y Control de la Contaminación Ambiental - Universidad del Valle, Colombia. E-mail:patricia.torres@correounivalle.edu.co

[4] PhD in Physical Sciences. Universidad de Zaragoza, España. PhD in Engineering, Martin-Luther Halle-Wittenberg University, Germany. Grupo de Investigación en Mecánica de Fluidos, Universidad Autónoma de Occidente, Colombia. E-mail:slain@uao.edu.co

different configurations and modifications in reactors (Stamou, 2008).

Owen (1992), quoted by Teefy (1996), carried out an instantaneous type of tracer study by observing San Diego WTP treatment (California), finding that real retention times in the clarifier were higher than theoretical time; such situation was associated with the presence of dead areas. A similar study by Hart, quoted by Teefy (1996), evaluated a secondary clarifier on a wastewater treatment plant, identifying high short-circuit levels.

The efficiency of two secondary clarifiers subjected to higher than design loads through tracer studies and using hydraulic performance indicators have been evaluated, giving 4.3 to 5.5 Morrill Index (MI) values in both clarifiers and 65% (clarifier 1) and 98% (clarifier 2) hydraulic efficiency for normal operational conditions as well as greater than design loads (Boyle *et al*., 2004).

Taebi-Harandy and Schroeder (2000) evaluated density current formation in a secondary clarifier, using tracer studies to determine existing hydraulics and their possible variation due to effect of density currents; they established that 0.2ºC differences between affluent temperature and the water contained in the clarifier promoted the formation of density currents which notably affected the flow pattern. Goula *et al.,* (2008) used computational fluid dynamics (CFD) determined that a 1ºC difference between the affluent and the water contained in the clarifier led to the formation of density currents, resulting in non-uniform solid distribution and, therefore, short circuits.

This study made a hydrodynamic evaluation of a hydraulic sludge blanket clarifier (flocculation and clarification) with sludge recirculation, by applying different hydraulic performance indicators and simplified flow models which were calculated from results obtained in two continuous tracer studies carried out within the reactor.

Experimental development

Sludge blanket clarifier description

This study was carried out at Cauca River WTP (CRWTP) in the city of Cali, Colombia, using a vertical hydraulic clarifier involving flocculation and clarification, with hydraulic sludge recirculation; it supplies 17% of the city's drinking-water needs. The CRWTP takes raw water from the Cauca River and treats an average 1.2 m^3/s flow. CRWTP treatment includes pre- chlorination, coagulation with aluminium sulphate, flocculation–clarification through sludge blanket clarifiers, filtration and disinfection with gaseous chlorine.

Figure 1 shows the clarifier distribution plant (3 hydraulic and 3 mechanical) and a profile of the vertical–circulator hydraulic clarifier.

The 6,000 m^3 clarifier consists of a hydro-ejector consisting of a conical piping nozzle and a diffuser to produce sludge recirculation acting as flocculation catalyst. Mixed water is forced, through a deflecting screen to descend to the settling zone; clarifier water is collected through twelve rectangular radial spouts.

Tracer studies for hydrodynamic evaluation

Two continuous-type tracer studies were carried out using 99.76% sodium chloride (NaCl), whose concentration was defined by injection time, water solubility at 25ºC (35 mg/l), sodium base concentration in raw water (7.4 mgNa/l) and the volume of water required for dilution. Injection time must be 2 to 3 theoretical residence times (t_o) and NaCl concentration 4 times higher than raw water base concentration (Teefe 1996). Table 1 lists the general test characteristics.

Table 1. Tracer study characteristics

Characteristic	Unit	Trial 1 (E1)	Trial 2 (E2)
Flow to clarifier inlet	m3/h	1,361	1,307
Theoretical HRT (to)	hours	3.8	4.02
Theoretical NaCl concentration to be added	mgNa/l	29.4	29.4
Volume of water required for solution	m3	2.13	1.44
Real NaCl solution concentration	mg/l	33	31
Duration of injection	hours	8.5	9
Total duration of trial	hours	18	18
Measurement frequency	minutes	variable	15

Conductivity data were expressed in terms of tracer concentration (mg/l NaCl) by interpolating previously prepared concentration on a calibration curve (conductivity *cf* NaCl). Samples were taken during and after tracer injection, ending the experiment once conductivity values near to those reported for clarifier water before injection were measured.

Hydrodynamic pattern indicators and simplified flow models

Accumulated residence distribution time curves F(t) were prepared during the injection (equation 1) and after this (equation 2) using the tracer concentration data at the clarifier outlet (Ci) and the maximum expected concentration (Co):

$$F(t) = \frac{C_i}{C_o} \qquad (1)$$

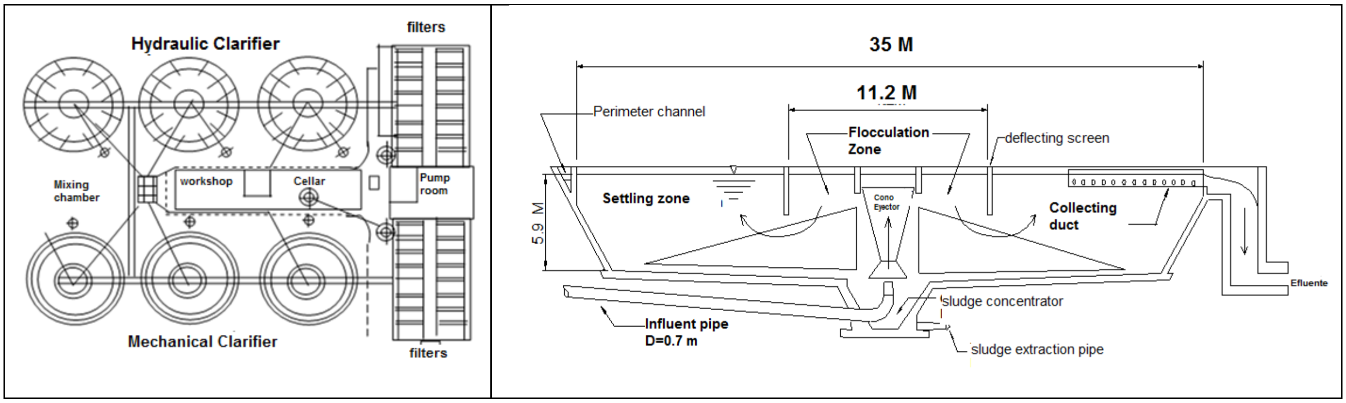


Figure 1. Diagram of the Cauca River WTP clarifier distribution plant and profile of the vertical–circulator hydraulic clarifier

$$F(t) = 1 - \frac{C_i}{C_o} \tag{2}$$

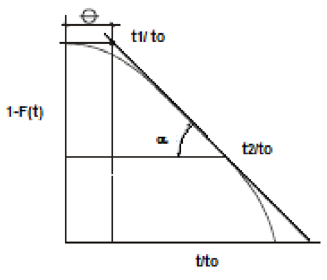
Determining hydraulic pattern indicators and simplified flow models was based on curves F(t), which are shown in Tables 2 and 3.

Table 2. Parameters, methodology and equations used for calculating indicators and flow models

Characteristic	Consideration	Equation
ti (hours)	Based on curve F(t) associated times were determined at different tracer outlet fractions, leading to calculating some hydraulic pattern indicators	Time of tracer appearance in the effluent
tm (hours)		Median time corresponding to 50% of tracer passage
t_{10} (hours)		Time representing 10% of total tracer amount passage
t_{90} (hours)		Time representing 90% of total tracer amount passage
t_o	Theoretical residence times were calculated based on flow to clarifier (Q) and clarifier volume (V)	$to = \frac{V}{Q}$
Morrill Index	Data t_{10} and t_{90} were used to calculate the Morrill Index	$MI = \frac{T_{90}}{T_{10}}$
E (t)	Curve E(t) was residence time distribution and was calculated from the mathematical equation previously obtained from curve F(t)	$E(t) = \frac{d}{dt}\left[\frac{C(t)}{Co}\right]$
HRTexp	Average residence time was determined from calculating the area under the curve t* E(t)	$HRTexp = \int_0^\infty tE(t)dt$
σ^2	Variance (σ^2) was calculated once E(t) and tm had been established	$\sigma^2 = \int_0^\infty (t - HRTexp)^2 E(t)dt$
Axial dispersion model	The degree of axial dispersion calculation was based on tm and σ^2	$\sigma_\theta^2 = 2\frac{D}{\mu L} - 2\frac{D}{\mu L} * (1 - e^{\left(-\frac{u}{LD}\right)})$
Series tanks model	The number of totally mixed tanks, representing the clarifier being studied (N) was based on variance (σ^2) and tm	$N = \frac{HRTexp^2}{\sigma^2}$

Adapted from Levenspiel (1999), Cepis (2006)

Table 3. Equations used for Wolf – Resnick application

The curve t/to *cf* 1 – F(t) traced on semi-logarithmic paper was used to determine: $\tan\alpha = \frac{1}{\frac{t_2}{to} - \theta}$ θ values and tan α it led to obtaining: $p = \frac{\theta\tan\alpha}{0.435 + \theta\tan\alpha}$ where p: plug flow fraction $M = 1 - p$ where M = mixed flow fraction $m = 1 - \frac{\theta}{p}$ where m = dead area fractions	**1-F(t) curve** 

Adapted from Cepis (2006)

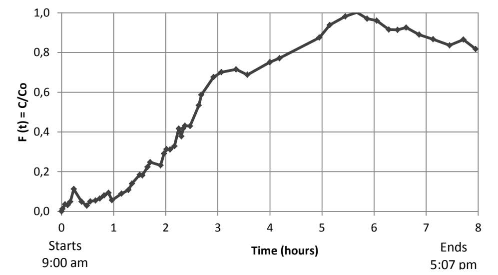
Temperature and turbidity

Effluent turbidity was measured (HACH 2100P turbidity meter) as well as the temperature at the inlet and within the clarifier to complement reactor hydrodynamic analysis during the tracer studies. This information was complemented with WTP operation data from the 2 days the trials lasted.

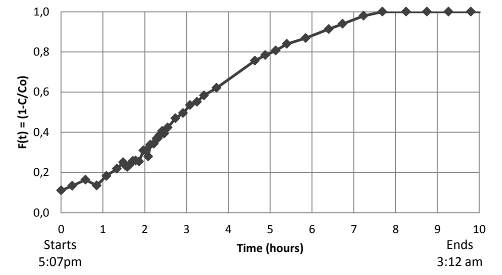
Results

Analysis of accumulated residence time distribution curve F(t)

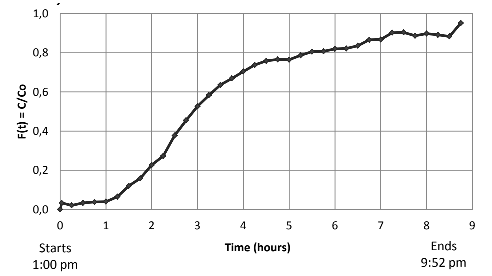
It was observed from curve F(t) (Figure 2) that curves E1A and E2A (representing injection pattern) were concave upwards and then changed during the first 2.5 hours' injection. This may have indicated that the clarifier had a dual or combined flow regime (plug-flow and complete mix) which would have been expected since a complete mix in the flocculation area and piston flow in the sedimentation area are expected when flocculation and sedimentation occur in the clarifiers.



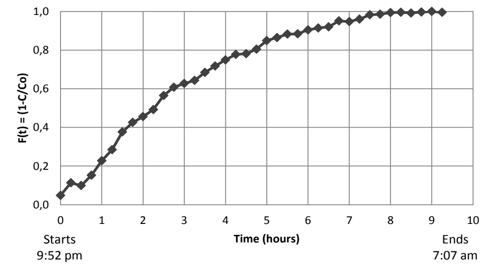
(a) E1A



(b) E1D



(c) E2A



(d) E2D

Figure 2. Curve F (t) obtained from tracer studies. a) Trial 1 during injection, b) Trial 1 after injection, c) Trial 2 during injection and d) Trial 2 after injection

Curves obtained after tracer injection (curves E1D and E2D) had a similar tendency to the theoretical curve reported by Levenspiel (1999) and Di Bernardo & Dantas (2005) for reactors having completely mixed flow predominance, which may have affected settling and, therefore, clarifier efficiency.

Hydraulic pattern indicators

Calculated HRT_{exp} based on residence time distribution function E(t) was close to 3 hours for both trials. Time less than theoretical HRT (t_o) (3.8 and 4 hours in trials 1 and 2, respectively) (Table 4). This pattern, according to Levenspiel (1999), is related to the displacement of curve E(t) towards the left from theoretical HRT, being typical of a reactor having dead areas.

Table 4. Results obtained by applying hydrodynamic pattern indicators

Parameter	Description[1]	Trial 1		Trial 2	
		E1A	E1D	E2A	E2D
HRT exp	Experimental retention time	3.0	3.4	2.8	31
HRT Theoretical (to)	Theoretical retention time	3.8		4.0	
ti/to	ti/to= 0 (complete mixed flow) ti/to= 1 (plugflow) ti/to< 0.3 (short circuits)	0.04	0.05	0.06	0.06
tm/to	tm/to< 1 (short circuits and/or dead zones) tm/to> 1 (undesired tracer or trial error accumulation)	0.7	0.8	0.5	0.6
t_{10}/t_o	t_{10}/t_o> 1 trial error $t_{10}/t_o \approx$ 0.3 – 0.6 clarifier characteristics	0.30	0.21	0.31	0.13
MI	MI ≈ 0 (plug flow predominance)	4.3	8.9	4.6	12.0

1. *Adapted from Pérez & Torres (2008), Cepis (2006) and van der Walt (2002)*

The t_i/t_o ratio obtained in all trials indicated complete mix predominance in the reactor with short circuits which was confirmed by tm/to ratio values being lower than 1, indicating the presence of short circuits and/or dead areas in the clarifiers. The t_{10}/t_o ratio was close to 0.3 during E1A and E2A; this value, according to Teefy (1996), is characteristic of clarifiers and settling equipment. This E1D and E2D ratio decreased and approached the values reported for a reactor without deflectors or drinking-water storage tanks.

The Morrill Index (MI) indicated that the reactor tended to dual flow; however, values higher than MI were present during E1D and E2D which, according to the Van Der Walt (2002), indicated a higher degree of mixing. MI shown on W1A and E2A was similar to that reported by Boyle *et al.*, (2004) for secondary clarifiers having good hydraulic efficiency. According to the clarifier system presented by Van Der Walt (2002) for evaluating the hydraulic efficiency of reactors from t_{10}, t_{90}, IM and dispersion number, the reactor had "acceptable" hydraulic efficiency during E1A and E2A while hydraulic efficiency was affected for E1D and E2D, having "poor" hydraulic efficiency.

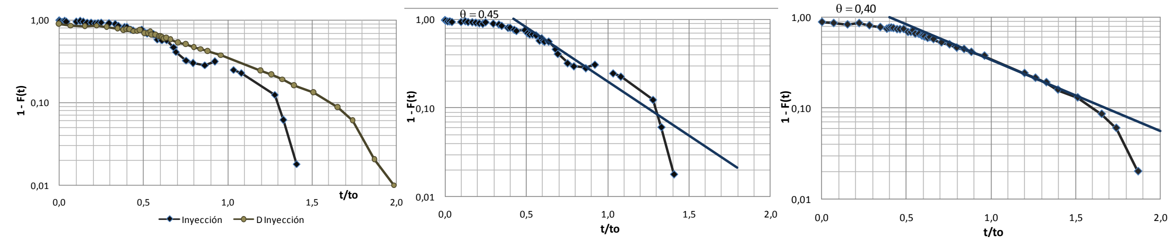
The indicators revealed short circuits and areas in the clarifier which, according to CEPIS (2006), may promote appreciable increases in surface loads or settling velocity in the settling units, thereby reducing the efficiency of removal from these treatment units to a great extent.

Flow Models

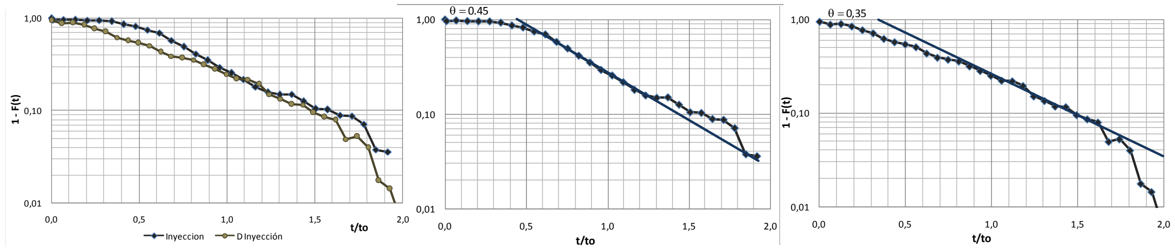
The values obtained in the axial dispersion model (Table 5) showed that the clarifier had an axial dispersion degree (D/µL) during both experiments considered "large" according to Levenspiel (1999) who indicated that the nearer to zero the dispersion number, the higher the plug flow at the reactor, thereby confirming that the reactor had a dual flow.

Table 5. Results of flow model applications

Characteristic	Variable	Trial			
		E1A	E1D	E2A	E2D
Variance (hours^2)	σ^2	8,440	18,995	9,076	9,495
Axial dispersion number	(D/µL)	0.06	0.06	0.12	0.10
Serial tanks model	N	4	4	3	4



Trial 1 (a) E1A (b) E1D



Trial 2 (c) E2A (b) E2D

Figure 3. Curve 1 – F(t) used for the Wolf – Resnick model

Regarding the results obtained in the serial tanks model, Table 5 shows that the clarifier had a pattern of 4 (E1A, E1D, E2D) and 3 (E2A) completely mixed serial tanks, matching the dispersion model according to which the clarifier had a dual flow. Di Bernardo & Dantas (2005) have reported that circular settling equipment with radial flow usually have N values equal to 2.

Figure 3 shows the 1 – F(t) curves obtained in the tracer studies where the Wolf – Resnick model was applied. These curves showed that trial 1 had divergence between curves when comparing the tendency presented by the curve during and after injection, a phenomenon according to Di Bernardo (2005) and Hudson (1981) which can occur in settling equipment due to thermal density currents. It was also observed that the E1A curve had deformation which, according to CEPIS (2006), represented turbulence in the reactor due to density currents.

The clarifier had dual flow hydraulic performance in both experiments according to the Wolf – Resnick model (Table 6), thereby agreeing with the findings for curve F(t) and the dispersion and serial tank models. However, a larger percentage of complete mixing in the reactor was observed for E1D and E2D (58%), confirming the tendency shown by the F (t) curve and some evaluated indicators.

Table 6. Results of applying Wolf – Resnick model

Characteristic	Variable	Trial			
		E1A	E1D	E2A	E2D
Plugflow fraction (%)	P	53.77	41.43	50.85	41.16
Complete mix fraction (%)	(1-p)	46.23	58.57	49.15	58.84
Dead area (%)	M	16.31	3.45	11.50	14.97

The clarifier had dead areas in both experiments, percentages ranging from 3.45% to 16.31%; however, the percentage found in the E1A trial may have affected clarifier hydraulics, in the same way as reported by Cobucci de Oliveira *et al*., (2007) who observed that settling equipment having close to 22% dead area had operational problems related to its hydrodynamic performance.

The presence of short circuits and dead areas in the clarifier, as well as predominant complete mix in E1D and E2D, may have been related to density currents within these treatment units (Krebs *et al*., 1995), causing the velocity vector to adopt an opposite direction regarding clarifier inlet and outlet as observed in different computational models evaluated by these authors.

Temperature and turbidity

Regarding clarifier average temperature and effluent turbidity, Figure 4 shows an inverse relationship, so that as temperature became reduced and stabilised, effluent turbidity increased; raw water temperature was higher than that of clarified water (between 0.2ºC and 1.1ºC) mainly at 4:00 pm in E1 and 6:00 pm in E2, after which turbidity increased. This pattern may have been associated with the formation of density currents as described by MWH (2005), confirming that hot water coming into a clarifier containing cold water results in hot water flow upwards, forming density currents that may invert clarifier content in the most critical cases.

Temperature differences between raw water and water contained in the clarifier ranged from 0.2ºC and 1ºC in both experiments, agreeing with that established by Taebi-Harandy & Schroeder (2000) and Goula *et al*., (2008) who observed that hot water coming into a clarifier containing cold water results in the formation of density currents; both groups of authors have reported flow pattern tendency similar to that described by MWH (2005).

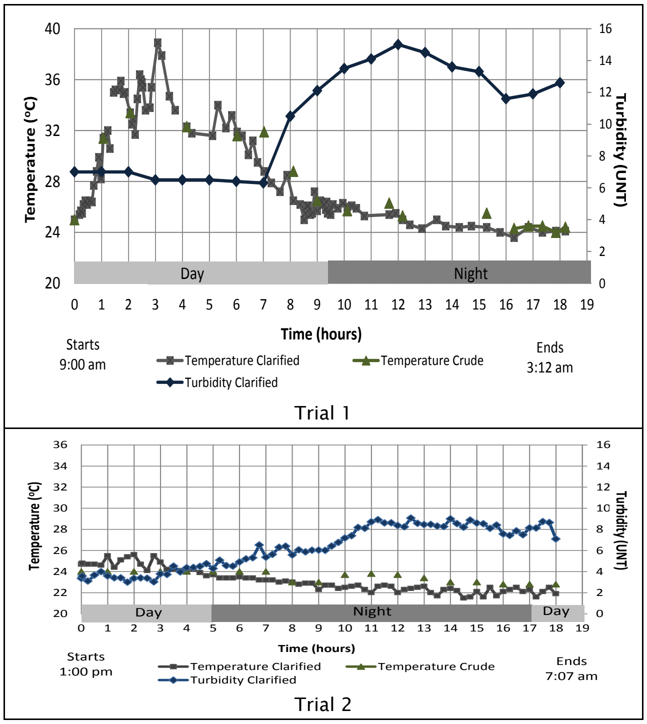


Figure 4. Effluent turbidity and water temperature pattern

Analysing the results of hydrodynamic evaluation and turbidity, raw water temperature and clarifier average temperature tendencies it was determined that incoming water at a temperature higher than that of the reactor's interior would promote the formation of density currents which might modify clarifier flow patterns, resulting in increased reactor mixing, as shown by the F(t) curve, hydraulic behaviour indicators and Wolf – Resnick model. This, in turn, results in settling material suspension and therefore increased turbidity in the effluent.

The short circuits shown by the hydraulic pattern indicators may have been associated with the aforementioned temperature differences which, according to MWH (2005), may create short circuits as hot water entering a reactor tends to displace to the surface and leave with the effluent in a nominal retention time. Otherwise, when cold water enters a reactor containing hot water, this tends to flow along the bottom, leaving the reactor faster. It should be stressed out that both situations were present during the tests.

Conclusions and Recommendations

The hydraulic clarifier hydrodynamic evaluation at the Cauca River water treatment plant indicated that it had a dual flow hydraulic regime with short circuits and dead areas, complete mix predominating during hours in which raw water temperature was higher than that of the water within the reactor. This pattern was associated with the formation of density currents by temperature action which affected reactor hydrodynamics and, therefore, effluent water quality in terms of turbidity.

Hydraulic pattern indicators and the Wolf – Resnick model had greater sensitivity to the effect of temperature on clarifier hydrodynamics, showing that the degree of mixing in the clarifier increased when affluent temperature was higher than that within the reactor. The contrary effect occurred with axial dispersion and tank series models, which were not sensitive to this effect, indicating that the reactor had dual flow for the four conditions evaluated here. This could have been related to the fact that continuous type tracer studies require calculating curve E(t) based on defining retention times (HRT_{exp}) and variance (σ^2) requires numerical derivations able to decrease the sensitivity and precision of models based on these two parameters.

Acknowledgements

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Nomenclature

Ci: tracer concentration data time i
Co: maximum concentration tracer
MI: Morrill Index
m: dead area fractions
M: mixed flow fraction
N: number completely mixed serial tanks
to: theoretical residence times
HRTexp: experimental hydraulic retention time
tm: median time
t_{90}: time that passes 90% of tracer
p: plug flow fraction
Q: flow
t: experimental time
V: volume
σ^2: variance
σ^2_θ: dimensionless variance
D/μL: axial dispersion number
ti: time of tracer appearance in the effluent.
t_{10}: time that passes 10% of tracer