PROSPECTIVE TEACHERS REPRESENTATIONS IN PROBLEM SOLVING OF SPECIAL ANGLE TRIGONOMETRY FUNCTIONS BASED ON THE LEVEL OF ABILITY

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Article Info	ABSTRACT
	One of the materials used as the basis for solving trigonometric function problems is special angle trigonometry. Prospective teachers' representation in problem-solving of trigonometric functions with special angles is thought to be influenced by prospective teachers' abilities. Therefore, this study aims to analyze the representations used by prospective teachers in problem-solving of special angle trigonometric function based on ability categories. This research is qualitative descriptive. The research subjects are prospective teachers of the mathematics education study program at a university in Malang. The data collected in this study are in the form of work results and observation. The research instrument consisted of the problem of the trigonometric function value of the special angle and the interview guide developed by the researcher. The analysis of prospective teacher work results was carried out by classifying the ability categories into low, medium, and high abilities. The work results of each of these categories are classified based on verbal, numeric, image, and algebraic representations. The analysis of the interview transcripts was carried out by coding the words or sentences which aims to determine prospective teachers' understanding of using representations. The results showed that prospective teachers with low ability use a lot of verbal representation, while prospective teachers with medium and high abilities use a lot of image representation in problem-solving of special angle trigonometric function. The implication of the results of this study is to teach special angle trigonometric function material based on appropriate representations.

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1. INTRODUCTION

As a prospective teacher, you should have mastered the concepts in trigonometric material. This is because trigonometric material is taught at the senior high school level,

where this trigonometric material is important material at the senior high school level (Cavanagh, 2008; Kamber & Takaci, 2018; Maknun et al., 2019; Nabie et al., 2018; Nejad, 2016; Siyepu, 2015; Tuna, 2013; Usman & Hussaini, 2017; Wongapiwatkul & Laosinchai, 2011). This trigonometry material is a prerequisite for vector analysis, calculus, and differential equations (Setiawan, 2021). Also, in this trigonometric material, there are concepts in mathematics that are important for students to master (Nabie et al., 2018). Therefore, a prospective teacher must master the concepts in the trigonometric material, to be able to teach correctly about the concepts in the trigonometric material to senior high school students.

Trigonometry is a part of mathematics that generally discusses triangles (Downing, 2009), and specifically discusses trigonometric functions (Lial et al., 2016). One of the important materials to study is the trigonometric function of special angles. Special angles consist of 30º, 45º, and 60º. The importance of studying the material on trigonometric functions is because this material is used as a basis for determining the value of the trigonometric function for non-acute and negative angles (Setiawan, 2021). For example, to find the value of sin 210º is to find the reference angle. The reference angle is defined as an acute angle that is not a quadrant angle in a standard position (Lial et al., 2016). Since 210º in quadrant III, the reference angle of 210º = 210º - 180º = 30º. So we can determine the value of sin 210º = - sin30º = - ½ (since 210º is in quadrant III, the value for sin is negative). So it can be said that this special angle trigonometric function material is important for students to learn, in order to solve problems about the non-acute angle and negative angle trigonometric functions.

However, the results showed that prospective teachers still experienced errors in determining the value of the trigonometric function for special angles. This can be seen from the results of research that show that prospective teachers experience errors in drawing cosine function graphs, errors in applying trigonometric comparisons, and errors in writing down trigonometric reduction identities to determine the value of the trigonometric function for special angles (Setiawan, 2021). The results also show that prospective teachers consider trigonometric material to be difficult and abstract (Nabie et al., 2018), so that conceptual understanding of prospective teachers is low in learning trigonometric material (Mustangin & Setiawan, 2021). This error is usually solved by introducing the prospective teachers to triangular trigonometry rather than circular trigonometry. However, prospective teachers have difficulties when introduced to triangular trigonometry, where the difficulties of prospective teachers include: (i) connecting triangular images with numerical relationships, (ii) determining trigonometric ratios, and (iii) manipulating symbols involved in trigonometric ratios (Wongapiwatkul & Laosinchai, 2011). So in general it can be said that prospective teachers still experience errors and difficulties in studying the material of special angle trigonometric functions.

One of the factors that influence the errors and difficulties of students in studying the material of special angle trigonometric functions is the representation used to understand the value of the special angle trigonometric function. Representations in mathematics are defined as representatives of abstract concepts (Bishop, 2000). The results show that students' understanding can be related to their schema structure and flexibility in using different representations (Trigueros & Martínez-planell, 2010). The results also show that representation can support children's understanding and reasoning in multiplication (Barmby et al., 2009). Therefore this representation is one of the keys to student success in learning mathematics. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations, such as physical objects, pictures, charts, graphs, and symbols (Joyner & Reys, 2000). Furthermore, the representation and symbolization of mathematical ideas is the heart of mathematics

related to mathematical activities (Kaput, 2008; Kaput et al., 2008). One must rely on concrete representations to make conclusions (Bishop, 2000). So it can be said that representation can affect a person in understanding mathematics, including mistakes and difficulties in learning mathematics. This is because representations can develop and deepen understanding of concepts, reasoning, and problem-solving.

Representations used by students in solving mathematical problems are also influenced by students' backgrounds. The results showed that the diagrammatic representation helped all students in solving trigonometric problems, but the illustration of trigonometric problems was influenced by the students' backgrounds (Cooper & Alibali, 2012). One of the backgrounds that influence a person in solving math problems is an ability (Clark et al., 2014; Perkins et al., 1993; Ron, 2001; Tishman & Andrade, 1995; Tishman et al., 1993). Ability is defined as the adequacy of a person's capacity to solve problems (Clark et al., 2014). The results showed that students who were able to prove had relevant basic knowledge (Setiawan, 2020a). The results of other studies show that students and students with low abilities often experience errors in solving problems (Mustangin & Setiawan, 2021; Setiawan, 2020b, 2020c, 2021; Setiawan et al., 2020). This means that representation can be influenced by the abilities students have in understanding the material or solving math problems.

Based on this description, it can be said that there are still errors made by students in determining the value of the special angle trigonometric function. These difficulties and errors are also influenced by the representations used in teaching the material for special angle trigonometric functions. Representations used by students in studying a material are also influenced by the abilities possessed by students in learning certain materials. The results of previous research have shown that diagrammatic representations have helped all students in solving trigonometric problems (Cooper & Alibali, 2012). However, this previous research did not consider the background in the form of students' abilities in solving trigonometric problems. Therefore, research is still needed on student representation in solving trigonometric problems based on the ability categories possessed by students. Based on this, this study aims to describe the various representations used by students in solving special angle trigonometric function value problems based on ability categories.

The theoretical benefit of the results of this study is to develop a theory of student representation in solving the problem of the value of the trigonometric function for special angles based on the ability category. The practical benefit of the results of this study is that it can be used by lecturers or teachers in teaching special angle trigonometric function values based on the level of ability of prospective teachers or students. If the representation used is in accordance with the student's ability level, then the student will better understand the material given. In the end, the representation used in learning can reduce the errors or difficulties of students in studying the material for the value of special angle trigonometric functions.

2. METHOD

By following the purpose of this study, namely describing the various representations used by students in determining the value of the trigonometric function of the special angle, this research method is a qualitative descriptive study with a case study approach to 12 research subjects who are students of a mathematics education study program at a university in Malang city.

The process of selecting subjects in this study consisted of three steps (see Figure 1). The first step is to ask 82 prospective teachers of semester 1 of the mathematics education

study program to solve six conceptual understanding questions, one of which is to determine the value from a special angle (see Figure 2) by providing a written explanation of the method they use. The second step is to correct student answers using the scoring guidelines developed by the researcher. Of the 82 students, the lowest score was 24 and the highest score was 97. From the scores obtained by 82 students, the students' abilities would be categorized into three categories, namely high, medium, and low. The categorization of students' abilities is based on the scores obtained when completing six questions of conceptual understanding. The determination of this category is based on the normative reference guidelines that exist at the Islamic University of Malang in the 2020/2021 academic year, namely: A value (score 80-100), B value (score 70-79), C value (score 55-69), and D value (score < 55). The results of categorizing the abilities of 82 students are obtained in Table 1.
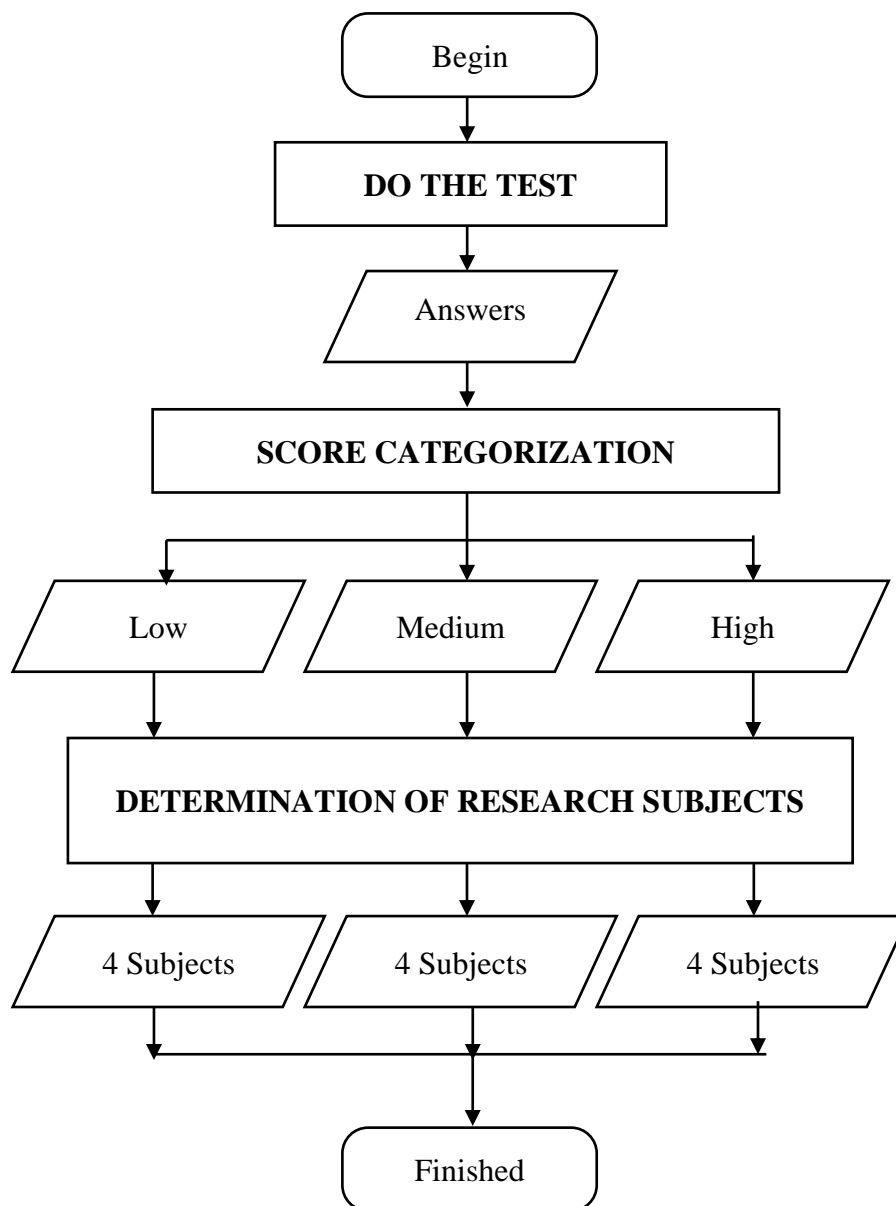

Figure 1. The process of selecting subjects

The third step is to classify students' answers from the high, medium, and low categories based on verbal, numeric, image, and algebraic representations (see Table 2). From the results of this classification, 4 students were taken from the high, medium, and low categories, so that the research subjects consisted of 12 prospective teachers. These subjects were chosen, because they can provide an explanation of the answers in detail.

Table 1. Results of prospective teachers ability categorization

No.	Score (x)	Category	Number of Prospective Teachers
1	$80 \leq x \leq 100$	High	23
2	$55 \leq x < 80$	Medium	40
3	$x < 55$	Low	19
Total			**82**

By following this type of research, the data collected in this study consisted of the results of the subject's work and transcripts of interview results. Data collection procedures in the form of subject work results are carried out by following the steps for selecting the subject. While the procedure for data collection of interview transcripts was carried out using two steps. The first step is to conduct interviews with research subjects through WhatsApp media. During the interview, an audio recording was also conducted. The second step is transcribing word for word so that a transcript of the results of the interviews with the research subjects is obtained.

By following the data collected in this study, the research instrument consisted of questions about the value of the trigonometric function for special angles (see Figure 2) and interview guidelines developed by the researcher. The instrument in Figure 2 has various representations that can be used in finding the value of the trigonometric function of a special angle. For example, using triangles, tables, graphs, circles, or algebraic representations of angular relations. Because this research instrument can be completed with various representations, this research instrument is valid to be used to identify the representations used by students.

Describe your method of determining the following angle values.
(a) $\sin 30^0 = \cdots$
(b) $\cos 30^0 = \cdots$

Figure 2. Research instruments

The analysis of student work results is classified based on the representation used by students in determining the value of the trigonometric function for special angles. The representation system consists of an internal representation system and an external representation system (Goldin & Shteingold, 2001). External representation systems range from conventional mathematical symbol systems (e.g., number symbols, formal algebraic notation, number lines, and cartesian coordinates) to structured learning environments (e.g. involving concrete manipulative material), whereas internal representation systems include student symbolization constructs and assignment of meanings to mathematical notation, students' natural language, visual imagery, and spatial representation, and problem-solving strategies (Goldin & Shteingold, 2001). This means that the external representation system is in the form of mathematical symbols that have been used by mathematicians which are then introduced to students in learning mathematics, while the internal representation is the

student's construction in learning mathematics. This study uses internal representations, namely constructs used by students in determining the value of the trigonometric function for special angles. The classification of representations in this study is carried out using four representations, namely: verbal, numerical, pictorial, and algebraic representations (Friedlander & Tabach, 2001). This is because the use the verbal, numeric, image, and algebraic representations have the potential to make the learning process of algebra meaningful and effective (Friedlander & Tabach, 2001). The framework for the classification of student representations in determining the value of the trigonometric function for special angles can be seen in Table 2.

Table 2. The framework of prospective teachers representation classification

Types of Representation	Indicators of Representation
Verbal	Verbal representations are usually used in posing problems and are needed in the interpretation of the final results obtained in solving problems. Verbal representations are used to understand the context and to communicate solutions. A student who can describe a situation verbally is not always able to represent it symbolically (Scher & Goldenberg, 2001). This means that someone's indicator in using verbal representations is that they can explain the final result verbally. Verbal representations in this study include rote memorization.
Numeric	Numerical representation is representing a geometric model through its numerical features, students can look for patterns, arrange experiments to test conjectures (Scher & Goldenberg, 2001). This numerical representation emphasizes the use of important numbers to gain an understanding of problem-solving. That is, someone's indicator in using numerical representations emphasizes the use of numbers.
Image	The graphical representation is very effective in providing a clear picture of the function of real number variables. Graphics are intuitive and very attractive to students who like a visual approach. Someone's indicator in using image representations is to use graphics, use geometric images, or other objects.
Algebra	Algebraic representation is effective in presenting mathematical patterns and models (Scher & Goldenberg, 2001). Manipulation of algebraic objects is sometimes the only method of justifying or proving general statements. Indicators of students using algebraic representations are using algebra or algebraic manipulation

Furthermore, the transcript analysis of the interview results was carried out by coding the words or sentences that showed the subject's understanding of the representations he used to determine the value of the trigonometric function. Thus it will be known that the subject understands the representation used. Through the analysis of the results of the work and the transcripts of the results of these interviews, it is hoped that the representation of students in solving the problem of the value of the trigonometric function for special angles can be identified.

3. RESULTS AND DISCUSSION

3.1. Results

The results of the research in the form of representation classification used by 82 prospective teachers in determining the size of the special angle based on the ability category can be seen in Figure 3. From Figure 3 it can be seen that prospective teachers with low ability use a lot of verbal representation, while prospective teachers with medium and high abilities use a lot of image representation. Furthermore, from each category of this ability level, four prospective teachers will be taken as research subjects to be further analyzed about the representations used by these prospective teachers.
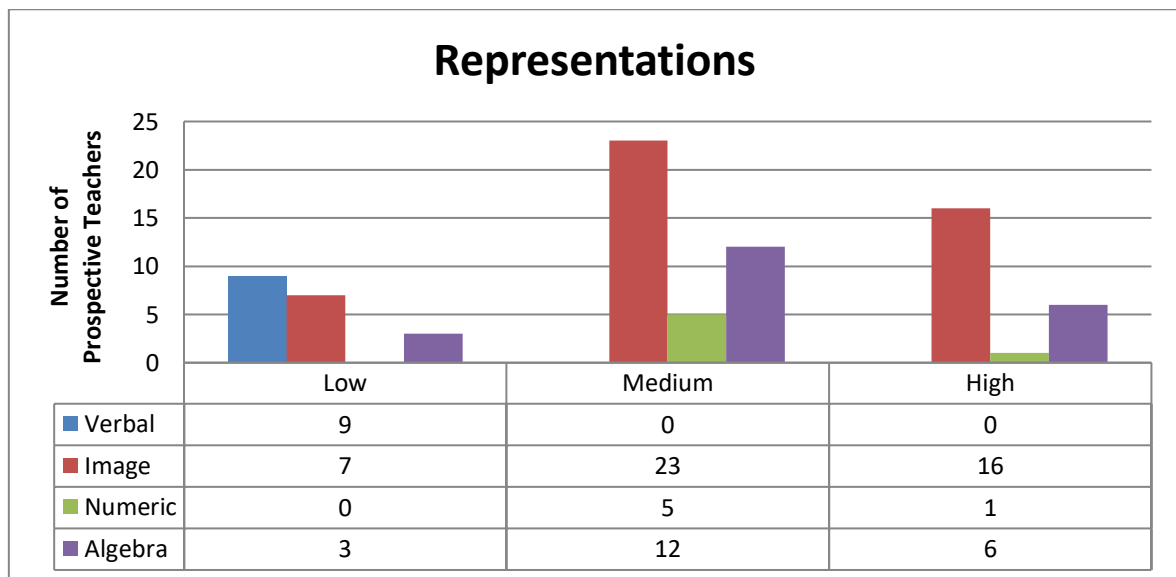

Representations

	Low	Medium	High
■ Verbal	9	0	0
■ Image	7	23	16
■ Numeric	0	5	1
■ Algebra	3	12	6

Figure 3. Prospective teachers representation diagram

3.1.1. Prospective Teachers Representation of Low Ability Categories

From Figure 3 it can be seen that of the 19 prospective teachers who have low abilities, it is obtained that 47% of prospective teachers use verbal representations (i.e. in the form of memorization), 37% of prospective teachers use image representations (consisting of six prospective teachers using triangles and one prospective teacher using graphs), and 16% uses algebraic representations (using angular relations) in determining the value of the acute angle trigonometric function. This means, in the low ability category, prospective teachers still use rote memorization to determine the value of the special angle trigonometric function. Furthermore, from the 19 prospective teachers, 4 prospective teachers were selected as the subjects of this study, each of which represented the representation used by the prospective teachers.

$$\text{Sin } 30° = \frac{1}{2} \text{ and } \text{Cos } 60° = -\frac{1}{2}$$

Figure 4. Verbal representation of the first subject

From Figure 4 it can be seen that the subject directly determines the value of the special angle trigonometric function. However, the subject experienced an error in

determining the value of the trigonometric function for the function cos 60°. Explanation of the subject using this method can be seen from the following interview excerpt.

R　:　*Try to explain how you solve the problem!*
S1　:　*Those are special angles, so I immediately determined by **using the memory from senior high school,** that is, sin 30 is equal to 1/2, and cos 60 is equal to -1/2.*

From the interview transcript, it can be seen that the subject only memorized in determining the value of the special angle trigonometric function. Furthermore, from the seventh prospective teachers who used image representations, two prospective teachers were taken as the second subject (S2) and the third subject (S3), each of which used a triangle representation (see Figure 5) and graphs (see Figure 6).
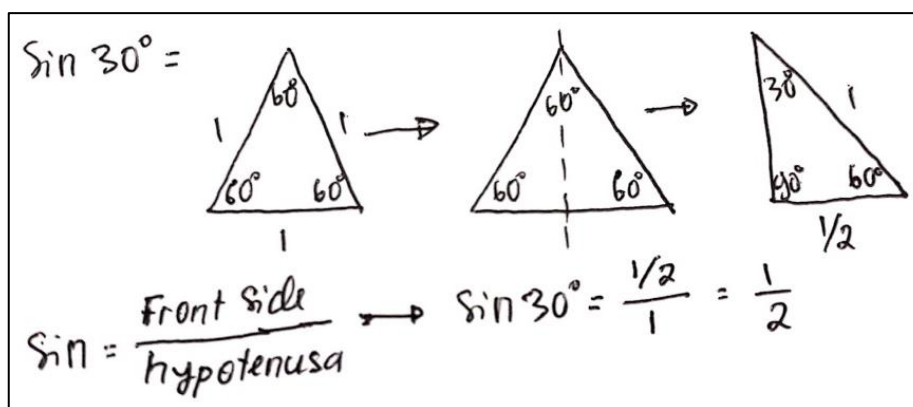

Figure 5. Triangular representation of the second subject

From Figure 5 it can be seen that the subject uses a triangular representation in determining the value of the trigonometric function for special angles. In using this triangular representation, the subject has succeeded in correctly determining the value of the privileged angle trigonometric function. The explanation of the method used by this subject can be seen from the following interview excerpt.

R　:　*Try to explain how you solve the problem!*
S2　:　*... The length of the sides of the base of the triangle, 1 divided by ½ into ½. Sin 30 **sin front side hypotenuse,** equal to ½. For cos 60 to be the same, cos 60 to ½. Cos is equal to the **side hypotenuse** so that ½ divided by 1 equals ½.*
R　:　*Why are you using this method?*
S2　:　*Because it is easier to **use right triangles,** namely **sindemi (sin = front/hypotenuse,** and **cosami (cos = side/hypotenuse).***

From the interview transcript excerpt, it can be seen that the subject was correct in using triangular comparisons to determine the value of the special angle trigonometric function. This method also according to the subject is the easiest way to determine the value of the special angle trigonometric function.
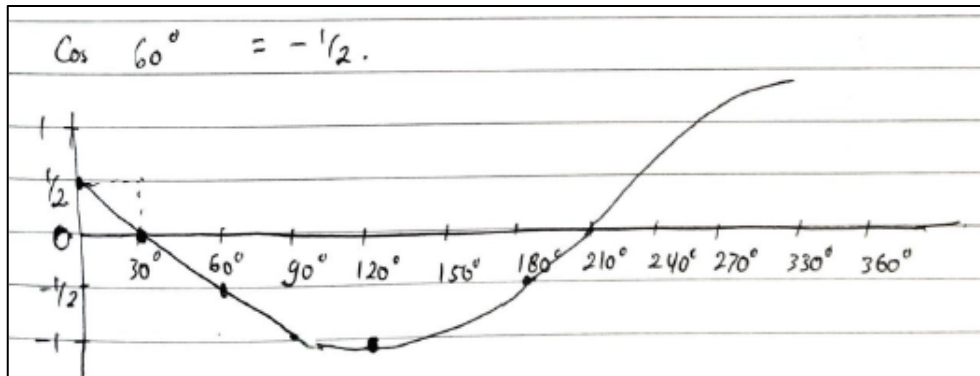
Figure 6. Graphical representations of the third subject

From Figure 6 it can be seen that the subject still experiences errors in drawing the graph of the trigonometric function for the cos function. Due to the error in drawing this graph, the subject experienced an error in determining the value of the special angle trigonometric function. The explanation of the method used by this subject can be seen from the following snippet of the interview transcript.

R : *Try to explain how you solve the problem!*
S3 : *... When I was at school the picture was also described, so I remember it like that. **I only remember that explanation in senior high school**, Sir. Because **I forgot a bit**, I was also given a picture like that.*
R : *Why are you using this method?*
S3 : *Because **that's what I remember** and that's what I think is **easy to use graphics**.*

From the interview transcript, it can be seen that the subject only remembers the graph of the trigonometric function. Because the subject forgot the correct graph, the subject experienced an error in determining the value of the trigonometric function when using a graphical representation.

Next, one prospective teacher was taken as the fourth subject (S4) out of 3 prospective teachers who used algebraic representations (namely angular relations) in determining the value of the special angle trigonometric function. The results of the fourth subject's work can be seen in Figure 7.

Figure 7. Representation using angular relations of the fourth subject

From Figure 7 it can be seen that the subject uses an algebraic representation in the form of an angular relation. However, the subject did not determine the value of his special angle trigonometric function. Therefore, the subject experienced an error in determining the value of the special angle trigonometric function. An explanation of the methods used by the subject can be seen from the following interview transcript excerpt.

R   :   *Try to explain how you solve the problem!*
S4 :   **... Sin 30^0 is equal to cos 60 degrees or cos 60 degrees is equal to sin 30 degrees.**
R   :   *Why are you using this method?*
S4 :   *Because I know it, sir*

From the interview transcript, it can be seen that the subject uses only angular relations (without determining the value of the trigonometric function). Also, the subject only knows that one way to determine the value of the trigonometric function of a special angle.

From the research results, it is found that the representation used by subjects with low abilities is the representation using a triangle. It can be seen that the subject using the triangle representation has used the correct concept. Meanwhile, subjects who use verbal representations, graphics, and angular relations tend to make mistakes. This error arose because the subjects only memorized the value of the special angle trigonometric function, memorized the graph of the trigonometric function, and also memorized the angle relation. Therefore, learning starts from the concept of a triangle to explain the value of the trigonometric function for special angles.

3.1.2. Prospective Teachers Representation of Medium Ability Categories

From Figure 3 it can be seen that of the 40 prospective teachers who have the medium ability, 58% of prospective teachers use image representations (22 use triangles and 1 use graphs), 12% of prospective teachers use numerical representations (tables of special angle trigonometric function values), and 30% of prospective teachers use algebraic representations (i.e. angular relations) in determining the value of the acute angle trigonometric function. For each of the types of representation used, one prospective teacher will be selected as a research subject who has medium abilities. So that the research subjects obtained for prospective teachers who have a medium ability category are four prospective teachers. The results of the fifth subject's work (S5) can be seen in Figure 8.
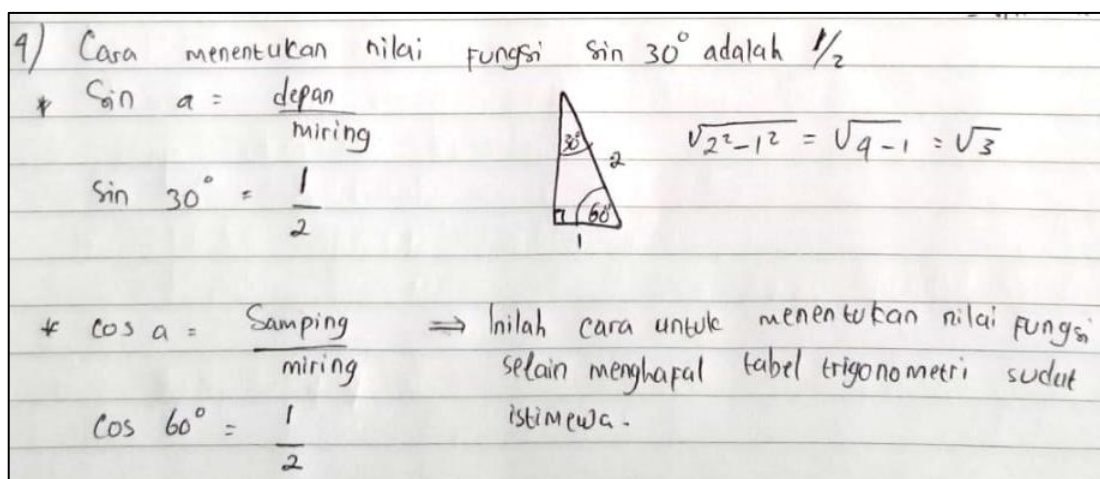

Figure 8. Representation using triangles of the fifth subject

From Figure 8 it can be seen that the subject is correct in determining the value of the trigonometric function for special angles using a triangle representation. An explanation of the method used by this subject can be seen from the following interview excerpt.

R   :   *Try to explain how you solve the problem!*
S5 :   *... I made a right triangle with the other angles 60^0 and 30^0. **Because it comes from an equilateral triangle, I take the side lengths 2 and 1**, then I find the other side lengths **using Pythagoras**. After that, I determined the values of sin 30 and cos 60 using the **sindemi (sin = front/hypotenuse, and **cosami (cos = side/hypotenuse)**.*
R   :   *Why are you using this method?*
S5 :   *Because this material is the same as the class XI material, **I still remember** the old material in senior high school.*

From the interview transcript excerpt, it can be found that the subject has understood how to determine the value of the trigonometric function of a special angle using a triangular representation. This method is obtained by remembering the material on special angle trigonometry.

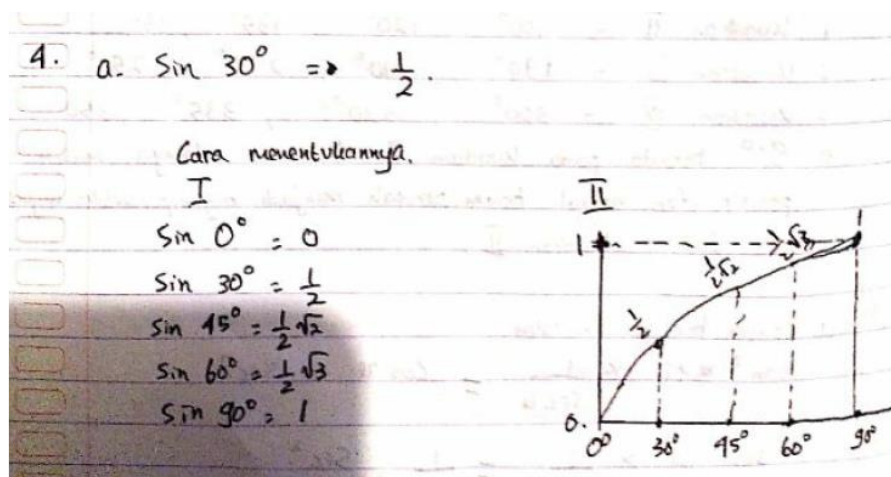Next is the sixth subject (S6) that uses graphics as can be seen in Figure 9.

Figure 9. Graphical representations of the sixth subject

From Figure 9 it can be seen that the subject can use the graphical representation correctly to determine the value of the special angle trigonometric function. The explanation of the method used by the subject can be seen from the following interview excerpt.

P   :   *Try to explain how you solve the problem!*
S6 :   *... I use a coordinate system. x and y coordinates. **when x is 1 and y is 1, then it is in sin 90 because sin starts from 0^0, the value is zero, then the upward value gets bigger,** so when **sin is 30^0 the value is half**. For cos, I use the opposite method for sin. When sin 0^0 equals cos 90^0, then the value of cos decreases in value. So when cos 60^0, the value equals sin 30^0, so cos 60^0 equals ½. ...*

From the interview transcript, it can be seen that the subject has understood the graph of the sin function and the graph of the cos function. Therefore, the subject answered correctly in determining the value of the special angle trigonometric function.

The second representation used by prospective teachers is a numerical representation in the form of a table of the values of the special angle trigonometric function. Of the 5 prospective teachers who used this representation, 1 prospective teacher was chosen as the seventh subject (S7) in this study. The results of the seventh subject's work can be seen in Figure 10.

Figure 10. Numeric representation of seventh subjects

From Figure 10 it can be seen that the table made by the subject still has errors, namely the value of cos 60º, while the value of sin 30º is correct. As a result, the subject experienced an error in determining the value of the trig function for cos 60º. The explanation of this method can be seen from the following interview transcript excerpt.

R   :   *Try to explain how you solve the problem!*
S7   :   *I use the method in Sir's books, namely **I make a table of the values of the trigonometric function of special angles using memorization**. Then I write down the values of the special angle trigonometric functions in the table. So that the value of sin 30^0 is ½ and the value of cos 60^0 is ½ the root of three.*

From the interview transcript excerpt, it can be seen that the subject still uses rote memorization in determining the values of the special angle trigonometric function which is then made a table. As a result, the trigonometric function value is still wrong, namely the value of cos 60º.

Next is the eighth subject (S8) which uses an algebraic representation in the form of angular relations. The results of the eighth subject's work can be seen in Figure 11.

Figure 11. Algebraic representation of the tenth subject

From Figure 11 it can be seen that the subject uses angle relations. However, the subject is wrong in writing the subtraction form of the cos function. As a result, the subject experienced an error in determining the value of the special angle trigonometric function. The explanation of the subject can be seen from the following interview excerpt.

R   :   *Try to explain how you solve the problem!*

S8   :   *... **Cos** 60^0 like that a, I also reduce it. So **cos 90 minus cos 30 then I break it down to be cos 90 multiplied sin 30 minus sin 90 multiplied cos 30.** Cos 90 is equal to zero multiplied ½ minus sin 90 1 multiplied cos 30 is $\frac{1}{2}\sqrt{3}$. Then minus $\frac{1}{2}\sqrt{3}$ like that*

R   :   *Why are you using this method?*

S8   :   *Because this material is the same as class XI material, **I still open old books in senior high school.***

From the interview transcript, it can be seen that the subject determines the value of the cosine function by describing it. However, the subtraction formula for the cos function is still wrong. This error is caused by only opening material during senior high school, without understanding how the formula is correct. Therefore, the subject experienced an error in determining the value of the special angle trigonometric function.

From the research results, it is found that the representation used by subjects with medium ability is the representation using the triangle and the representation using the sin and cos function graphs. It can be seen that the subject using the triangle representation and the graphical representation of the sin and cos functions has used the correct concept. Meanwhile, there are subjects who use numerical and algebraic representations who still experience errors in determining the value of trigonometric functions. Because the subject only memorized the value of the special angle trigonometric function and memorized the formula for the angle relation. Therefore, learning for medium category prospective teachers can be done with the concept of triangles and graphs to explain the value of the trigonometric function for special angles.

3.1.3. Prospective Teachers Representation of High Ability Categories

From Figure 3 it can be seen that out of 23 prospective teachers who have high abilities, 70% of prospective teachers use image representations (i.e. 15 prospective teachers use triangles and 1 prospective teacher use circles), 4% of prospective teachers use numerical representations (i.e. use tables), and 26% of prospective teachers use algebraic representations (i.e. angular relations). From each of these representations, one student will be selected as the research subject. So that the number of research subjects for students with high ability categories there are 4 prospective teachers.
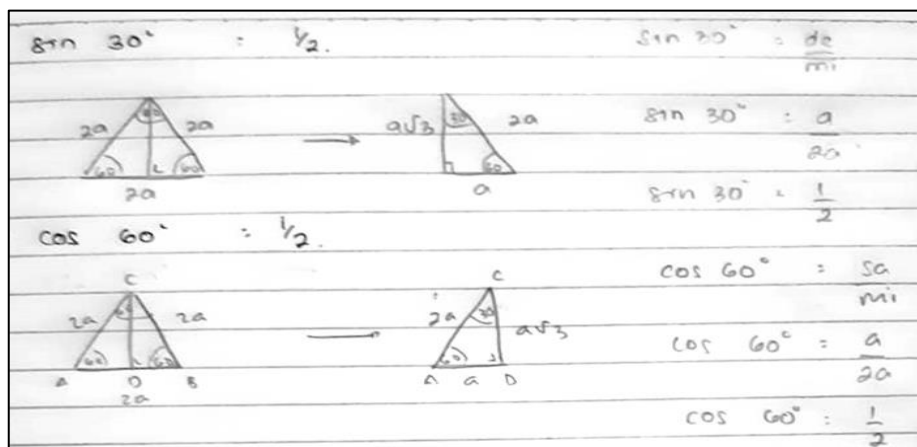

Figure 12. Triangular representation of the ninth subject

From Figure 12 it can be seen that the subject uses the correct triangle representation to determine the value of the special angle trigonometric function. An explanation of the method used by the subject can be seen from the following snippet of the interview transcript.

R	:	*Try to explain how you solve the problem!*
S9	:	*First **I made an equilateral triangle by assuming the side length is 2a**, then I divided the triangle into two so that **two congruent right triangles** were obtained. Then I determine the value of sin 30^0, that is, the **sindemi** is $\frac{a}{2a} = \frac{1}{2}$. Next, I determine the value of cos 60, that is, **cosami** is $\frac{a}{2a} = \frac{1}{2}$. So we get sin $30^0 = \frac{1}{2}$ and cos $60^0 = \frac{1}{2}$.*
R	:	*Why are you using this method?*
S9	:	*Because in my opinion, this method is easy to use.*

From the interview transcript excerpt, it can be seen that the subject already understands how to use triangular representations, namely sindemi (sin = front/hypotenuse) and cosami (cos = side/hypotenuse) in determining the value of the special angle trigonometric function.

Furthermore, the tenth subject (S10) uses a circular image representation to determine the value of the special angle trigonometric function. The results of the tenth subject's work can be seen in Figure 13.
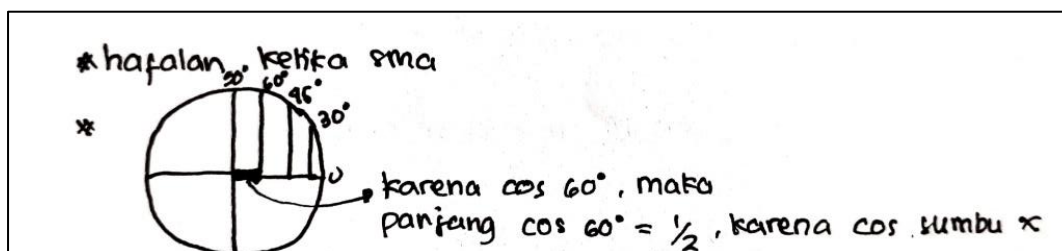

Figure 13. Representation of circles of tenth subject

From Figure 13 it can be seen that the subject uses a circle to determine the value of the trig function. In this case, the subject managed to answer correctly. The explanation of the subject in using the circle can be seen from the following interview transcript excerpt.

R	:	*Try to explain how you solve the problem!*
S10	:	*... I use a circle to determine the value of the sin and cos functions, **where the y-axis is the sin value and the x-axis is the cos value.** So I wrote that the result of sin 30^0 is equal to ½ because the value of ½ is on the straight y-axis at an angle of 30^0 and the result of cos 60^0 is ½ because the value of ½ is on the x-axis.*
R	:	*Why are you using this method?*
S9	:	*Because besides memorizing, **I also learned about circles to determine the value of trigonometric functions.***

From the interview excerpt, it can be seen that the subject has an understanding that the x-axis is the value of the cos function and the y-axis is the sin value. This understanding is by following the concept of the unit circle. Where the unit circle has a radius of 1 unit. If we are going to find the value $\sin \theta = \frac{y}{r} = \frac{y}{1} = y$ and $\cos \theta = \frac{x}{r} = \frac{x}{1} = x$. So subject with high abilities can understand the representation of the circle to determine the value of the trigonometric function of special angles.

Next is the eleventh subject (S11) who uses a numeric representation in the form of a table in determining the value of the special angle trigonometric function. The results of the eleventh subject's work can be seen in Figure 14.

Figure 14. Table Representation of the Eleventh Subject

From Figure 14 it can be seen that the subject determines the values of the trig function correctly so that the subject can determine the value of the functions sin 30° and cos 60° correctly. An explanation of the methods used by the subject can be seen from the following interview transcript excerpt.

R   :   *Try to explain how you solve the problem!*
S11   :   *I'm **using a table of values for sin, cos, and tan**. So I immediately determined the result of* $\sin 30^0 = \frac{1}{2}$ *and* $\cos 60^0 = \frac{1}{2}$.
R   :   *Where did you get your values in the table?*
S11   :   *I obtained the values in the table **using a right triangle** with angles* 30^0, 45^0, *and* 60^0, *while the angles* 0^0 *and* 90^0 *are quadrant angles.*

From the interview excerpt, it can be seen that the subject obtains the values in the table by using a right triangle. However, the subject did not write down the triangle method, where the subject focused more on obtaining the trigonometric ratio values which were then written in the table. This means that the subject can use the right triangle representation and the subject can also use the numerical representation with a focus on the numerics generated from the right triangle.

Next is the twelfth subject (S12) which uses algebraic representations in the form of angular relations. The work of the twelfth subject can be seen in Figure 15.

Figure 15. Representation of angular relations of the twelfth subject

From Figure 15 it can be seen that the subject uses the angular relation, which is the relation between sin and cos. The explanation of the method used by the subject can be seen from the following interview transcript excerpt.

R   :   *Try to explain how you solve the problem!*
S12 :   *I write* $\sin 30^0 = \sin(90 - 60)$, *because* $\sin(90 - \alpha) = \cos \alpha$, *we get* $\sin 30^0 = \cos 60^0 = \frac{1}{2}$. *Then for* $\cos 60^0 = \cos(90 - 30)$, *because* $\cos(90 - \alpha) = \sin \alpha$, *we get* $\cos 60^0 = \sin 30^0 = \frac{1}{2}$.
R   :   *Why are you using this method?*
S12 :   **Because this method can be used to determine the values of acute angles**.

From the interview transcript excerpt, it can be seen that the subject has a good understanding of the angular relation, namely the relation between the function sin and cos at an acute angle. Therefore, the subject managed to answer correctly.

From this explanation, it can be concluded that the representations used by highly skilled subjects are representations using triangles, representations using circles, representations using tables, and representations using angular relations. It can be seen that subjects who use triangular representations, circle representations, table representations, and angle relation representations have used the correct concept. There are no students who experience errors in determining the value of trigonometric functions. This is because the subjects have understood from the representations they use to determine the value of the special angle trigonometric function. Therefore, learning for high category prospective teachers can be done with the concept of triangles, circles, tables, and angle relations to explain the value of the special angle trigonometric function.

From the explanation of the results of this study, in general, a description of the prospective teachers representation in problem-solving the value of the trigonometric function for special angles based on the ability category can be seen in Table 3.

Table 3. Description of prospective teachers representations in solving special angle trigonometric function problems

Ability Category	Type of Representation			
	Verbal	**Image**	**Numeric**	**Algebra**
Low, namely prospective teachers who get a concept comprehension test score below 55.	The verbal representation used is still based on memorization and there are errors in memorizing the value of the special angle trigonometric function.	The image representation that has been successfully used is a right triangle, while the graph representation of the sin and cos functions is still experiencing errors.	Numerical representations are not used.	Algebraic representations that use angular relations still have errors.
Medium, namely prospective teachers who get a concept comprehension test score between 55 to 80.	Verbal representations in the form of rote are not used.	The image representation in the form of a right triangle and the graph of the sin and cos functions is used correctly.	The numerical representation in the form of a table of trigonometric function values has an error.	The algebraic representation in the form of an angular relation still has errors.

Ability Category	Type of Representation			
	Verbal	Image	Numeric	Algebra
High, namely prospective teachers who get a concept comprehension test score above 80.	The verbal representation in the form of rote is used correctly.	The image representation in the form of right triangles and circles is used correctly.	The numerical representation in the form of a table of trigonometric function values can be used correctly.	Algebraic representations in the form of angular relations can be used correctly.

3.2. Discussion

The results of this study contribute to the representation theory used by prospective teachers in problem-solving the value of the trigonometric function of special angles based on the background category of concept understanding ability. The results showed that prospective teachers who are in the low ability category can only use the right triangle representation to determine the value of the special angle trigonometric function. Prospective teachers in the medium ability category can use right triangle representations and graphs. Meanwhile, prospective teachers in the high ability category can correctly represent right triangles, unit circles, tables, and angular relations. The results of this study are in accordance with the results of previous studies which show that representation can help students solve math problems (Byers, 2010; Cooper & Alibali, 2012; Özsoy, 2018). However, the results of this study expand on the results of previous studies by explaining the representations used by prospective teachers in solving special angle trigonometric function problems based on ability categories.

The first is that prospective teachers with low ability categories can only use image representations in the form of right triangles correctly in solving the problem of special angle trigonometric functions. This can be seen when prospective teachers use verbal representations in the form of memorization, image representations in the form of graphics, and algebraic representations in the form of angular relations still experiencing errors. The errors that arise are generally caused by students only memorizing the methods used and not understanding the methods used. Previous research results also showed that prospective teachers still experienced errors in drawing trigonometric function graphs (Jaelani, 2017; Setiawan, 2021). The results of other studies also show that prospective teacher's understanding of concepts in trigonometric material is still lacking (Mustangin & Setiawan, 2021; Nabie et al., 2018; Tuna, 2013). However, what is new from the results of this study is that all prospective teachers with low abilities can use the triangle representation correctly so that they can successfully solve the problem of special angle trigonometric functions. Therefore, learning material on this special angle trigonometric function should be started by using a right triangle representation.

Second, prospective teachers with the medium ability category can only use image representations in the form of right triangles and graphs of the sin and cos functions. The results of this study indicate that prospective teachers in this category are successful in using the representation of right triangles and graphs. This is because prospective teachers already understand the concept of right triangles and the graphs used to determine the value of a special angle trigonometric function. Meanwhile, prospective teachers who use numerical and algebraic representations still experience errors. This error is caused due to inaccuracy in writing the values of trigonometric functions in tables and inaccuracy in writing the formula for angular relations. Previous research results also show that this accuracy is

directly proportional to the abilities possessed (Byers, 2009; Hästö et al., 2019). The results also show that the result of this inaccuracy is that students experience errors in solving math problems (Setiawan, 2020b, 2020c, 2021; Setiawan et al., 2020). The results of other studies also show that students can correct wrong answers by reexamining the wrong answers (Setiawan, 2020e, 2020f). The results showed that this accuracy is also important in using certain representations in solving mathematical problems.

      The third is that prospective teachers with high ability categories can use verbal, pictorial, numerical, and algebraic representations in solving special angle trigonometric function problems correctly. This can be seen from the results of research which show that prospective teachers in this category have understood how to use right triangles, unit circles, tables of special angle trigonometric function values, and algebraic relations correctly in determining the value of trigonometric functions. This means that a person's ability can influence the representation they use to solve problems. The results of this study are in accordance with the results of previous studies which show that students who can prove it are students who have relevant knowledge (Setiawan, 2020a). Even this ability is an important component of thinking disposition (Perkins et al., 1993; Ron, 2001; Setiawan, 2020d; Tishman & Andrade, 1995; Tishman et al., 1993). Ability in the context of problem-solving is defined as the adequacy of knowledge that a person has in solving problems. Therefore, prospective teachers who have high abilities have knowledge of visual, image, numerical, and algebraic representations in solving special angle trigonometric function problems.

      So in general it can be said that representation is influenced by the ability in the form of sufficient knowledge capacity in using the representation. For example, low-ability prospective teachers are successful in using the right triangle representation because they have good knowledge about the use of this right triangle representation. Therefore it is important to equip prospective teachers with knowledge of various representations when solving the problem of special angle trigonometric functions. One of the methods proposed by the researcher based on the results of this study is to teach various representations according to the prospective teacher's ability level. Thus, the main implication of the results of this study is that it contributes to the learning of special angle trigonometric functions using various representations based on ability categories (see Figure 16).
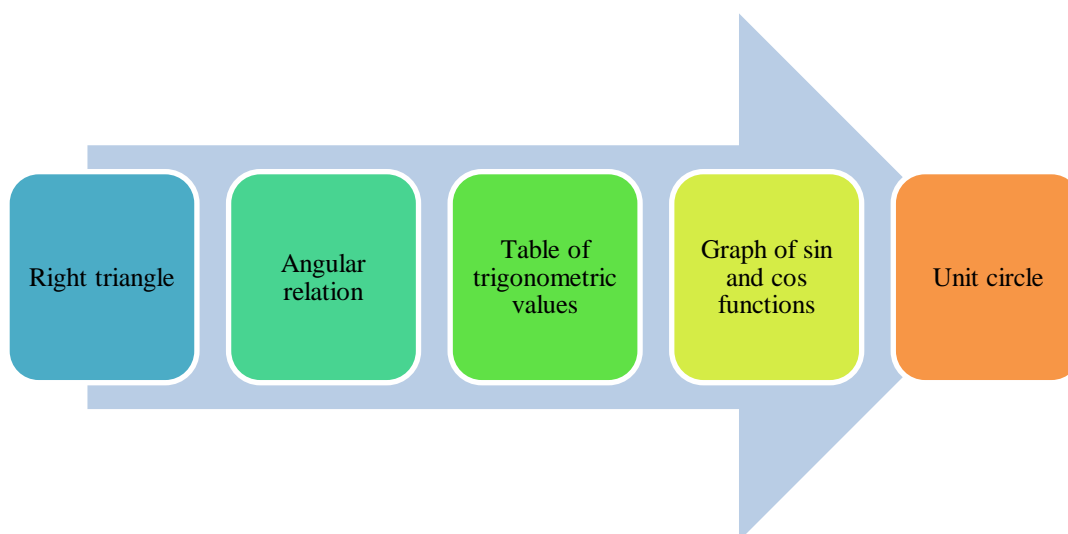

Figure 16. Representation in learning trigonometric functions special angle

      From Figure 16 it can be seen that the first representation used in learning the value of a special angle trigonometric function is a right triangle using the acronyms SINDEMI

(sin = front/hypotenuse), COSAMI (cos = side/hypotenuse), and TANDESA (tan = front/side). The results of this study indicate that all prospective teachers of a low, medium, and high ability can use this right triangle representation. Therefore, as a basis for learning the value of the trigonometric function for special angles is to use a right triangle. The second representation is the angular relation. Although the results of this study indicate that there are prospective teachers with the medium ability category who experience errors in using angular relations. However, many prospective teachers use this angular relation. Therefore, it is important to learn about the correct angular relation knowledge in determining the value of the trigonometric function for special angles. The third representation is to use a table of the values of the trigonometric function for special angles. This is because after understanding right triangles and angular relations in determining the value of the trigonometric function for special angles, the resulting values of right triangles are presented in tabular form. The results of this study also showed that prospective teachers with medium abilities used tables a lot. The fourth representation is by using the graph of the sin and cos functions. This is because the values in the table have a relationship if they are placed in a Cartesian field. This relationship will form a graph, which is a graph of the sin and cos functions. Therefore, this fourth representation is to use the graph of the sin and cos functions in learning the value of the special angle trigonometric function. The fifth representation is using the unit circle. By providing an understanding that the graph of the sin and cos functions is a circular shape with a radius of one unit. Therefore, the representation of the unit circle is closely related to the graph of the sin and cos functions. Through learning these five representations, it is hoped that prospective teachers can understand the material value of the trigonometric function for special angles properly and correctly. So that the problems or difficulties of prospective teachers in studying the material for the value of trigonometric functions can be reduced.

4.   CONCLUSION

From the research results, it can be concluded that the background abilities possessed by prospective teachers affect the representation used in determining the value of the special angle trigonometric function. Prospective teachers with low abilities can use triangular representations, prospective teachers with medium ability can use triangular and graphical representations, while prospective teachers who have high abilities can use representations of triangles, unit circles, tables of values of special angle trigonometric functions, and angular relations in determining the value of the acute angle trigonometric function. This research is only limited to special angle material which is only based on the background of the ability to understand the concept. The next research can investigate the representation used by students or prospective teachers in determining the value of the trigonometric function for non-acute angles. The results of this study will contribute to an understanding of the methods used by students or prospective teachers in determining non-acute angles, namely whether they tend to use graphical representations, unit circles, tables of trigonometric function values, reference angles, and angle relations. Researchers also recommend lecturers or teachers use a sequence of representations in teaching special angle trigonometric functions based on ability.

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