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Propositional Relevance[1]

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I. Introduction

The purpose of this paper is to ascertain the nature of one kind of relevance and irrelevance that one proposition can have to another. I shall defend the following definitions, which, in simpler versions, were rejected over sixty-five years ago:

> Proposition '*p*' is relevant to proposition '*q*'[2] if and only if, considering only '*p*' and '*q*', the probability of '*q*' conditional on '*p*' is greater or less than ½; and '*p*' is irrelevant to '*q*' if and only if, considering only '*p*' and '*q*', the probability of '*q*' conditional on '*p*' is ½.[3]

These definitions do not capture all of the senses in which one proposition may be relevant or irrelevant to another. For instance, they do not capture the sense in which, because they share common subject-matter, '*p*' is relevant to the conjunction '*p* and *q*' and 'Some philosophers are left-handed' is relevant to 'Some philosophers are fathers'.[4] But they do, I think, define the senses of 'relevant' and 'irrelevant' that we have in mind when, in evaluating an argument, we say that its premises are either relevant or irrelevant to its conclusion.

Those senses are exemplified below, in propositions (1)-(6a). (1) is relatively uncontroversial. It says that relevance and irrelevance, in the senses to be defined, are contrary (*i.e.*, mutually exclusive) relational properties.

1. If '*p*' is relevant to '*q*', then it is false that '*p*' is irrelevant to '*q*'; and, consequently, if '*p*' is irrelevant to '*q*', then it is false that '*p*' is relevant to '*q*'.

Now consider the following data, all of which assume normal, non-paradoxical circumstances:[5]

2a. If '*p*' makes '*q*' certain, then '*p*' is relevant to '*q*'. (For example, 'Some philosophers are fathers' makes 'Some fathers are philosophers' certain, and so the first of these propositions is relevant to the second.)

3a. If '*p*' makes '*q*' probable, then '*p*' is relevant to '*q*'. (For example, 'It is probable that some philosophers are fathers' makes 'Some fathers are philosophers' probable, and so the first of these propositions is relevant to the second.)

4a. If '*p*' makes '*q*' improbable, then '*p*' is relevant to '*q*'. (For example, 'It is improbable that some philosophers are fathers' makes 'Some fathers are philosophers' improbable, and so the first of these propositions is relevant to the second.)

5a. If '*p*' makes '*q*' impossible, then '*p*' is relevant to '*q*'. (For example, 'It is impossible that some philosophers are fathers' makes 'Some fathers are philosophers' impossible, and so the first of these propositions is relevant to the second.)[6]

6a. If '*p*' makes '*q*' neither certain, nor probable, nor improbable, nor impossible, then '*p*' is irrelevant to '*q*'. (For example, 'Some philosophers are left-handed' makes 'Some fathers are philosophers' neither certain, nor probable, nor improbable, nor impossible, and so it is irrelevant to the second proposition.)[7]

Relevance, in the sense here exemplified, may be either favorable (or positive), as in (2a) and (3a), where 'p' makes 'q' certain or probable; or unfavorable (or negative), as in (4a) and (5a), where 'p' makes 'q' improbable or impossible.

II. Criticism of the received definitions

The received definitions of relevance and irrelevance, given by John Maynard Keynes,[8] are—

7. 'p' is relevant to 'q' conditional on the background evidence 'h' if the probability of 'q' conditional on the conjunction of 'h' and 'p' is not the same as its probability conditional on 'h' alone. And 'p' is irrelevant to 'q' conditional on 'h' if the probability of 'q' conditional on the conjunction of 'h' and 'p' is the same as its probability conditional on 'h' alone.[9]

These definitions do not define the kinds of relevance and irrelevance exhibited in propositions (2a)-(6a). For suppose the background evidence, 'h', makes 'q' certain. Suppose also that 'p' by itself makes 'q' certain. Since the probability of 'q' conditional on 'h' is the same with as without 'p', according to (7) 'p' is irrelevant to 'q' conditional on 'h'—despite its making 'q' certain.[10] Similar comments might be made concerning degrees of probability lower than certainty.[11]

(7) also has the consequence that the time when a proposition comes to be known can determine whether it is relevant to another proposition. Suppose that 'p' and 'q' come to be known simultaneously, that each makes 'r' certain, but that their common background information, 'h_1', does not make 'r' certain. Then, according to (7), both 'p' and 'q' are relevant to 'r'. But now suppose that 'p' comes to be known so much earlier than 'q' that, by the time 'q' is known, 'p' has become part of 'q's background information, 'h_2'. Under this supposition, (7) leads to the conclusion that 'q' is irrelevant to 'r', since the probability of 'r' conditional on the conjunction of 'q' and 'h_2' is the same as the probability of 'r' conditional on 'h_2' alone. Hence, according to (7), the time when 'q' comes to be known makes a difference to whether it is relevant to 'r'—which disagrees with the senses of 'relevant' and 'irrelevant' exemplified in (2a)-(6a).

There are two features of (7) that prevent it from dealing adequately with the data (2a)-(6a): first, it makes relevance depend on a *difference* in a proposition's conditional probabilities; and second, it does not consider the probability of 'q' conditional only on 'p' but instead makes relevance always involve the background evidence, 'h'.[12] In data (2a)-(6a), however, whether 'p' is relevant or irrelevant to 'q' depends only on how probable it, and it alone, makes 'q'. Whatever senses of 'relevant' and 'irrelevant' (7) may correctly define, it does not correctly define the senses exemplified in the propositions (2a)-(6a).

III. The proposed definitions

Let us now try to work out definitions of relevance and irrelevance that can account for (2a)-(6a). The first step is to consider the following definitions:

2b. A proposition is certain if and only if its probability is 1.[13]

3b. A proposition is probable if and only if its probability is less than 1 but greater than ½.

4b. A proposition is improbable if and only if its probability is less than ½ but greater than 0.

5b. A proposition is impossible if and only if its probability is 0.

6b. A proposition is neither certain, nor probable, nor improbable, nor impossible if and only if its probability is ½.

Bearing these definitions in mind, and remembering that if a proposition's probability belongs to it, not intrinsically, but with reference to at least one other proposition, then that probability is *conditional* on the other proposition, we see that all of the relations included in the data (2a)-(6a) are expressible in terms of conditional probability:[14]

2c. 'p' makes 'q' certain if and only if the probability of 'q' conditional on 'p' is 1. That is to say, 'q' is validly deducible from 'p' if and only if the probability of 'q' conditional on 'p' is 1.

3c. 'p' makes 'q' probable if and only if the probability of 'q' conditional on 'p' is less than 1 but greater than ½.

4c. 'p' makes 'q' improbable if and only if the probability of 'q' conditional on 'p' is less than ½ but greater than 0.

5c. 'p' makes 'q' impossible if and only if the probability of 'q' conditional on 'p' is 0.

6c. 'p' makes 'q' neither certain, nor probable, nor improbable, nor impossible if and only if the probability of 'q' conditional on 'p' is ½.[15]

So, in the language of conditional probability, the data (2a)-(6a) are:

2d. If the probability of 'q' conditional on 'p' is 1, then 'p' is relevant to 'q'.

3d. If the probability of 'q' conditional on 'p' is less than 1 but greater than ½, then 'p' is relevant to 'q'.

4d. If the probability of 'q' conditional on 'p' is less than ½ but greater than 0, then 'p' is relevant to 'q'.

5d. If the probability of 'q' conditional on 'p' is 0, then 'p' is relevant to 'q'.

6d. If the probability of 'q' conditional on 'p' is ½, then 'p' is irrelevant to 'q'.

So translated, data (2d)-(6d) entail the following statement of sufficient conditions for one proposition to be relevant or irrelevant to another:

8. If the probability of 'q' conditional on 'p' is greater or less than ½, then 'p' is relevant to 'q'; but if that probability is equal to ½, then 'p' is irrelevant to 'q'.

And from (8) and (1) follow necessary and sufficient conditions for one proposition to be relevant or irrelevant to another:

9. 'p' is relevant to 'q' if and only if the probability of 'q' conditional on 'p' is greater or less than ½; and 'p' is irrelevant to 'q' if and only if the probability of 'q' conditional on 'p' is ½.[16]

(9) defines relevance and irrelevance.

This definition accords well—certainly better than (7)—with the lexical definition of relevance as "affording evidence tending to prove or disprove the matters at issue or under discussion."[17] And it captures, I think, the senses in which we say that an argument's premises are either relevant or irrelevant to its conclusion. For when we affirm or deny, say, that an argument's premises are favorably relevant to its conclusion, we mean either that they do or that they do not make it probable or certain—*i.e.*, either that they do or that they do not confer on it a probability greater than ½—; and when we say that an argument's premises are irrelevant to its conclusion, we mean that they make the conclusion neither certain, probable, improbable, nor impossible—*i.e.*, that they confer on it a probability of ½.

The sense of conditional probability employed in (9) is not that defined by the probability calculus, for the following reasons. In the probability calculus the definition of conditional probability is—

$$10.\ \Pr(q \mid p) = \frac{\Pr(q \,\&\, p)}{\Pr(p)}$$

—, where '$\Pr(q \mid p)$' is read 'the probability of "q" conditional on, or given, "p"'. One way in which 'p' may be relevant to 'q' is that 'q' may be validly deducible from 'p'. In that case, according to (2c) above, $\Pr(q \mid p) = 1$. Now, two problems arise concerning contradictions. First, it is possible validly to deduce a contradiction from premises: we do so in the course of a *reductio ad absurdum.* In such a case, 'q' is a contradiction, and $\Pr(q \mid p)$ should equal 1. But according to (10) it cannot. For since 'q' is a contradiction, its own probability is 0. Hence, the numerator of the fraction on the right-hand side of (10) is 0, since the conjunction of any proposition with a contradiction is no more probable than the contradiction alone. Consequently, in this case $\Pr(q \mid p)$ is not 1, as it should be. Second, it is possible validly to deduce a conclusion from a contradiction. In that case, 'p' is a contradiction, and, according to (2c), $\Pr(q \mid p)$ should equal 1. But again according to (10) it cannot. For since 'p' is a contradiction, its own probability is 0. Hence, the denominator of the fraction on the right-hand side of (10) is 0, so that in this case $\Pr(q \mid p)$ is not 1, as it should be, but no value at all; for division by 0 is undefined.

To avoid these absurd results, we must exclude the probability calculus' definition of conditional probability from (9). In its place we employ a familiar but as yet unmathematized concept of conditional probability. When in everyday life we attempt to ascertain the degree of probability that one proposition confers on another, it is not (10)—a mathematical operation on previously ascertained probabilities—that we use but an assessment according either to the propositions, or meanings, involved or to their forms. For instance, when we judge that the premise of the argument—

11. *Premise:* It is probable that some philosophers are fathers.
Conclusion: Some fathers are philosophers.

—makes its conclusion probable, we assess the probability of the conclusion 'Some fathers are philosophers', conditional on the premise 'It is probable that some philosophers are fathers'; and we do so, not by first ascertaining the probability of the conjunction 'It is probable that some philosophers are fathers, and some fathers are philosophers' and then dividing by the probability of 'It is probable that some philosophers are fathers' (for ordinarily we do not know these probabilities; nor, I suspect, would we consult them if we did know them), but by considering the (perhaps formal) relation between the two propositions themselves.[18] Similarly, when we judge that 'This object is red all over' is inconsistent with 'This object is blue all over', we do so by consulting the *propositions,* or meanings, involved, rather than, as the probability calculus' definition of conditional probability would dictate, their *probabilities*. Whatever profitable use the probability calculus may make of its own concept of conditional probability as defined in (10) above, that concept does not seem to be either the only one possible or the one that we ordinarily use in evaluating arguments—particularly those whose premises or conclusions are contradictions.

IV. Replies to objections

That is (9) and the evidence in its favor. Let us next examine criticisms, some of which will require a modification in it.

Objection 1, which is adapted from Keynes, goes like this.[19] Consider the three contrary (*i.e.*, mutually exclusive) propositions 'This book is red', 'This book is blue', and 'This book is black'. To each of them the proposition 'This book weighs twelve ounces' is irrelevant, since the weight of a book is irrelevant to its color. Hence, by (9), the probability of each, conditional on 'This book weighs twelve ounces', is ½. Now, according to the probability calculus' Special Addition Rule, where 'q', 'r', and '*s*' are contrary propositions, the probability of the disjunction 'Either "*q*" or "*r*" or "*s*"' conditional on '*p*' is equal to the probability of '*q*' conditional on '*p*', added to both the probability of '*r*' conditional on '*p*' and the probability of '*s*' conditional on '*p*'. Therefore, (9) leads to the conclusion that the probability of the disjunction 'Either this book is red, or it is blue, or it is black' conditional on 'This book weighs twelve ounces' is ½ + ½ + ½, or 1½. But this is impossible, since the highest probability-value is 1. So, Objection 1 concludes, (9) is false.[20]

Objection 2, which also comes from Keynes, is this. An object's weighing twelve ounces is irrelevant to its being red. And so, according to (9), the probability of 'Object *a* is red' conditional on 'Object *a* weighs twelve ounces' is ½. But likewise, an object's weighing twelve ounces is irrelevant to its being a red book. Therefore, according to (9), the probability of the conjunction 'Object *a* is red, and object *a* is a book' conditional on 'Object *a* weighs twelve ounces' is also ½. These two probabilities, each being equal to ½, are equal to each other. That is, the probability of 'Object *a* is red' conditional on 'Object *a* weighs twelve ounces' equals the probability of the conjunction 'Object *a* is red, and object *a* is a book' conditional on 'Object *a* weighs twelve ounces'. Now, the only way that '*p*'s probability conditional on '*q*' can equal the probability of '*p* and *r*', again conditional on '*q*', is for '*p*' to make '*r*' certain. For, unless '*p*' entails '*r*', the probability of the conjunction '*p* and *r*', conditional on '*q*', must always be less than the probability of '*p*' alone, conditional on '*q*'. Proposition (9), then, leads to the consequence that the proposition 'Object *a* is red' must entail the proposition 'Object *a* is a book'. Since this consequence is obviously false, (9), which leads to it, is false as well.[21]

The initial plausibility of these two objections may be dispelled by making explicit a restriction already implicit in (2a)-(6a). That restriction is that consideration be limited to the propositions '*p*' and '*q*'. Incorporating this restriction into our definitions, we have—

12. '*p*' is relevant to '*q*' if and only if, considering only the propositions '*p*' and '*q*', the probability of '*q*' conditional on '*p*' is greater or less than ½; and '*p*' is irrelevant to '*q*' if and only if, considering only the propositions '*p*' and '*q*', the probability of '*q*' conditional on '*p*' is ½.

This restriction is not *ad hoc* but implicit in our initial data, (2a)-(6a), because when we judge that '*p*' is relevant to '*q*' in that '*p*' makes '*q*' certain, we are considering only the propositions '*p*' and '*q*'. No other propositions enter into our judgment. Similarly, when we judge that '*p*' is relevant to '*q*' because '*p*' makes '*q*' probable, we consider only '*p*' (on which the probability is conditional) and '*q*' (on which the probability is conferred). Even if '*q*' is made certain, probable, improbable, or even impossible by propositions other than '*p*', those other propositions do not enter into our judgment that '*p*' is relevant to '*q*' because it makes '*q*' probable. And so on with the remainder of the data. In each case, when we judge that '*p*' is relevant or irrelevant to '*q*' because it stands in some probability-relation to '*q*', it is implicit that we are restricting our attention to '*p*' and '*q*'.

Let us see how this restriction will permit us to avoid Objection 1. When we judge that the probability of 'This book is red' is ½, conditional on 'This book weighs twelve ounces', because the latter proposition is irrelevant to the former, we restrict our consideration to the two propositions 'This book weighs twelve ounces' and 'This book is red'. This accords with our restriction, as do our similar judgments that 'This book weighs twelve ounces' confers a probability of ½ on 'This book is blue' and on 'This book is black'. Now, according to the Special Addition Rule, the probability of the disjunction 'Either this book is red, or it is blue, or it is black' conditional on 'This book weighs twelve ounces' equals the probability of 'This book is red' conditional on 'This book weighs twelve ounces' plus the probability of 'This book is blue' conditional on 'This book weighs twelve ounces' plus the probability of 'This book is black' conditional again on 'This book weighs twelve ounces'. But the condition under which the probability of 'This book is red' conditional on 'This book weighs twelve ounces' is ½ is inconsistent with the condition under which the probability of 'This book is blue' conditional on 'This book weighs twelve ounces' is ½; and both of these conditions are inconsistent with that under which the probability of 'This book is black' conditional on 'This book weighs twelve ounces' is ½. For the first probability is ½ if we consider only the two propositions 'This book is red' and 'This book weighs twelve ounces'; the second is ½ if we consider only the two propositions 'This book is blue' and 'This book weighs twelve ounces'; and the third is ½ if we consider only the two propositions 'This book is black' and 'This book weighs twelve ounces'. These three conditions are mutually inconsistent, so that we cannot consistently substitute the values ½, ½, and ½ as the three probabilities to be added together to equal 1½. Consequently, we cannot reach the absurd conclusion that the probability of the disjunction 'Either this book is red, or it is blue, or it is black' conditional on 'This book weighs twelve ounces' is greater than 1. The restriction thus blocks Objection 1.

It also blocks Objection 2. For, observing the restriction, we infer that since 'Object *a* weighs twelve ounces' is irrelevant to 'Object *a* is red', if we consider only the two propositions 'Object *a* is red' and 'Object *a* weighs twelve ounces', the probability of 'Object *a* is red' conditional on 'Object *a* weighs twelve ounces' is ½. Likewise, since 'Object *a* weighs twelve ounces' is irrelevant to the conjunction 'Object *a* is red, and object *a* is a book', if we consider only the propositions 'Object *a* is red, and object *a* is a book' and 'Object *a* weighs twelve ounces', the probability of 'Object *a* is red, and object *a* is a book', conditional on 'Object *a* weighs twelve ounces', is also ½. But from this we cannot further infer that the probabilities are equal to each other, because they are both equal to ½. For they are equal to ½ under mutually incompatible conditions: the probability of 'Object *a* is red' conditional on 'Object *a* weighs twelve ounces' is ½ under the condition that we consider only the propositions 'Object *a* is red' and 'Object *a* weighs twelve ounces'; the probability of 'Object *a* is red, and object *a* is a book' conditional on 'Object *a* weighs twelve ounces' is ½ under the condition that we consider only the propositions 'Object *a* is red, and object *a* is a book' and 'Object *a* weighs twelve ounces'; and these conditions are obviously incompatible with each other. This inference, then, is not permissible; and without it Objection 2 fails.

The added restriction also permits an answer to Objections 3 and 4, which employ the probability calculus' Special Multiplication Rule.

Objection 3. Contrary to (12), irrelevance cannot be correctly defined in terms of a conditional probability of ½. For it is sometimes the case that a proposition, '*p*', that is irrelevant to each of two other propositions, '*q*' and '*r*', is irrelevant to their conjunction as well. For example, 'Roses are red' is irrelevant not only to 'Socrates is mortal' and to 'Edinburgh is north of London' but also to their conjunction, 'Socrates is mortal, and Edinburgh is north of London'. But according to (12), if '*p*' is irrelevant to '*q*', then the probability of '*q*' conditional on '*p*' is ½; and, similarly, if '*p*' is irrelevant to '*r*', then the probability of '*r*' conditional on '*p*' is ½. Now, according to the Special Multiplication Rule, assuming that '*q*' and '*r*' are mutually independent, the probability of the conjunction of '*q*' and '*r*' conditional on '*p*' is always equal to the probability of '*q*' conditional on '*p*', multiplied by the probability of '*r*' conditional on '*p*'. Hence, where '*q*' and '*r*' are independent of each other, if '*p*' is irrelevant to '*q*' and to '*r*', the probability of the conjunction of '*q*' and '*r*' conditional on '*p*' is equal to ½ multiplied by ½, or ¼. This means that if '*p*' is irrelevant both to '*q*' and to '*r*', then it is never irrelevant, but always unfavorably relevant, to the conjunction of '*q*' and '*r*'. Therefore, Objection 3 concludes, (12) is false because it has the false consequence that, for instance, although 'Roses are red' is irrelevant both to 'Socrates is mortal' and to 'Edinburgh is north of London', it is unfavorably relevant to their conjunction.

Objection 4. (12) does not correctly define relevance. For, if '*p*' is favorably relevant both to '*q*' and to '*r*', then it is never unfavorably relevant to the conjunction '*q* and *r*'. But, if relevance is defined in accordance with (12), then, because of the Special Multiplication Rule, it will sometimes be the case that, although '*p*' is favorably relevant both to '*q*' and to '*r*' (which are assumed to be independent of each other), it is unfavorably relevant to their conjunction. For instance, assuming that '*q*' and '*r*' are independent of each other, if the probability of '*q*' conditional on '*p*' is 0.70, and the probability of '*r*' also conditional on '*p*' is 0.60, then the probability of the conjunction '*q* and *r*' conditional on '*p*' is 0.70 multiplied by 0.60, or 0.42. So, although '*p*' is favorably relevant both to '*q*' and to '*r*', it is unfavorably relevant to their conjunction. Since (12) has this false consequence, Objection 4 concludes, it is false.[22]

The answer to Objections 3 and 4 is the same: they violate the restriction added to (12). Concerning Objection 3, the restriction implies that the probability of '*q*' conditional on '*p*' is ½ if we consider only the propositions '*p*' and '*q*'; it also implies that the probability of '*r*' conditional on '*p*' is ½ if we consider only the propositions '*p*' and '*r*'. But these conditions are mutually incompatible: we cannot both consider only the propositions '*p*' and '*q*' and consider only the propositions '*p*' and '*r*', where '*q*' and '*r*' are different propositions. Hence, we cannot simultaneously substitute the value ½ both for the probability of '*q*' conditional on '*p*' and for the probability of '*r*' conditional on '*p*'. Consequently, we cannot use the Special Multiplication Rule to multiply ½ by ½ to get ¼. Objection 3, then, fails.

And concerning Objection 4, the restriction implies that the probability of '*q*' conditional on '*p*' is some value greater than ½ if we consider only the propositions '*p*' and '*q*'; and it also implies that the probability of '*r*' conditional on '*p*' is some value greater than ½ if we consider only the propositions '*p*' and '*r*'. But we cannot simultaneously consider only the propositions '*p*' and '*q*' and only the propositions '*p*' and '*r*', where '*q*' and '*r*' are different propositions. So, we cannot simultaneously substitute those two values for the probability of '*q*' conditional on '*p*' and for the probability of '*r*' conditional on '*p*'. This prevents our employing the Special Multiplication Rule to multiply the two values to obtain a new value for the probability of the conjunction '*q* and *r*' conditional on '*p*'. Objection 4, then, also fails. Thus the new restriction permits us to avoid both these objections employing the Special Multiplication Rule.

Objection 5. It is possible for a piece of evidence both to confer a probability of ½ on, and yet to be relevant to, some proposition. For instance,

> . . . imagine a coin which has been carefully examined and found to be exactly symmetrical in its weight distribution. Furthermore, extensive tests on it and similarly weighted coins have revealed a percentage of roughly 50 per cent heads in tosses by the kind of mechanism we are employing; we may also imagine we have extensive knowledge of the workings of our tossing mechanism. Here we obviously have evidence relevant to competing hypotheses about the toss of a coin—indeed no information could be more relevant—but our evidence is equally balanced between the two hypotheses, supporting the hypothesis

> of heads no more or less than it supports the hypothesis of tails.[23]

In other words, the evidence is relevant to each of the hypotheses, and yet it confers on each a probability of ½. Consequently, (12) is wrong when it says that '*p*'s conferring a probability of ½ on '*q*' is a sufficient condition of its being irrelevant to '*q*'.

The answer to Objection 5 is that the evidence in the example is not relevant to the same propositions on which it confers a probability of ½. It confers a probability of ½ on, and is irrelevant to, the competing hypotheses 'This toss of the coin will yield heads' and 'This toss of the coin will yield tails'; but it is relevant to, and confers a probability greater than ½ on, the proposition that the probabilities of the two hypotheses are equal.[24]

Objection 6. The concept of conditional probability substituted for the probability calculus' definition is both obscure and unnecessary. It is obscure because we are given no definition of it, mathematical or otherwise. And it is unnecessary because we can avoid the conflict between the probability calculus' definition of conditional probability, (10), and (2c) by making in the latter an exception for contradictory premises.[25]

First, a concept is not obscure merely because it is undefined. And even if the concept of conditional probability employed in (12) is obscure, its obscurity does not prevent its common and profitable employment outside the confines of the probability calculus, of which employment I have already given examples. And second, making an *ad hoc* exception in (2c) for contradictory premises, in order to prevent its conflict with (10), is objectionable on two grounds. The first is that (2c), as it stands, is more obviously right than (10). And the second is that making such an exception will not solve the previously mentioned problem that (10) does not permit $\Pr(q \mid p)$ to be 1 when '*q*', rather than '*p*', is the contradiction.

Objection 7. (12) is wrong, because '*q*' may be highly probable conditional on '*p*', although '*p*' is irrelevant to '*q*'. Let '*q*' be a proposition whose unconditional probability—*i.e.*, whose own probability, independent of any evidence—, is high (*e.g.*, 'There are not exactly 2,000,001 angels dancing on the head of this pin'). Now let '*p*' be some proposition irrelevant to '*q*' (*e.g.*, 'Some sheep are black'). Because '*q*'s unconditional probability is high, so is its probability conditional on '*p*'; for, according to (10), as the probability of '*q*' approaches 1, the probability of the conjunction '*q* & *p*' approaches the probability of '*p*' alone, so that the probability of '$q \mid p$' also approaches 1. So, contrary to (12), one proposition may be irrelevant to another which it renders highly probable.[26]

The claim that 'Some sheep are black' makes highly probable 'There are not exactly 2,000,001 angels dancing on the head of this pin' is so counterintuitive—not to say plainly false—that its acceptance depends on the prior acceptance of (10) as the definition of conditional probability. Consequently, the admission of an alternative conception of conditional probability relieves us of any reason to accept such a claim, without which the present objection fails.

Objection 8. Proposition (6a) is false, because it is possible for one proposition to be relevant to another without making the second certain, probable, improbable, or impossible. For instance, let '*p*' be 'Aspirin tends to cause stomach bleeding' and '*q*' 'You shouldn't take aspirin unless you really need it'. In this case, '*p*' is relevant to '*q*'—it is a reason for '*q*'—, although it does not make '*q*' certain, probable, improbable, or impossible.[27]

'*p*'s being a reason for, and relevant to, '*q*' depends on the truth of a third proposition, '*r*'—*e.g.*, 'Stomach bleeding is bad' or 'You shouldn't take anything that tends to cause stomach bleeding unless you really need it'. For if '*r*' were false—*e.g.*, if stomach bleeding were neither good nor bad, so that there were no reason to avoid taking something that tends to cause it—, '*p*' would not be a reason for, or relevant to, '*q*'. Hence, it is not '*p*' alone but the conjunction of '*p*' and '*r*' that is a reason for, and relevant to, '*q*'. And that conjunction does make '*q*' probable if not certain.[28]

Objection 9. Definition (12) is an instance of *obscurum per obscurius,* because it defines the obscure concepts of relevance and irrelevance in terms of the obscurer concept of probability. Therefore, we should reject (12).

Granted, (12) defines relevance and irrelevance in terms of probability, which is a concept possibly no clearer than relevance and irrelevance themselves. But it does not follow from this that we should reject (12). If it did follow, then we should reject (7) as well, for it too defines relevance and irrelevance in terms of probability. The reasons why we should reject neither (7) nor (12) on these grounds are, first, that our principal goal is not merely clarity but truth; and, second, that (7) or (12) may bring us nearer that goal. Similar reasoning leads us to tolerate the introduction of a problematic concept like truth into definitions of such other concepts as knowledge and truth-function.

Moreover, however obscure the concept of probability may be, it is clear enough to permit us to understand what is required for the purposes of this paper—namely, that propositions (1), (2a)-(6a), (2b)-(6b), (2c)-(6c), and hence (2d)-(6d) are true; that proposition (7) is inconsistent with at least some of the propositions (2d)-(6d) if it means the same thing by 'relevance' and 'irrelevance' that they do; that proposition (9) is deducible from (1) and (2d)-(6d) and amendable to (12); and that (12) is defensible against other objections.

Objection 10. (12) is wrong, because it permits the paradoxes of strict implication, which show that '*q*' may have a probability of 1 conditional on '*p*', although '*p*' is irrelevant to '*q*'. There are two paradoxes to consider. First, let '*p*' be the contradiction 'Some sheep are black, and it is false that some sheep are black', and let '*q*' be any proposition—say, 'Some fathers are philosophers'. Then '*q*' is deducible from '*p*' by means of acceptable rules of inference, and so '*q*' has a probability of 1 conditional on '*p*', although '*p*' is irrelevant to '*q*'. Second, let '*q*' be the tautology 'Either some sheep are black, or it is false that some sheep are black' and let '*p*' be 'Some fathers are philosophers'. Then again '*q*' is deducible from '*p*' by means of acceptable rules of inference, and so '*q*' has a probability of 1 conditional on '*p*', although '*p*' is irrelevant to '*q*'.[29]

It is true that (12) permits the two paradoxes of strict implication. To some readers this may warrant its dismissal. But it seems to me that the following three reasons collectively warrant at least its continued consideration. First, as I have shown above, (12) is deducible from propositions (1), (2a)-(6a) (suitably restricted to consideration of '*p*' and '*q*'), and (2c)-(6c). So, if it is false, one or more of them must be so too. But they all seem obviously true. Second, if we reject (12) because it permits the paradoxes, we must evenhandedly reject the received theory, (7), as well; for it too permits the first paradox. And third, although neither (12) nor (7) blocks both paradoxes, it is possible that neither must do so in order to be an acceptable theory of relevance. For it is possible that no otherwise acceptable theory of relevance can block both paradoxes.

V. Conclusion

In this paper I have tried to show, first, that the received definitions of relevance and irrelevance do not define the kinds of relevance or irrelevance exhibited in propositions (2a)-(6a); second, that those propositions, when properly translated and conjoined with proposition (1), entail different definitions of relevance and irrelevance; and, finally, that those definitions, when properly amended, are defensible against objections.[30]

Notes

1 I have benefited from the criticisms of Robert Paul Churchill, Alec Fisher, James B. Freeman, John E. Nolt, John Woods, and *Informal Logic*'s referees.

2 For simplicity's sake, I treat 'p' and 'q' throughout as though they were simple propositions. But they may, of course, be complex. In particular, 'p' may be the conjunction of the premises of an argument.

3 These seem to be the definitions of relevance and irrelevance assumed by Wayne Grennan in "A 'Logical Audit' Scheme for Two-premise Arguments", *Informal Logic*, Vol. viii, No. 3 (Fall 1986), p. 126. Thomas E. Gilbert and I employ simplified versions of these definitions in our unpublished textbook, *Everyday Logic*.

4 See, among others, Richard L. Epstein, "Relatedness and Implication," *Philosophical Studies*, Vol. 36 (1979), pp. 137-173. For additional senses, see Douglas N. Walton, *Topical Relevance in Argumentation* (Amsterdam: John Benjamins Publishing Company, 1982), pp. 51-53, 70-73.

5 As I shall assert below, (2a)-(6a) are not fully explicit without the condition that only 'p' and 'q' are considered.

6 From (5a) there follows 'If "p" is irrelevant to "q", then "p" is consistent with "q"', which makes impossible any definition of irrelevance in terms of an undefined conditional probability.

7 *Cf.* John Patrick Day, *Inductive Probability* (New York: The Humanities Press, 1961), p. 314: "We must rather say that p and q are mutually relevant if and only if p entails q or conversely, or they are inconsistent, or p would be evidence for or against q or conversely."

8 F. G. Benenson, *Probability, Objectivity and Evidence* (London: Routledge & Kegan Paul, 1984), p. 240: ". . . Keynes' 'simplest definition of irrelevance' . . . still forms the basis of the theory of relevance . . ." But it is not the only definition proposed, and so I owe the reader some explanation why I neglect others.

Carl G. Hempel's definition of relevance is: ". . . an empirical finding is relevant for a hypothesis if and only if it constitutes either favourable or unfavourable evidence for it; in other words, if it either confirms or disconfirms the hypothesis." ("Studies in the Logic of Confirmation" [I], *Mind*, Vol. LIV, No. 213 [January 1945], p. 3.) This definition is too narrow for our purposes, in that it is concerned not with all propositions but only with empirical data and scientific hypotheses. Although probably meant to be an elaboration of (7), it is actually compatible with either (7) or (12).

Nuel D. Belnap Jr. claims that, ". . . for A to be relevant to B in the required sense, a necessary condition is that A and B have some propositional variable in common . . ." ("Entailment and Relevance", *The Journal of Symbolic Logic*, Vol. 25, No. 2 [June 1960], p. 144). This gives only a necessary, not also a sufficient, condition of relevance; and so it is not a definition of relevance. But it seems to be consistent with both (7) and (12).

John Woods offers the definition that "A is irrelevant$_3$ to B, if, and only if, the truth-value of B remains constant independently of whether A or its contradictory is true." ("Relevance", *Logique et Analyse*, Nouvelle Serie, 7^e Annee, Vol. 27 [October 1964], p. 135). This entails the false proposition that 'It is probable that some philosophers are fathers' is irrelevant to 'Some fathers are philosophers', because the truth-value of the latter proposition "remains constant" whether the former is true or false.

Carl Wellman's definition of relevance and irrelevance (*Challenge and Response: Justification in Ethics* [Carbondale: Southern Illinois University Press, 1971], p. 110) is: "The distinction between 'relevant' and 'irrelevant' considerations is the distinction between those considerations which continue to make a difference to our acceptance or rejection after criticism and those which do not affect our critical conviction." But considerations do (or do not) continue to make a difference to our acceptance or rejection after criticism because we *think* them relevant (or irrelevant), and we may be mistaken in so thinking. Moreover, even if we thought something relevant (or irrelevant) if and only if it actually is so, Wellman's definition would not give the *nature* of relevance or irrelevance. For a consideration would (or would not) continue to make a difference to our acceptance or rejection after criticism because it was relevant (or irrelevant); it would not be relevant (or irrelevant) because it did (or did not) make such a difference.

Ralph H. Johnson and J. Anthony Blair (*Logical Self-Defense*, Second Edition [Toronto:

McGraw-Hill Ryerson Limited, 1977, 1983], pp. 39-40) offer not one, as they seem to intend, but two characterizations of relevance. The first says that Q is relevant to T only if it provides "some basis for judging that T is true, or that T is false" (p. 39). This gives only a necessary condition for relevance and otherwise differs from proposition (12) only in that it does not explain "providing some basis for judging that" in terms of probability. The second says that Q is relevant to T only if either (i) Q increases the probability that T is true, (ii) Q increases the probability that T is false, (iii) the falsity of Q increases the probability that T is true, or (iv) the falsity of Q increases the probability that T is false (p. 40). This gives only necessary conditions for relevance; it fails to distinguish the relevance of Q to T from the relevance of the negation of Q to T; and it is vulnerable to at least some of the same criticisms as (7), because it too makes relevance depend on a difference in probabilities.

B. J. Copeland's pragmatic definition of relevance ("Horseshoe, hook, and relevance", *Theoria*, Vol. 50, No. 2-3 [1984], pp. 152-54) defines the relevance of one proposition to another in terms of their shared relevance to some context, whereas the relevance or irrelevance of 'p' to 'q' in data (2a)-(6a) is independent of their shared relevance to any context.

Trudy Govier (*A Practical Study of Argument*, Second Edition [Belmont, California: Wadsworth Publishing Company, 1985, 1988], pp. 98-99) characterizes relevance thus: "The statement A will be relevant to the statement B if A either counts toward establishing B as true or counts against establishing B as true." This gives only sufficient conditions for relevance and is compatible with either (7) or (12).

And Stephen Read (*Relevant Logic* [Oxford: Basil Blackwell, 1988], p. 133) says that A is logically relevant to B if neither the fusion of A with not-B nor the fusion of B with not-A can be true, where fusion is a special (and to me obscure) kind of conjunction. But according to this definition, A is logically relevant to B only if A entails B, and that is too narrow for the sense of relevance defined here, since it excludes data (3a)-(5a). Moreover, this definition makes relevance a symmetric relation, which is not true of the sense defined here: although 'Most dogs have fleas' is relevant to 'My dog has fleas', the latter is not relevant to the former, since 'My dog has fleas' makes 'Most dogs have fleas' neither certain, probable, improbable, nor impossible.

[9] John Maynard Keynes, *The Collected Writings of John Maynard Keynes, Volume VIII: A Treatise on Probability* (New York: St. Martin's Press, 1921, 1973), Ch. 4, §14, pp. 58-59. For simplicity's sake, I present the looser of Keynes' statements of his definitions of 'irrelevance' and 'relevance'. Substituting the stricter statement would not avoid the criticism about to be advanced.

[10] For instance, let 'q' be 'Someone was a philosopher', 'h' 'Socrates was a philosopher', and 'p' 'Plato was a philosopher'. According to (7), although 'p' entails 'q', 'p' is not relevant to 'q', given 'h', since 'h' alone entails 'q', so that the probability of 'q' conditional on 'p and h' equals the probability of 'q' conditional on 'h' alone. (Adapted from Wesley C. Salmon, "Confirmation and Relevance", in *Induction, Probability and Confirmation, Minnesota Studies in the Philosophy of Science*, Vol. 6, ed. Grover Maxwell and Robert M. Anderson, Jr. [Minneapolis: University of Minnesota Press, 1975], p. 7.)

Revisions of (7) by Peter Gärdenfors (in "On the Logic of Relevance", *Synthese*, Vol. 37, No. 3 [March 1978], pp. 351-367) and George N. Schlesinger (in "Relevance", *Theoria*, Vol. LII [1986], pp. 57-67) are similarly vulnerable to counterexamples. Part of Gärdenfors' definition of irrelevance (p. 362) says that p is irrelevant to r on evidence e if $\Pr(r \mid p \,\&\, e)$ is equal to $\Pr(r \mid e)$ and for all sentences q, if $\Pr(r \mid q \,\&\, e)$ is equal to $\Pr(r \mid e)$ and $\Pr(p \,\&\, q \,\&\, e)$ is not equal to 0, then $\Pr(r \mid p \,\&\, q \,\&\, e)$ is equal to $\Pr(r \mid e)$. And part of Schlesinger's (p. 64) says that p is irrelevant to r on e if $\Pr(r \mid p \,\&\, e)$ is equal to $\Pr(r \mid e)$ and there is no true sentence q such that both $\Pr(r \mid q \,\&\, e)$ is not equal to $\Pr(r \mid p \,\&\, q \,\&\, e)$ and $\Pr(r \mid q \,\&\, e)$ is not equal to $\Pr(r \mid p \,\&\, q \,\&\, e)$. From each of these definitions it follows that 'Plato was a philosopher' ('p') is irrelevant to 'Someone was a philosopher' ('r') on the evidence 'Socrates was a philosopher' ('e'), despite the fact that 'Plato was a philosopher' entails 'Someone was a philosopher'.

[11] See, for example, Peter Achinstein, "Concepts of Evidence", in *The Concept of Evidence*, ed. Peter Achinstein (Oxford: Oxford University Press, 1983), pp. 152-53. Although the author's topic is evidence rather than relevance, his discussions of the paradox of ideal evidence and the second lottery case apply to relevance as well.

[12] As William Kneale (*Probability and Induction* [London: Oxford University Press, 1949], p. 128) emphasizes.

[13] *Objection.* (2b) and (5b) are false, because a proposition's having a probability of 1 is not a sufficient condition of its being certain, or necessary; nor is its having a probability of 0 a sufficient condition of its being impossible. For suppose the number of possible outcomes of a given experiment—say, a person's height being within the interval from 42 to 86 inches—is uncountably infinite.

> If we were to form elementary events consisting of single outcomes, it would be mathematically impossible to assign probabilities to each of the simple events so that their sum would be 1 unless we assigned the probability 0 to all except a finite or countably infinite number of them. (John P. Hoyt, *A Brief Introduction to Probability Theory* [Scranton, Pennsylvania: International Textbook Company, 1967], p. 38.)

But assigning a probability of 0 to an event does not mean that the event is impossible.

> For example, in our height-measuring experiment, the probability that a man chosen at random will have a height of 72 in. is 0, but that does not mean that the man cannot have a height of 72 in. Similarly, the event complementary to the event that the man's height is 72 in. has a probability of 1, but this does not mean that the complementary event is sure to occur. (*Ibid.*, p. 39. Jonathan Adler brought this objection to my attention in a private communication of May 14, 1990.)

Reply. This objection has the consequences, first, that the probability of something that is possible is sometimes exactly the same as the probability of something that is impossible and, second, that the probability of something that is necessary is sometimes exactly the same as the probability of something that is not necessary. These consequences seem intuitively to be absurd.

[14] Below I shall give reasons for employing a notion of conditional probability other than that defined by the probability calculus.

[15] This assumes that the probability of '*q*' conditional on '*p*' is defined—*i.e.*, that there is such a probability-value. This assumption seems secure for the following reasons. The probability of '*q*' conditional on '*p*' would be undefined if and only if we employed a definition of conditional probability that, like the definition given by the probability calculus, involved division (so that the divisor might sometimes be 0). I shall argue shortly that we should not employ any such definition. If that argument is sound, then the probability of '*q*' conditional on '*p*' will always be defined.

[16] Proposition (9) is not the Principle of Indifference espoused by adherents of the Classical Theory of Probability. Unlike that Principle, (9) provides a means of ascertaining, not the initial probability of a proposition, but the relevance or irrelevance of one proposition to another; and it deals only with conditional probability. But there is some relation between (9) and the Principle of Indifference, since (9) sufficiently resembles the Principle that I have been able without difficulty to adapt for use against (9) one of Keynes' criticisms (Objection 1, below) of the Principle; and Keynes directed another criticism (Objection 2, below) against something like (9) as an implication of the Principle.

[17] *Webster's Third New International Dictionary of the English Language*, ed. Philip Babcock Gove (Springfield, Massachusetts: G. & C. Merriam Company, 1966), p. 1917, 'relevant', def. 1.

[18] *Cf.* Everett J. Nelson, "Intensional Relations" (*Mind*, New Series, Vol. XXXIX, No. 156 [October 1930], pp. 440-453). For example, "Entailment, not being defined in terms of truth-values, is a necessary connexion between meanings" (445) and "The 'implication' of ordinary discourse is, like consistency, essentially relational: it depends upon the meaning of *both* propositions" (446).

[19] Although Keynes (*op. cit.*, Ch. 4, §2, p. 45) acknowledges his debt for several criticisms of the Principle of Indifference to Johannes von Kries (*Die Principien der Wahrscheinlichkeits-Rechnung: Eine logische Unterschung* [Freiburg: 1886]), he took neither Objection 1 nor Objection 2 from that source.

[20] Adapted from Keynes, *op. cit.*, Ch. 4, §4, pp. 45-46.

[21] *Ibid.*, p. 47.

[22] The third through fifth sentences of this paragraph are adapted from Keith Lehrer, "Knowledge and Probability", *The Journal of Philosophy*, Vol. LXI, No. 12 (June 11, 1964), p. 369.

[23] Benenson, *op. cit.*, p. 252.

[24] *Ibid.*, p. 255.

[25] Brian Skyrms appears committed to making this exception, since he says (*Choice and Chance: An Introduction to Inductive Logic*, Second Edition [Belmont, California: Dickenson Publishing Company, 1975], p. 140, n. 2-3): "When p is a self-contradiction, then for any statement *q* there is a deductively valid argument from *p* to *q* *and* a deductively valid argument from *p* to -*q*. In such a case, Pr(*q given p*) has no value." If there is a deductively valid argument from *p* to *q*, although the probability of *q* conditional on *p* is not 1, then clearly we cannot define a valid deduction as one in which the probability of the conclusion is 1 conditional on the premises.

John Nolt and Dennis Rohatyn (*Theory and Problems of Logic* [New York: McGraw-Hill Book Company, 1988], p. 29, n. 6) make this exception explicitly: ". . . arguments with inconsistent premises are exceptions to the rule that the inductive probability of a deductive argument is 1. (They are the only exceptions.)"

[26] Freely adapted from Nolt and Rohatyn, *op. cit.*, pp. 28-30.

[27] Trudy Govier, private communication, June 27, 1988, p. 1.

[28] We wouldn't want to say that if the conjunction of '*p*' and '*r*' is relevant to '*q*', then '*p*' alone is relevant to '*q*'. For that would lead to such absurdities as that 'Some sheep are black' is relevant to 'You shouldn't take aspirin unless you really need it', because the conjunction 'Some sheep are black, and you shouldn't take anything unless you really need it' is relevant to 'You shouldn't take aspirin unless you really need it'.

[29] The paradoxes are set forth in Clarence Irving Lewis and Cooper Harold Langford, *Symbolic Logic*, Second Edition (New York: Dover Publications, Inc., 1959), pp. 250-251. Their use against (12) is adapted from Nolt and Rohatyn, *op. cit.*, pp. 28-29.

[30] It may prevent misunderstanding to reiterate here that I reject (7) not unqualifiedly but only as definitions of the kinds of relevance and irrelevance exhibited in (2a)-(6a), that likewise I reject (10) not unqualifiedly but only as the kind of conditional probability used to define those same kinds of relevance and irrelevance, and that I impose on conditional probability the restriction that consideration be confined to '*p*' and '*q*' not unqualifiedly but only when such probability is used to define those same kinds of relevance and irrelevance.

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