International Journal of Production Management and Engineering

doi:10.4995/ijpme.2015.3314
Received 2014-10-13 Accepted: 2014-12-12

Integrating forward and reverse logistics network for commercial goods management. An integer linear programming model proposal

Eva Ponce-Cueto[i] and Melisa Molenat Muelas[ii]

Industrial Engineering and Logistics Research Group. Department of Industrial Engineering, Business Administration and Statistics. Escuela Técnica Superior de Ingenieros Industriales de Madrid, Universidad Politécnica de Madrid
José Gutiérrez Abascal, 2, 28006 Madrid, SPAIN

[i] *eva.ponce@upm.es*

[ii] *melisa.molenat@gmail.com*

Abstract: In this paper, an optimization model is formulated for designing an integrated forward and reverse logistics network in the consumer goods industry. The resultant model is a mixed-integer linear programming model (MILP). The objective is to minimize the total costs of the closed-loop supply chain network. It is important to note that the design of the logistics network may involve a trade-off between the total costs and the optimality in commercial goods management. The model comprises a discrete set as potential locations of unlimited capacity warehouses and fixed locations of customers' zones. It provides decisions related to the facility location and customers' requirements satisfaction, all of this related with the inventory and shipment decisions of the supply chain. Finally, an application of this model is illustrated by a real-life case in the food and drinks industry. We can conclude that this model can significantly help companies to make decisions about problems associated with logistics network design.

Key words: Forward/Reverse Logistics Network, Network Design, Mixed-Integer Linear Programming, Facility Location, Commercial Goods.

1. Introduction

Different authors examine the interdependence between facility location, transportation and inventory issues in a distribution network design problem (Jayaraman, 1998). In this network design problem, warehouses location plays an important role. Teo and Shu (2004) study the distribution network design problem focusing on warehouse location by integrating transportation and inventory cost function, but considering only forward logistics. To determine how many warehouses to set up, where to locate them and how to serve many points of sales using these warehouses, reducing costs and maintaining a good service level are not trivial problems. In addition, and particularly if we focus on commercial goods management (such as modular counters, display cabinets, frozen food fridges or retail displays), reverse logistics activities need to be considered in the problem, creating a closed loop system, where those activities are included encompassing the transportation and recollecting of commercial products.

Few authors have however addressed such problem (Jayaraman *et al.*, 2003; Lu and Bostel, 2007; Srivastava, 2008; Yongsheng and Shouyang, 2008) and further research on the topic is still required (Cardoso *et al.*, 2013).

The aim of this paper is to develop a logistics network model for helping companies to configure their warehouse network of commercial goods in the consumer goods industry (food and beverage). A mixed integer linear programming model in which reverse logistics activities are considered simultaneously with forward supply chain activities, will be proposed. To determine the optimal number of warehouses that satisfy customers' requirements with minimum costs is the main contribution of this paper.

The paper is structured as follows: in section 2, the main problem characteristics are detailed. In section 3, the mathematical formulation is presented. In section 4, an application and analysis to a real life case study of consumers' goods company is presented. Finally, in section 5, conclusions and future works are discussed.

2. Method

2.1. Problem definition

In this paper, a supply chain network design problem for commercial goods is considered, following the mathematical model for the reverse distribution problem proposed in the article written in 2003 by Jayaraman *et al.* It is important to point out that their model can be adapted for different type of products such as end of life and commercial returns, considering reverse functions like recycling, remanufacturing, reuse...

The supply chain of the present paper is formed by several warehouses, which store products and deliver them directly to markets. The points of sale (POSs) are the last echelon in the considered supply chain.

Each POS demands commercial goods (such as modular counters, display cabinets, frozen food fridges or retail displays) at the beginning of each summer season. Warehouses integrate the received orders and transform them into an assembly service. At the end of the summer season, the product flow is reverse: products are collected by warehouses from POS, in order to store and reuse them in the next summer season. In the reverse product flow, it is assumed that disassembly services consist of the sum of: (1) non-recovered products that are a fraction of the products supply to markets at each time period, which will never be collected at the end of the time period, (2) returned products to be sent to warehouses, to be stored and reuse in a next time period and (3) end of life products to be sent to disposal sites, to be disassembled and recycled. The amount of each type of these products is calculated as a percentage of the total number of disassembly services in each time period. To resume, in this problem there are four services: assembly, disassembly, maintenance (from a technical or a commercial point of view) and storage. The total amount of each service is associated with a percentage of the installed-base of products in the entire market, based on the number of services done in previous years.

The fact that the commercial goods are recovered from the forward supply chain and sent again to the warehouses for a new use configures a closed-loop supply chain.

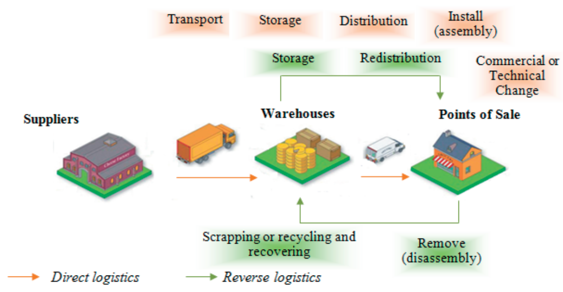

Figure 1. A closed-loop supply chain is configured.

The time horizon is a month, in order to consider the evolution of the consumer's demand over a year.

The problem is modelled through a multi-product mixed integer linear programming (MILP) model.

The warehouses of the supply chain will be located in a number of pre-determined candidate locations. As stated before, the exact location of POS is known. The aim of this problem is to determine the optimal network structure of the supply chain that minimises the total logistics cost. The problem formulation is detailed in section 2.2.

2.2. Problem formulation

Having in consideration the problem characteristics described previously and following the example proposed by Jayaraman *et al.* (2003), a MILP model is developed and formulated. The nomenclature, indices and parameters used are shown in the Table 1.1, Table 1.2, Table 2.1 and Table 2.2.

2.2.1. Variables

A continuous variable is used to define the total cost related to the assembly services of the commercial goods in the points of sale: AssemblyC.

A continuous variable is used to define the total cost related to the disassembly services of the installed commercial goods in the points of sale: DisassemblyC.

A continuous variable is used to define the total cost related to the maintenance services to repair goods installed in the points of sale: TecChangeC.

A continuous variable is used to define the total cost related to the commercial changes: CommercialExchangeC.

As shown in the Table 2.1, in the continous variables defined previously, the number of goods that have to be assembled, disassembled or changed for

Table 1.1 Sets and parameters used in the model.

Type	Identifier	Definition
Set	$w \in W$	Set of potential locations for locating warehouses.
	$z \in Z$	Set of customers' zones.
	$p \in P$	Set of products families in function of the dimensional characteristics of each commercial good.
Parameters	$PercenRepresent_p$	Percentage of each products family $p \in P$.
	$Km_{w,z}$	Kilometric distance between the warehouses w and the centre of gravity of the customers' zone $z \in Z$.
	NumWarehouses	Pre-determined number of warehouses considered to design the supply chain network.
	$InstalledBase_z$	Quantity of installed commercial goods in a customers' zone $z \in Z$.
	$InstalledBaseToHibernate_z$	Quantity of installed commercial goods to hibernate associated to a customers' zone $z \in Z$.
	PercenAssembly	Percentage of assembled commercial goods of the installed goods.
	PercenAssembliesFailed	Percentage of the assemblies which cannot be done in the predicted moment.
	PercenDisassembly	Percentage of disassembled commercial goods of the installed goods.
	PercenDisassembliesFailed	Percentage of the disassemblies which cannot be done in the predicted moment.
	PercenDisassembliesLost	Percentage of the disassemblies which cannot be done because the good has disappeared (robbery...).
	PercenDisassembliesDisposal	Percentage of the disassemblies which are going to be disposed.
	PercenTecChange	Percentage of technical changes in the installed commercial goods.
	PercenTecChangesFailed	Percentage of the technical changes which cannot be repaired.

a commercial or technical reason in a POS by a warehouse $w \in W$ is converted in a kilometric distance multiplying by the distance between the warehouse $w \in W$ and the gravity center of the costumers' zone $z \in Z$. In theory, each assembly, disassembly, technical or commercial change means a round trip, and that is why the kilometric distance is multiplying by 2. But in reality, in some cases the technical service cannot do the activity in the predicted moment (because the POS is closed or because the responsible of the POS is not agreed). In those cases, the technical service has to come back in another moment. That is why the kilometric distance is multiplying in these particular situations by 4. In addition synergies are considered transporting as much goods as it is possible, depending on the size of the box truck and on the dimensions of the items. Finally, the total kilometrics are converted in a cost multiplying by a unit kilometric cost.

A continuous variable is used to define the total cost related to the storage of commercial goods in the warehouses: StorageC.

A continuous variable is used to define the total cost related to the opening of the warehouses: OpeningC.

A binary variable is used to define the location of warehouses (equal to 1 if the warehouse $w \in W$

Table 1.2 Sets and parameters used in the model.

Type	Identifier	Definition
Parameters	PercenComChange	Percentage of commercial changes of the installed commercial goods.
	PercenComChangesFailed	Percentage of commercial changes which cannot be done in the predicted moment.
	PercenSecurityStock	Percentage of the security stock considered.
	PercenToHibernate	Percentage of the installed commercial goods to hibernate.
	TruckBoxSize	Surface of the truck box which can be used to transport commercial goods.
	ProdFamilySizep	Size of the products family $p \in P$.
	PalletsOccupiedp	Number of pallets occupied by the products family $p \in P$.
	TransportUC	Variable cost per unit of kilometric distance.
	WarehouseUFC	Fixed cost of opening a warehouse.
	WarehouseUVC	Variable cost per unit of pallets occupied by the stored products family $p \in P$.

Table 2.1 Variables used in the model.

Identifier	Definition
AssemblyC	TransportUC·$\Sigma_w\Sigma_z$ ((InstalledBase$_z$·PercenAssembly·AllocationZW$_{w,z}$· KmWZ$_{w,z}$·(2· (1-PercenAssembliesFailed) + 4·PercenAssembliesFailed))/(TruckBoxSize/ Σ_p(PercenRepresent$_p$·ProdFamilySize$_p$))
DisassemblyC	TransportUC·$\Sigma_w\Sigma_z$ ((InstalledBase$_z$·PercenDisassembly·AllocationZW$_{w,z}$· KmWZ$_{w,z}$·(2·(1-PercenDisassembliesFailed) + 4·PercenDisassembliesFailed))/(TruckBoxSize/ Σ_p(PercenRepresent$_p$·ProdFamilySize$_p$))
TecChangeC	TransportUC·$\Sigma_w\Sigma_z$ ((InstalledBase$_z$·PercenTecChange·AllocationZW$_{w,z}$· KmWZ$_{w,z}$·(2·(1-PercenTecChangesFailed) + 4·PercenTecChangesFailed))/(TruckBoxSize/ Σ_p(PercenRepresent$_p$·ProdFamilySize$_p$))
CommercialExchangeC	TransportUC·$\Sigma_w\Sigma_z$ ((InstalledBase$_z$·PercenComChange·AllocationZW$_{w,z}$· KmWZ$_{w,z}$·(2·(1-PercenComChangesFailed) + 4·PercenComChangesFailed))/(TruckBoxSize/ Σ_p(PercenRepresent$_p$·ProdFamilySize$_p$))

is located in potential location w and equal to 0 otherwise): Opening$_w$.

A binary variable is used to define the customers' zone service (equal to 1 if the customers' zone is served by the warehouse w ∈ W and equal to 0 otherwise): AllocationZW$_{w,z}$

The objective function minimizes the total logistics cost:

$$\text{Min(C)=AssemblyC+DisassemblyC+TecChangeC+ComChangeC+StorageC+OpeningC} \quad (1)$$

The total cost of the supply chain is the sum of the transportation costs and the facilities costs. The transportation costs are those related to assembly services, disassembly services and technical and commercial maintenance services. The facilities costs are those related to warehouses location and operation. (includes in equation 1 as StorageC and OpeningC. See Table 2.2 for variables definition).

Subject to:

$$\Sigma_w \text{Opening}_w = \text{NumWarehouses} \quad (2)$$

$$\Sigma_w \text{AllocationZW}_{w,z} = 1, \forall w \in W \quad (3)$$

$$\text{Opening}_w \geq \text{AllocationZW}_{w,z}, \forall w \in W, \forall z \in Z \quad (4)$$

Table 2.1 Variables used in the model.

Identifier	Definition
OpeningC	Σ_w WarehouseUFC$_w$·Opening$_w$
Opening$_w$	Binary variable equal to 1 if the warehouse w ∈ W is located in potential location w and equal to 0 otherwise
AllocationZW$_{w,z}$	Binary variable equal to 1 if the customers' zone is served by the warehouse w ∈ W and equal to 0 otherwise.
StorageC	Months of the summer season: Σ_w WarehouseUVC$_w$·Σ_z ((InstalledBase$_z$·PercenAssembly·AllocationZW$_{w,z}$) + (InstalledBase$_z$·PercenTecChange·AllocationZW$_{w,z}$)+(InstalledBase$_z$·PercenComChange·AllocationZW$_{w,z}$) + (InstalledBase$_z$·PercenAssembly·AllocationZW$_{w,z}$)·PercenSecurityStock – (InstalledBase$_z$·PercenDisassembly·AllocationZW$_{w,z}$)·(1-PercenDissembliesDisposal – PercenDisassembliesLost) · (Σ_p(PercenRepresent$_p$· PalletsOccupied$_p$)) Months of the winter season: Σ_w WarehouseUVC$_w$·Σ_z ((InstalledBase$_z$·PercenAssembly·AllocationZW$_{w,z}$) +(InstalledBase$_z$·PercenTecChange·AllocationZW$_{w,z}$)+ (InstalledBase$_z$·PercenComChange·AllocationZW$_{w,z}$)+ (InstalledBaseToHibernate$_z$·PercenToHibernate·AllocationZW$_{w,z}$)–((InstalledBase$_z$·PercenDisassembly·AllocationZW$_{w,z}$)·(1-PercenDisassembliesDisposal–PercenDisassembliesLost)·(Σ_p(PercenRepresent$_p$·PalletsOccupied$_p$))

$Opening_w \in \{0,1\}$, $AllocationZW_{w,z} \in \{0,1\}$ $\forall w \in W$, $\forall z \in Z$ (5)

AssemblyC, DisassemblyC, TecChangeC, ComChangeC, StorageC, OpeningC ≥ 0 (6)

The equation (2) is a logical constraint ensuring that the number of open warehouses is exactly equal to the number of warehouses defined. The equation (3) ensures that all customers' zones are taken into account and that each one is served by a unique warehouse. Inequality (4) ensures that before allocating a customers' zone to a warehouse, the warehouse has to be opened. Finally, constraint (5) is for binary variables and constraint (6) the non-negative variables constraint.

3. Case study application

In this section, the formulated model is applied to a real-life industrial case study (based on a consumer goods company, which operates in the food and drink industry).

In particular, the case study includes the supply chain network of fridges for ice creams.

Figure 2. Example of commercial goods considered in the case study.

Specifically, the company manages the supply chain of goods, controlling the storage, the delivery, the maintenance and repair, the recovery and, if needed, the scrapping of those items.

The total of warehouses considered varies between 1 (centralized supply chain network) and 47 (decentralized supply chain network). 47 is the maximum number of warehouses considered since there is only one candidate location for locating warehouses in each Spanish province.

In the supply chain network more than 140,000 points of sale are considered and aggregated per Spanish provinces. The exact location of each point of sale cannot be described due to confidentiality company reasons.

The average amount of commercial goods installed in Spain in 2013 was 173,461.

In this case study, 8 products families are considered.

Table 3. Characteristics of the products family.

Products Family	% Of Each Products Family	Number Of Pallets Occupied By The Products Family
1	22,03%	1
2	5,08%	1
3	6,78%	1
4	1,69%	1
5	38,98%	1
6	3,39%	1
7	3,39%	2
8	18,64%	1

The percentages introduced in the model are the same as the month percentages of the last year.

It is assumed that these percentages are the same in all the provinces, without geographical distinction.

Table 4.1 Month percentages considered with respect to all goods installed in POS.

Month	%Assemblies	%Disassemblies
January	6,6%	9,5%
February	6,5%	10,7%
March	**13,7%**	**9,9%**
April	15,4%	8,4%
May	14,1%	7,6%
June	13,6%	7,1%
July	9,0%	6,7%
August	4,7%	4,8%
September	**3,6%**	**12,0%**
October	7,4%	12,6%
November	2,1%	6,7%
December	3,2%	4,0%

As shown in Table 4.1, the end of the summer season is in August because the disassemblies percentage increases and the assemblies percentage decreases in September.

So, to calculate the amount of commercial goods to hibernate, the information related to the amount of goods installed in Spain in August has to be considered. Specifically, 8.3% of goods installed in POS in August have to be storage during the hibernation period (from September to February).

Table 4.2 Month percentages considered with respect to all goods installed in POS.

Month	%Commercial Changes	%Technical Changes
January	4%	5%
February	10%	11%
March	14%	11%
April	17%	11%
May	15%	7%
June	12%	10%
July	10%	10%
August	6%	10%
September	5%	9%
October	3%	6%
November	2%	5%
December	2%	5%

Table 4.3 Month percentages considered with respect to all goods installed in POS.

%Assemblies failed	10%
%Disassemblies failed	10%
%Technical changes failed	15%
%Commercial changes failed	12%
%Disassemblies lost	5%
%Disassemblies disposal	8%

The kilometric distances considered are those defined among capitals of provinces.

In order to estimate the transportation cost, the database of the infraestructure Ministry of Spain, ACOTRAM 2.4.0 was consulted.

Table 5. Cost parameters settings in example (Source: ACOTRAM 2.4.0).

Description	Value
TransportUC	0.93 €/km
WarehouseUFC	18000 €/warehouse
WarehouseUVC	3.0 €/palet·month

In this study, the surface of the truck box which is used to transport commercial goods is equal to 16.17 square meters.

4. Results

Considering all numerical parameters defined in the previous section, the model formulated was solved. The tool used to solve the model is AIMMS v. 3.13 and the solver CPLEX.

The reported computational results were obtained on a 1.65 GHz AMD E Series Processor, 3 GB DDR3-SDRAM RAM, 1333 MHz Memory Speed and 320 GB SATA Hard Drive, running Windows XP Professional. The computational time needed to find the results was lower than 1 minute.

The results of the model shows the influence of the warehouses number in the annual total cost:

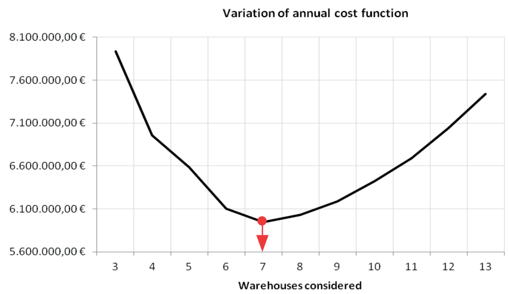

Figure 3. Relationship between the annual total cost and the number of warehouses considered in the model.

The minimum point of the annual total cost curve defines the optimal number of warehouses that has to be considered in the developed model: 7 warehouses.

In addition, the result of the model shows the location of these 7 warehouses.

Figure 4 shows, in a schematic way, the locations of all warehouses and the consumers' zone allocation.

Figure 4. Model results: warehouse's locations and consumers' zones allocation (AIMMS 3.13).

Finally, the results of the model specifies the amount of the total logistics costs for this particular case study:

- Costs related with the transport:
    - Assemblies: 462,437.0€
    - Disassemblies: 586,741.0€
    - Technical changes: 1,456,208.0€
    - Commercial changes: 591,765.0€
- Costs related with warehouses:
    - Fixed costs for opening warehouses: 337,645.0€
    - Variable costs for storage products: 1,412,744.0€
- Annual total cost: 4,847,540.0€.

As shown, in the developed model, the annual total transport cost represents the 63.9% and the annual total cost related with warehouses represents the 36.1%.

Figure 5 shows the current situation of the company. 97 warehouses are spread out throughout the Spanish geography.

Each warehouse serves in its zone. So there is in total 97 consumers' zones defined in this situation.

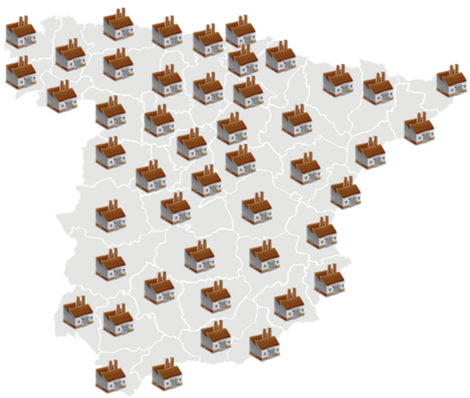

Figure 5. Current situation in the company.

The current total logistics costs for the current situation is:

- Costs related with the transport:
    - Assemblies: 217,617.9€
    - Disassemblies: 275,727.8€
    - Technical changes: 669,812.5€
    - Commercial changes: 672,451.9€
- Costs related with warehouses:
    - Fixed costs for opening warehouses: 4,678,795.0€
    - Variable costs for storage products: 2,132,487.1€
- Annual total cost: 8,646,892.2€

In that model, the annual total transportation cost represents 21.9% and the annual total cost related with warehouses supposes 78.1%.

Comparing the results obtained with the model developed with the current situation implemented by the company nowadays, the annual total transportation cost has increased 1.69 times, but the installation cost of warehouses has been considerably reduced (from 6,811,282€ to 1,750,389€), so the annual total costs have descreased in a 44%.

5. Discussion and conclusions

In the present work, it has been proposed an optimization model for the design of a supply chain that integrates forward and reverse flows with application in the consumers' goods industry. The model developed was applied to the optimization of the supply chain of a Spanish company, where cabinets for ice creams were studied. The model application shows that the situation proposed results in a decrease of the total costs (including transportation, fixed costs for opening warehouses and variable costs for storage products in the warehouses). More specifically, in comparison with the current situation, the total costs have decreased in a 44%.

Considering all its results, it has been shown that the situation described as the solution tends to create a more profitable, simple and efficient supply chain network.

As future work, an improvement of model formulation will be explored. Having in mind that, in the present work, the level defined by plants and the one defined by infrastructures for used and recovered products which life is ended, are not considered. Multi-objective approaches should be then addressed as an extension of the present study.

References

ACOTRAM. (2014). *Asistente para el Cálculo de Costes del Transporte de Mercancías por Carretera*. Published by the Infrastructure Ministry of Spain.

Cardoso, S.R., Barbosa-Povoa, A.P., Relvas, S. (2013). Design and planning of supply chains with integration of reverse logistics activities under demand uncertainty. *European Journal of Operational Research*, 226(3): 436-451. doi:10.1016/j.ejor.2012.11.035

Jayaraman, V. (1998). Transportation, facility location and inventory issues in distribution network design: An investigation. *International Journal of Operations & Production Management*, 18(5): 471-494. doi:10.1108/01443579810206299

Jayaraman, V., Patterson, R.A., Rolland, E. (2003). The design of reverse distribution networks: Models and solution procedures. *European Journal of Operational Research*, 150(1): 128-149. doi:10.1016/S0377-2217(02)00497-6

Lu, Z., Bostel, N. (2007). A facility location model for logistics systems including reverse flows: The case of remanufacturing activities. *Computers & Operations Research*, 34(2): 299-323. doi:10.1016/j.cor.2005.03.002

Srivastava, S.K. (2008). Network design for reverse logistics. *Omega*, 36(4): 535-548. doi:10.1016/j.omega.2006.11.012

Yongsheng, Z., Shouyang, W. (2008). Generic Model of Reverse Logistics Network Design. Journal of transportation systems engineering and information technology, 8(3): 71-78. doi:10.1016/S1570-6672(08)60025-2

Teo, C.P., Shu, J. (2004). Warehouse-Retailer Network Design Problem, *Operations Research,* 52(3): 396-408. doi:10.1287/opre.1030.0096

