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Study of Mathematical Reasoning Ability for Mathematics Learning in Schools: A Literature Review

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A B S T R A C T S

This article aims to examine students' mathematical reasoning abilities. This study is in the form of using approaches and strategies for improving students' abilities. Differences in mathematical reasoning abilities were revealed from a gender perspective, teaching materials oriented towards mathematical reasoning, teacher perceptions, and design of teacher action characteristics to improve the process of students' mathematical reasoning. Scientific articles are studied to collect information about students' mathematical reasoning. The study results indicate: 1) learning strategies such as open-ended, visual basic application for excel, adversity question, and argument-driven inquiry could be used to improve students' mathematical reasoning abilities. 2) the development of male students' mathematical reasoning was significantly better than female students. 3) teachers' perceptions of mathematical reasoning differ from the perceptions of experts. 4) the quality of students' mathematical reasoning is still dominated by imitative reasoning. 5) The ability to generalize and justify will emerge if the teacher designs a challenging lesson for students followed by activities to guide students. This research is expected to be helpful in education, especially mathematics learning in schools, where it can be used as a reference for choosing strategies and teacher reading materials to improve students' mathematical reasoning abilities.

A R T I C L E   I N F O

1. INTRODUCTION

Mathematics is a branch of science that has a very important role in the development of science and technology. The important role of mathematics is recognized by Wijaya *et al.* (2020) which states that at every level of education, one of the most important subjects in mathematics. Mathematics is important learning because it is always used in everyday life. Mathematics is one of the disciplines that can improve the ability to think and argue so that it contributes to solving everyday problems. Therefore, mathematics learning must be centered on the basic concepts of mathematics. So that students can apply the basic concepts of mathematics to everyday life.

In the Regulation of the Minister of National Education of the Republic of Indonesia Number 20 of 2006 concerning Content Standards, it is stated that mathematics learning aims to make students have the following abilities: 1) Understand mathematical concepts, explain the relationship between concepts and apply concepts or algorithms in a flexible manner, accurate, efficient, and precise in solving problems; 2) Using reasoning on patterns and properties, performing mathematical manipulations in making generalizations, compiling evidence, or explaining mathematical ideas and statements; 3) Solve problems which include the ability to understand problems, design mathematical models, solve models and interpret the solutions obtained; 4) Communicating ideas with symbols, tables, diagrams, or other media to clarify the situation or problem; 5) have an attitude of appreciating the usefulness of mathematics in studying problems, as well as being resilient and confident in problem solving. Mathematics learning includes five basic mathematical abilities: problem-solving, reasoning, communication, connection, and representation. If we look closely, the objectives of mathematics subjects from the 2006 National Education Ministerial Regulation and Grouws show that mathematics learning is structured to pay attention to aspects of developing students' mathematical reasoning abilities.

However, the results of the Program for international student assessment (PISA) in 2015 show that the reasoning ability of Indonesian students is still below the average compared to 75 other countries. The results of the 2018 Program for International Student Assessment (PISA) show that the quality of Indonesian learning is ranked 75 out of 80 countries, with the PISA score in each field decreasing, for mathematics decreasing from 386 to 379. Other results are also shown by Trends in International Mathematics and Science Study (TIMMS), Indonesia is ranked 44 out of 49 countries. The results of the 2015 TIMMS on achievement in mathematics show 54% low, 15% moderate, and 6% high. From the PISA and TIMMS results, it can be concluded that the quality of mathematics learning in Indonesia is very low. The National Center for Education Statistics, publishing the abilities of Indonesian students referring to PISA in 2012 shows that almost all Indonesian students only master subject matter up to level 4, while many other countries have reached levels 5 and 6. Organization for Economic Cooperation and Development (OECD) explained that in 2015 the ability to think at levels 5 and 6 Indonesian students was only 0.8% of the participants. On the other hand, 20% of the participants are at level 2. This means that the thinking ability of Indonesian students is still dominated by low-order thinking (LOT).

The low math scores in the results of the PISA, TIMMS, and OECD surveys indicate that the objectives of mathematics have not been fully achieved. The low score of mathematics is related to students' reasoning abilities because one of the objectives of the mathematics subject, as stated by the 2006 Ministry of National Education, states that students can use reasoning on patterns and traits, perform mathematical manipulations in making generalizations, compile evidence, or explain mathematical ideas and statements. Demeter

(2019) stated that mathematical reasoning is the foundation for the construction of mathematical knowledge. This means that mathematical reasoning skills are the foundation for gaining mathematical knowledge. Reasoning is an activity, a process, a thinking activity to draw conclusions or make a statement whose truth has been previously proven or assumed. Therefore, with good reasoning, a person will be able to make conclusions or decisions related to everyday life. A person with low reasoning abilities will always have difficulty in dealing with various problems because of the inability to relate facts to conclusions. Thus, reasoning should be developed in each individual. Broadly speaking, the structural aspects of mathematical reasoning consist of deductive reasoning, inductive reasoning, and abductive reasoning. Meanwhile, aspects of the mathematical reasoning process are processes related to the search for similarities and differences and processes related to validation (Marasabessy & Hasanah, 2021).

Several studies on student reasoning have been conducted. Sumartini (2015) found that students' ability in reasoning was not what was expected. Many students still have difficulty in thinking. Wahyudin's research results found five weaknesses in students, among others: lack of good prerequisite knowledge, lack of ability to understand and recognize concepts (Fuadi, *et al*., 2016). Basic mathematical concepts (axioms, definitions, rules, theorems) related to the subject being discussed, lacks the ability and accuracy in listening to a problem or math problems related to a particular subject, cannot listen back to the answers obtained, and cannot reason logically in solving mathematical problems or problems. Responding to the difficulties faced by students in learning, teachers tend to interpret it as a result of students' efforts that have not been maximal in learning or a limitation of students in learning teaching material. The difficulties experienced by students are the result of a learning process in which there is an interaction between teachers, students, and teaching materials. The difficulties faced by students in learning are not the result of the students themselves but can come from the way the teacher presents the material or teaching material used when learning occurs. This was expressed by Bachelard and Piaget (Brousseau, 2006) that the difficulties faced by students were not only due to delays and changes as expressed by the views of empiricism and behaviorism but also the result of previous knowledge that was considered appropriate but now revealed as something wrong or not applicable in the present context.

Seeing that there are many learning barriers related to developing mathematical reasoning abilities, these learning barriers must be overcome immediately so that students' mathematical reasoning abilities develop properly. Thus, researchers are interested in studying students' mathematical reasoning abilities for learning mathematics in schools. This study is in the form of tracing the results of research in 10 international journals. This research is expected to be helpful in the field of education, especially mathematics teaching, where follow-up and handling of problems related to student reasoning found in schools can be carried out.

2. METHODS

Literature study is a method used in writing this article. According to Knopf (2006), Literature Review is a critical and in-depth evaluation of previous research. We can conclude that literature review does not only mean reading literature but instead leads to an in-depth and critical evaluation of previous research on a topic. Meanwhile, according to Creswell (2017), searching, selecting, weighing, and reading literature is the first job in any research project. The literature study is critical in conducting research; this is because research cannot be separated from the scientific literature. The method for reviewing journals is done by

searching and collecting literature studies with the keywords of reasoning, mathematical reasoning, and reasoning in school learning on Google Scholar, Taylor, and Francis, publish or perish 7, and international journal websites. The four criteria for selecting articles were access, completeness, novelty, and authenticity. A total of 100 articles were found according to these keywords and then screened, then 36 full-text articles were assessed for feasibility. At the end of the process, ten international articles constitute literature. The author examines the ideas, opinions, or findings contained in the literature to provide theoretical information regarding mathematical reasoning abilities in learning in schools.

3. RESULTS AND DISCUSSION

Ten articles are collected in **Table 1**.

In articles 1, 2, 7, and 9 discuss the use of approaches and strategies to improve students' mathematical reasoning abilities. These articles explain that students are not proficient in mathematical concepts because they assume that learning mathematics only remembers formulas. Therefore, an alternative is needed to improve students' mathematical skills. In addition, students need vital explanations to generalize some examples to be used in everyday life by enhancing students' mathematical reasoning abilities. To help improve mathematical reasoning skills, learning media such as 1 PowerPoint with Visual Basic PowerPoint are needed, assisted by an open-ended approach. Learning with the Open-Ended approach ends with using the Visual Basic PowerPoint application better than the class using the usual learning method. In contrast to 1, article 2 uses Visual Basic Applications for Excel to improve students' mathematical reasoning skills.

Table 1. Articles about students' mathematical reasoning.

No	Title / Author	Result
1	Improve student mathematic reasoning ability with an open-ended approach using VBA for PowerPoint (Benard & Chotimah, 2018)	Learning with the Open-Ended approach ends using the Visual Basic Powerpoint application, which is better than the class using the usual way of learning. And the open-ended approach using the Visual Basic Application for Powerpoint can be used as an alternative to improve students' reasoning skills in mathematics learning, especially in the number theory course in solving theory proof problems and helping to explain definitions in number theory.
2	The contextual approach using VBA learning media to improve students' mathematical displacement and disposition ability (Chotimah, *et al.,* 2018)	The achievement and improvement of students' mathematical reasoning abilities and dispositions using a contextual approach supported by VBA (Visual Basic Application for Excel) learning media are better than students who receive conventional learning
3	Developing teaching material based on realistic mathematics and oriented to the mathematical reasoning and mathematical communication (Habsah, 2017)	That mathematics teaching materials with a communication-oriented realistic mathematics approach and students' mathematical reasoning abilities that have been developed are valid, practical, and effective in terms of mathematical reasoning and communication abilities.

Table 1 (Continue). Articles about students' mathematical reasoning.

No	Title / Author	Result
4	Gender and mathematical reasoning ability (Kadarisma, *et al*., 2019)	There is no significant difference in mathematical reasoning abilities between male and female students after using the problem-based learning approach in their learning, meaning that the problem-based learning approach can reduce differences in the mathematical reasoning abilities of male students and female students.
5	Analysis of students' mathematical reasoning (Sukirwan, *et al*., 2018)	The results showed that students in general still experienced problems in reasoning. Students tend to use imitative reasoning, which means students tend to use routine procedures when dealing with reasoning.
6	A framework for primary teachers' perceptions of mathematical reasoning (Herberta, *et al*., 2015)	Teachers' perceptions of reasoning differ from those of mathematicians and curriculum writers.
7	Improving students' creative mathematical reasoning ability students through adversity quotient and argument-driven inquiry learning (Hidayat, *et al*., 2018)	(1) The increase in mathematical creative reasoning abilities of students who are prospective mathematics teachers receiving Argument-Driven Inquiry (ADI) learning is better than students who receive direct learning. (ADI) and direct learning are reviewed based on the type of Adversity Quotient (Low Quitter / AQ, Champion / Medium AQ, and Climber / High AQ); (3) The learning factor and the type of Adversity Quotient (AQ) affect the improvement of students' mathematical creative reasoning abilities. In addition, there is no interaction effect between learning and AQ together in developing students' mathematical creative reasoning abilities; (4) The mathematical creative reasoning ability of prospective mathematics teacher students has not been achieved optimally in the novelty indicator.
8	Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification (Mata-Pereira & da Ponte, 2017)	This article provides a set of design principles and characterization of teacher actions that enhance students' mathematical reasoning processes such as generalization and justification.
9	Enhancing an Ability Mathematical Reasoning through Metacognitive Strategies (Lestari & Jailani, 2018)	The performance measure of reasoning ability consists of three parts: making assumptions, providing arguments, and observing patterns. The results showed that students who were exposed to metacognitive strategies in collaborative learning (COLAB + META) significantly outperformed their peers who were exposed to collaborative learning without a metacognitive strategy (COLAB). This work provides evidence of the advantages of using metacognitive strategies to empower mathematical reasoning. Furthermore, the findings indicate a positive effect of the COLAB + META method in both higher and lower achievers.
10	Age- And Gender-Related Change in Mathematical Reasoning Ability and Some Educational Suggestions (Erdem & Soylu, 2017)	The analysis shows that with increasing age mathematical reasoning develops and male students perform significantly better than female students in mathematical reasoning. It is imperative to (a) take encouraging steps to ensure that women are attracted to mathematics rather than hopeless in society, and (b) expose students to higher-level problems in an open format with no choice of answers to grades to increase their mathematical reasoning.

The results of 2 are not much different from 1, which explains an increase and achievement of mathematical reasoning abilities of students whose learning with a contextual approach is supported by Visual Basic Application for Excel learning media. This learning is better than students who get conventional education without assistance. VBA for excel. In article 7, they are using Adversity Quotient (AQ) and Argument-Driven Inquiry (ADI) learning to improve students' reasoning skills. The results of article 7 show that learning with ADI is better than students who get direct learning; there is no difference in the increase in the mathematical reasoning ability of students who receive learning with ADI and AQ. What distinguishes articles 7 from 1 and 2 is the difference in research subjects, 1 and 2 using students in secondary schools while 7 using student teacher candidates. This contrasts with article 9 that compared the effects of collaborative learning with or without metacognitive strategies on higher and lower-achieving students in mathematical reasoning. 9 explained that the performance measure of reasoning ability consists of three parts, namely making assumptions, providing arguments, and observing patterns. The results showed that students who were exposed to metacognitive strategies in collaborative learning (COLAB + META) significantly outperformed their peers who were exposed to collaborative learning without a metacognitive strategy (COLAB). This work provides evidence of the advantages of using metacognitive strategies to empower mathematical reasoning. Furthermore, the findings indicate a positive effect of the COLAB + META method in both higher and lower achievers.

From articles 1, 2, 7, and 9, we can find strategies that can be used to improve students' mathematical reasoning abilities. Articles 4 and 10 discuss differences in mathematical reasoning abilities seen from a gender perspective. 4 explained that there was no significant difference between the mathematical reasoning abilities of male and female students. This is different from the results obtained 10; the article explains that with increasing age, students' mathematical reasoning will increase, and male students perform significantly better than female students. Furthermore, 10 demonstrated, with this result, it is hoped that teachers will take specific steps to ensure that female students are more interested in mathematics instead of showing a hopeless attitude, and it is expected that teachers will provide HOTS questions more often to improve mathematical reasoning skills. Students. In connection with the difference in mathematical reasoning abilities between male and female students, article 4 provides a solution to using a problem-based learning approach in the learning process. This is because this approach is proven to reduce differences in mathematical reasoning abilities between male students and female students.

Article 3 produces teaching materials in mathematical textbooks based on realistic mathematics oriented towards mathematical reasoning. This article contributes to teaching materials that are valid, practical, and effective in improving students' reasoning abilities. 3 explained that teaching material is said to be useful if the expert's assessment is categorized as 'good,' teaching material is classified as practical if the minimum evaluation of teachers and students is classed as 'good.' Meanwhile, teaching materials are said to be effective if at least 75% of the students' scores are categorized as good in the mathematical reasoning test. It was also explained that books/teaching materials developed as a result are more effective than e-books for schools from the government. This shows that directing students to realistic mathematics will affect students' reasoning abilities. In article 6, it describes teachers' perceptions of mathematical reasoning. In paper 6, it is shown that teachers' perceptions of mathematical reasoning differ from those of mathematicians and curriculum writers. According to the teacher, reasoning is considered a very private thought that is carried out independently. Suppose Sonya (10 years of experience teaching grades 3 and 4) states that reasoning is the process that children go through to solve problems and assignments. And

according to Sonya, it is their thought process and how they solve it. Similar to Olive's opinion (8 years of experience, teaching grades 3 & 4), according to Olive, the reasoning is thinking about thinking. So kids analyze what they think. From the opinions of the two teachers, I can conclude that they consider reasoning to be personal and involve making choices that affect personal analysis and reflection. This is different from reasoning, according to experts who state that reasoning is a process of concluding and a method of giving reasons. In this case, there is no indication that reasoning is something that can be shared with others.

Article 6 also provides suggestions for teachers to take part in professional learning to improve teachers' reasoning knowledge. If article 6 explains the teacher's perceptions of mathematical reasoning, paper 5 focuses on analyzing students' mathematical reasoning. The purpose of this article is to determine the quality of students' mathematical reasoning based on the Lither perspective. Lither see how the environment affects mathematical reasoning. In this connection, Lither makes two perspectives, namely imitative reasoning and creative reasoning. 5 Students still experience problems when dealing with the reasoning in general, and the quality of students' mathematical reasoning is still dominated by imitative reasoning, where the problematic situations faced by students are fixated on implementing routines from daily learning. This shows that students tend to use routine procedures when contending with reasoning problems. The research begins with giving a mathematical reasoning test. The test results in student answers are then analyzed and categorized using reasoning criteria, namely memorized reasoning, algorithmic reasoning, and creative reasoning. The next stage is to conduct interviews. The purpose of this interview is to confirm students' answers to verify the mathematical reasoning grouping carried out in the previous stage.

Article 8 discusses a set of design principles for characterizing teacher actions that improve students' mathematical reasoning processes, such as generalization and justification. In this article, consider inviting, informing/suggesting, supporting / guiding, and challenging students in group discussions. This article also provides interventions directed at dealing with the reasoning process. The results of this article show that the ability to generalize and justify will emerge if the teacher designs a challenging lesson for students, followed by guiding students. Intervention activities that the teacher can carry out are that the teacher is involved in all student activities; in this case, the teacher invites questions that can direct students to solve mathematical reasoning problems. Next, the teacher provides suggestions and guides students in discussions. Familiarize students with conversations with their peers because this will indirectly improve students' reasoning abilities. Then take advantage of the environment in the learning process or other words, every time you teach, it is always associated with the student's world of reality. The last is to provide challenging questions for students, followed by a process of guidance from the teacher. This will indirectly affect improving students' reasoning abilities.

Another thing that needs to be considered in the design of the learning flow. Teachers must provide a systematic learning flow to make it easier for students to master mathematical concepts. This learning flow is a kind of concept map or another name for this process is Learning Trajectory, a topic that you want to teach.

3.1. Reasoning in Learning Mathematics

According to Henningsen & Stein (1997) in building reasoning and strategic thinking in mathematics learning, teachers must pay attention to which types of mathematical thinking suit students, for example, the kinds of teaching materials, class management, the role of teachers, and students' autonomy in thinking and move. The thinking characteristics

expressed by Henningsen & Stein (1997) can be used as a reference in compiling and developing teaching materials that are according to the curriculum's demands, student development, teacher abilities, and environmental conditions. Bernard & Chotimah (2018) underlined that to develop students' knowledge in reasoning, learning should be directed at Open-Ended, and the solution process given must be open; the final answer to the problem is available, and how to solve it is available.

The research results conducted by Shimizu (2000) revealed that teachers have a very central role in the learning process through disclosure, encouragement, and the development of students' thinking processes. In addition, the teacher's questions during learning activities can effectively lead students' thought processes towards correct completion. The leading questions given by the teacher will effectively help students' thinking activities and representations to reach the right answer. Sumarmo shows that for students' mathematical reasoning and thinking abilities to develop optimally, students must have very open opportunities to think and be active in solving various problems. Thus, giving the broadest possible autonomy to students in thinking to solve problems can develop students' abilities in reasoning and thinking optimally.

In learning mathematics, teachers should pay attention to and develop deductive and adaptive reasoning skills. Adaptive reasoning deals with the capacity to think logically about the relationship between concepts and situations. This reasoning process is declared accurate and valid if it is the result of careful observation of various alternatives and using knowledge to provide explanations and justification of conclusions. In mathematics, adaptive reasoning is the glue for the integration of different student abilities that are encouraged and as a learning guide. One uses adaptive reasoning to organize various facts, procedures, concepts, and ways and analyzes that they are all intertwined in a precise path. One of the manifestations of adaptive reasoning is to justify the process and results of a job. The justification here is intended as an instinct to provide sufficient reasons, for example, in a mathematical proof.

Not a few conceptions of mathematical reasoning are used as the basis for formal proof or other forms that require deductive reasoning. Deductive reasoning in mathematics can be used to show the truth of uncertainty. An answer can be believed to be true because it is based on correct assumptions and through a series of logical analyzes. According to [29] the human ability to find analogical correspondences is a powerful reasoning mechanism. The definition of deductive reasoning not only concerns justification but also includes intuition and inductive reasoning based on patterns, analogies, and metaphors. This is in line with what was stated by English (2013) analogical reasoning, metaphors, and mental and physical representations are thinking tools that are often a source of inspiration for hypotheses, problem-solving, and learning aids.

Piaget (1964) stating that the reasoning ability of children under 12 years is still limited, including if they are asked how to add a fraction to arrive at an answer. However, this fact does not guarantee that children aged 12 and over will not have problems with their reasoning abilities. Research results found by Rosnawati (2013) show that the reasoning ability of Indonesian junior high school students is very low. For example, students were asked to complete TIMSS 2011 questions that measured the cognitive domain at the lowest level, namely knowledge related to the numeric content domain, namely 42.65 + 5.748. The problem involved the problem of adding the decimal number to two places and three decimal places. Judging from the average correct answer of international students is 73% of the eighth-grade participants. In many countries, more than 80% of students answered correctly,

including six East Asian countries, namely Singapore, Malaysia, Hong Kong, Taiwan, Korea, and Japan. However, only 57% of Indonesian students can answer correctly.

This question is not classified as complicated, but the low percentage of Indonesian students answering this problem is generally due to an inadequate understanding of place values. In the junior high school mathematics curriculum, the knowledge of place values are included in the topic of numbers, but the notion of numbers, especially decimal numbers such as place values, as a prerequisite for carrying out the operation of calculating decimal numbers is not getting enough attention, it can be shown that students often read 42.65 with 'forty-two. Point sixty-five '. Incorrect understanding of place values will result in errors in addition to operations.

Learning the topic of decimal numbers is often seen as a simple topic. The delivery of material tends to focus on developing skills for performing arithmetic operations that involve decimal numbers. Usually, the rules for simplifying arithmetic operations in decimal numbers are given by relating the rules that apply to integer operations without being given a reason why the rule applies. Most students (57%) chose the answer 100.13 to answer questions 42.65 + 5,748. Arithmetic operations are basic skills that students must master to be able to master higher mathematical skills. Therefore, if the student's potential for reasoning is not developed optimally, the students' reasoning abilities cannot develop properly.

To become skilled problem solvers, students need to learn how to form mental representations of problems, detect mathematical relationships, and find new methods when needed. The fundamental characteristic required in the problem-solving process is flexibility. This flexibility develops through expanding and deepening the knowledge required to solve non-routine problems. Because in solving routine problems, students already know how to solve them based on their experience. When faced with common problems, students only need to think reproductively because they only need to reproduce and apply known procedures to calculate 412 × 32. Most junior high school students are used to this problem, so they can do it. Another case, if students are faced with questions that are not routine. Intermittent problems are problems that are not immediately known how to solve, require productive thinking because students must first understand the problem, find a way to get a solution, and solve it. Examples of non-routine problems are as follows: 'At the time of the exam, Ical was given 20 multiple-choice questions. If Ical answered correctly, he was given a score of 5, if he answered incorrectly, he was given a score of -2, and if he did not answer, he was given a score of 0. If it is known that Ical's score was 44 with some unanswered questions, how many questions did Ical not answer?'

One way of thinking to solve it is to do a trial (trial-error), if Ical answered 10 correct questions then the score obtained by Ical is 10 × 5 = 50, the possibility of the number of wrong questions is 3 questions so that 3 × -2 = - 6, while the unanswered questions are 7 questions (20-13), thus the score on supposition I am 44. Another way that students can think of is to suppose that 12 questions are correct so that the score obtained by Ical is 12 × 5 = 60, the possibility of questions What is wrong is 8 questions so that 8 × -2 = -16, while the questions that are not answered are 0 questions (20- (12 + 8)), thus the score obtained by Ical in supposition II is 44. Based on this supposition, it is known that Two possibilities can be used as the answer, namely the questions that Ical did not answer, there were 7 questions or none of the questions were not answered. Because in the questions it was stated that there were questions Ical did not answer, so many questions Ical did not answer, there were 7 questions.

The wise way is of course using the algebraic approach, but if from the non-routine questions we assume there are three variables, namely questions that were answered correctly (x), questions that were answered but incorrectly (y), and questions that were not

answered (z), then by using the concept of algebra obtained $x + y + z = 20$ and $5x - 2y + 0z = 44$. Generally, to solve the form of a three-variable equation, 3 linear equations are needed. However, we only have 2 equations, so we need another strategy to solve it (for example trial-error). The trial-error steps to solve the problem are 1) take the possibility where if the number of correct questions × 5 results in a score greater than 44. 2) determine the number of wrong questions × (-2) results in a score of 44, and 3) determine the number of questions that are not answered.

Students who have good strategic competence will be able to solve non-routine problems in various ways but must be able to have flexible abilities in choosing strategies, such as the trial-error method, using analogies, finding patterns, geometric forms, algebraic methods, or other methods. Appropriate to answer the problem according to the request and the situation. The ability to use this flexible approach is a significant cognitive skill needed to solve non-routine problems.

4. CONCLUSION

Learning strategies such as open-ended, visual basic application for excel, adversity quotient, and argument-driven inquiry can be used to improve students' mathematical reasoning abilities. Seen from a gender perspective, male students' mathematical reasoning is significantly better than female students. Teachers' perceptions of mathematical reasoning differ from those of experts. The quality of students' mathematical reasoning is still dominated by imitative reasoning. The ability to generalize and justify will emerge if the teacher designs a challenging lesson for students followed by activities. In learning mathematics, teachers should pay attention to and develop deductive and adaptive reasoning skills. Adaptive reasoning is the glue for integrating various student abilities that are encouraged and as a learning guide. One uses adaptive reasoning to organize different facts, procedures, concepts, and ways and analyzes that they are all intertwined in a precise path. One of the manifestations of adaptive reasoning is to justify the process and results of a job. The justification here is intended as an instinct to provide sufficient reasons, for example, in a mathematical proof.

Meanwhile, deductive reasoning in mathematics can be used to show the truth of uncertainty. An answer can be believed to be true because it is based on correct assumptions and through a series of logical analyzes. To become skilled problem solvers, students need to learn how to form mental representations of problems, detect mathematical relationships, and find new methods when needed. The fundamental characteristic required in the problem-solving process is flexibility. This flexibility develops through expanding and deepening the knowledge needed to solve non-routine problems. Because in solving routine problems, students already know how to solve them based on their experience. When faced with common problems, students only need to think reproductively because they only need to reproduce and apply known procedures. Students who have good strategic competence will be able to solve non-routine problems in various ways but must be able to have flexible abilities in choosing strategies, such as the trial-error method, using analogies, finding patterns, geometric forms, algebraic methods, or other methods. Appropriate to answer the problem according to the request and the situation. The ability to use this flexible approach is a significant cognitive skill needed to solve non-routine problems.

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