

## Modelling the Effect of Road Excitation on Vehicle Suspension System

Robert Emmanuel Jonjo and Sahr Tamba Nyalloma



Received: 19 March 2020  
Accepted: 29 March 2020  
Published: 30 March 2020  
Publisher: Deer Hill Publications  
© 2020 The Author(s)  
Creative Commons: CC BY 4.0

### ABSTRACT

The vehicle suspension system serve a dual purpose – to provide passenger comfort and good road holding. In the design of a vehicle suspension system, these two contradictory criteria must be balanced out. Road irregularities are also a major source of anxiety amongst drivers and passengers alike. This research was undertaken to investigate the effect road irregularities will have on the vehicle structure especially the suspension system. In this study, the responses of different linear vehicle models are studied for step road input. The mathematical models considered are: a two degrees-of-freedom system (quarter car model) and a four degrees-of-freedom system (half car model). The equations of motion for both models were obtained using Newton's method. These models are analysed using SIMULINK/Matlab. Different response parameters such as the acceleration of the vehicle body and the travel of the suspension are investigated for a passive suspension system. The responses of the vehicle suspension due to changes in parameters such as suspension stiffness and damping coefficients are investigated. The results show that road irregularities affect the vehicle structures and the response of the suspension system is dependent on the suspension parameters. Passive suspension systems do not satisfy road holding and passenger comfort at the same time.

### 1 INTRODUCTION

The vehicle Suspension consists of springs, shock absorbers and other linkages that separates the car body from the car wheel. The functions of the automotive suspension on a vehicle includes; Isolating a car body from road irregularities, providing good handling by ensuring that roll and pitch motion are minimized and supporting the vehicle static weight. Vehicle suspension system should therefore isolate the vehicle body from road disturbances in order to give maximum passenger ride comfort while retaining continuous road-wheel contact so as to provide excellent road holding (Changzil and Rouhani 2011). It is therefore a challenge to design a vehicle to satisfy both ride comfort and road holding. Suspension systems are generally classified as passive, semi-active and active suspensions (Ghazaly and Moazz 2014, Tejas 2016)

- Passive suspensions: Passive suspension systems usually consist of a non-controlled spring and a damper with fixed parameters (Agharkakli .A 2012). They lack enough energy absorption capability to sustain the load input into the vehicle systems (Gao et al 2016). They lack stability when compared with other suspension systems (Dahunsi 2011).
- Semi-active suspensions: Semi active suspensions on the other hand provide a rapid change in rate of springs and damping coefficients. They usually have a fixed spring rate and a variable shock absorber. They do not provide any energy into suspension system. Compared to the active suspensions, these suspensions use less energy to operate and they are cheaper (Williams 1994).
- Active suspensions: The active vehicle suspension system employs electronic control systems that monitor the operation of the suspension elements. In active suspension systems, the shock absorber is replaced by a force actuator. They reduce car body accelerations by allowing the suspension to absorb wheel accelerations using an actuator (Pedro and Ekoru 2013). Unlike passive and semi-active suspensions, active suspensions give dynamic compensation (Kashem et al. 2012). They are Superior in performance than passive and semi-active suspension systems.

### 2 MATHEMATICAL MODELING

The three common vehicle models found in literature are the quarter car model (two degrees of freedom), the half car model (four degrees of freedom) and the full car model (seven degrees of freedom). Roll and pitch motions are

---

R. E. Jonjo and S. T. Nyalloma ✉  
Mechanical and Maintenance Engineering Department  
Fourah Bay College, University of Sierra Leone  
Barham Road, Southern Central, 00232  
Freetown, Sierra Leon  
E-mail: sahr.nyalloma@usl.edu.sl

Reference: Jonjo, R. E. and Nyalloma, S. T. (2020). Modelling the Effect of Road Excitation on Vehicle Suspension System. *International Journal of Engineering Materials and Manufacture*, 5(1), 19-28.

ignored in the quarter car model. A half car model on the other hand considers either roll motion or pitch motion. The full car model considers roll, yaw and pitch motions.

## 2.1 QUARTER CAR MODEL

The quarter car model shown in Figure 1 is simple and widely used for dynamic performance analysis. It is a two degrees of freedom model. The quarter car model consists of one-fourth of the body mass, suspension components and one wheel. It represents the suspension system at any of the four wheels of the vehicle and the degrees-of-freedom are displacement of axle and displacement of the vehicle body at a particular wheel. The assumptions of a quarter car modelling are as follows:

- The tire, spring and damper are modelled as a linear spring without damping,
- Rotational motion in wheel and body are ignored,
- The tire remains in contact with the road surface at all times
- Effect of friction between road and wheel is neglected.

Consider the free body diagram of the forces acting on the sprung mass as shown in Figure 2 with  $Z_s > Z_u > Z_r$ . The equation of motion of the sprung mass

$$M_s \ddot{Z}_s + C_s (\dot{Z}_s - \dot{Z}_u) + K_s (Z_s - Z_u) = 0 \quad (i)$$

Equation (i) can be rewritten as

$$\ddot{Z}_s = \frac{1}{M_s} \{ C_s (\dot{Z}_u - \dot{Z}_s) + K_s (Z_u - Z_s) \} \quad (ii)$$

Also, the free body diagram of the forces acting on the unsprung mass is shown in Figure 3. The equation of motion of the unsprung mass becomes:

$$M_u \ddot{Z}_u - C_s (\dot{Z}_s - \dot{Z}_u) - K_s (Z_s - Z_u) + K_t (Z_u - Z_r) = 0 \quad (iii)$$

Also, equation (iii) becomes;

$$\ddot{Z}_u = \frac{1}{M_u} \{ -C_s (\dot{Z}_u - \dot{Z}_s) - K_s (Z_u - Z_s) - K_t (Z_u - Z_r) \} \quad (iv)$$

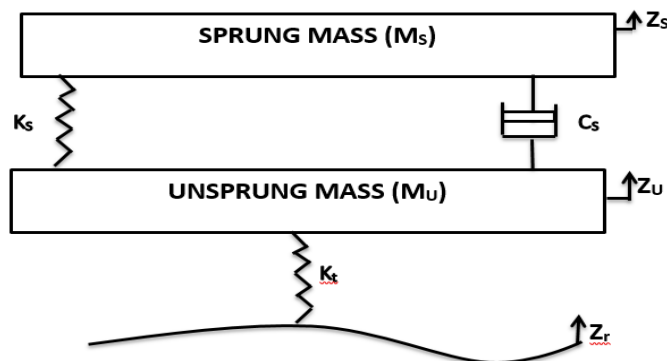


Figure 1. Quarter car model

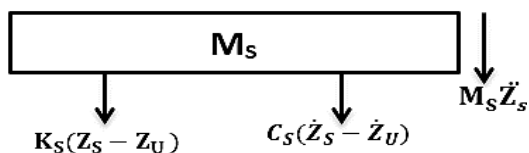


Figure 2. Forces acting on sprung mass for Quarter car model

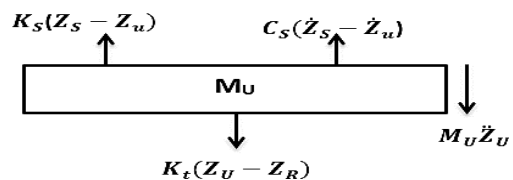


Figure 3. Forces acting on the unsprung mass for the Quarter car model

## 2.2 HALF CAR MODEL

The half car suspension model is a four degree of freedom system shown in Figure 4. This model considers both pitch and bounce motion of the vehicle. In this model, the following assumptions are made;

- The vehicle undergoes both pitch and bounce motion separately.
- The half car model can be a side, front or rear car model depending on the section of the vehicle considered. In this study, the side half car model will be considered.
- The tire, springs and dampers are modelled as a linear spring.
- The wheel is in contact with the road at all times.

The free body diagram for both the sprung and unsprung masses with bounce and pitch motion is shown in Figures 5 and 6. The equation of motion of the sprung mass exhibiting bounce motion is:

$$M_s \ddot{Z}_s + C_{sr}(\dot{Z}_s - \dot{Z}_{ur} - C\dot{\theta}) + C_{sf}(\dot{Z}_s - \dot{Z}_{uf} + b\dot{\theta}) + K_{sr}(Z_s - Z_{ur} - C\theta) + K_{sf}(Z_s - Z_{uf} + b\theta) = 0 \quad (v)$$

The equation of motion of the sprung mass exhibiting pitch motion is

$$I\ddot{\theta} - C_{sr}C(\dot{Z}_s - \dot{Z}_{ur} - C\dot{\theta}) + C_{sf}b(\dot{Z}_s - \dot{Z}_{uf} + b\dot{\theta}) - K_{sr}C(Z_s - Z_{ur} - C\theta) + K_{sf}b(Z_s - Z_{uf} + b\theta) = 0 \quad (vi)$$

The equation of motion for both the front unsprung mass and rear unsprung mass are given respectively as,

$$M_{uf}\ddot{Z}_{uf} - C_{sf}(\dot{Z}_s - \dot{Z}_{uf} + b\dot{\theta}) - K_{sf}(Z_s - Z_{uf} + b\theta) + K_{tf}(Z_{uf} - Z_{rf}) = 0 \quad (vii)$$

$$M_{ur}\ddot{Z}_{ur} - C_{sr}(\dot{Z}_s - \dot{Z}_{ur} - C\dot{\theta}) - K_{sr}(Z_s - Z_{ur} - C\theta) + K_{tr}(Z_{ur} - Z_{rr}) = 0 \quad (viii)$$

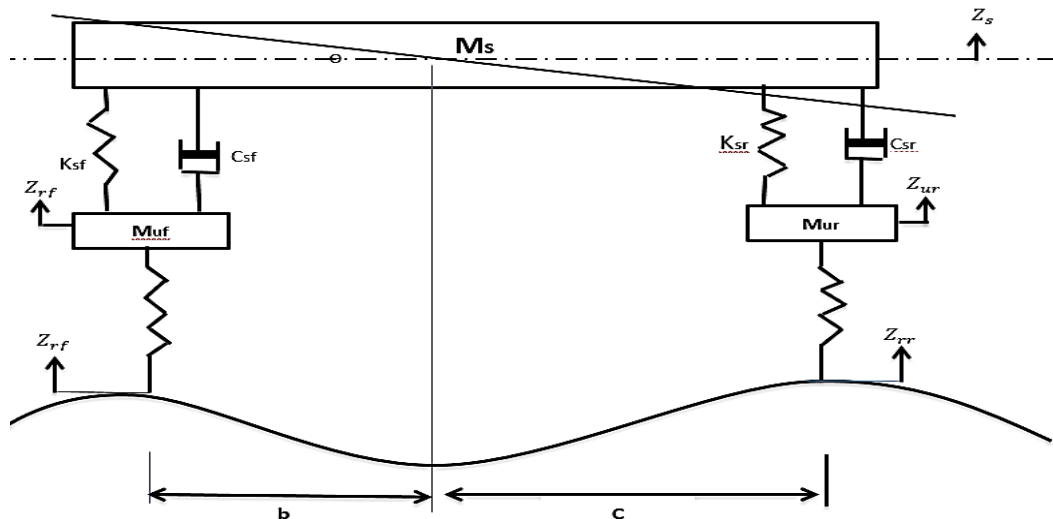


Figure 4. Half car model

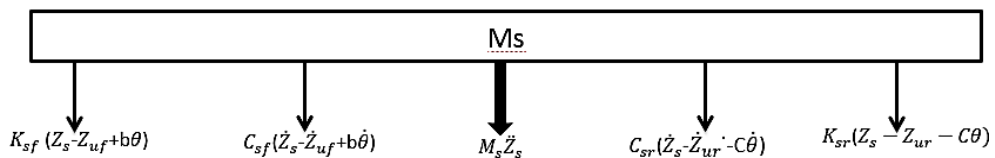


Figure 5. Free body for sprung mass exhibiting bounce motion

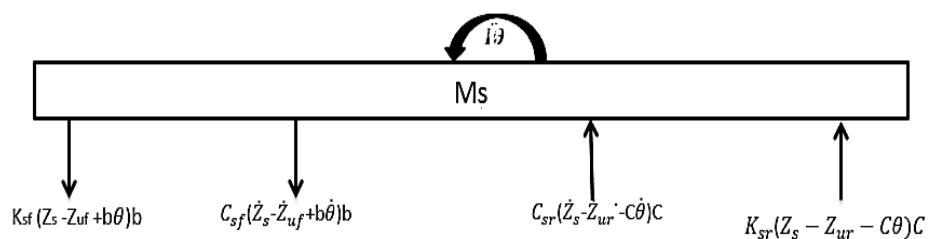


Figure 6. Free body diagram for sprung mass exhibiting (pitch motion)

### 3 SIMULATION

#### 3.1 Simulation Parameters

In the current research vehicle parameters to undertake simulation of the quarter car and half car models were obtained for a passenger sedan as presented by (Agostinacchio et al. 2014) and presented in Tables one and two respectively. Road excitation input is a step signal of amplitude 10 cm.

### 3.2 Simulation and Results

The Simulink block diagram for the quarter car and half car models are shown in Figures 8 and 9 respectively. Both Figures represent the differential equations of motion for the two models investigated, that is, equations 2 and 4 for the quarter car model and equations 5, 6, 7 and 8 for the half car model.

Using the parameters presented in Tables 1 and 2, simulation of the system models was undertaken and the results presented in graphical format. The displacement of the vehicle body, displacement of the unsprung mass(s), deflection of the wheels, suspension travel, acceleration of the car body in the vertical direction are the parameters that were investigated. The effect of the changes of various passive suspension parameters are investigated by keeping all other system parameters fixed and gradually varying the parameter being investigated.

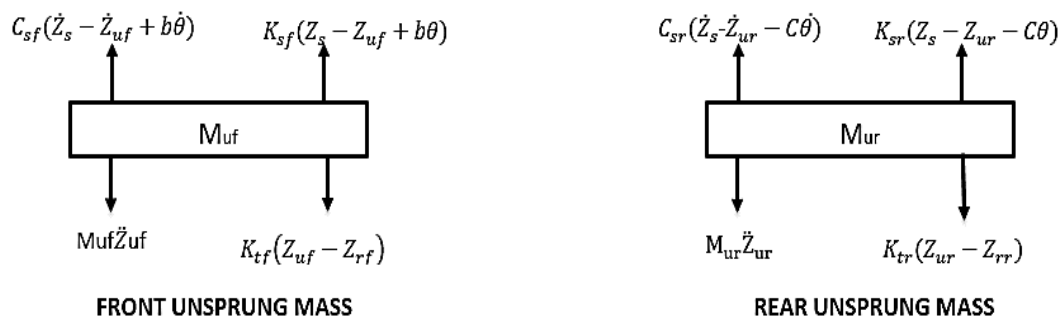


Figure 7. Free body diagrams for front and rear unsprung mass

Table 1. Simulation parameters for Quarter car model

Parameter	Symbol	Values	Unit
Sprung mass of the vehicle	$M_s$	400	Kg
Unsprung mass of vehicle	$M_u$	40	Kg
Stiffness of tire	$K_t$	190000	N/m
Spring constant of axle	$K_s$	21000	N/m
Damping coefficient of axle	$C_s$	1000	Ns/m

Table 2. Simulation parameters for half car model

Parameter	Symbol	Values	Unit
Sprung mass of the vehicle	$M_s$	400	kg
Moment of inertia of vehicle	$I_y$	1100	kgm <sup>2</sup>
Unsprung mass of front axle	$M_{uf}$	40	kg
Unsprung mass of rear axle	$M_{ur}$	40	kg
Stiffness of front tire	$K_{tf}$	150000	N/m
Stiffness of rear tire	$K_{tr}$	150000	N/m
Spring constant of front axle	$K_{uf}$	21000	N/m
Spring constant of rear axle	$K_{ur}$	21000	N/m
Damping coefficient of front axle	$C_{sf}$	1500	Ns/m
Damping coefficient of rear axle	$C_{sr}$	1500	Ns/m
Front body length from centre of gravity	$c$	1.47	m
Rear body length from centre of gravity	$b$	1.4	m

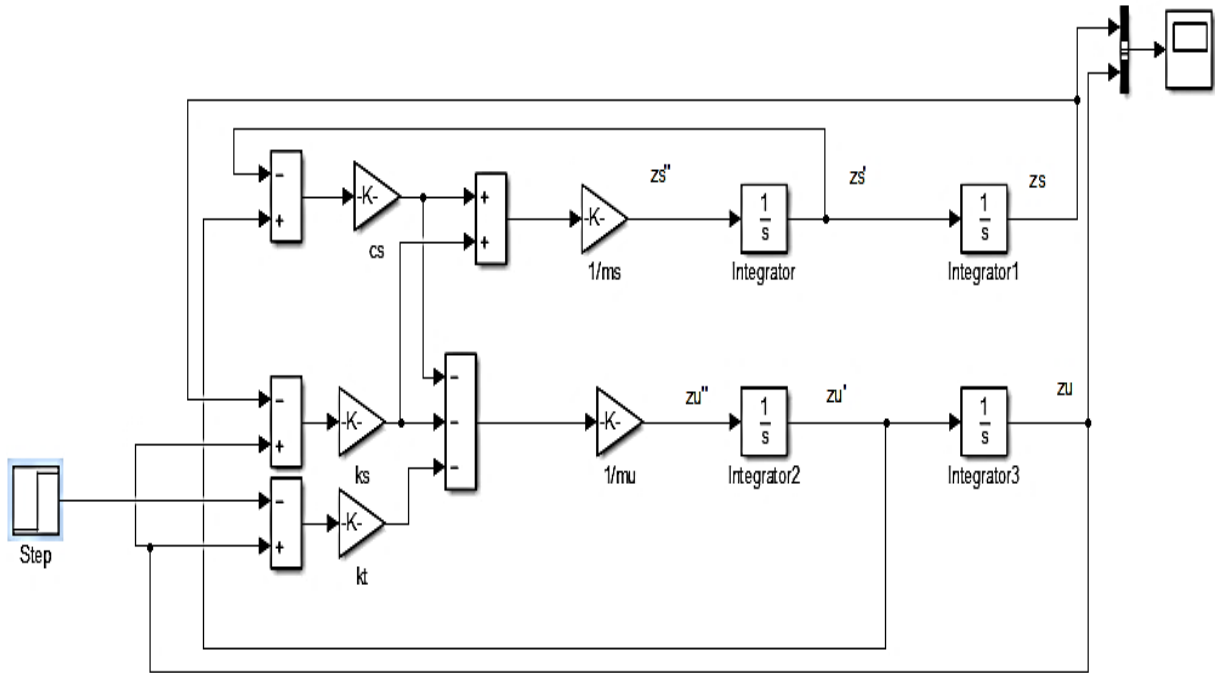


Figure 8. Simulink block diagram for quarter car model

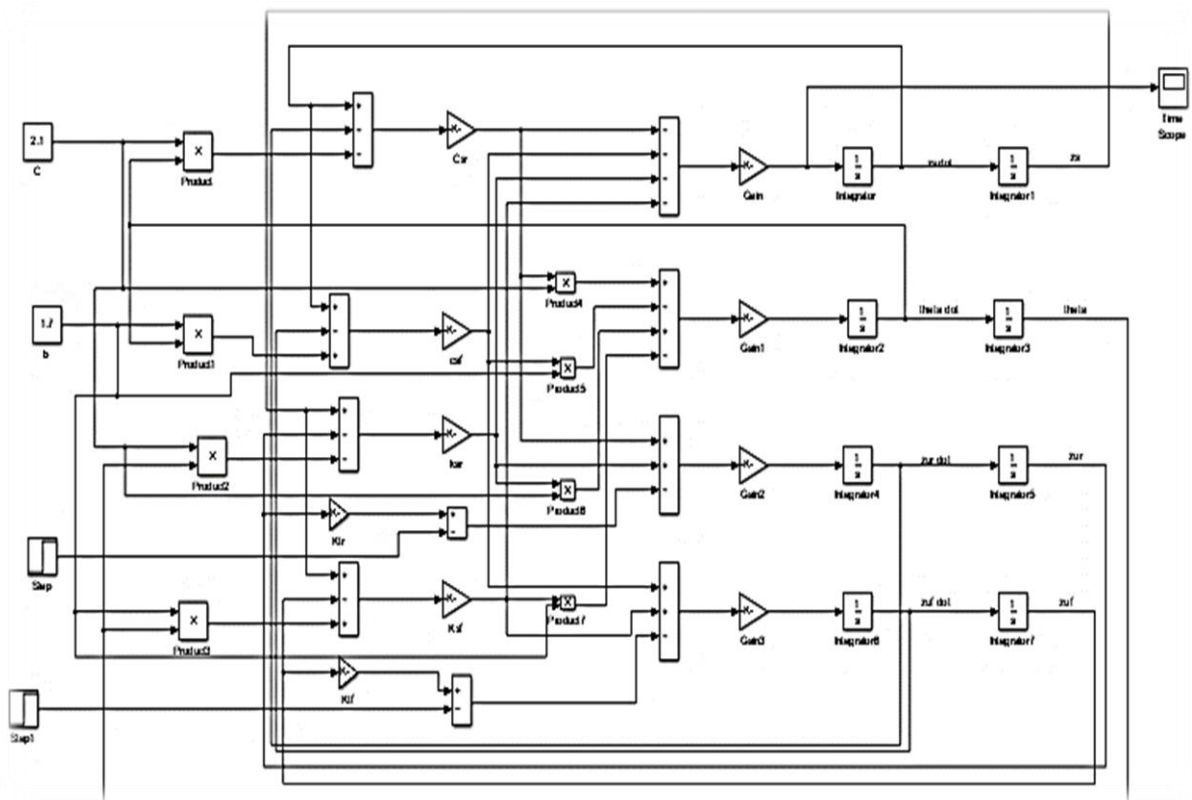


Figure 9. Simulink block diagram for half car model

### 3.2.1 Simulation for Quarter Car Model

The displacement of the sprung and unsprung masses, acceleration of the sprung and unsprung masses, suspension travel and tire deflection are presented in Table 3. Figures 10, 11 and 12 show the displacement of the sprung mass, acceleration and displacement of the unsprung mass for the quarter car model.

**Table 3.** Simulation results for quarter car model

No	Parameters	Values	Maximum Sprung Mass Displacement	Maximum Unsprung Mass Displacement	Sprung Mass Acceleration		Suspension Travel		Wheel Deflection		Rms Acceleration
1	Sprung Mass	400	0.1569	0.1268	16.776	-2.969	0.050	-0.115	0.0268	-0.100	0.5829
2		500	0.1601	0.1258	13.569	-2.439	0.053	-0.113	0.0258	-0.100	0.472
3		1000	0.1686	0.1238	6.920	-1.329	0.059	-0.1175	0.0238	-0.100	0.253
4		1500	0.1730	0.1231	4.643	-0.923	0.064	-0.1189	0.0231	-0.100	0.178
5	Unsprung Mass	50	0.1576	0.1309	15.796	-3.436	0.049	-0.1125	0.0309	-0.100	0.582
6		100	0.1584	0.1433	13.045	-5.056	0.051	-0.1161	0.0433	-0.100	0.586
7		150	0.1599	0.1461	11.452	-5.420	0.062	-0.1180	0.0426	-0.100	0.597
8		200	0.1631	0.1539	10.622	-6.335	0.067	-0.112	0.0534	-0.100	0.593
9	Suspension Stiffness	21000	0.1569	0.1268	16.776	-2.959	0.050	-0.1115	0.0268	-0.100	0.5829
10		30000	0.1646	0.1217	17.858	-4.346	0.054	-0.1059	0.0217	-0.100	0.639
11		40000	0.1709	0.1164	19.010	-5.892	0.055	-0.099	0.0164	-0.100	0.704
12		60000	0.1750	0.1197	20.139	-7.330	0.056	-0.095	0.0197	-0.100	0.791
13	Damping Coefficient	1500	0.1569	0.1268	16.776	-2.959	0.050	-0.115	0.0268	-0.100	0.5829
14		2000	0.1492	0.1188	18.998	-3.200	0.043	-0.099	0.0188	-0.100	0.659
15		2500	0.1421	0.1120	20.023	-3.195	0.037	-0.091	0.0120	-0.100	0.749
16		3000	0.1381	0.1100	22.478	-3.542	0.038	-0.0828	0.0108	-0.100	0.786
17	Tire Stiffness	150000	0.1569	0.1268	16.776	-2.959	0.050	-0.1115	0.0268	-0.100	0.5829
18		200000	0.1555	0.1310	19.880	-5.183	0.050	-0.1205	0.031	-0.100	0.629
19		250000	0.1542	0.1388	23.220	-7.252	0.050	-0.1278	0.0388	-0.100	0.680
20		300000	0.1535	0.1439	26.143	-8.821	0.049	-0.1324	0.0439	-0.100	0.747

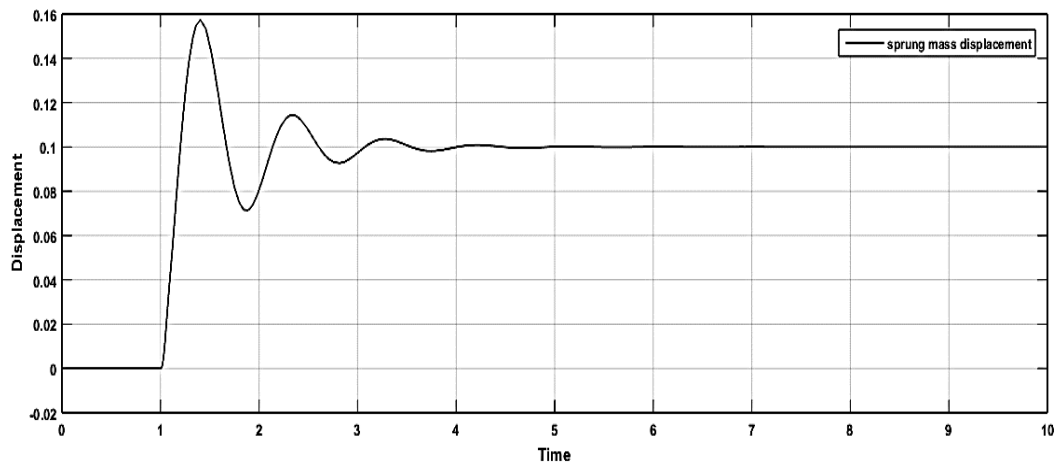


Figure 10. Displacement of the sprung mass Quarter car model

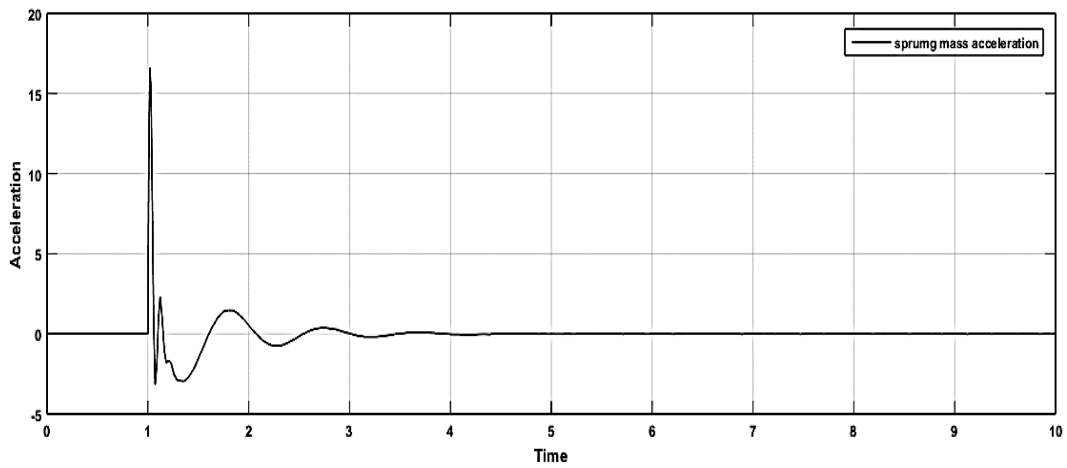


Figure 11. Acceleration of sprung mass Quarter car model

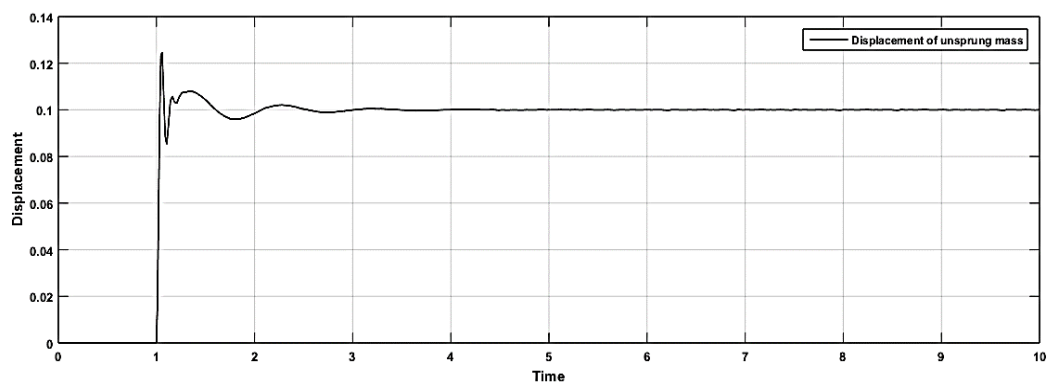


Figure 12. Displacement of the unsprung mass

### 3.2.2 Simulation for Half Car Model

The simulation results for the Half- Car model is presented in Table 4. Figures 13, 14 and 15 show the displacement of the sprung mass, and the displacement of the unsprung masses for the half - car model.

## 4 RESULTS

Simulations were carried out for the Quarter car and Half- car models. Different response properties were investigated such as vehicle sprung mass displacement, unsprung mass displacement, suspension travel, wheel deflection. From the simulation results presented in Tables 3 and 4, it is observed that the vehicle experiences a vertical acceleration as it passes over a road irregularity. The following general observations are made from the simulation results.

From the simulation results it is evidently clear that an increase in the sprung mass leads to a lower vertical acceleration of the car body, an increase in the vertical displacement of the car body and an increase in the suspension travel. If all other parameters are kept constant while the unsprung mass(es) is gradually increased, similar trends are observed as in the case of the sprung mass. Vertical acceleration is reduced but suspension travel is increased.

Increasing damping coefficient reduces car body displacement and suspension travel considerably but vertical acceleration increases whereas an increase in the stiffness of vehicle suspension leads to an increase in vertical acceleration. The suspension travel shows an increase and decrease pattern.

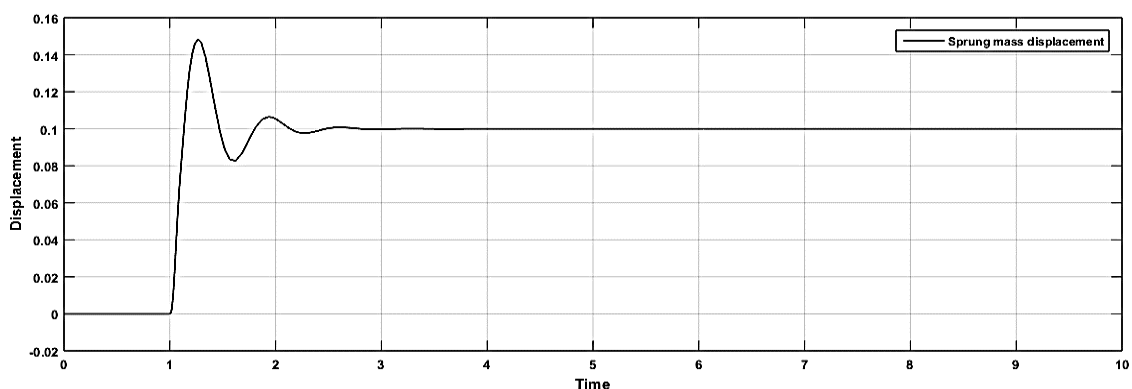


Figure 13. Displacement of sprung mass of half car model

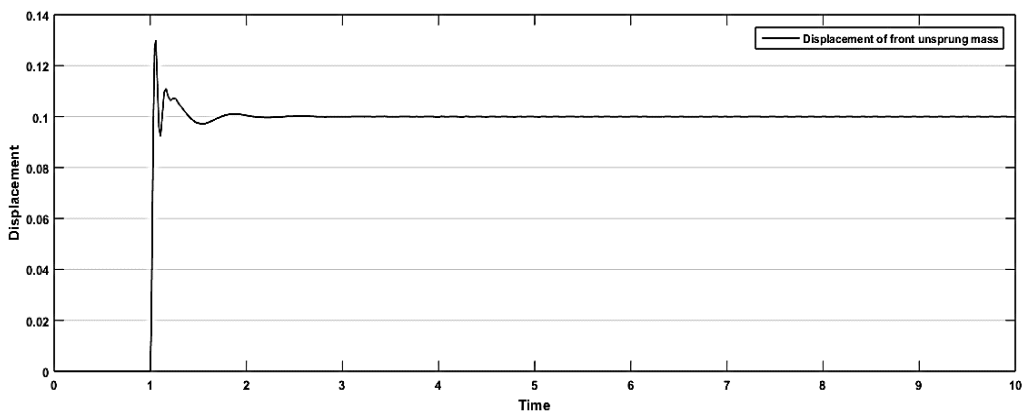


Figure 14. Displacement of front unsprung mass

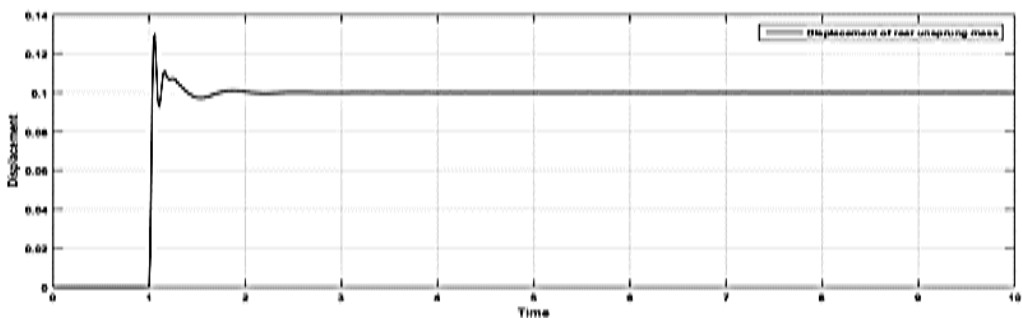


Figure 15. Displacement of rear unsprung mass



**Table 4.** Simulation results for half car model

No	Parameters	Values	Sprung Mass Displace	Unsprung Mass Displacement		Sprung Mass Acceleration		Suspension Travel				Wheel Deflection		RMS Acceleration
				Front	Rear			Front		Rear		Front	Rear	
1	Sprung Mass	400	0.148	0.1299	0.1301	32.034	-10.685	0.0415	-0.1045	0.0416	-0.1046	0.0299	0.030	3.80
2		500	0.151	0.1277	0.1278	26.057	-7.295	0.0443	-0.1074	0.0444	-0.1075	0.0278	0.0278	3.080
3		1000	0.1602	0.1231	0.1235	13.539	-2.434	0.0525	-0.1137	0.0527	-0.1138	0.0234	0.0236	1.595
4		1500	0.1652	0.1225	0.1226	9.129	-1.697	0.0570	-0.1159	0.0573	-0.1160	0.0225	0.02258	1.102
5	Front unsprung mass	50	0.148	0.1361	0.1299	30.318	-10.178	0.0419	-0.1023	0.0421	-0.1055	0.0361	0.0299	3.748
6		100	0.150	0.1485	0.1294	24.453	-7.746	0.0471	-0.0989	0.0434	-0.1080	0.0485	0.0294	3.430
7		150	0.148	0.1567	0.1297	23.178	-10.807	0.0613	-0.0930	0.0424	-0.1100	0.0568	0.0297	3.226
8		200	0.148	0.1607	0.1288	21.392	-12.042	0.0747	-0.0880	0.0438	-0.1110	0.0677	0.0288	3.090
9	Rear unsprung mass	50	0.1482	0.1298	0.1360	30.292	-10.172	0.0421	-0.1054	0.0419	-0.1020	0.0298	0.0362	3.748
10		100	0.1501	0.1292	0.1486	25.460	-7.748	0.0432	-0.1087	0.0474	-0.0989	0.0292	0.0486	3.436
11		150	0.1480	0.1295	0.1569	23.179	-10.823	0.0423	-0.1090	0.0613	-0.0939	0.0295	0.0569	3.227
12		200	0.1506	0.1286	0.1609	21.402	-12.052	0.0438	-0.1114	0.0747	-0.0889	0.0286	0.0609	3.090
13	Front suspension stiffness	21000	0.1483	0.1299	0.1301	32.034	-10.685	0.0415	-0.1045	0.0416	-0.1046	0.0299	0.0300	3.801
14		30000	0.1523	0.1257	0.1312	32.306	-9.342	0.0421	-0.1000	0.0452	-0.1036	0.0257	0.0312	3.917
15		40000	0.1547	0.1206	0.1315	33.043	-9.524	0.0412	-0.0958	0.0479	-0.1028	0.0206	0.0315	4.056
16		50000	0.1565	0.1200	0.1317	33.924	-10.805	0.0393	-0.0919	0.0506	-0.1023	0.0200	0.0317	4.179
17	Rear suspension stiffness	21000	0.1483	0.1299	0.1301	32.034	-10.085	0.0415	-0.1045	0.0416	-0.1046	0.0299	0.0300	3.801
18		30000	0.1521	0.1316	0.1267	32.203	-9.607	0.0449	-0.1014	0.0422	-0.0985	0.03163	0.0267	3.967
19		40000	0.1545	0.1316	0.1212	32.461	-9.477	0.0477	-0.1022	0.0408	-0.0956	0.03164	0.0212	4.071
20		50000	0.1559	0.1317	0.1202	33.609	-10.772	0.0501	-0.1019	0.0389	-0.0919	0.0317	0.0202	4.188
21	Front damping coefficient	1500	0.1483	0.1299	0.1301	32.034	-10.085	0.0415	-0.1045	0.0416	-0.1046	0.0299	0.0300	3.801
22		2000	0.1440	0.1254	0.1318	33.159	-10.669	0.0387	-0.0922	0.0372	-0.1011	0.0254	0.0318	4.102
23		2500	0.1406	0.1230	0.1320	34.671	-11.702	0.0366	-0.0847	0.0336	-0.0997	0.0230	0.0322	4.272
24		3000	0.1375	0.1230	0.1314	35.598	-12.669	0.0349	-0.0800	0.0310	-0.0986	0.0231	0.0314	4.498
25	Rear damping coefficient	1500	0.1484	0.1299	0.1300	32.034	-10.685	0.0415	-0.1045	0.0416	-0.1046	0.0299	0.0300	3.807
26		2000	0.1440	0.1317	0.1255	33.148	-10.684	0.0370	-0.1011	0.0387	-0.0922	0.03170	0.0255	4.100
27		2500	0.1405	0.1319	0.1233	34.548	-11.761	0.0335	-0.0995	0.0367	-0.085	0.0317	0.0255	4.274
28		3000	0.0317	0.0319	0.1234	35.656	-12.700	0.0308	-0.0985	0.0350	-0.0802	0.0317	0.0255	4.495

## 5 CONCLUSIONS

In this work the effect of road excitations on a passive suspension for a passenger car was investigated. Mathematical modelling has been performed using a two and four degrees of freedom models. The differential equations of motion were simulated with Simulink/Matlab. The results from the simulation were used to analyse the performance of vehicle dynamics for step input road profile. The following conclusions are drawn. On the basis of the results, it can be concluded that road excitations affect the vehicle suspension which determines the ride comfort and stability of the vehicle.

## REFERENCES

- Agharkakli A., Ghobad Shafiei Sabet, Armin Barouz (2012). Simulation and Analysis of Passive and Active Suspension System using Quarter Car Model for Different Road Profile. *International Journal of Engineering Trends and Technology*, 3 (5).
- Agostinacchio, M., Ciampa D. and Olita, S. (2014). The Vibration Induced by Surface Irregularity in Road pavements– a MATLAB Approach. *Springer*, 6, 267-275.
- Changizi, N. and M. Rouhani, (2011). Comparing PID and Fuzzy Logic Control of a Quarter Car Suspension System. *The Journal of Mathematics and Computer Science*, 2(3), 559-564.
- Gao, H; Xue,S; Yin,S; Qiu, J; Wang C. Output feedback control of multirate sampled-data systems with frequency specifications. *IEEE Trans. control syst. technol.* 2016
- Ghazaly, N. M. and A. O. Moazz, (2014). The Future Development and Analysis of Vehicle Active Suspension System. *IOSR Journal of Mechanical and Civil Engineering*, 11(5), 19-25.
- Kashem,S.B.A; Ektesabi, M.; Nagarajah, R. Comparison between different sets of suspension parameters and introduction of a new modified skyhook control strategy incorporating varying road condition. *Vehicle Systems Dynamics*. 2012.
- Pedro, J. O. and J. E. D. Ekoru (2013). Proportional-integral derivative control of nonlinear half-car electro-hydraulic suspension systems. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, 14(6), 401-416.
- Turakhia T. P, Prof. M. J. Modi (2016). Mathematical Modeling and Simulation of a Simple Half – Car Vibration Model. *International Journal for Scientific Research & Development*, 4(2), 2321-0613.
- Williams R.A, (1994). Electronically controlled automotive suspensions. *Computing & Control Engineering Journal*, 5(3), 143-148.

## LIST OF ABBREVIATIONS

- $M_s$  = sprung mass of vehicle  
 $M_{uf}$  = unsprung mass of front axle  
 $M_{ur}$  = unsprung mass of rear axle  
 $K_{uf}$  = suspension stiffness of front axle  
 $K_{ur}$  = suspension stiffness of rear axle  
 $C_{sf}$  = damping coefficient of front axle  
 $C_{sr}$  = damping coefficient of rear axle  
 $K_{tf}$  = stiffness of front tire  
 $K_{tr}$  = stiffness of rear tire  
L = length of wheelbase  
b = distance to front axle from centre of gravity  
C = distance to rear axle from centre of gravity  
 $\theta$  = pitch angle  
 $Z_{rf}$  = road excitation at the front wheel  
 $Z_{rr}$  = road excitation at the rear wheel