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Hedging Strategy Using Copula and Nonparametric Methods: Evidence from China Securities Index Futures[1]

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ABSTRACT: Calculating accurately the optimal hedge ratio plays an important role in the futures market for both practitioners and academicians. In this paper, we combine copula and nonparametric technique, where marginal setting is modeled by nonparametric technique and bivariate is linked by dynamic Patton (2006)'s SJC copula function, to estimate the parameters of optimal hedge ratio. Various types of GARCH models to fit the marginal distribution are also compared. Furthermore, model specification for marginal setting is investigated by Hong and Li (2005)'s statistics, which test the i.i.d. and U(0,1) simultaneously. The empirical results show that transformed residuals generated by nonparametric technique are i.i.d. U(0,1), while most of one generated by popular GARCH-type are not. For hedging effectiveness, our methods perform better than traditional copula-GARCH models. The robust test also supports the results.

Keywords: Hedge Strategy; Optimal Hedge Ratio; Nonparametric Estimation; Patton (2006)'s SJC-Copula; Hong and Li (2005)'s Statistics; CSI 300 Index Futures.
JEL Classifications: C49; G10; G15

1. Introduction

The fundamental function of futures market is hedging. The most important issue is to determine the optimal hedge ratio in hedging with related futures products. The optimal hedge ratio consists of the optimized objective function and the ways of parametric estimation. Chen, Lee, and Shrestha (2003) do an excellent review for hedge ratio based on various objective functions. This paper only focuses on the estimation of the optimal hedge ratio rather than the design of optimized objective function. The conventional method to calculate the optimal hedge ratio is the coefficient of regression the spot on the futures, named ordinary least squares (OLS) approach (Ederington, 1979; Malliaris and Urrutia, 1991; Benet, 1992). However, the optimal hedge ratio is constant over time. Grammatikos and Aunder (1983) extend to time-varying hedge ratio by random coefficient technique. Bivariate GARCH-type models are also popular to be modeled the dynamic optimal hedge ratio (Cecchetti et al., 1988; Baillie and Myers, 1991; Sephton, 1993; Park and Switzer, 1995; Choudhry, 2003). The drawback of these models cannot capture the phenomenon of the symmetry and nonlinear dependence in the returns, which is common style in financial market (Cont, 2001; Longin and Solnik, 2001; Ang and Chen, 2002; Patton, 2006; Hong et al., 2007; Pan et al., 2014). The issue of dimension curse arises since many parameters are needed to estimate. Thanks to the Sklar's theorem, these problems have been solved to some extent. For example, copula-GARCH model is constructed to consider the style of returns, which marginal distribution is modeled by GARCH-class models with skewed-t distribution and nonlinear dependence is measured by various copula functions (Patton, 2006; Hsu et al., 2008; Lai et al., 2009; Wei et al., 2011). As mentioned by Patton (2006), before linking the bivariate using copula function, checking the i.i.d.U(0,1) of the transformed residuals is required. Most of papers take i.i.d.

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test and U(0,1) test apart. However, according to Hong and Li (2005), testing hypothesis of i.i.d.U(0,1) jointly is not equal to testing individually.

We propose a copula and nonparametric models for the optimal hedge ratio and find some evidence which is not presented in the previous literatures. First, using the nonparametric technique models the marginal distribution. Comparing to the GARCH-class methods, our methods are model-free, which avoids the risk of model misspecification. Second, test the null hypothesis of i.i.d.U(0,1) jointly by Hong and Li (2005)'s statistics. To our knowledge, most of test is apart, i.e., the null hypothesis of i.i.d is checked by Ljung-Box Q-test for first moment correlation and Engle test for higher moment correlation, and the standard Uniform distribution is tested by traditional Kolmogorov-Smirnov test. Therefore, our joint testing procedure is more accurate than other procedure. Third, the empirical results show that the transformed residuals generated by nonparametric technique pass the Hong and Li (2005)'s statistics but many popular GARCH-class models do not at 5%significant level. The hedging effectiveness based our approach is superior to one based popular copula-GARCH approach.

The remainder of this paper is as follows. Section 2 introduces the optimal hedge ratio based Minimum-Variance function and hedging effectiveness. Section 3 proposes a copula and nonparametric models to calculate the optimal hedge ratio, take traditional copula-GARCH approach as benchmark and sketch Hong and Li (2005)'s statistics. Application for CSI 300 index futures are presented in Section 4. Summary and conclusion are in section 5.

2. Optimal Hedge Ratio and Hedging Effectiveness

Based on various objective functions, the optimal hedge ratios show slightly different. Chen, Lee and Shrestha (2003) do a comprehensive review for these differences. This paper focuses on the most widely used Minimum-Variance hedge ratio, which minimize the portfolio variance. Minimum-Variance hedge ratio is first proposed by Johnson (1960) and adopted widely in the recent literature. Lai et al. (2009), Wei et al. (2011) and among others apply this hedge ratio to test their strategies. Following the spirit of Minimum-Variance objective function, we define the optimal hedge ratio as follows,

$$h_t^* = \frac{\mathrm{cov}(r_{s,t}, r_{f,t})}{\mathrm{var}(r_{f,t})} = \rho_t \frac{\sigma_{s,t}}{\sigma_{f,t}} \ , \qquad (2.1)$$

Where h_t^* is the optimal hedge ratio at time t, which means per unit value of a long spot position needs to short h_t^*units in the futures markets. $r_{s,t}$ and $r_{f,t}$ denote the logarithmic returns of the spot and futures, respectively. $\sigma_{s,t}$ and $\sigma_{f,t}$ are the standard deviations of $r_{s,t}$ and $r_{f,t}$ at time t, respectively. ρ_t denotes the time-varying correlation coefficient.

To measure the performance of hedge strategy, following Ederington (1979), Park and Switzer (1995), Wei, Wang and Huang (2011) and among others, the hedging effectiveness is given by:

$$E = \frac{\sigma_{r_s}^2 - \sigma_{r_h}^2}{\sigma_{r_s}^2} = 1 - \frac{\sigma_{r_h}^2}{\sigma_{r_s}^2} \qquad (2.2)$$

Where subscript r_h is the returns of hedged portfolio, i.e.,$r_h = r_s - hr_f$. $\sigma_{r_h}^2$ and $\sigma_{r_s}^2$ denote the variance of the hedged portfolio and only holding spot, respectively. From equation (2.2), as E reaching to 1, the hedging effectiveness is more sufficient.

3. Hedge Strategies

Under the framework describing the optimal hedge ratio in Section 2, the crucial task is how to accurately measure the dependence coefficient ρ_t, standard deviation $\sigma_{s,t}$ and $\sigma_{f,t}$, respectively. Comparing to widely-used GARCH-Copula approach, our parameters $\sigma_{s,t}$ and $\sigma_{f,t}$ is model-free based on nonparametric method and the dependence coefficient ρ_t is estimated by copula function.

3.1 Copula and nonparametric model

Assume both the logarithmic returns of the spot and futures are driven by below equations,

$$r_{k,t} = \mu_{k,t} + \sigma_{k,t}\epsilon_{k,t} \qquad (3.1)$$

$$\hat{\mu}_{k,t} = \frac{\sum_{i=1}^{n} K_b(r_{k,i} - r_{k,t}) r_{k,i}}{\sum_{i=1}^{n} K_b(r_{k,i} - r_{k,t})} \qquad (3.2)$$

$$\hat{\sigma}_{k,t}^2 = \frac{\sum_{i=1}^n K_b(r_{k,i} - r_{k,t}) r_{k,i}^2}{\sum_{i=1}^n K_b(r_{k,i} - r_{k,t})} - \hat{\mu}_{k,t}^2 \qquad (3.3)$$

where $k \in \{s, f\}$. Equation (3.2) is well-known Nadaraya-Watson estimator of μ_k , which is an consistent estimator and is applied in time series models by Chen and Gao (2007). Equation (3.3) is also kernel estimate of the volatility function appearing in Chen and Gao (2007) and Zheng (2008). Unlike parametric models, Equation (3.2) (3.3) propose a nonparametric estimate without assuming the function form. In this paper, Let the kernel $K_b(u)$ be Gaussian, i.e., $K_b(u) = \frac{1}{\sqrt{2\pi}}\exp(-u^2/(2b^2))$, where b is a bandwidth. Like Scott (1992), Hong and Li (2005) and among others, chooseb $= r_z n^{-1/6}$, where r_z is the sample standard deviation of$\{r_i\}_{i=1}^n$, and n is sample size.

After modeling the marginal distribution, we need to specify the dependence between the logarithmic returns of the spot and futures. The asymmetric dependence in financial market is demonstrated by Longin and Solnik (2001), Ang and Chen (2002), Patton (2006), Hong, Tu and Zhou (2007) and Pan, Zheng and Chen (2014). Therefore, we use the asymmetrized Joe-Clayton (SJC) copula, which nests symmetric one as a special case, modified by Patton (2006) to capture the asymmetric dependence of the spot and futures. The SJC copula is given below:

$$C_{SJC}(u_t, v_t | \tau_t^U, \tau_t^L) = 0.5\left(C_{JC}(u_t, v_t | \tau_t^U, \tau_t^L) + C_{JC}(1 - u_t, 1 - v_t | \tau_t^U, \tau_t^L) + u_t + v_t - 1\right) \qquad (3.4)$$

Where

$$C_{JC}(u_t, v_t | \tau_t^U, \tau_t^L) = 1 - (1 - ((1 - (1 - u_t)^\kappa)^{-\gamma}) + (1 - (1 - v_t)^\kappa)^{-\gamma} - 1)^{-\frac{1}{\gamma}})^{\frac{1}{\kappa}}$$

$$\kappa = \frac{1}{\log_2(2 - \tau_t^U)}$$

$$\lambda = -\frac{1}{\log_2(\tau_t^L)}$$

Where $\tau_t^U \in (0,1), \tau_t^L \in (0,1)$. The SJC copula has two parameters $\tau_t^U$and τ_t^L, which measure the dependence of upper and lower tail dependence, respectively. Since the situation of economic change over time, then time-varying dependence should be rational at model specification. Thus, let tail dependence term τ_t^U and τ_t^L be dynamic, which similar to Patton (2006),

$$\tau_t^U = \Xi\left(\alpha_0^U + \alpha_1^U \tau_{t-1}^U + \alpha_2^U \frac{1}{10}\sum_{i=1}^{10} |u_{t-1} - v_{t-i}|\right) \qquad (3.5)$$

$$\tau_t^L = \Xi\left(\alpha_0^L + \alpha_1^L \tau_{t-1}^L + \alpha_2^L \frac{1}{10}\sum_{i=1}^{10} |u_{t-1} - v_{t-i}|\right) \qquad (3.6)$$

Where $\Xi(x) = \frac{1}{1+\exp(-x)}$ is the logistic function. Obviously, the values of dependence parameters τ_t^U and τ_t^L fall in (0, 1) at all times. For the equations (3.5)(3.6), the upper and lower tail dependence are driven by an AR(1) term and a forcing variable which we use formula $\frac{1}{10}\sum_{i=1}^{10}|u_{t-1} - v_{t-i}|$ as.

To sum up, the copula and nonparametric models are constructed by nonparametric marginal distribution and dynamic copula function. The joint distribution of returns on the spot and futures is given by:

$$\begin{aligned} H(r_{s,t}, r_{f,t} | \Theta, \Omega_{t-1}) &= C_{SJC}\left(F_{r_s}(r_{s,t} | \Theta, \Omega_{t-1}), G_{r_f}(r_{f,t} | \Theta, \Omega_{t-1}) | \Theta, \Omega_{t-1}\right) \\ &= C_{SJC}\left(F_{\epsilon_s}(\epsilon_{s,t} | \Theta, \Omega_{t-1}), G_{\epsilon_f}(\epsilon_{f,t} | \Theta, \Omega_{t-1}) | \Theta, \Omega_{t-1}\right) \qquad (3.7) \end{aligned}$$

Where $F_{\epsilon_k}(\epsilon) = \frac{1}{n+1}\sum_{i=1}^n I(\epsilon_{k,t} \leq \epsilon)$, and $\epsilon_{k,t} = \frac{r_{k,t} - \mu_{k,t}}{\sigma_{k,t}}, k \in (s, f)$

Where $\Theta = (\tau_t^U, \tau_t^L)$, Ω_{t-1} is information set at time t-1. Since Markov assumption, $\Omega_{t-1} = \{r_{s,t-1}, r_{f,t-1}\}$. Further, we let $r_{s,t}$ only depend on itself lag term rather than other variable lag term. This assumption is also appearing in Patton (2006). C_{SJC} is SJC copula defined in Equation (3.4). Let $F_{r_s}, G_{r_s}, F_{\epsilon_s}$ and G_{ϵ_s} be the conditional cumulative distribution function for $r_{s,t}, r_{f,t}, \epsilon_{s,t}$ and $\epsilon_{f,t}$, respectively. I(A) denote an indicator function, i.e., I(A)=1 if even A holds, otherwise I(A)=0. $\mu_{k,t}$and$\sigma_{k,t}$ are estimated by Equation (3.2) and (3.3), respectively. Then parameter $\sigma_{k,t}$ corresponds to the standard deviation of optimal hedge ratio in Equation (2.1), and the dependent coefficient is defined by $\rho = \max(\tau_t^U, \tau_t^L)$.

3.2 Dynamic copula-GARCH strategy: benchmark

For the purpose of examine the performance of the nonparametric marginal distribution, we take the GARCH-class as a benchmark and let copula function be similar to Section 3.1. In numerous empirical works, the excess kurtosis and skewness styles are found in univariate distribution of many economic variables. And character of volatility cluster becomes common view for financial variables. Therefore, GARCH (p,q)-skewed t model could capture these characters mentioned above (see Patton (2004), Patton(2006)).

$$r_{k,t} = \mu_k + \sum_{j=1}^{p} \phi_{k,j} r_{k,t-j} + \eta_{k,t} \qquad (3.8)$$

$$\eta_{k,t} = \sqrt{h_{k,t}}\epsilon_{k,t} \qquad (3.9)$$

$$\epsilon_{k,t} \sim \text{skewed } t(\nu, \varrho),\ \ k \in (s, f) \qquad (3.10)$$

where term $\mu_k + \sum_{j=1}^{p} \phi_{k,j} r_{k,t-j}$ capture the mean value of r_k, where lag parameter p is generally determined by Akaike information criterion (AIC). From our empirical analysis, p=1 is superior to others. The residual innovations $\epsilon_{k,t}$ follow a skewed student t distribution to model the skewness and kurtosis. Term h_k is usually adopted to capture the phenomenon of volatility cluster. In this paper, we compare six most cited h_k processes as follows:

Model 1. The GARCH(1,1) model (see Bollerslev ,1986)

$$h_{k,t} = \omega_k + \alpha_k \eta_{k,t-1}^2 + \beta_k h_{k,t-1} \qquad (3.11)$$

Model 2. The Nonlinear ARCH(1,1) model (denoted NARCH) (see Engle and Bollerslev, 1986)

$$h_{k,t} = \omega_k + \alpha_k |\eta_{k,t-1}|^{\delta} + \beta_k h_{k,t-1} \qquad (3.12)$$

Model 3. The GJR(1,1) model (see Glosten, Jagannathan and Runkle, 1993)

$$h_{k,t} = \omega_k + \alpha_k \eta_{k,t-1}^2 + \gamma_k I(\eta_{k,t-1} < 0)\eta_{k,t-1}^2 + \beta_k h_{k,t-1} \qquad (3.13)$$

Model 4. The EGARCH(1,1) model (see Nelsen, 1991)

$$\log(h_{k,t}) = \omega_k + \alpha_k\left(\frac{|\eta_{k,t-1}|}{\sqrt{h_{k,t-1}}} - \sqrt{\frac{2}{\pi}}\right) + \gamma_k \frac{\eta_{k,t-1}}{\sqrt{h_{k,t-1}}} + \beta_k \log(h_{k,t-1}) \qquad (3.14)$$

Model 5. The Asymmetric GARCH(1,1) model (denoted AGARCH)(see Engle, 1990)

$$h_{k,t} = \omega_k + \alpha_k(\eta_{k,t-1} + \theta_k)^2 + \beta_k h_{k,t-1} \qquad (3.15)$$

Model 6. The VGARCH(1,1) model (see Engle and Ng, 1993)

$$h_{k,t} = \omega_k + \alpha_k\left(\frac{\eta_{k,t-1}}{\sqrt{h_{k,t-1}}} + \theta_k\right)^2 + \beta_k h_{k,t-1} \qquad (3.16)$$

Following Hansen (1994), the density of skewed student t distribution is given by:

$$\text{skewed } t(\epsilon|\nu, \varrho) = \begin{cases} bc\left(1 + \frac{1}{\nu - 2}\left(\frac{b\epsilon + a}{1 - \varrho}\right)^2\right)^{-\frac{\nu+1}{2}}, & \epsilon < -a/b \\ bc\left(1 + \frac{1}{\nu - 2}\left(\frac{b\epsilon + a}{1 + \varrho}\right)^2\right)^{-\frac{\nu+1}{2}}, & \epsilon \geq -a/b \end{cases}$$

where$2 < \nu < \infty$ and $-1 < \varrho < 1$. And the constant a, b and c are defined as follows:

$$a = 4\varrho c\left(\frac{\nu - 2}{\nu - 1}\right), b^2 = 1 + 3\varrho^2 - a^2, c = \frac{\Gamma((\nu + 1)/2)}{\sqrt{\pi(\nu - 2)}\Gamma(\nu/2)}$$

The skewed student's t distribution nests the student's t distribution by letting $\varrho = 0$. Parameter ϱ measure the degree of skew to zero, i.e., $\varrho > 0$ means the right skewness, and vice-versa when $\varrho < 0$. And parameter ν capture the kurtosis of density.

Further, we need to check whether model (3.11) is suitable for the logarithmic returns$r_{k,t}$. The intuitive judgement is that if model (3.11) is appropriate, then the probability integral transforms of residual innovation $\epsilon_{k,t}$ will be independent and identical distribution, i.e., Uniform (0,1). This results is powerful, is rigid proven by Rosenblatt (1952), and is applied to calculate density forecasts (Diebold, Gunther and Tay, 1998; Hong, 2002) and construct statistics for model specification (Hong and Li, 2005; Bai, 2003; Corradi and Swanson, 2006). To our knowledge, most of paper take the test of null hypothesis of i.i.d.U(0,1) apart. i.e., apply Ljung-Box Q-test for first moment and Engle test for higher moment to check independent and identical distribution, and the Kolmogorov-Smirnov (K-S) statistic for standard Uniform distribution, respectively. The existing problem of apart procedure is that it is easy to miss the situation of non-i.i.d.U(0,1). Particularly, the critical value of Kolmogorov-Smirnov statistic cannot be adopted directly since this statistic is not parameter estimation free but the asymptotic distribution of K-S statistic does not take this into account. Thus, Hong and Li (2005) propose a Nonparametric omnibus test to check the joint hypothesis of independent and identical distribution and Uniform (0,1). As we all known, there has not been paper using nonparametric omnibus test (Hong and Li, 2005) to check the GARCH class for building copula model so far. In this paper, we sketch the nonparametric omnibus test proposed by Hong and Li (2005). For univariate series $\{R_\tau\}_{\tau=1}^{n}$, let $g_j(r_1, r_2)$ be the joint density of $\{R_\tau, R_{\tau-j}\}$. The spirit of nonparametric omnibus test is to comparing the estimator $\hat{g}_j(r_1, r_2)$ of$g_j(r_1, r_2)$with 1, which is the product of two i.i.d. Uniform (0,1). The joint density $g_j(r_1, r_2)$ is estimated through kernel method given below:

$$\hat{g}_j(r_1, r_2) = \frac{1}{n-j}\sum_{\tau=j+1}^{n} K_b^H(r_1, R_\tau)\, K_b^H(r_2, R_{\tau-j})$$

Where where$K_b^H(r_1, r_2)$ is a boundary modified by Hong and Li (2005). Then Hong and Li (2005) construct a statistics $\hat{Q}(j)$ as follows:

$$\hat{Q}(j) = \frac{\left[(n-j)b\int_0^1\int_0^1\left[\hat{g}_j(r_1, r_2) - 1\right]^2 dr_1 dr_2 - bA_b^0\right]}{V_0^{1/2}}, \quad j = 1, 2, \cdots$$

where A_b^0 and V_0 are the non-stochastic centering and scaling parameters defined below:

$$A_b^0 = \left[(b^{-1} - 2)\int_{-1}^{1} k^2(u)du + 2\int_0^1\int_{-1}^{v} k_v^2(u)dudv\right]^2 - 1$$

$$V_0 = 2\left[\int_{-1}^{1}\left[\int_{-1}^{1} k(u+v)k(v)dv\right]^2 du\right]^2$$

Where $k_v(x) = k(x)/\int_{-1}^{v} k(y)dy$. Under $\{R_\tau\}_{\tau=1}^{n}$ is i.i.d. Uniform (0,1), Hong and Li (2005) show that,

$$\hat{Q}(j) \to N(0,1), \qquad \text{as } n \to \infty.$$

As mentioned in Hong and Li (2005), it is the most important and information when j=1. Thus, we choose j=1 to test the i.i.d. Uniform (0,1) jointly[2].

4. Application

Empirical analysis is investigated in this section. We calculate the optimal hedge ratio for CSI 300 index futures using copula and nonparametric methods we have just described. To check the robustness of these results, robust test is also considered.

4.1 Optimal hedge ratio for CSI 300 index futures

We focus on the daily data from wind database containing the prices of the China Securities Index 300 (CSI 300) spot and index futures, from 19 April, 2010 to 20 April, 2012, yielding 487 observations. In order to check the robustness of our results, we take the CSI 100 and CSI 200, which consists of the largest 100 stocks in CSI 300 and the rest of CSI 300, as substitution for CSI 300. The main reason for choosing CSI 100 and CSI 200 is that there exist many funds based on these two indices in China Stock Exchange market[3]. Figure 1 plots the prices and returns of the CSI 300 index futures, CSI 300 index, CSI 100 index and CSI 200 index, respectively. Figure 2 depicts the correlations between CSI 300 and CSI 300 index futures are higher when the market goes down than it goes up, which is consistent with Ang and Chen (2002)'s findings in the stock market. But our ranging from 0.82 to 0.96 is smaller than theirs.

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[2] Notes: As shown by Hong and Li (2005) in P46, if model specification is wrong, then statistic $\hat{Q}(j) \to \infty$. Unlike traditional two-sided test, $\hat{Q}(j)$ statistic is upper-tailed test since negative values of $\hat{Q}(j)$ statistic appear only under null hypothesis as $n \to \infty$.

[3] Now, there are only two funds based on CSI 300, launched on 7 May, 2012. For more detail see www.csindex.com.cn

Figure 1. The first column depicts the prices for CSI 300 index futures, CSI 300 index, CSI 100 index and CSI 200 in sequence; and the second column shows the returns.

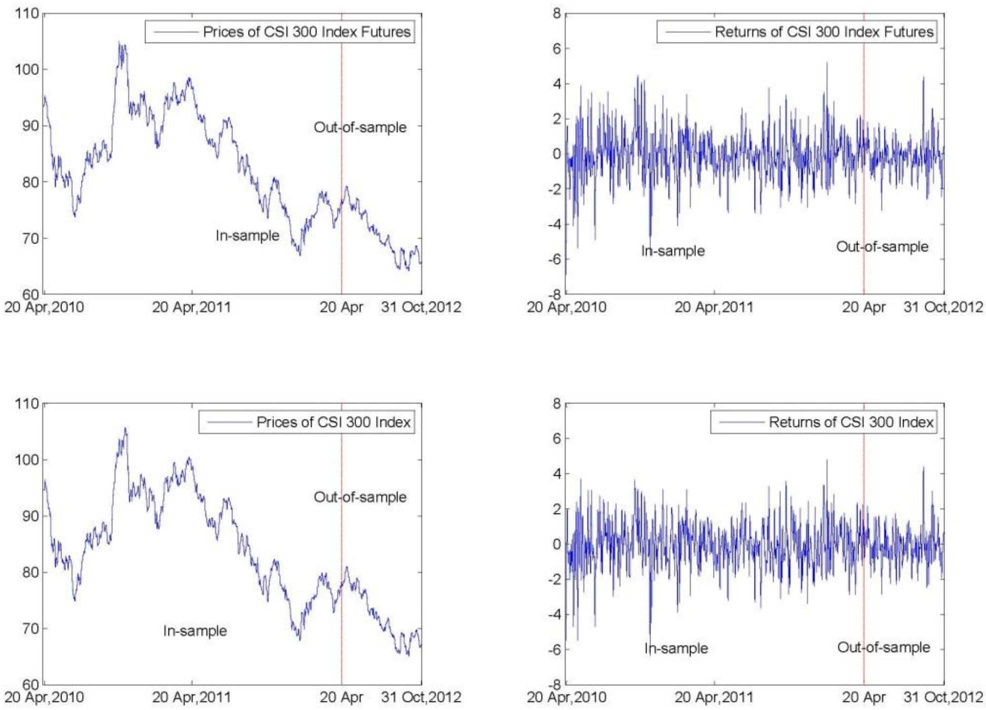

Figure 2. Correlations between returns on CSI 300 index and CSI 300 index futures. The horizontal axis is the quantile, and the vertical one is the value of correlation under that both exceed that quantile. Correlations for CSI 100 index and CSI 200 index with CSI 300 index futures, respectively, not be exhibited, but available upon request.

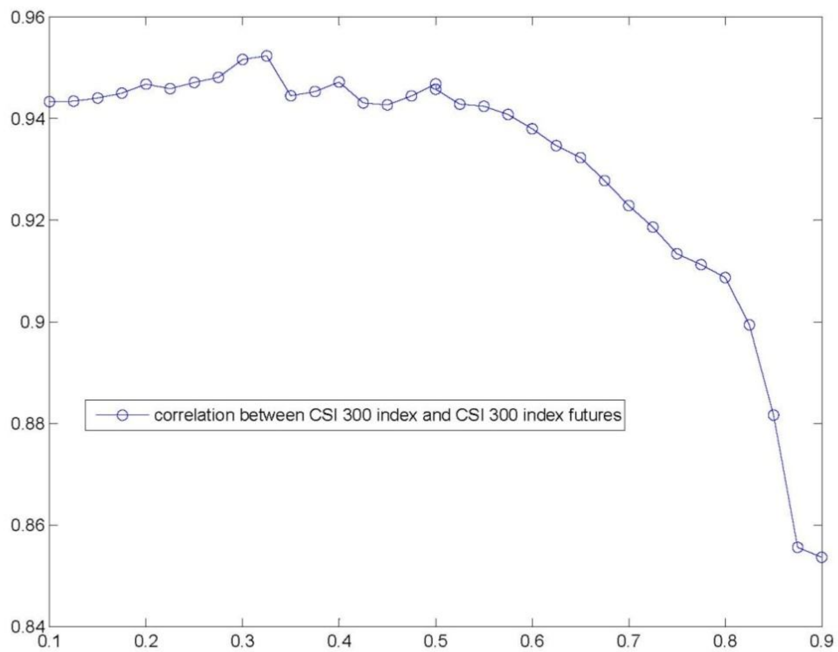

Table 1 provides Descriptive statistics for the data. Means of the four indices returns exhibit negative, and small relative to counterpart standard deviation. The higher moment shows the four time series are negative skewness and excess kurtosis. The value of Jarque-Bera test is larger enough to reject the normality hypothesis at significant level 1%. The Ljung-Box Q test indicates there is serial autocorrelation in all time series and autoregression term should be included in mean processes. The augmented Dickey-Fuller test for unit root shows that the null hypothesis of having a unit root is rejected at the significant level 1%, indicating that these time series are stationary and can be modeled directly.

Table 1. Description of CSI 300 Index and CSI 300 Index Futures

	Index futures	CSI300	CSI100	CSI200
Mean(%)	-0.054	-0.050	-0.038	-0.040
Std dev. (%)	1.522	1.483	1.444	1.656
Skewness	-0.164	0.306	-0.163	-0.309
Kurtosis	4.831	4.368	4.366	3.958
JB test	70.1910***	45.5580***	40.0097***	26.3621***
ADF test	-23.903**	-22.656***	-22.968***	-21.388***
Correl	--------	0.975	0.968	0.926

JB test denotes Jarque-Bera test of the null hypothesis that the sample comes from a normal distribution. ADF test assesses the null hypothesis of a unit root in a univariate time series. Correl measures the correlation for CSI 300 index, CSI 100 index and CSI 200 index with CSI 300 index futures. *** indicates the rejection at significant level 1%.

Table 2 and Table 3 present the estimation of types of GARCH models (AR(1)- GARCH (1,1), AR(1)-NARCH(1,1), AR(1)-GJR(1,1), AR(1)-EGARCH(1,1), AR(1)-AGARCH(1,1), AR(1)-VGARCH (1,1)) for CSI 300 index futures data and CSI 300, respectively. There are common features among various GARCH models. The log-likelihood value is almost the same, implying no GARCH model is clearly better than others. Relative to ARCH parameter α, most of the GARCH parameter β is larger up to 0.9, indicating that the volatility is high persistent dependence. Figure 3 and Figure 4 also depicts the high persistent behavior for various GARCH models, while nonparamtric volatility performs low persistent.

Table 2. Coefficient of CSI 300 index futures under GARCH-class models

	GARCH	NARCH	GJR	EGARCH	AGARCH	VGARCH
μ	0.0438	0.0429	0.0450	0.0552	0.0513	-0.0438
ϕ	-0.0589	-0.0591	-0.0590	-0.0572	-0.0612	-0.0619
ω	0.0311	0.0604	0.0297	0.0165	0.0100	0.0370
α	0.0283	0.0078	0.0257	0.0679	0.0258	0.0372
β	0.9590	0.9520	0.9600	0.9867	0.9667	0.9628
δ	---	3.0258	---	---	---	---
θ	---	---	0.0043	-0.0234	0.5747	-0.3890
ϱ	0.0391	0.0441	0.0386	0.0290	0.0346	0.0406
ν	4.5629	4.4791	4.5473	4.4963	4.5473	4.9530
loglik	862.4942	861.7935	862.4758	864.2407	862.1388	863.6359

Table 4 is model specification using Hong and Li (2005)'s statistic. The second column is statistics value of Hong and Li (2005)'s statistic for CSI 300 index futures and CSI 300 index. They are both negative, and pass the test at the significant level 5% according to the rule of Hong and Li (2005)s statistics. However, from column 3 to column 7, all of the statistics value is positive, which differ from nonparametric case, and All GARCH-class models are fall at significant level 10%. In particular, all GARCH-class models are also fall for CSI 300 data at significant level 1%. Therefore, modeling the marginal distribution using GARCH-class models faces the risk of model specification.

Table 3. Coefficient of CSI 300 index under GARCH-class models

	GARCH	NARCH	GJR	EGARCH	AGARCH	VGARCH
μ	0.0330	0.0370	0.0358	0.0153	0.0417	-0.0355
ϕ	0.0190	0.0186	0.0175	0.0202	0.0207	-0.0199
ω	0.0355	0.6409	0.0278	1.1521	8.8450e-4	5.5568e-4
α	0.0157	0.1627	0.0027	-0.1970	0.0147	0.0235
β	0.9671	0.6317	0.9742	-0.5082	0.9773	0.9765
δ	---	0.0501	---	---	---	---
θ	---	---	0.0181	0.0435	0.9999	0.0181
ϱ	-0.0058	4.58E-04	-0.0053	0.013	-0.0116	-0.0048
ν	6.8313	4.4791	6.6696	6.3101	6.7171	6.943
loglik	863.8759	865.9224	863.4197	863.7231	862.625	862.875

Table 4. Hong and Li's(2005) test for model specification

CSI300	Nonparam	GARCH	NARCH	GJR	EGARCH	AGARCH	VGARCH
Futures	-2.7983	1.3866*	1.5238*	1.3789*	1.3194*	1.3608*	1.9027&**
Spot	-0.6463	4.5092***	3.9127***	4.5398***	2.5009***	4.6315****	4.5616***

Notes: Hong and Li (2005)'s statistic is upper-tail test. ***, **, * denote the rejection at significant level 1%, 5% and 10%, respectively.

Figure 5 describes the time-varying tail dependence based on SJC-copula and dynamic optimal hedge ratios in Figure 6. Figure 5 shows the tail dependence for different marginal distribution. The time-varying tail dependence appears steady during the period. The critical reason may be our sample is not enough large and the integrated market trend to fall since 2008 in China security market. From the Nonparametric Tail Dependence figure, the lower tail dependence is larger than the upper one, which is consistent with Ang and Chen (2002)'s findings. However, AR(1)-NARCH(1,1) and AR(1)-VGARCH(1,1) marginal distribution exhibit upper tail dependence is larger than lower one, which confirms nonparametric marginal distribution is superior to GARCH-class ones. Figure 6 depicts the optimal hedge ratio. It is similar to volatility picture in Figure 3 and Figure 4. Obviously, owing to the tail dependence is steady. Thus, the optimal hedge ratio only depends on the counterpart volatility.

Figure 3. From top to bottom, the volatility of CSI 300 index is estimated by nonparametric model, AR(1)-GARCH(1,1), AR(1)-NARCH(1,1), AR(1)-GJR(1,1), AR(1)-EGARCH(1,1), AR(1)-AGARCH(1,1), AR(1)-VGARCH(1,1).

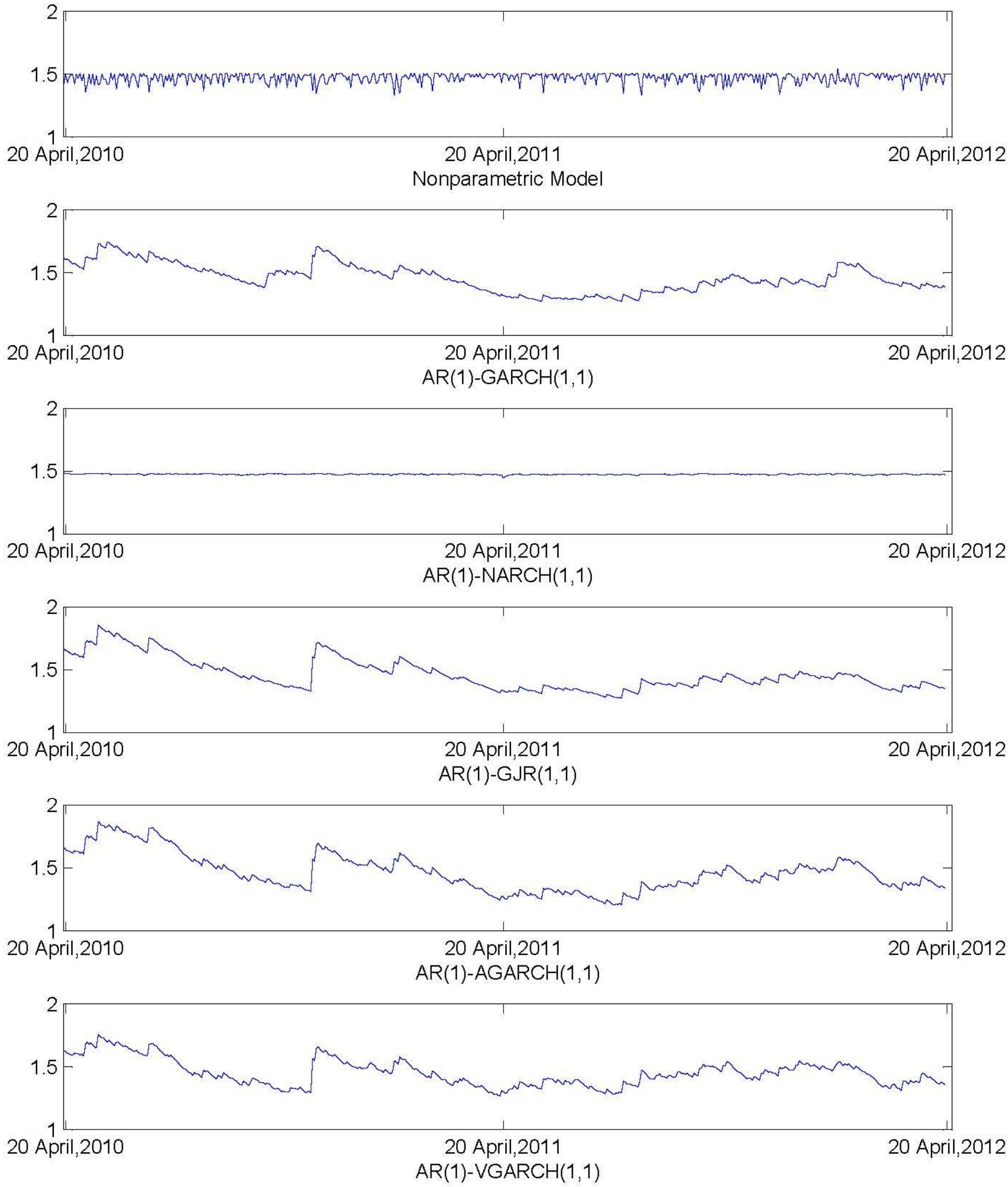

Figure 4. From top to bottom, the volatility of CSI 300 index Futures is estimated by nonparametric model, AR(1)-GARCH(1,1), AR(1)-NARCH(1,1), AR(1)-GJR(1,1), AR(1)-EGARCH(1,1), AR(1)-AGARCH(1,1), AR(1)-VGARCH(1,1).

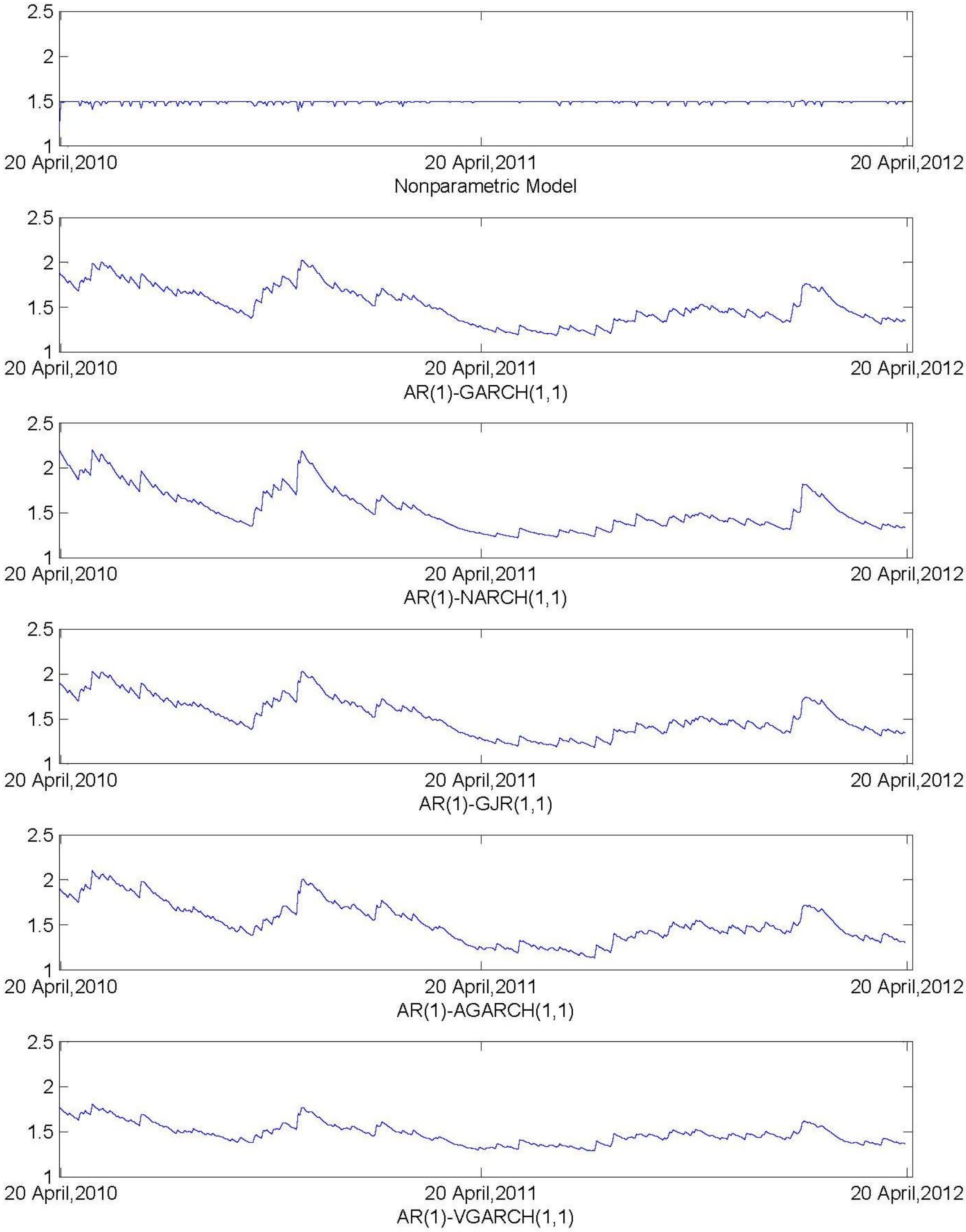

Figure 5. The dynamic tail dependence is estimated by SJC-copula, but marginal distribution is set by nonparametric model, AR(1)-GARCH(1,1), AR(1)-NARCH(1,1), AR(1)-GJR(1,1),AR(1)-EGARCH(1,1), AR(1)-AGARCH(1,1), AR(1)-VGARCH(1,1) from top to bottom.

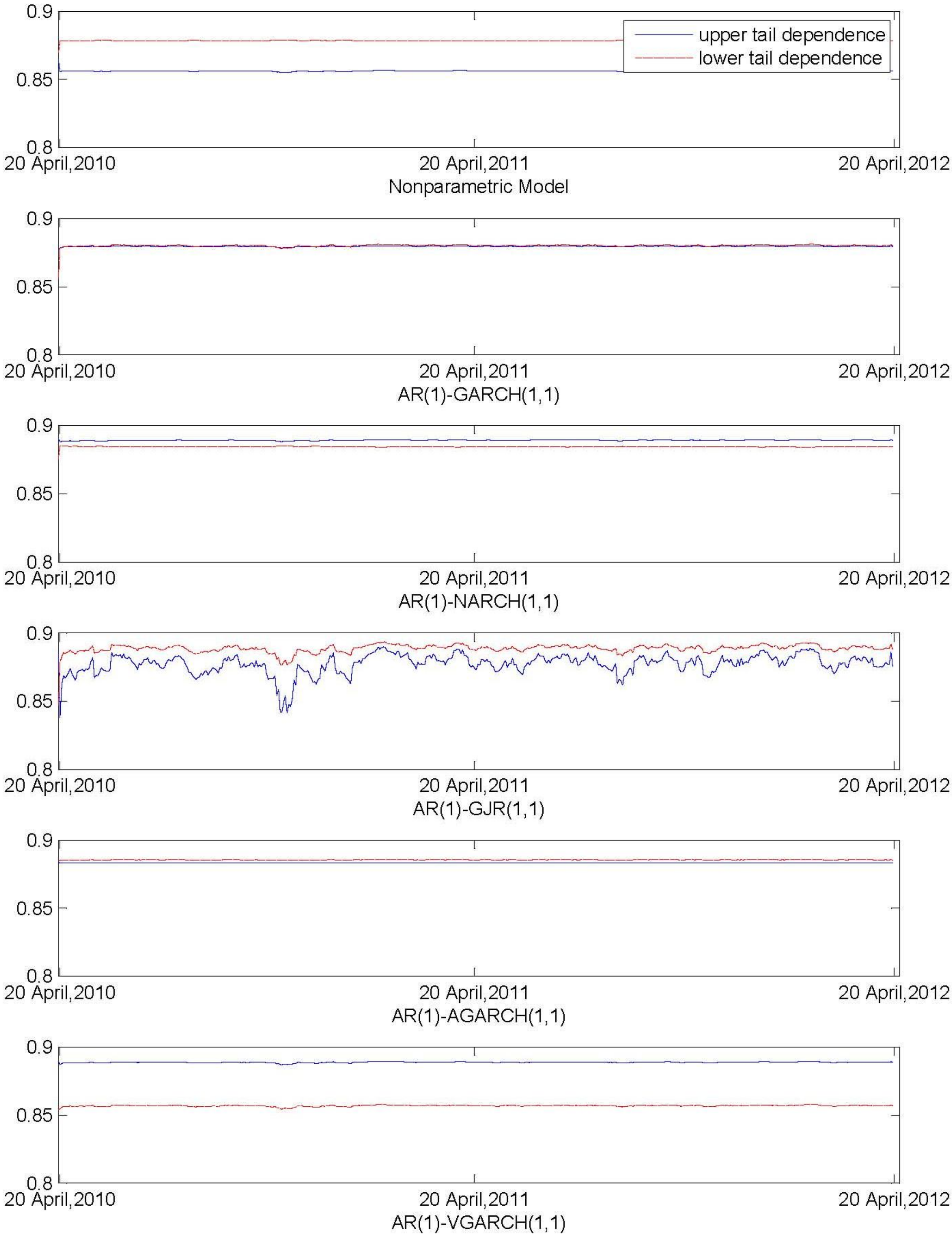

Table 5 presents the hedge ratio and hedging effectiveness. Mean of hedge ratio of nonparametric marginal distribution is larger than others as well as the standard deviation, ranging from 0.3104 to 1.5501. One may argue that the larger the standard deviation, the more transaction cost. Indeed, but transaction cost is less in futures market than one in stock market. Anyway, the transaction cost should be considered but left to a future research. The last column shows the hedging effectiveness. The value of nonparametric SJC-copula is larger than other marginal distributions, implying our approach dominates all GARCH-class models and our nonparametric marginal distribution to capture the behavior of the univariate is more sufficient. And GARCH-class models may exist the risk of model misspecification which just shown in Hong and Li (2005)'s statistics and tail dependence behavior in Figure 5.

Figure 6. The dynamic optimal hedge ratios is estimated by equation \ref{HE}, and marginal distribution is set by nonparametric model, AR(1)-GARCH(1,1), AR(1)-NARCH(1,1), AR(1)-GJR(1,1), AR(1)-EGARCH(1,1), AR(1)-AGARCH(1,1), AR(1)-VGARCH(1,1) from top to bottom.

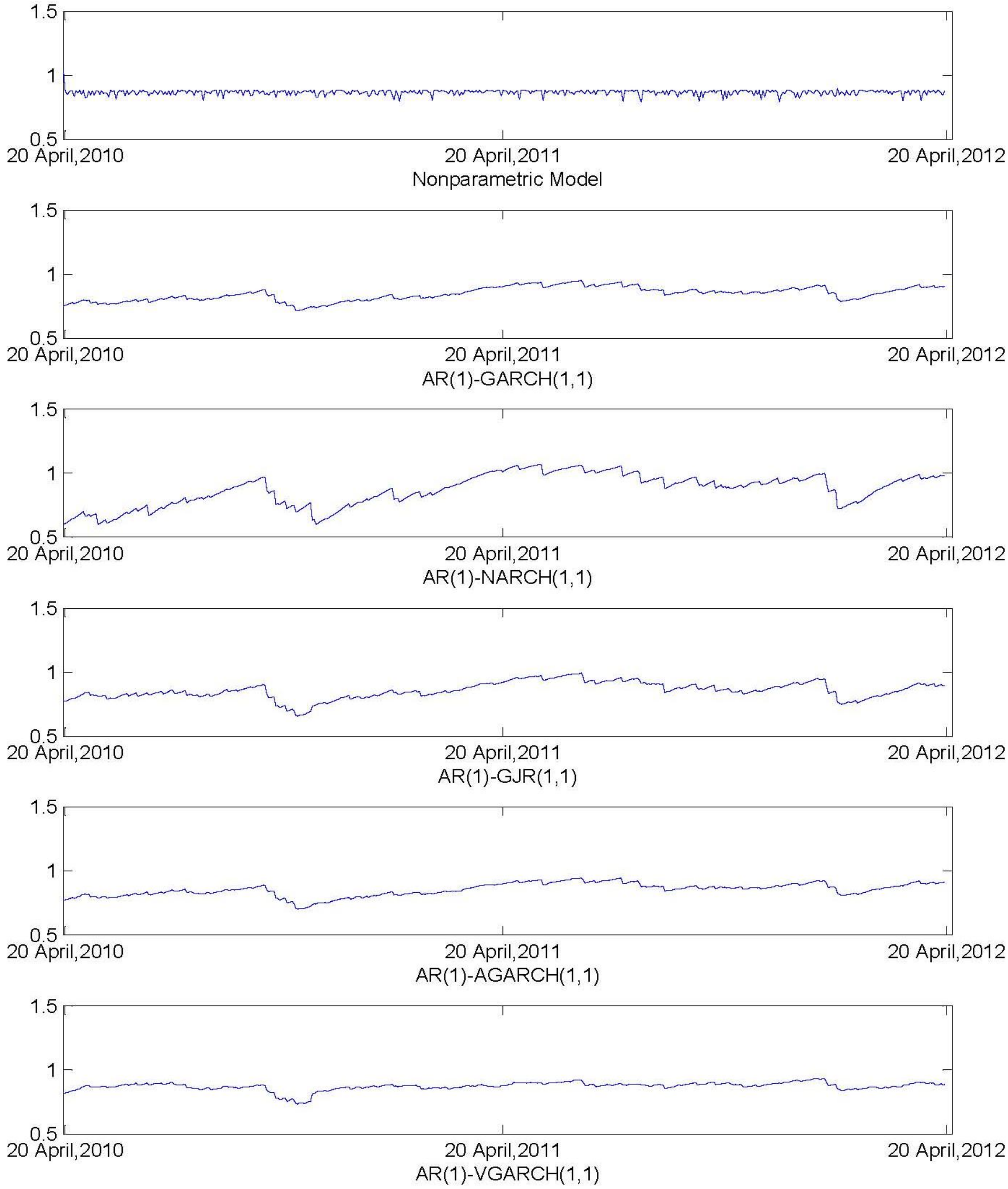

Table 5. Hedge ratio and hedging effectiveness

Marginal setting	Mean	Std	Min	Max	E
Nonparametric	0.8826	0.1436	0.3104	1.5501	0.9664
GARCH	0.8482	0.0532	0.7151	0.9548	0.9365
NARCH	0.8705	0.1224	0.5986	1.0674	0.9307
GJR	0.8584	0.0645	0.6568	0.9956	0.9392
EGARCH	0.8649	0.1328	0.451	1.2117	0.9296
AGARCH	0.8533	0.0506	0.7003	0.9453	0.9377
NAGARCH	0.8719	0.0317	0.7318	0.9328	0.9438

Notes: E is calculated by equation 2.2, and the closer the E get to 1, the more sufficient hedging effectiveness is

4.2 Robust test

To check whether copula and nonparametric models are robust, we substitute CSI 100 index and CSI 200 index for CSI 300 index, which many funds have been exchanged based on them in China security market.

We neither present the estimation of various GARCH-class models for CSI 100 index and CSI 200 index, nor is figures of volatility, tail dependence and hedge ratios. But all the related results are available on request. From Table 6, the nonparametric marginal distribution is not rejected at significant level 5%. However, GARCH-class models are rejected at significant level 5%, except AR(1)-EGARCH(1,1) marginal distribution for CSI 200 index, which also imply nonparametric technique is more suitable for modeling marginal distribution than GARCH-class models.

Table 6. Hong and Li's(2005) test for model specification

CSI	Nonparam	GARCH	NARCH	GJR	EGARCH	AGARCH	VGARCH
100	-0.5812	2.9976***	2.7958 ***	2.9903** *	2.7939 ***	3.0199 ***	3.0635 ***
200	0.6480	2.6512***	2.5644***	2.2403***	1.1265 ***	2.2626 ***	1.8246 ***

Notes: see Table 4.

Table 7 presents the hedging effectiveness for CSI 100 index and CSI 200 index, respectively. The main results are same as CSI 300, where the mean of hedge ratio for nonparametric marginal distribution is larger than others, as well as standard deviation. The hedging effectiveness of nonparametric marginal distribution is the closest to 1, indicating the degree of hedge is the best. Comparing to the CSI 300 index, the value of hedging effectiveness tend to be small. The solution should be found in Table 1. The value of correlation with CSI 300 index futures is CSI 300 index, CSI 100 index and CSI 200 index in sequence sorted by descending. It also reveals that traded underlying asset pay a critical role in hedging effectiveness. Fortunately, two funds based on CSI 300 index are launch on 7 May, 2012 that will improves efficiency in China stock market.

Table 7. Hedge ratio and hedging effectiveness

Marginal setting	Mean	Std	Min	Max	E
CSI 100 Index					
Nonparametric	0.8826	0.16	0.3177	1.7473	0.9493
GARCH	0.8249	0.0428	0.7265	0.9095	0.9283
NARCH	0.83	0.117	0.5706	1.0195	0.9143
GJR	0.8218	0.0432	0.7109	0.9165	0.9242
EGARCH	0.79	0.1376	0.5307	1.3427	0.9079
AGARCH	0.8256	0.0439	0.7113	0.9241	0.9245
VGARCH	0.8316	0.0233	0.7399	0.8693	0.9272
CSI 200 Index					
Nonparametric	1.0209	0.2951	0.2814	2.4118	0.9098
GARCH	0.87	0.0566	0.7061	0.9778	0.8395
NARCH	0.8695	0.0651	0.6842	0.9836	0.8381
GJR	0.8884	0.0894	0.6369	1.0802	0.8467
EGARCH	0.892	0.1539	0.5243	1.2961	0.8459
AGARCH	0.889	0.0917	0.7012	1.0878	0.8437
VGARCH	0.9167	0.0857	0.7407	1.1825	0.8546

Notes: see Table 5.

5. Conclusions

Nonparametric technique have rapid developed in many fields of statistics and econometrics. Especially, there exists many concerns nonparametric method in financial literature. In this paper, we propose a copula and nonparametric models to estimate the parameters of optimal hedge ratios. I.e., the marginal distribution is modeled by nonparametric technique and bivariate is linked by Patton (2006)'s SJC-copula. To highlight this method, we use our approach to do an empirical analysis for CSI 300 index futures. Meanwhile, comparing with most cited GARCH-class models, the findings show that our approach obtains the best hedging effectiveness. Hong and Li (2005)'s statistic shows that contrary to the popular parametric models such as GARCH model, nonparametric technique can be a robust tool for model specification of financial data.

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