

## Elastic, mechanical and thermodynamic properties of zinc blende III-X (X= As, Sb): ab-initio calculations

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**Abstract** – In this work, density functional theory plane-wave full potential method, with local density approximation (LDA) are used to investigate the structural, mechanical and thermodynamic properties of zincblende III-X (X= As, Sb) compounds. Comparison of the calculated equilibrium lattice constants and experimental data shows very good agreement. The elastic constants were determined from a linear fit of the calculated stress-strain function according to Hooke's law. From the elastic constants, the bulk modulus B, shear modulus G, Young's modulus E, Poisson's ratio  $\sigma$ , anisotropy factor A, the ratio B/G and the hardness parameter H for zincblende III-X (X= As, Sb) compound are obtained. Our calculated elastic constants indicate that the ground state structure of III-X (X= As, Sb) is mechanically stable. The sound velocities and Debye temperature are also predicted from elastic constants.

**Keywords:** Mechanics, Mechanics, energy, DFT, LDA

Received: 03/03/2019 – Accepted: 15/05/2019

### I. Introduction

The method developed by Charpin (modified by Ferenc Karsai) and integrated in WIEN2k code [1] has been used to obtain elastic constants of considered binary compounds. The knowledge of elastic parameters of solids is very important because they provide important information about the stability and mechanical properties of solids such as sound velocities, load deflection, fracture toughness, thermoelastic stress and internal strain etc.

### II. Elastic and mechanical properties

The elastic constants  $C_{klmn}$  (where the letter k, l, m, n refer to Cartesian components) are defined by the help of a Taylor expansion of the total energy of the system,  $E(V, \epsilon)$ , in accordance with a small strain  $\epsilon$  of the lattice (V is the volume of the system). The energy  $E(V, \delta)$  fit curve versus strain,  $\delta$ , for the three different types of strains, namely the volume conserved, tetragonal and rhombohedral shear strains, are plotted in Figure 1 (a)-(c), respectively, for the studied binary compounds. The

total energy has been calculated for five to seven different distortions for each of the three different deformations of the lattice. There are 21 independent elastic constants  $C_{ij}$ , but symmetry of the cubic lattice reduces this number to only 3 independent constants ( $C_{11}, C_{12}$ , and  $C_{44}$ ) for cubic lattices. The calculated values of elastic constants are summarized in Table 1. The calculated elastic constant values of studied binary compounds are in good agreement with the results of other calculations [2, 4] and the available experimental data [3, 5]. The obtained values for the elastic tensor constants satisfy the mechanical stability restrictions for cubic unit cells  $C_{11} - C_{12} > 0$ ,  $C_{11} + 2C_{12} > 0$  and  $C_{12} < B < C_{11}$  [6]. Resistance to shear distortions of a cubic crystal is best characterized by the two moduli the tetragonal shear constant  $C' = (C_{11} - C_{12})/2$  and  $C_{44}$ . The elastic constant  $C_{44}$  is related to an orthorhombic deformation whereas  $C'$  is related to a tetragonal deformation. At any volume V, the bulk modulus B for a cubic crystal is related to elastic constants by  $B_0 = (C_{11} + 2C_{12})/3$  [7]. The  $C_{11}$  and  $C_{12}$  can be obtained from the calculated bulk modulus and  $C'$ .

The Kleinmann parameter [8],  $\zeta$ , is an important parameter describing the piezoelectric effect of solids [9]. It is given by the following relation:

$$\zeta = \frac{C_{11} + 8C_{12}}{7C_{11} + 2C_{12}}$$

The obtained values of  $\zeta$  for the materials are between 0.575 and 0.716 as shown Table 1. The obtained results for studied binary compounds fairly coincide with previous first-principles calculations [2, 10]. We have also found the anisotropy factor  $A = 2C_{44}/(C_{11} - C_{12})$ . For an isotropic crystal, A is equal to 1, while any value smaller or larger than 1 indicates anisotropy. From Table 1, it is clearly seen that the calculated anisotropy factor for these compounds deviate from 1. The magnitude of the deviation from 1 is a measure of the degree of elastic anisotropy possessed by the crystal.

The isotropic bulk modulus  $B_0$ , which is related to  $C_{11}$  and  $C_{12}$ , and shear modulus (G) are determined by the calculated elastic constants [11]. However, there is no distinct expression for the polycrystal-averaged shear modulus with respect to the  $C_{ij}$ , but one can evaluate approximate averages of the lower and upper bounds given by various theories [12]. Voight [13] found upper bounds, while Reuss [14] discovered lower bounds for all lattice.

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The upper bound due to Voight is calculated as

$$G_V = (C_{11} - C_{12} + 3C_{44})/5$$

and the lower bound due to Reuss reads as

$$5/G_R = 4/(C_{11} - C_{12}) + 3/C_{44}$$

According to Hill [15], the arithmetic average of the Voight and Reuss values can be used as an estimated of the average shear modulus  $G=1/2 (G_V+G_R)$ . Another important mechanical parameter that is directly correlated to the ductility is the Poisson's ratio and given by

$$\nu = \frac{3B - 2G}{2(3B + G)}$$

where Y is the Young's modulus and is related to the bulk and shear moduli

$$Y = \frac{9BG}{3B + G}$$

Y and  $\nu$  are frequently measured for polycrystal materials when investigating their hardness. Young's modulus is a measure of the stiffness of a given material, whereas Poisson's ratio is the ratio (when a sample is stretched) of the contraction or transverse strain to the extension or axial strain. The calculated average shear modulus (G), Young's modulus (Y) and Poisson's ratio ( $\nu$ ) are given in Table 1. The obtained results for studied binary compounds fairly coincide with previous first-principles calculations [2, 10] and experimental results [16, 17]. Materials with high G are likely to be hard materials. In studied compounds, GaAs exhibits the largest value of G (54.551 GPa) being the most incompressible of all. The Young modulus (Y) determines the stiffness of the material, i.e., the larger value of Y, the stiffer is the material [18] and the stiffer solids have covalent bonds [19]. It can be seen from Table 1 that the largest value of Y (130.388 GPa) being the most stiffer of all occurs for GaAs implying it to be more covalent in nature as compared to other studied compounds.

The proportion between bulk modulus and average shear modulus ( $B_0/G$ ) has been proposed by Pugh [20] to roughly determine the ductile or brittle character of a material. The critical value which separates ductile and brittle material is 1.75; i.e., if  $B_0/G$  is smaller than 1.75, then the material behaves in a brittle manner; otherwise it

will be of ductile nature [21]. The  $B_0/G$  ratio of studied materials is presented in Table 1.

The Cauchy pressure is another interesting elastic parameter which describes the angular characteristic of atomic bonding in a material can be calculated by using the following relation [23]:

$$C' = C_{12} - C_{44}$$

Table 1 Calculated elastic constants ( $C_{11}$ ,  $C_{12}$  and  $C_{44}$ ) and tetragonal shear constant ( $C'$ ), Kleinman's internal-strain parameter ( $\xi$ ), shear modulus anisotropy ( $A$ ), the average shear modulus ( $G$ ), Young's modulus ( $Y$ ) and Poisson's ratio ( $\nu$ ), and  $B_0/G$  ratio of studied materials and the comparison of these quantities with available theoretical and experimental data.

Comp.	C11 (GPa)	C12 (GPa)	C44 (GPa)	C'	B0 (GPa)	$\xi$	A	G (GPa)	Y (GPa)	$\nu$	B0/G
AlAs	110,442	56,982	78,132	26,730	74,802	0,638	2,923	50,867	124,402	0,22	1,471
	113.1 [2]	55.5 [2]	54.7 [2]	28.8 [2]		0.592 [2]	1.899 [2]	77.14 [10]	175.39 [10]	30.329 [2]	1.043 [10]
	119.9 [3]	57.5 [3]	56.6 [3]			0.481 [10]				0.136 [10]	
GaAs	113,589	50,115	78,401	31,737	71,273		2,470	54,551	130,388	0,195	1,307
	115,1 [4]	51.5 [4]	56.8 [4]	36.4 [2]	72.7[4]	0,575	1.742 [2]	32.6 [16]	85.5 [16]	0.293 [2]	
	118.1 [5]	53.2 [5]	62.0 [5]			0.506 [2]				0.31 [17]	
InAs	81,547	49,809	60,838	15,869	60,388	0,716	3,834	35,683	89,433	0,253	1,692
	92.2 [2]	46.5 [2]	44.4 [2]	22.9 [2]		0.598 [2]	1.943 [2]			0.335 [2]	
	83.3 [5]	45.3 [5]	39.6 [5]								
AlSb	84,615	42,727	60,737	20,944	56,690	0,629	2,900	39,665	96,490	0,216	1,429
	85.5 [2]	41.4 [2]	39.9 [2]	22.1 [2]		0.601 [2]	1.81 [2]			0.326 [2]	
	89.4 [5]	44.3 [5]	41.6 [5]								
GaSb	84,169	38,939	59,841	22,615	54,016	0,593	2,646	40,517	97,238	0,200	1,333
	92.7 [2]	38.7 [2]	46.2 [2]	27 [2]		0.530 [2]	1.711 [2]			0.295 [2]	
	88.4 [5]	40.3 [5]	43.2 [5]								
InSb	65,062	38,348	50,121	13,357	47,253	0,699	3,752	29,636	73,535	0,241	1,594
	72.0 [2]	35.4 [2]	34.1 [2]	18.3 [2]		0.603 [2]	1.863 [2]			0.487 [2]	
	66.7 [5]	36.5 [5]	30.2 [5]								

It is clearly seen from this table that  $B_0/G$  ratio of considered structures should be classified as brittle character.

To obtain the stiffness of these compounds, the microhardness parameter ( $H$ ) is also calculated using the following equation [22]:

$$H = \frac{(1 - 2\nu)Y}{6(1 + \nu)}$$

The calculated  $H$  values are 9.40GPa, 11.09GPa, 5.87GPa, 7.50GPa, 8.10GPa and 5.12 for AlAs, GaAs, InAs, AlSb, GaSb and InSb at zero pressure, respectively.

The positive value of Cauchy pressure is responsible for a ionic bonding while a negative Cauchy pressure, however, requires an angular or directional character in the bonding (covalent bonding). The more negative the Cauchy pressure, the more directional and of lower mobility the bonding. Moreover, a material with more negative value of Cauchy pressure will have more brittle nature. The calculated values of  $C'$  are summarized in Table 2, which indicate that the sign of the Cauchy pressure is negative for all studied materials. The kind of bonds can be also determined by means of the value of Poisson's ratio ( $\nu$ ). The value of Poisson's ratio is nearly 0.25 or more for a typical ionic material, while it is much less than 0.25 (around 0.1) for a typical covalent

compound [24]. As indicated Table 1, our calculation shows that  $\nu < 0.25$  for all studied materials. Consequently, our Cauchy pressure calculations are consistent with our Poisson's ratio values.

Table 2. Calculated microhardness parameter (H), Cauchy pressure ( $C''$ ), and 1<sup>st</sup> and 2<sup>nd</sup> Lamé constants ( $\lambda$ ,  $\mu$ ), wave velocities ( $v_t$ ,  $v_l$  and  $v_m$ ), Debye temperature ( $\theta_D$ ), melting point ( $T_m$ ) and the minimum thermal conductivity ( $\kappa_{min}$ ) of studied materials.

Comp.	H (GPa)	$C''$ (GPa)	$\lambda$ (GPa)	$\mu$ (GPa)	$v_t$ (m/s)	$v_l$ (m/s)	$v_m$ (m/s)	$\theta_D$ (K)	$T_m$ (K)±300	$\kappa_{min}$ (WmK <sup>-1</sup> )
AlAs	9,40	-21,150	40,891	50,867	3680,935	6163,647	4073,853	428,576	1205,712	1,769
GaAs	11,09	-28,286	34,905	54,551	3193,605	5188,860	3523,838	371,173	1224,314	1,534
InAs	5,87	-11,029	36,599	35,683	2501,806	4351,768	2778,513	273,101	1034,942	1,053
AlSb	7,50	-18,009	30,247	39,665	3036,484	5046,935	3358,187	325,907	1053,074	1,241
GaSb	8,10	-20,901	27,005	40,517	2678,678	4374,136	2957,205	288,857	1050,442	1,107
InSb	5,12	-11,773	27,496	29,636	2258,360	3864,231	2504,483	230,170	937,519	0,830

The other interesting elastic parameters are Lamé constants ( $\lambda$ ,  $\mu$ ) which depend on a material and its temperature. These parameters are related to the Young modulus and Poisson's ratio by using the following equations:

$$\lambda = \frac{Y\nu}{(1+\nu)(1-2\nu)} \text{ and } \mu = \frac{Y}{2(1+\nu)}$$

where Y is the Young's modulus and  $\nu$  Poisson's ratio. Our calculated values of  $\lambda$  and  $\mu$  are summarized in Table 2. The two parameters together constitute a parametrization of the elastic moduli for homogeneous isotropic media,  $\lambda$  is known as Lamé's first constant and  $\mu$  is Lamé's second constant. The Lamé's first modulus,  $\lambda$ , is related to a fraction of Young's modulus. For an isotropic system one can show that  $\lambda=C_{12}$  and  $\mu=C'$  [25]. As shown from Table 1 (from anisotropy factor, A) the studied materials are strongly anisotropic character; therefore our obtained results do not satisfy the later relations, which are valid only for the isotropic systems, which is in agreement with the obtained results.

### III. Thermodynamic properties

The Debye temperature ( $\theta_D$ ) which is a significant fundamental parameter closely related to many physical properties such as elastic constants, specific heat and melting point can be obtained from the average sound velocity ( $v_m$ ) by the following classical relation [26]:

$$\theta_D = \frac{\hbar}{k_B} \left[ \frac{6\pi^2 n}{V_0} \right]^{1/3} v_m$$

where  $V_0$  is atomic volume and  $v_m$  the average wave velocity in the polycrystalline material is approximately calculated from the following equation [26]:

$$v_m = \left[ \frac{1}{3} \left( \frac{2}{v_t^3} + \frac{1}{v_l^3} \right) \right]^{-1/3}$$

constant. The scatter of all the different points falls within plus or minus 300 K of the following equation for  $T_m$  in units of K [28]:

$$T_m = 553 \text{ K} + (591/\text{Mbar}) C_{11} \pm 300 \text{ K}$$

The calculated wave velocities ( $v_t$ ,  $v_l$ ,  $v_m$ ), Debye temperature ( $\theta_D$ ) and melting point ( $T_m$ ) of studied binary compounds and ternary alloys estimated from elastic constants are listed in Table 2. It is clearly seen from Table 2, the Debye temperature of the considered compounds decrease with increasing atomic number. The high value of the Debye temperature for AlAs implies that its thermal conductivity is to be higher than other studied compounds. The sound velocities are related to the elastic moduli. Therefore, for a material having larger elastic moduli means higher sound velocity. Thermal conductivity,  $\kappa$ , is the property of a material that indicates its ability to conduct heat. So, in order to know if material is a potential candidate for thermal barrier coating application, its thermal conductivity need to be investigated.

Based on the Debye model, Clarke [29] suggested that the theoretical minimum thermal conductivity can be calculated after replacing different atoms by an equivalent atom with a mean atomic mass  $M/n$ :

$$\kappa_{min} = 0.87 k_B N_A^{2/3} \frac{n^{2/3} \rho^{1/6} Y^{1/2}}{M^{2/3}}$$

where  $k_B$  is the Boltzmann's constant, M is the molecular mass and n is the number of atoms per molecule,  $N_A$  the Avogadro's number,  $\rho$  the density. The calculated minimum thermal conductivity of studied materials is summarized in Table 2. Table 2 Indicates that the value of minimum thermal conductivity decreases when one moves from Al to In in the compound XAs (Sb) (X=Al, Ga and In). The reduction can be attributed mainly to the difference in Young's modulus, which is a measure of the second derivative of the bonding energy at the

equilibrium interatomic distance  $x_0$ , between the studied binary compounds.

Where  $v_t$  and  $v_l$  are the transverse and longitudinal elastic wave velocities, respectively, obtained using the shear modulus  $G$  and the bulk modulus  $B$  from Navier's equations [27]:

$$v_t = \sqrt{\frac{G}{\rho}}$$

$$v_l = \sqrt{\frac{3B + 4G}{3\rho}}$$

where  $\rho$  is the density.

Fine et al. [28] have studied many cubic metals and compounds and have obtained an approximate empirical.

#### IV. Conclusion

In this study, the structural, mechanical and thermodynamic properties of III-X ( X= As, Sb) compounds have been investigated by means of the DFT within Wien2k code. Our results for the optimized lattice parameters (a) and (c) are in good. Agreement with the available experimental data. The elastic constants  $C_{ij}$ , and related polycrystalline mechanical parameters such as bulk modulus  $B$ , shear modulus  $G$ , Young's modulus  $E$  and Poisson coefficient  $\sigma$  are calculated using Voigt–Reuss–Hill approximations.

The III-X (X= As, Sb) compound is mechanically stable according to the elastic stability criteria, while no experimental results of elastic moduli for comparison. The calculated Zener factor indicates that III-X (X= As, Sb) compound is elastically anisotropic. The values of the ratio  $B/G$  and Cauchy pressure ( $C_{12}-C_{44}$ ) show a ductile manner for the III-X ( X= As, Sb) compound. The polycrystalline III-X ( X= As, Sb) turns out to be a low stiff material according to the calculated hardness parameter ( $H$ ). Finally, from the knowledge of the elastic constants and the average sound velocities, the Debye temperature has been predicted successfully.

#### References

- [1] P. Blaha, K. Schwarz, G. Madsen, D. Kvasnicka, J. Luitz, "An Augmented Plane Wave plus Local orbital program for Calculating the Crystal Properties", revised edition WIEN2k\_13.1, pp.132-135.
- [2] S. Q. Wang and H. Q. Ye, "Phys. Stat". Sol B 1, 2003, pp 45.
- [3] Wei Li and Markus Pessa, "Lattice parameter in GaNAs epilayers on GaAs: Deviation from Vegard's law", Appl. Phys. Lett. 78, 2864, 2001.
- [4] G. Stenuit and S. Fahy, "First-principles calculations of the mechanical and structural properties of  $GaN_xAs_{1-x}$ : Lattice and elastic constants", Phys. Rev. B 76, 035201, 2007.
- [5] R. Weil and W. O.Groves, "Elastic Constants of Gallium Phosphide", J. Appl. Phys. Vol 39, 1968, pp 4049.
- [6] D.C. Wallace, "Thermodynamics of Crystals," John Wiley, New York, 1972.
- [7] D. Connetable, O. Thomas, "First-principles study of the structural, electronic, vibrational, and elastic properties of orthorhombic NiSi", Phys. Rev. B 79, 2009, pp 094101.
- [8] L. Kleinmann, "Deformation Potentials in Silicon. I. Uniaxial Strain", Phys. Rev. Vol 128, 1962, pp 2614
- [9] W. Cady, "Piezoelectricity (McGraw-Hill)", New York, 1946.
- [10] M. Sajjad, S.M. Alay-e-Abbas, H.X. Zhang, N.A. Noor, Y. Saeed, Imran Shakir, A. Shaukat, "First principles study of structural, elastic, electronic and magnetic properties of Mn-doped AlY (Y=N, P, As) compounds", J Mag Mater, Vol 390, 2015, pp 78-86.
- [11] W.A. Harrison, "Elastic Structure and Properties of Solids, Dover Publications", Inc., New York, 1980.
- [12] V. I. Razumovskiy, A. V. Ruban, P. A. Korzhavyi, "First-principles study of elastic properties of Cr- and Fe-rich Fe-Cr alloys", Phys. Rev. B 84, 2011, pp 024106-8.
- [13] W. Voight, Ann. Phys. (Leipzig) 38 (1889) 573.
- [14] A. Reuss, Z. Math. Mech. Vol 9, 1929, pp 49.
- [15] R. Hill, "The Elastic Behaviour of a Crystalline Aggregate", Proc. Phys. Soc., London, Sect. A 65, 1952, pp 349.
- [16] J. S. Blakemore, "Semiconducting and other major properties of gallium arsenide", J App. Phys. Vol 53, 1982, R123.
- [17] S. M. Sze, "Semiconductor sensors". New York: John Wiley & Sons, 1994.
- [18] V. Mankad, N. Rathad, S. D. Gupta, S. K. Gupta, P. K. Jha, Mater. Chem. Phys. Vol 129, 2011, pp 816.
- [19] C. H. Jenkins, S. K. Khanna, "Mechanics of Materials", ISBN 0-12-383852-5, 2005, pp. 62-72.
- [20] S.F. Pugh, "Relations between the elastic moduli and the plastic properties of polycrystalline pure metals", Philos. Mag. Vol 45, 1954, pp 823-843.
- [21] Z. Dang, M. Pang, Y. Mo, Y. Zhan, "Theoretical prediction of structural, elastic and electronic properties of Si-doped TiCu Geintermetallics", Current Appl. Physics, Vol 13, 2013, pp 549-555.
- [22] E.S. Yousef, A. El-Adawy, N. El-Khesh Khany, "Effect of rare earth (Pr2O3, Nd2O3, Sm2O3, Eu2O3, Gd2O3 and Er2O3 ) on the acoustic properties of glass belonging to bismuth–borate system", Solid State Commun. Vol 139, 2006, pp 108.
- [23] D. G. Pettifor, "Theoretical predictions of structure and related properties of intermetallics", Mater. Sci. Technol. 8, 1992, pp 345.
- [24] M. Jamal, S. J. Asadabadi, I. Ahmad, H. A: R. Aliabad, "Elastic constants of cubic crystals", Comput. Mater. Sci. Vol 95 , 2014, pp 592-599.
- [25] [http://en.wikiversity.org/wiki/Introduction\\_to\\_Elasticity/](http://en.wikiversity.org/wiki/Introduction_to_Elasticity/)
- [26] O. L. Anderson, "A simplified method for calculating the debye temperature from elastic constants", J. Phys. Chem. Solids 24, 1963, pp 909-917.
- [27] E. Schreiber, O.L. Anderson, N. Soga, "Elastic Constants and Their Measurements", 3th ed., McGraw-Hill, New York, 1973.
- [28] M.E. Fine, L.D. Brown, H.L. Marcus, "Elastic Constants Versus Melting Temperature in Metals", Scripta Metall., 18, 1984, pp 951-956.
- [29] D. R. Clarke, "Materials selection guidelines for low thermal conductivity thermal barrier coatings", Surf. Coat. Tech. 163, 2003, pp 67-74.