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Stochastic Fuzzy Algorithms for Impairment of Assets Management

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Abstract

The present paper aims to analyze the impairment of tangible assets with the help of artificial intelligence. Stochastic fuzzy numbers have been introduced with a dual purpose: on one hand to estimate the cash flows generated by tangible assets exploitation and, on the other hand, to ensure the value ranges stratifications that define these cash flows. Estimation of cash flows using stochastic fuzzy numbers was based on cash flows generated by tangible assets in previous periods of operation. Also, based on the Lagrange multipliers, were introduced: the objective function of minimizing the standard deviations from the recorded value of the cash flows generated by the tangible assets, as well as the constraints caused by the impairment of tangible assets identification according to which the cash flows values must be equal to the annual value of the invested capital. Within the determination of the impairment value and stratification of the value ranges determined by the cash flows using stochastic fuzzy numbers, the impairment of assets risk was identified. Information provided by impairment of assets but also the impairment risks, is the basis of the decision-making measures taken to mitigate the impact of accumulated impairment losses on company's financial performance.

Keywords: fuzzy logic-based methods, tangible assets, impairment, management tool, risk, fuzzy stochastic number, cash flows, performance.

1 Introduction

Impairment of assets plays an important role in ensuring the financial performance of companies in the medium and long term, but also in identifying those tangible assets that no longer generate economic benefits for the company. Impairment expenses may have high values at different times which may affect the financial stability of the company, with direct consequences on the financial results but also on the remuneration of the company's shareholders.

This is the reason why, managers and shareholders are interested in developing management tools that provide solutions for early identification of possible impairment of tangible assets but also of the possible impairment risks. IAS 36 - Impairment of assets, offer an algorithm for testing the impairment of tangible assets. According to this algorithm, a tangible asset is considered to be impaired when its recoverable amount is lower than its carrying amount. Therefore, the fundamental equation of impairment of assets is based on the two values, the carrying amount obtained after deducting from the book value the accumulated depreciation and the recoverable amount of the tangible assets. On the other side, as IAS 36 defines, an asset's recoverable amount is the higher value of its fair value less disposal costs and its value in use. Most often the fair value less disposal costs is the net selling price, for those tangible assets for which there is an active market.

The net selling price, as a component part of the impairment test of tangible asset is obtained according to the market rules. In the situation where on the market there is a tangible asset traded according to the market rules, on the basis of a sale contract, then the sale price from that contract will be taken into account for the impairment test. In case such a sale contract does not exist, then the best estimate of the net sale price is the market price from which the costs of disposing of the asset are deducted. Finally, if there is neither sale price in the market, nor sales contract, then the International Accounting Standard recommends the use of the latest sale prices resulting from transactions with similar tangible assets in the same economic branch [23]. Therefore, the net sale price of the tangible assets taken into account for the impairment test must comply with the requirement for its best estimate, as well as the requirement regarding the source of information regarding the formation of the net sale price of the tangible asset.

Concerning the asset's value in use, this can be determined in two different stages. In the first stage, cash flows are estimated for the entire asset's life cycle, as well as for its residual value. In the second stage, cash flows can be updated with an adequate discount rate to allow estimations for future capitalization generated by tangible assets in present values, using the discounting techniques. When evaluating the updated cash flows that are the base for computing the value in use of the tangible asset, the following will be considered: the best estimates established on the basis of the best economic life conditions existing for the remaining life of the tangible asset, five years forecasts approved by the company, exploration of previous forecasts adjusted with certain growth / decrease rates, etc. The value in use of the tangible asset although it requires complex calculations, however, is most often used in performing the impairment tests.

As this impairment test is a rational algorithm based on comparing the carrying amount of the tangible assets with the recoverable value of the tangible assets, however, it has a number of disadvantages resulting from the mechanism of performing the impairment test, namely:

1. it is based on estimated values of the cash flows generated by the asset's exploitation, although the depreciation is based on the actual cash flows generated by the asset's exploitation;
2. the impairment test is carried out at different periods of time which cannot surprise in real time whether the asset is impaired or not, or if there are real indications that the tangible asset has started to lose value;
3. does not provide to company decision makers the operational information about impairment of assets, in order to take measures to mitigate the impact of the financial loss.

The purpose of this paper is to provide solutions to the above-mentioned problems with the help of stochastic fuzzy numbers and Langrange multipliers to identify in real-time the loss of tangible assets value, in order to determine the value of the impairment, but also to provide an appropriate tool expense management with impairment. This is because any loss of tangible assets value means an expense to the company.

The introduction of fuzzy stochastic numbers in the study of impairment of tangible assets has a double impact: on one hand they allow the estimation of the cash flows for future periods based on the actual cash flows resulting from the exploitation of the tangible assets, and on the other hand the fuzzy numbers allow stratification. Of the value ranges resulting from estimating future cash flows in order to accurately identify those intervals where the impairment of tangible assets risk is high.

At the same time, the Lagrange multipliers method allows the determination of the impairment value, respectively of the loss of value for the tangible assets in the conditions of establishing the objective function as the one minimizing the deviation of the cash flows values from their planned value and of introducing the condition that the value of the annual cash flows is equal to the annual value of the capital invested in tangible assets.

The innovative nature of the research paper is thus the introduction of stochastic fuzzy numbers in the management of expenses generated by the impairment of tangible assets but also in identifying the value ranges for which the impairment of the tangible assets risk is the highest.

2 Literature review

The impairment of tangible assets represents an important area of research, due to the fact that affects in a direct way the financial performance of companies in the medium and long term and identify those tangible assets that no longer generate economic benefits. Over time, researchers from all over the world have devoted their attention for studying this problem, in present being recorded 388 ISI scientific papers in the WoS database that include in their topic the key words: "impairment of assets".

The first articles published in this area of research appeared in 1993, and over time, their number has grown steadily, as shown in the Figure 1.

Most of these falls into the following areas of research: finance (30%), management (15%), economics (15%) and business (8%). Regarding the country of origin, USA ranks first with 35% of all published scientific studies, followed by China (13%), Australia (10%), England (8%), Italy (5%), Germany (4%), Canada (4%), Romania (3%), France (3%) and Portugal (3%).

In this section most important related articles conducted on this topic were reviewed. Thus, Andre et al.(2015)[3] has analysed the effects of mandatory IFRS adoption on conditional conservatism in Europe. They demonstrated that firms which do not record asset impairment when evidence suggests the probable need to do so, will show an important reduction in conditional conservatism.

The conservative accounting practices were targeted also by Lawrence et al.[19]. They have modelled the impact of extant accounting rules on the accounting rules, on conservative accounting rules, especially those that require assets to be written down when their fair values drop sufficiently below their book values.

Feng et al. [10] has analysed the correlations between the internal control over the financial reporting and the operating activity of a company. Their findings support the general hypothesis that internal control over financial reporting has an economically significant effect on firm operations.

Glaum et al.[13] has analysed compliance for a large sample of European companies mandatorily applying IFRS, targeting the disclosures required by IFRS 3 Business Combinations and IAS 36 Impairment of Assets.

The results have showed substantial non-compliance, both at the companies and country level, indicating that accounting traditions and other specific factors play an important role despite the use of common reporting standards under IFRS. Riedl [22] realized an examination of long-lived asset impairments. His results reveal that economic factors have a weaker association with write-offs reported after SFAS no. 121.

Among the most relevant papers that studied the impairment of assets, was noticed the study of [2] Agarwal et al. (2009) which analysed the financial decision over the life cycle and the implications for regulation, focusing on the financial mistakes and regulatory strategies that might help avoid these mistakes. Gox and Wagenhofer [15] studied the optimal impairment rules and the optimal precision of the accounting information, in order to provide comparative static results that lead to predictions on the determinants of impairment rules.

The impact of impairment of assets in the bank sector was studied by Badertscher et al. [4]. Analysing the fact that fair value provisions in U.S. accounting rules enabled the financial crisis by depleting banks' regulatory capital, which triggered asset sales, their results have shown that fair value accounting losses had minimal effect on regulatory capital and these provisions did not affect the commercial banking industry in the ways commonly alleged.

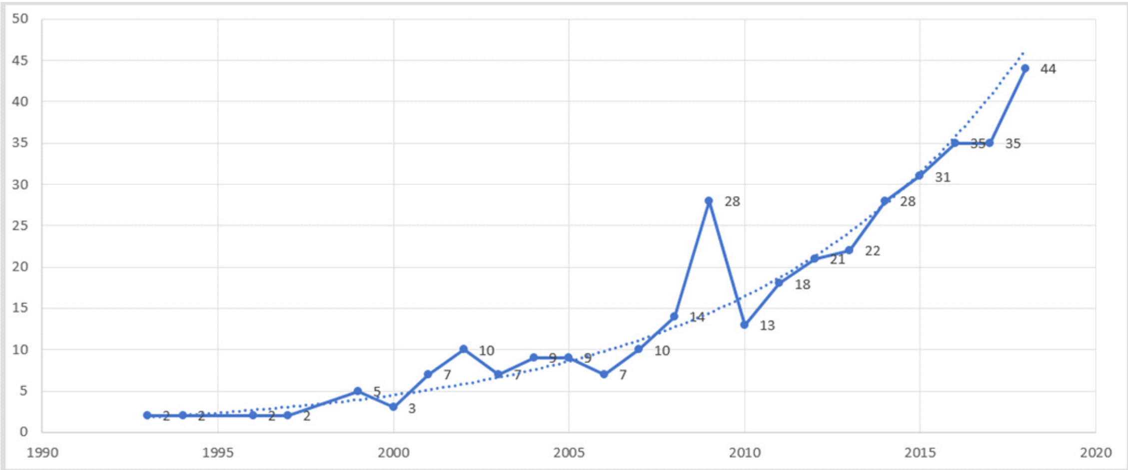

Figure 1: Number of ISI articles targeting the impairment of assets

Chalmers et al. [8] analysed whether the adoption of IFRS can be associated with a loss of potentially useful information about intangible assets. Their results showed that there is a negative correlation between accuracy and dispersion of analysts' earnings forecasts and aggregate reported intangibles after IFRS adoption, especially for firms with high levels of underlying intangible assets.

Carlin and Finch [7] studied the connection between the discounted cash flow modelling and the asset's recoverable amount. The reporting entities consider the rate selection as a factor of influence for the outcomes of the impairment testing process conducted under IFRS. Their study showed the existence of variances between independently generated risk-adjusted discount rates and those disclosed as having been used by a sample of large listed companies.

The effects of managerial discretion and stock market reactions after adopting the IFRS Standards were studied by Hamberg [17]. Their results have shown that by replacing the historical cost with fair value measures, the amount of capitalized goodwill increased, due to the fact that goodwill impairments under IFRS are considerably lower than goodwill amortizations and impairments made under GAAP. The conclusions revealed that the adoption of IFRS increased reported earnings.

Godfrey and Koh [14] examined whether goodwill impairment write-offs reflect firms' investment opportunities during the first years of the US goodwill impairment accounting regime. The results supported the International Accounting Standards Board and Financial Accounting Standards Board statement that an impairment test regime can reflect firms' underlying economic attributes and also can reflect that managers use discretion to reduce contracting costs.

Recently, goodwill impairment was the most studied issue in the accounting literature, as we found in AbuGhazaleh et al. [1], Hamberg and Beisland [16], Giner and Pardo [11], Korosec et al. [18], Schatt et al. [24].

Bolos et al. [5], analysing the investment decisions optimisation process within a company, created a fuzzy logic managerial decision tool for assets acquisition that combines, with very good results, the acquisition cost of assets with their economic performance. After explaining and testing their proposed model, they concluded that the fuzzy logic multi-criteria algorithm becomes a tool that provides for any company, regardless the number of assets and the number of selection criteria, the most convenient combination of assets cost and their economic and technical performance.

This is relevant, as Matemilola and Ahmad [20] highlighted, because most times tangible fixed assets are more preferred by firms when they want to obtain long term debt financing. Another study developed by Bolos et al. [6], targeted the differences registered between the projected assets production capacity and the volume of finished goods and services that are sold to consumers. The company is exposed to cash-flow deficits risk, which makes it difficult for the company to pay the debts to its financial creditors. In these terms, Mamdani fuzzy logic system was developed and the probabilistic values of the influence factors in the credit process are identified, in order to subsequently detect the risks due to probabilistic cash-flow deficits.

To date, there is no indexed article in the WoS database, which use stochastic fuzzy algorithms to analyse the impairment of assets. Fuzzy intelligence is used especially in the following areas of research: computer science, artificial intelligence, engineering electrical electronic, automation control systems, operation research and management science.

In business intelligence, however, applications of fuzzy technology are still rather scarce, even though the original motivation and justification of fuzzy intelligence was to model and improve human communication (see Meyer and Zimmermann [21]).

Only 2% of the total ISI articles targets the economic field, but none of them can be framed in the accounting domain. Only 2% of the total ISI articles targets the economic field, but none of them can be framed in the accounting domain.

The purpose of the research paper is to provide a tool needed to manage the impairment expenses of the tangible assets by introducing the fuzzy stochastic numbers and Lagrange's multipliers method, in order to determine the value of the impairment but also to determine the value ranges in which the impairment of tangible assets risk is higher.

The research methodology involved the following stages:

Stage I - Theoretical foundation of stochastic fuzzy numbers;

Stage II - Modeling the impairment value of an asset, two assets and N-assets using fuzzy stochastic numbers;

Stage III - Testing the methods for computing the value of the impairment and the impairment risk for one asset, respectively for two assets.

3 Stochastic elements and the impairment of assets

As noted, the impairment of an asset is recognised when the carrying amount (V_{CA}) exceeds the recoverable amount of the asset (V_{RA}).

The recoverable amount is defined as the largest value of the net fair value of an asset (fair value less costs of disposal) and the asset's value in use. The fundamental equation of asset impairment is always described by the existence of the inequality between its carrying amount (V_{CA}) and recoverable amount (V_{RA}), with two possible situations, namely:

(a) If $V_{CA} > V_{RA}$ then the asset is impaired and the company will record an impairment expense that will affect its financial performance. The causes that lead to such situations are diverse and depend most on the behaviour of the assets when they are used in the company's operational activity. As we know, the causes of impairment can be both internal and external. In case of internal causes, we considered that the maintenance and revision costs of tangible assets contribute significantly to the impairment of assets, as they could have an exponential evolution as their useful life increases;

(b) If $V_{CA} < V_{RA}$ then the asset is not impaired and the company will continue to benefit from its operation. During this period, the evolution of the operating revenues generated by the asset's operations must be monitored.

These revenues are dependent on the production capacity of the asset (Cp) and the possibility that the products/services are sold on the market ($CA_{p,s}$).

Also, the operating expenses of the asset that can completely deplete the operational revenues with the passage of time, must be monitored. In this case, the impairment test can not be avoided. The amount of value in use of an asset is dependent on the amount of cash flows (Cf_A) generated each year of its useful life.

Theorem 1. *If the cash flow value is a constant over the asset's useful life ($Cf_A = c$), with $c \in R$ and the asset's useful life is $Lf_A \in [0; t]$, then the updated value (Cf_u) at the discounted rate (d_r) of the cash flows over the life of the asset will be as follows:*

$$Cf_u = c \int_0^t e^{-d_r t} dt = \frac{c}{d_r} \left(1 - e^{-d_r t}\right) \tag{1}$$

Proof. Let A be the asset that generates a constant cash flow of the form $Cf_A = c$ with $c \in R$ and with $Lf_A \in [0; t]$, then the present value of the future cash flows generated by the use of the asset will be:

1. For the base year of asset's life span $Cf_A(0) = c\frac{1}{(1+d_r)^0}$
2. For the first year of asset's life span: $Cf_A(1) = c\frac{1}{(1+d_r)^1}$;
3. For the second year of asset's life span: $Cf_A(2) = c\frac{1}{(1+d_r)^2}$;
4. For year t of asset life: $Cf_A(t) = c\frac{1}{(1+d_r)^t}$;

Throughout the life of the asset $Lf_A \in [0;t]$ the aggregate form of cash flows will be quantified as follows:

$$Cf_A(Lf_A) = \sum_{i=0}^{t} c\frac{1}{(1+d_r)^i};\ Lf_A \in [0;t] \tag{2}$$

If the economic life of the asset is indefinite, $Cf_A(Lf_A) \to \infty$, then we can write:

$$Cf_A(Lf_A) = \lim_{i\to\infty} c\left[\left(1+\frac{d_r}{i}\right)^{\frac{i}{d_r}}\right]^{-r_a i} = ce^{-d_r t} \tag{3}$$

If cash flows occur continuously over the life of the asset, then they shall be determined as follows:

$$Cf_A(Lf_A) = \int_0^t ce^{-d_r t}dt = \frac{c}{d_r}\left(1-e^{-d_r t}\right) \tag{4}$$

□

Theorem 2. *The cash flows generated over the life of the asset $Lf_A \in [0;t]$ as a stochastic process with standard normal distribution ($\mu = 0$ and $\sigma = 1$) have the dynamic equation of the form:*

$$\Delta Cf_A = \mu Cf_A \Delta t + \sigma Cf_A \Delta z \tag{5}$$

Proof. The asset's life span is shared in time intervals of the form $t_0 \le t_1 \le t_2 \le \ldots \le t_{t-1} \le t_t$ so that any increase/change in the cash flows value generated by the company's asset can be written as follows:

$$\frac{Cf_A(t_t)}{Cf_A(t_0)} = \frac{Cf_A(t_1)}{Cf_A(t_0)} \times \frac{Cf_A(t_2)}{Cf_A(t_1)} \times \ldots \times \frac{Cf_A(t_t)}{Cf_A(t_{t-1})} \tag{6}$$

By logarithmizing of the above expression, it is obtained:

$$ln\frac{Cf_A(t_t)}{Cf_A(t_0)} = ln\sum_{t=1}^{n} \frac{Cf_A(t_t)}{Cf_A(t_{t-1})} \tag{7}$$

According to the central limit theorem in the probability theory, it is known that for very high values of n, when $n \to \infty$,

we will obtain that $\sum_{t=1}^{n} \frac{Cf_A(t_t)}{Cf_A(t_{t-1})} \to 0$, which means that the sum of the ratios of the cash flow formula, between two consecutive moments, has the mean $\mu = 0$ and the distribution $\sigma = 1$.

This will mean that the ratio $\frac{Cf_A(t_t)}{Cf_A(t_{t-1})}$ has a log-normal distribution, $ln\frac{Cf_A(t_t)}{Cf_A(t_{t-1})}$ is normally distributed, and $ln\sum_{t=1}^{n} \frac{Cf_A(t_t)}{Cf_A(t_{t-1})}$ the terms of the sum are random, independent and finite variation variables.

To write the cash flows variations (ΔCf_A) as a stochastic process, is used the expression of the asset's growth rate that generated cash flows over its entire lifetime (RCf_A):

$$RCf_A = \frac{\Delta Cf_A}{Cf_A} = \frac{Cf_A(t_t) - Cf_A(t_0)}{Cf_A(t_0)} = \frac{Cf_A(t_t)}{Cf_A(t_0)} - 1 \tag{8}$$

The above relation can be rewritten as:

$$RCf_A + 1 = \frac{Cf_A(t_t)}{Cf_A(t_0)} \tag{9}$$

By logaritmizing, the above expression becomes:

$$ln(RCf_A + 1) = ln\frac{Cf_A(t_t)}{Cf_A(t_0)} \tag{10}$$

For lower values of RCf_A, the expression $\ln(RCf_A+1) = RCf_A$ will be: $ln\frac{Cf_A(t_t)}{Cf_A(t_0)} = RCf_A$.

Because $ln\frac{Cf_A(t_t)}{Cf_A(t_0)}$ has a normal distribution with the arithmetic mean $\mu = 0$ and the variance $\sigma = 1$, we can write that $\frac{\Delta Cf_A}{Cf_A} = \mu\Delta t + \sigma\Delta z$.

The above expression can be rewritten as such:

$$\Delta Cf_A = \mu Cf_A \Delta t + \sigma Cf_A \Delta z \tag{11}$$

where: $\Delta z = \varepsilon\sqrt{\Delta t}$ with ε – random variable with standard normal distribution respectively $E(\varepsilon) = 0$ and $\sigma_\varepsilon^2 = E(\varepsilon^2) - [E(\varepsilon)]^2 = E(\varepsilon^2) = 1$. □

Theorem 3. *The probability that the cash flows $Cf_A(Lf_A)$ take the values $[Cf_A min(Lf_A); Cf_A max(Lf_A)]$ throughout the assets lifetime, as they evolve after a logarithmic function, is established with the relation:*

$$Cf_A(t_0)\, e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t - \sigma\sqrt{t_t}} \leq Cf_A(t_t) \leq e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t + \sigma\sqrt{t_t}} Cf_A(t_0) \tag{12}$$

Proof. The cash flows generated by companies' tangible assets are the basis for the assets' value in use determination. Their evolution is logarithmic as the operating revenues begin to decrease as their useful life increase.

The asset's value in use (Vu_A) and the cash flows generated over its lifetime $(Cf_A(Lf_A))$ establishes the relation: $Vu_A = lnCf_A(Lf_A)$.

The equation of stochastic dynamics was described in Theorem 2, according to which:

$$\Delta Cf_A = \mu Cf_A \Delta t + \sigma Cf_A \Delta z \tag{13}$$

According to Ito's lemma between Vu_A and $Cf_A(Lf_A)$ the following relation is established:

$$dVu_A = \left(\frac{\partial Vu_A}{\partial t} + \frac{\partial Vu_A}{\partial Cf_A}\mu Cf_A + \frac{1}{2}\frac{\partial^2 Vu_A}{\partial {Cf_A}^2}\sigma^2 {Cf_A}^2\right)dt + \frac{\partial Vu_A}{\partial Cf_A}\sigma Cf_A dz \tag{14}$$

The calculation of derivatives leads to the following results, namely:

$$\frac{\partial Vu_A}{\partial t} = 0; \frac{\partial Vu_A}{\partial Cf_A} = \frac{1}{Cf_A}; \frac{\partial^2 Vu_A}{\partial {Cf_A}^2} = -\frac{1}{Cf_A^2}.$$

The results obtained are replaced in the equation above and we obtain:

$$dVu_A = \left(\frac{1}{Cf_A}\mu Cf_A - \frac{1}{2}\sigma^2\frac{1}{{Cf_A}^2}{Cf_A}^2\right)dt + \sigma\frac{1}{Cf_A}Cf_A dz \tag{15}$$

or

$$d(\ln Cf_A) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dz \tag{16}$$

From Theorem 2 can be noticed that $ln\frac{Cf_A(t)}{Cf_A(t_0)}$ is normally distributed with the following distribution: $\ln\frac{Cf_A(t_t)}{Cf_A(t_0)} \approx \theta\left(\left(\mu - \frac{1}{2}\sigma^2\right)t_t, \sigma\sqrt{t_t}\right)$.

For standard normal distribution we apply the formula of probabilities and it is obtained:

$$P(\mu - \alpha\sigma \leq z \leq \mu + \alpha\sigma) = 2N(\alpha) - 1 \tag{17}$$

It results that:

$$P\left(\left(\mu - \frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t} \leq \ln\frac{Cf_A(t_t)}{Cf_A(t_0)} \leq \left(\mu - \frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t}\right) = 2N(\alpha) - 1 \tag{18}$$

Solving the above inequation it results that:

$$e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t}} \leq \frac{Cf_A(t_t)}{Cf_A(t_0)} \leq e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t + \alpha\sigma\sqrt{t_t}} \tag{19}$$

or

$$Cf_A(t_0)\, e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t}} \leq Cf_A(t_t) \leq e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t + \alpha\sigma\sqrt{t_t}} Cf_A(t_0) \tag{20}$$

□

In conclusion, it can be argued that in regard to impairment of tangible assets, the elements of stochasticity that play an important role in the determination of the asset's value in use are the cash flows that are generated over the asset's useful life.

If the cash flows are constant over the useful life of the asset, then they have the value equal to: $Cf_A\left(Lf_A\right)=\int_0^t ce^{-d_r t_t}dt=\frac{c}{d_r}\left(1-e^{-d_r t_t}\right)$.

If the cash flows have a stochastic evolution, after a normally distributed allocation function, over the asset's useful life, then the likelihood that the cash flows will take certain values within a certain range will be:

$$Cf_A\left(t_0\right)e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t-\alpha\sigma\sqrt{t_t}}\leq Cf_A\left(t_t\right)\leq e^{\left(\mu-\frac{1}{2}\sigma^2\right)t_t+\alpha\sigma\sqrt{t_t}}Cf_A\left(t_0\right) \tag{21}$$

4 The use of stochastic fuzzy numbers in modelling discounted future cash flows generated by the use of tangible assets

Companies' managers are required to make assumptions and judgements regarding the estimated future cash flows and discount rate. Because estimates of future cash flows and determination of discount rate often require significant estimates, judgments, and assumptions to be made, they are vulnerable to potential misstatement. As reflected in IAS 36, estimated future cash flows should be based on appropriately detailed underlying assumptions, which include changes in working capital and capital expenditure. Overhead costs relating to the day-to-day use of asset, as well as future overheads costs, are only included to the extent that they can be attributed directly, or allocated on a reasonable and consistent basis. As we found by Ginevičius and Gudačiauskas [12] value of the discounted net cash flows was calculated for brand valuation. We can observe that most evaluation models of estimated future cash flows used for other non-current assets, both tangible and intangible, are based on discounting rate in order to calculate an updated value.

As Dzitac et al. highlighted in [9], when we assess, judge or decide we usually use a natural language in which the words do not have a clear, definite meaning. Consequently, we need fuzzy numbers to express linguistic variables, to describe the subjective judgement of a decision maker in a quantitative manner, and the most often used are triangular fuzzy numbers. The use of stochastic fuzzy numbers allows the modelling of estimated future cash flows on the range of values and time that result from probability calculations when the cash flow distribution function is normally distributed, with the mean $\mu=0$ and the square mean variance $\sigma=1$. The purpose of modelling using stochastic fuzzy numbers is to identify the impairment of asset risk, respectively, if in the future, the company will record an expense that may cause deterioration in financial performance. The triangular fuzzy numbers will also be introduced for modelling the estimated future cash flows generated by the use of assets.

Definition 4. *Let $a\in R$ and b, c two positive real numbers. We say about the variable $\left(Cf_A\right)$ that is a triangular fuzzy number and we denote $\tilde{A}^{\Delta}_{Cf_A}=(a,b,c)$*
if the fuzzy subset $\tilde{A}^{\Delta}_{Cf_A}\in R$ and its function of membership is given by the relation:

1. *If $b,c>0$, then the membership function has the form:*

$$\tilde{A}^{\Delta}_{Cf_A}\left(Cf_{Ax}\right)=\begin{cases} 1-\frac{a-Cf_{Ax}}{a-b}\ for\ any\ \ b\leq Cf_{Ax}\leq a \\ 1-\frac{Cf_{Ax}-a}{c-a}\ for\ any\ a\leq Cf_{Ax}\leq c \\ 0\ in\ the\ rest \end{cases} \tag{22}$$

2. *If $b=c$ then $\tilde{A}^{\Delta}_{Cf_A}(a)=1$ and $\tilde{A}^{\Delta}_{Cf_A}\left(Cf_{Ax}\right)=0$ for any $Cf_{Ax}\neq a$, where: Cf_{Ax} is the cash-flow value at a certain time.*

The level sets of the fuzzy triangle number described above are: $\left[\tilde{A}^{\Delta}_{Cf_A}\right]^{\alpha}=\left[a_1(\alpha);a_2(\alpha)\right]$, where $a_1(\alpha)=b+(a-b)\alpha$ and $a_2(\alpha)=c-(c-a)\alpha$ with $\alpha\in[0,1]$.

Definition 5. *The triangular fuzzy number is called the stochastic fuzzy number if for any $a \in R$ and for any two positive real numbers b, c, the sub-set $\tilde{A}^{\Delta}_{Cf_A}$ fulfils cumulatively the following conditions:*
The membership function for any $b, c > 0$ is:

$$\tilde{A}^{\Delta}_{Cf_A}\left(Cf_{Ax}\right) = \begin{cases} 1 - \frac{a - Cf_{Ax}}{a-b} \ for\ any\ b \leq Cf_{Ax} \leq a \\ 1 - \frac{Cf_{Ax} - a}{c-a} \ for\ any\ a \leq Cf_{Ax} \leq b \\ \quad 0\ in\ the\ rest \end{cases} \tag{23}$$

For $b = c$ it results according to Definition 4 that $\tilde{A}^{\Delta}_{Cf_A}(a) = 1$ and $\tilde{A}^{\Delta}_{Cf_A}(Cf_{Ax}) = 0$ for any $Cf_{Ax} \neq a$.

The values of the variable $Cf_A\left(Lf_A\right)$ on the interval $[b, c]$ are given by the relation:

$$Cf_A\left(t_0\right) e^{\left(\mu - \frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t}} \leq Cf_A\left(t_t\right) \leq e^{\left(\mu - \frac{1}{2}\sigma^2\right)t_t + \alpha\sigma\sqrt{t_t}} Cf_A\left(t_0\right) \tag{24}$$

thus:

$$b = \ Cf_A\left(t_0\right) e^{\left(\mu - \frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t}}$$

and

$$c = Cf_A\left(t_0\right) e^{\left(\mu - \frac{1}{2}\sigma^2\right)t_t + \alpha\sigma\sqrt{t_t}};$$

The set of membership degree determined by the membership function will be:

$$\left[\mu\left(Cf_A\left(t_0\right) e^{\left(\mu - \frac{1}{2}\sigma^2\right)t_t - \alpha\sigma\sqrt{t_t}}\right); \mu\left(Cf_A\left(t_0\right) e^{\left(\mu - \frac{1}{2}\sigma^2\right)t_t + \alpha\sigma\sqrt{t_t}}\right)\right],$$

corresponding to the variation level considered $[b, c]$;

The variable $Cf_A\left(Lf_A\right)$ is a random variable with density distribution function
$f\left(Cf_A\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(Cf_a - \mu\right)^2}{2\sigma^2}}$
and with the distribution function written as primitive of the density distribution function
$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(Cf_a - \mu\right)^2}{2\sigma^2}} dt = 1.$

The fuzzy stochastic triangular number $\tilde{A}^{\Delta}_{Cf_A}\left(Cf_{Ax}\right)$ introduced above is used to model the estimated future cash flows generated by the normal company's asset operation.

The company purpose is to maximize the revenues obtained from asset's operation within a reasonable period of time, in order to ensure the payments recovery related to the invested capital in the asset $\left(K_{inv}\right)$, but also to cover the payments generated by its running costs.

The major advantages of using stochastic triangular fuzzy numbers is that allow the setting of cash flows value ranges for future periods of time and an easy processing of these flows with fuzzy numbers.

5 Modeling the impairment of assets risk with stochastic fuzzy numbers

5.1 Stochastic fuzzy modelling for impairment of assets risk for one asset

Theorem 6. *Is considered an asset A_1 whose contribution to company sales is $\left(w_1\right)$, generating cash flows Cf_{A1} expressed with the stochastic fuzzy numbers, $\tilde{A}^{\Delta}_{Cf_A} ef : X \rightarrow [0, 1]$.*

The impairment of the asset value (V_{imp}) will be established as follows:

$$V_{imp} = w_1 \tilde{A}^{\Delta}_{Cf_A} ef - r_1 K_{inv1}; \tag{25}$$

with:

$$w_1 = \frac{r_1 K_{inv1}}{\tilde{A}^{\Delta}_{Cf_A} ef}$$

Proof. When the company owns only one asset (A_1), the following assumptions apply:

The asset generates inflows from the obtained revenues modelled with the stochastic fuzzy triangular numbers $\tilde{A}^{\Delta}_{Cf_{A1}}\left(Rev_{A1}\right)$ and outflows generated by the operating expenses of the asset, also modelled with stochastic triangular fuzzy numbers $\tilde{A}^{\Delta}_{Cf_{A1}}\left(Exp_{A1}\right)$ of the form:

$$w_1 Cf_{A1} = w_1 \frac{1}{d_r}\left(1 - e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A}\left(Rev_{A1}\right) - \tilde{A}^{\Delta}_{Cf_A}\left(Exp_{A_1}\right)\right] = w_1 \tilde{A}^{\Delta}_{Cf_A} ef \tag{26}$$

The generated cash flow must be at least equal to the part of the invested capital in the asset, over the reference period in which the impairment analysis is performed, denoted $(r_i K_{invi})$.

Also, to avoid impairment and expenses that affect the financial performance, the company sets as its objective the minimization of the deviation between the expected cash flows and the effective ones realized during the period of the asset's operation after a relation of the form:

$$min\ w_1^2 \left\{\frac{1}{d_r}\left(1 - e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef\right]\right\}^2 \tag{27}$$

The Lagrangian function is:

$$L = \ min\ w_1^2 \left\{\frac{1}{d_r}\left(1 - e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef\right]\right\}^2 - \lambda\left(w_1 \tilde{A}^{\Delta}_{Cf_A} ef - r_1 K_{inv1}\right) \tag{28}$$

for which the optimum conditions are:

$$\begin{cases} \frac{\partial \mathrm{L}}{\partial w_1} = 0 \\ \frac{\partial \mathrm{L}}{\partial \lambda} = 0 \end{cases} \tag{29}$$

By carrying out the calculations, we obtain:

$$\begin{cases} 2w_1 \left\{\frac{1}{d_r}\left(1 - e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef\right]\right\}^2 - \lambda \tilde{A}^{\Delta}_{Cf_A} ef = 0 \\ w_1 \tilde{A}^{\Delta}_{Cf_A} ef - r_1 K_{inv1} = 0 \end{cases} \tag{30}$$

We note $\alpha = 2\frac{1}{d_r}\left(1 - e^{-d_r t}\right)$ and we obtain that:

$$w_1 = \frac{\lambda \tilde{A}^{\Delta}_{Cf_A} ef}{\left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef\right]^2} \tag{31}$$

By replacing the expression of w_1 in the last equation, we obtain:

$$\frac{\lambda \tilde{A}^{\Delta}_{Cf_A} ef}{\left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef\right]^2} \tilde{A}^{\Delta}_{Cf_A} ef = r_1 K_{inv1} \tag{32}$$

Hence, it results the calculation formulae for parameter λ of the form:

$$\lambda = \frac{r_1 K_{inv1}\left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef\right]^2}{\tilde{A}^{\Delta}_{Cf_A} ef^2} \tag{33}$$

By replacing the parameter λ in the formula obtained for w_1 the way to calculate w_1 is obtained:

$$w_1 = \frac{r_1 K_{inv1}}{\tilde{A}^{\Delta}_{Cf_A} ef} \tag{34}$$

The value of impairment of the asset (V_{imp}) will be given by the formula:

$$V_{imp} = w_1 \tilde{A}^{\Delta}_{Cf_A} ef - r_1 K_{inv1} \tag{35}$$

In conclusion, it can be said that the asset impairment value depends on the asset's contribution to sales (w_1), but also on the volume of cash flows $\left(\tilde{A}^{\Delta}_{Cf_A} ef\right)$ as well as the invested capital $(r_1 K_{inv1})$. □

5.2 Stochastic fuzzy modelling of assets' impairment risk for a 2-asset group

Theorem 7. *There are considered two assets* $(A_1,\ A_2)$ *participating in the sales volume with the weights* (w_1, w_2), *each generating cash flow* (Cf_{A1}, Cf_{A2}), *modelled based on stochastic triangular fuzzy numbers,* $\tilde{A}^{\Delta}_{Cf_A}ef : X \to [0,1]$.

The impairment of the assets value (V_{imp}) *will be given by the relation:*

$$V_{imp} = w_1\tilde{A}^{\Delta}_{Cf_A}ef + w_2\tilde{A}^{\Delta}_{Cf_A}ef - (r_1K_{inv1} + r_2K_{inv2}) \tag{36}$$

with values for (w_1) *and* (w_2) *established below:*

$$w_1 = \frac{(r_1K_{inv1} + r_2K_{inv2})\left[\tilde{A}^{\Delta}_{Cf_A}pl(2) - \tilde{A}^{\Delta}_{Cf_A}ef(2)\right]^2}{\tilde{A}^{\Delta}_{Cf_A}ef(1)^2\left[\tilde{A}^{\Delta}_{Cf_A}pl(2) - \tilde{A}^{\Delta}_{Cf_A}ef(2)\right]^2 + \tilde{A}^{\Delta}_{Cf_A}ef(2)^2\left[\tilde{A}^{\Delta}_{Cf_A}pl(1) - \tilde{A}^{\Delta}_{Cf_A}ef(1)\right]^2} \tag{37}$$

$$w_2 = \frac{(r_1K_{inv1} + r_2K_{inv2})\left[\tilde{A}^{\Delta}_{Cf_A}pl(1) - \tilde{A}^{\Delta}_{Cf_A}ef(1)\right]^2}{\tilde{A}^{\Delta}_{Cf_A}ef(1)^2\left[\tilde{A}^{\Delta}_{Cf_A}pl(2) - \tilde{A}^{\Delta}_{Cf_A}ef(2)\right]^2 + \tilde{A}^{\Delta}_{Cf_A}ef(1)^2\left[\tilde{A}^{\Delta}_{Cf_A}pl(1) - \tilde{A}^{\Delta}_{Cf_A}ef(1)\right]^2} \tag{38}$$

Proof. For modelling estimated future cash flows using stochastic triangular fuzzy numbers, is assumed that the company holds a minimum of two assets denoted symbolically by $(A_1,\ A_2)$ that generate cash flows throughout their useful life
$(Cf_{A1} = r_1k_{inv1}, Cf_{A2} = r_2k_{inv2})$.

The cash flows generated during the time frame that is the object of fuzzy modelling, are composed of inflows modelled with the stochastic triangular fuzzy numbers, $\tilde{A}^{\Delta}_{Cf_{Ai}}(Rev_{Ai})$ and outflows, also modelled with stochastic triangular fuzzy numbers, $\tilde{A}^{\Delta}_{Cf_{A1}}(Exp_{Ai})$.

For each asset owned by the company, the fuzzy equation describing the cash flow is as follows:

$$Cf_{Ai} = \frac{1}{d_r}\left(1 - e^{-d_rt}\right)\left[\tilde{A}^{\Delta}_{Cf_A}(Rev_{Ai}) - \tilde{A}^{\Delta}_{Cf_A}(Exp_{A_i})\right] = \tilde{A}^{\Delta}_{Cf_A}ef(i) \tag{39}$$

In addition, each of the assets (A_k) held by the company makes a certain contribution to the company's sales that are determined by the production capacity of each (Cp_{Ak}), but also by the possibility of selling the products obtained on the market. We note this asset contribution (A_k) for making sales with (w_k).

So, we will have:

For asset A_1: an effective cash flow is generated
$w_1(Cf_{A1})$=$w_1\left\{\frac{1}{d_r}\left(1 - e^{-d_rt}\right)\left[\tilde{A}^{\Delta}_{Cf_{A1}}(Rev_{A1}) - \tilde{A}^{\Delta}_{A1}(Exp_{A1})\right]\right\} = w_1\tilde{A}^{\Delta}_{Cf_{A1}}ef\,(1)$
and the objective is to minimize the deviation of the effective cash flows values $\tilde{A}^{\Delta}_{Cf_A}ef\ (1)$ from the planned cash flows $\tilde{A}^{\Delta}_{Cf_A}pl\,(1)$, so that the use value of the asset does not change significantly and impair, respectively $min\ w_1^2\left\{\frac{1}{d_r}\left(1 - e^{-d_rt}\right)\left[\tilde{A}^{\Delta}_{Cf_A}pl(1) - \tilde{A}^{\Delta}_{Cf_A}ef(1)\right]\right\}^2$.

The value of the cash flows generated by asset (A_1) must be equal to the part allocated from the invested capital for the year of analysis denoted by (r_1k_{inv1}), so the asset not to be affected by cash-flows fluctuations generated by it, so the usefulness value begin to diminish.

For asset A_2: a cash flow of the following form is generated:

$w_2(Cf_{A2}) = w_2\left\{\frac{1}{d_r}\left(1 - e^{d_rt}\right)\left[\tilde{A}^{\Delta}_{Cf_A}(Rev_{A2}) - \tilde{A}^{\Delta}_{Cf_A}(Exp_{A2})\right]\right\} = w_2\tilde{A}^{\Delta}_{Cf_A}ef$

and the objective is also to minimize the deviations of the effective cash flows $\widetilde{A}^{\Delta}_{Cf_A}ef$ from the planned cash flows $\tilde{A}^{\Delta}_{Cf_A}pl\,(2)$, so that the use value of the asset does not change significantly and impair, respectively, $min\ w_2^2\left\{\frac{1}{d_r}\left(1 - e^{-d_rt}\right)\left[\tilde{A}^{\Delta}_{Cf_A}pl - \tilde{A}^{\Delta}_{Cf_A}ef\right]\right\}^2$.

The amount of the cash flow generated by asset A_2 must be equal to (r_2K_{inv2}), so that the asset's value of use is not affected.

This creates an optimization problem where the aim is to minimize the deviations of the cash flows from their expected value:

$$\begin{aligned} & min\ w_1^2\left\{\frac{1}{d_r}\left(1-e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A}pl(1)-\tilde{A}^{\Delta}_{Cf_A}ef(1)\right]\right\}^2 \\ & + min\ w_2^2\left\{\frac{1}{d_r}\left(1-e^{-d_r t}\right)\ \left[\tilde{A}^{\Delta}_{Cf_A}pl(2)-\tilde{A}^{\Delta}_{Cf_A}ef(2)\right]\right\}^2 \end{aligned} \tag{40}$$

The problem restrictions are that the amount of cash flows generated by each asset is equal to the share of the invested capital for each asset, for the reference year for which the analysis is performed:

$$w_1\tilde{A}^{\Delta}_{Cf_A}ef(1)+w_2\tilde{A}^{\Delta}_{Cf_A}ef(2)=r_1K_{inv1}+r_2K_{inv2} \tag{41}$$

The Lagrangian of the optimization problem is:

$$\begin{aligned} L = \ & min\ w_1^2\left\{\frac{1}{d_r}\left(1-e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A}pl(1)-\tilde{A}^{\Delta}_{Cf_A}ef(1)\right]\right\}^2 + \\ & min\ w_2^2\left\{\frac{1}{d_r}\left(1-e^{-d_r t}\right)\ \left[\tilde{A}^{\Delta}_{Cf_A}pl(2)-\tilde{A}^{\Delta}_{Cf_A}ef(2)\right]\right\}^2 - \\ & \lambda\left(w_1\tilde{A}^{\Delta}_{Cf_A}ef(1)+w_2\tilde{A}^{\Delta}_{Cf_A}ef-(r_1K_{inv1}+r_2K_{inv2})\right) \end{aligned}$$

We establish the optimization conditions for the situation above and we obtain:

$$\begin{cases} \frac{\partial L}{\partial w_1}=0 \\ \frac{\partial L}{\partial w_2}=0 \\ \frac{\partial L}{\partial \lambda}=0 \end{cases}$$

Thus, after making the calculations we will obtain the following system of equations:

$$\begin{cases} 2\ w_1\left\{\frac{1}{d_r}\left(1-e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A}pl(1)-\tilde{A}^{\Delta}_{Cf_A}ef(1)\right]\right\}^2-\lambda\tilde{A}^{\Delta}_{Cf_A}ef(1)=0 \\ 2\ w_2\left\{\frac{1}{d_r}\left(1-e^{-d_r t}\right)\left[\tilde{A}^{\Delta}_{Cf_A}pl(2)-\tilde{A}^{\Delta}_{Cf_A}ef(2)\right]\right\}^2-\lambda\tilde{A}^{\Delta}_{Cf_A}ef(2)=0 \\ w_1\tilde{A}^{\Delta}_{Cf_A}ef(1)+w_2\tilde{A}^{\Delta}_{Cf_A}ef(2)=r_1K_{inv1}+r_2K_{inv2} \end{cases} \tag{42}$$

From the equations above we take out w_1 and w_2 and we obtain the following:

$$w_1=\frac{\lambda\tilde{A}^{\Delta}_{Cf_A}ef(1)}{\alpha\left[\tilde{A}^{\Delta}_{Cf_A}pl(1)-\tilde{A}^{\Delta}_{Cf_A}ef(1)\right]^2} \tag{43}$$

Where α is the constant $\alpha= 2\frac{1}{d_r}\left(1-e^{-d_r t}\right)$.

$$w_2=\frac{\lambda\tilde{A}^{\Delta}_{Cf_A}ef(2)}{\alpha\left[\tilde{A}^{\Delta}_{Cf_A}pl(2)-\tilde{A}^{\Delta}_{Cf_A}ef(2)\right]^2} \tag{44}$$

By replacing w_1 and w_2 in the last equation of the system above, we obtain:

$$\begin{aligned} & \frac{\lambda\tilde{A}^{\Delta}_{Cf_A}ef\,(1)}{\alpha\left[\tilde{A}^{\Delta}_{Cf_A}pl\,(1)-\tilde{A}^{\Delta}_{Cf_A}ef\,(1)\right]^2}\tilde{A}^{\Delta}_{Cf_A}ef(1)+ \\ & +\frac{\lambda\tilde{A}^{\Delta}_{Cf_A}ef\,(2)}{\alpha\left[\tilde{A}^{\Delta}_{Cf_A}pl\,(2)-\tilde{A}^{\Delta}_{Cf_A}ef\,(2)\right]^2}\tilde{A}^{\Delta}_{Cf_A}ef(2) \\ & =(r_1K_{inv1}+r_2K_{inv2}) \end{aligned} \tag{45}$$

$$\begin{aligned} & \lambda\alpha\left[\tilde{A}^{\Delta}_{Cf_A}ef(1)^2\right]\left[\tilde{A}^{\Delta}_{Cf_A}pl(2)-\tilde{A}^{\Delta}_{Cf_A}ef(2)\right]^2+\tilde{A}^{\Delta}_{Cf_A}ef(2)^2\left[\tilde{A}^{\Delta}_{Cf_A}pl(1)-\tilde{A}^{\Delta}_{Cf_A}ef(1)\right]^2 \\ & =(r_1K_{inv1}+r_2K_{inv2})\ \alpha\left[\tilde{A}^{\Delta}_{Cf_A}pl(1)-\tilde{A}^{\Delta}_{Cf_A}ef(1)\right]^2\left[\tilde{A}^{\Delta}_{Cf_A}pl(2)-\tilde{A}^{\Delta}_{Cf_A}ef(2)\right]^2 \end{aligned}$$

From the equation above the value of parameter λ is obtained:

$$\lambda = \frac{(r_1 K_{inv1} + r_2 K_{inv2}) \left[\tilde{A}^{\Delta}_{Cf_A} pl(1) - \tilde{A}^{\Delta}_{Cf_A} ef(1)\right]^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef(2)\right]^2}{\tilde{A}^{\Delta}_{Cf_A} ef(1)^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef(2)\right]^2 + \tilde{A}^{\Delta}_{Cf_A} ef(2)^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(1) - \tilde{A}^{\Delta}_{Cf_A} ef(1)\right]^2} \tag{46}$$

By replacing the expression obtained above for parameter λ in the expressions of w_1 and w_2 is obtained:

$$w_1 = \frac{\lambda \tilde{A}^{\Delta}_{Cf_A} ef(1)}{\alpha \left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef(1)\right]^2}$$

and respectively

$$w_2 = \frac{\lambda \tilde{A}^{\Delta}_{Cf_A} ef(2)}{\alpha \left[\tilde{A}^{\Delta}_{Cf_A} pl - \tilde{A}^{\Delta}_{Cf_A} ef(2)\right]^2}$$

$$w_1 = \frac{(r_1 K_{inv1} + r_2 K_{inv2}) \left[\tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef(2)\right]^2}{\tilde{A}^{\Delta}_{Cf_A} ef(1)^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef(2)\right]^2 + \tilde{A}^{\Delta}_{Cf_A} ef(2)^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(1) - \tilde{A}^{\Delta}_{Cf_A} ef(1)\right]^2} \tag{47}$$

$$w_2 = \frac{(r_1 K_{inv1} + r_2 K_{inv2}) \left[\tilde{A}^{\Delta}_{Cf_A} pl(1) - \tilde{A}^{\Delta}_{Cf_A} ef(1)\right]^2}{\tilde{A}^{\Delta}_{Cf_A} ef(1)^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef(2)\right]^2 + \tilde{A}^{\Delta}_{Cf_A} ef(1)^2 \left[\tilde{A}^{\Delta}_{Cf_A} pl(1) - \tilde{A}^{\Delta}_{Cf_A} ef(1)\right]^2} \tag{48}$$

□

Conclusion: The impairment of assets depends on the size of the invested capital ($r_i K_{invi}$), the contribution of the companies' assets to the sales volume (w_1, w_2), but also on the value of cash flows modelled by the stochastic fuzzy numbers

$$\tilde{A}^{\Delta}_{Cf_A} pl\,(1) - \tilde{A}^{\Delta}_{Cf_A} ef\,(1)\,,\ \tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef.$$

5.3 Stochastic fuzzy modelling for impairment of assets risk for a group of N assets

Theorem 8. *Is considered that the company own N-assets* $(A_1, A_2 \ldots\ldots A_N)$ *holding the weights in total sales* $(w_1, w_2 \ldots w_N)$.

Each of these assets generates cash flows of the form $(Cf_{A1}, Cf_{A2}, \ldots\ldots Cf_{AN})$ *modelled by stochastic fuzzy numbers* $\tilde{A}^{\Delta}_{Cf_{A1}} ef\,(1)\,, \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2)\,, \ldots\ldots\ \tilde{A}^{\Delta}_{Cf_{An}} ef\,(N)\,.$

Let us say that the impairment value of all assets denoted by (V_{imp}) *is a matrix equation:*

$$\begin{pmatrix} V_{imp1} \\ V_{imp2} \\ \cdots \\ V_{impN} \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_N \end{pmatrix} \begin{pmatrix} \tilde{A}^{\Delta}_{Cf_A} ef(1) \\ \tilde{A}^{\Delta}_{Cf_A} ef(2) \\ \cdots \\ \tilde{A}^{\Delta}_{Cf_A} ef(N) \end{pmatrix} - \begin{pmatrix} rk_{inv1} \\ rk_{inv2} \\ \cdots \\ rk_{invN} \end{pmatrix} \tag{49}$$

where:

$$W = \frac{1}{\propto} \frac{RK_{inv}}{\left[\tilde{A}^{\Delta}_{Cf_A} ef\right]}$$

Proof. The assets held by the company $(A_1, A_2 \ldots\ldots A_N)$ generate cash flows $(Cf_{A1}, Cf_{A2}, \ldots\ldots Cf_{AN})$ modelled by stochastic fuzzy numbers $\tilde{A}^{\Delta}_{Cf_{A1}} ef\,(1)\,, \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2)\,, \ldots\ldots\ \tilde{A}^{\Delta}_{An} ef\,(N)$.

The system of equations formed for testing the impairment of the assets will be:

$$\begin{cases} min \sum_{k=1}^{N} w_k^2 \alpha \Big[\tilde{A}^{\Delta}_{Cf_A} pl(k) - \tilde{A}^{\Delta}_{Cf_A} ef(k)\Big]^2 \\ \sum_{k=1}^{N} w_k \tilde{A}^{\Delta}_{Cf_A} ef(k) = \sum_{k=1}^{N} r_k k_{invk} \end{cases} \tag{50}$$

The Lagrangian of the problem becomes:

$$L = \ min \sum_{k=1}^{N} w_k^2 \alpha \Big[\tilde{A}^{\Delta}_{Cf_A} pl(k) - \tilde{A}^{\Delta}_{Cf_A} ef(k)\Big]^2 - \lambda \left(\sum_{k=1}^{N} w_k \tilde{A}^{\Delta}_{Cf_A} ef(k) - \sum_{k=1}^{N} r_i k_{invi} \right) \tag{51}$$

with the conditions of optimum:

$$\begin{cases} \frac{\partial \mathrm{L}}{\partial w_k} = 0 \\ \frac{\partial \mathrm{L}}{\partial \lambda} = 0 \end{cases} \tag{52}$$

After calculations, we will have:

$$\begin{cases} \sum_{k=1}^{N} 2 w_k \alpha \Big[\tilde{A}^{\Delta}_{Cf_A} pl(k) - \tilde{A}^{\Delta}_{Cf_A} ef(k)\Big]^2 - \lambda \sum_{k=1}^{N} \tilde{A}^{\Delta}_{Cf_A} ef(k) = 0 \\ \sum_{k=1}^{N} w_k \tilde{A}^{\Delta}_{Cf_A} ef(k) - \sum_{k=1}^{N} r_k k_{invk} = 0 \end{cases} \tag{53}$$

We write in the matrix form the equations from above which will become:

$$\alpha \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{pmatrix} \begin{pmatrix} \tilde{A}^{\Delta}_{Cf_A} pl(1) - \tilde{A}^{\Delta}_{Cf_A} ef(1) \\ \tilde{A}^{\Delta}_{Cf_A} pl(2) - \tilde{A}^{\Delta}_{Cf_A} ef(2) \\ \dots \\ \tilde{A}^{\Delta}_{Cf_A} pl(N) - \tilde{A}^{\Delta}_{Cf_A} ef(N) \end{pmatrix}^2 = \lambda \begin{pmatrix} \tilde{A}^{\Delta}_{Cf_A} ef(1) \\ \tilde{A}^{\Delta}_{Cf_A} ef(2) \\ \dots \\ \tilde{A}^{\Delta}_{Cf_A} ef(N) \end{pmatrix} \tag{54}$$

And respectively:

$$\begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{pmatrix} \begin{pmatrix} \tilde{A}^{\Delta}_{Cf_A} ef(1) \\ \tilde{A}^{\Delta}_{Cf_A} ef(2) \\ \dots \\ \tilde{A}^{\Delta}_{Cf_A} ef(N) \end{pmatrix} = \begin{pmatrix} r_1 k_{inv1} \\ r_2 k_{inv2} \\ \dots \\ r_N k_{invN} \end{pmatrix} \tag{55}$$

Or the matrix form of the equation system

$$\begin{cases} \propto W \Big[\tilde{A}^{\Delta -}_{Cf_A}\Big]^2 = \lambda \tilde{A}^{\Delta}_{Cf_A} ef \\ W \tilde{A}^{\Delta}_{Cf_A} ef = RK_{inv} \end{cases} \tag{56}$$

From the equation system from above results that:

$$W = \frac{1}{\propto} \frac{\breve{} \tilde{A}^{\Delta}_{Cf_A} ef}{\Big[\tilde{A}^{\Delta -}_{Cf_A}\Big]^2} \tag{57}$$

By replacing the expression of parameter W in the matrix equation we obtain:

$$\frac{1}{\propto} \frac{\lambda \Big[\tilde{A}^{\Delta}_{Cf_A} ef\Big]^2}{\Big[\tilde{A}^{\Delta -}_{Cf_A}\Big]^2} = \ RK_{inv} \tag{58}$$

Hence it results that:

$$\lambda = \frac{RK_{inv} \Big[\tilde{A}^{\Delta -}_{Cf_A}\Big]^2}{\Big[\tilde{A}^{\Delta}_{Cf_A} ef\Big]^2} \tag{59}$$

By replacing the expression of λ we will obtain:

$$W = \frac{1}{\propto}\frac{\lambda \tilde{A}^{\Delta}_{Cf_A} ef}{\left[\tilde{A}^{\Delta-}_{Cf_A}\right]^2} = \frac{1}{\propto}\frac{\frac{RK_{inv}\left[\tilde{A}^{\Delta-}_{Cf_A}\right]^2}{\left[\tilde{A}^{\Delta}_{Cf_A} ef\right]^2}\tilde{A}^{\Delta}_{Cf_A} ef}{\left[\tilde{A}^{\Delta-}_{Cf_A}\right]^2} = \frac{RK_{inv}}{\left[\tilde{A}^{\Delta}_{Cf_A} ef\right]} \tag{60}$$

Expression W can be written in matrix form thus:

$$\begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_N \end{pmatrix} = \frac{\begin{pmatrix} r_1 k_{inv1} \\ r_2 k_{inv2} \\ \cdots \\ r_N k_{invN} \end{pmatrix}}{\begin{pmatrix} \tilde{A}^{\Delta}_{Cf_A} ef(1) \\ \tilde{A}^{\Delta}_{Cf_A} ef(2) \\ \cdots \\ \tilde{A}^{\Delta}_{Cf_A} ef(N) \end{pmatrix}} \tag{61}$$

The impairment of asset value will be written in matrix form, thus:

$$\begin{pmatrix} V_{imp1} \\ V_{imp2} \\ \cdots \\ V_{impN} \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_N \end{pmatrix}\begin{pmatrix} \tilde{A}^{\Delta}_{Cf_A} ef(1) \\ \tilde{A}^{\Delta}_{Cf_A} ef(2) \\ \cdots \\ \tilde{A}^{\Delta}_{Cf_A} ef(N) \end{pmatrix} - \begin{pmatrix} r_1 k_{inv1} \\ r_2 k_{inv2} \\ \cdots \\ r_N k_{invN} \end{pmatrix} \tag{62}$$

□

Conclusion: It can be argued that in the previous modelling situation, for a group of N assets, the impairment value depends on the value of the invested capital $(r_k k_{invk})$, the size of cash flows generated by each asset $\left(\tilde{A}^{\Delta}_{Cf_A} ef\,(k)\right)$ and the contribution that each asset has to the company sales $(\mathrm{w_k})$.

6 Testing the efficiency of the stochastic fuzzy numbers in asset impairment testing

6.1 Single asset case

In order to simulate the efficiency of the stochastic fuzzy numbers in asset impairment testing, were considered as indicators: the initial investment cost $(\mathrm{K_{inv1}})$, the lifetime of the asset $(\mathrm{Lf_1})$, an investment rate for the 10% impairment calculation and respectively $\mathrm{r_1K_{inv1}}$. The monthly average expected future cash flows for each year of the tangible asset life (10 years) and also the cash flows generated by the tangible asset for 10 months period, are illustrated in the Figure 2.

It was determined whether in the next period of the tangible asset operation the asset will be influenced by the impairment process, for which the company will incur an impairment expense. In the first stage, based on the data of the cash flows generated over the lifetime of the asset, the mean μ and variance σ shall be determined using the statistical formulas $\mu = \frac{1}{10}\sum_{i=1}^{10} Cf_{Ai}$ and respectively $\sigma = \frac{1}{N-1}\sqrt{\sum_{i=1}^{N}\left(Cf_i - \overline{Cf}\right)^2}$. By making the calculations we obtain $\mu = 1,98\%$ and the value of $\sigma = 3,04\%$. With the two determined statistical parameters, namely the mean $\mu = 1,98\%$ and the value $\sigma = 3,04\%$, the expected cash flows for the next operation period is estimated to be between $(Cf_{min}; Cf_{max})$ as follows:$1,2\ mil.\ euro\ \leq Cf \leq 1,8\ mil.\ euro$. In this interval, three triangular fuzzy numbers are constructed that characterize the cash flows generated by the asset's operation in the form of:

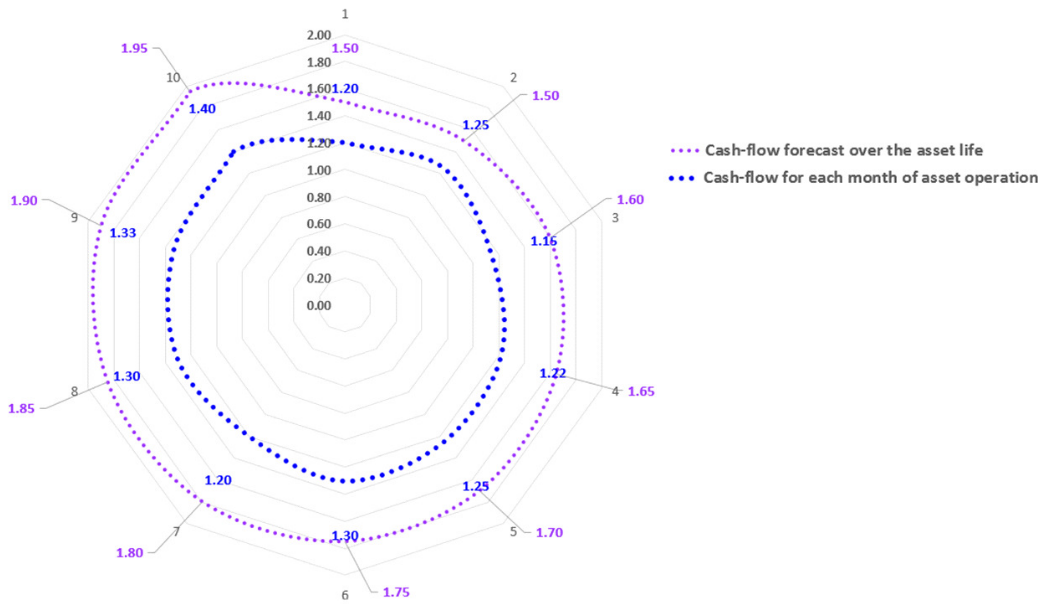

Figure 2: Cash-flow forecast over the asset life and for each month of asset operation

$$\begin{cases} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(1) = [1,2\;1,3\;1,4] \\ \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(2) = [1,4\;1,5\;1,6] \\ \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(3) = [1,6\;1,7\;1,8] \end{cases} \tag{63}$$

The triangular fuzzy number $\tilde{A}^{\Delta}_{Cf_{A1}} ef\,(1) = [1,2\;1,3\;1,4]$ characterizes the cash flows of the tangible asset with high risk of impairment,
the fuzzy triangular number $\tilde{A}^{\Delta}_{Cf_{A1}} ef\,(2) = [1,4\;1,5\;1,6]$ characterizes the cash flows of the asset with medium risk of impairment, while the triangular fuzzy number $\tilde{A}^{\Delta}_{Cf_{A1}} ef\,(3) = [1,6\;1,7\;1,8]$ characterizes the asset cash flows with low risk of impairment.

According to Theorem 6, the value of the impairment of assets will be determined with the formula

$$V_{impA1} = w_1 \tilde{A}^{\Delta}_{Cf_{A1}} ef - r_1 K_{inv1} \tag{64}$$

where:

$$w_1 = \frac{r_1 K_{inv1}}{\tilde{A}^{\Delta}_{Cf_{A1}} ef} \tag{65}$$

After calculations, we will have:

$$V_{imp} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(1) = [1,2\;1,4] - 1,5 = \;[-0,3 - 0,1]$$

$$V_{imp} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(2) = [1,4\;1,6] - 1,5 = \;[-0,1 + 0,1]$$

$$V_{imp} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(3) = [1,6\;1,8] - 1,5 = \;[+0,1 + 0,3]$$

Conclusion: It is observed that within the range delimited by the first triangular fuzzy number $V_{imp} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(1) = [1,2\;1,4]$ the tangible asset is completely impaired and it is necessary to record the expense for impairment of the asset whose maximum value is – 0,3 mil € and the minimum value is – 0,1 mil. euro.

Also, in the range delimited by the second fuzzy number $V_{imp} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(2) = [1,4\;1,6]$ the value of the impairment to be recognized is of minimum – 0,1 mil. euro , while for the interval delimited by the third fuzzy number $V_{imp} \tilde{A}^{\Delta}_{Cf_{A1}} ef\,(3) = [1,6\;1,8]$ it is not necessary to recognize the impairment.

Full analysis of impairment was made after a period of time and the cash flows generated by the use of the asset during the operation period were monitored.

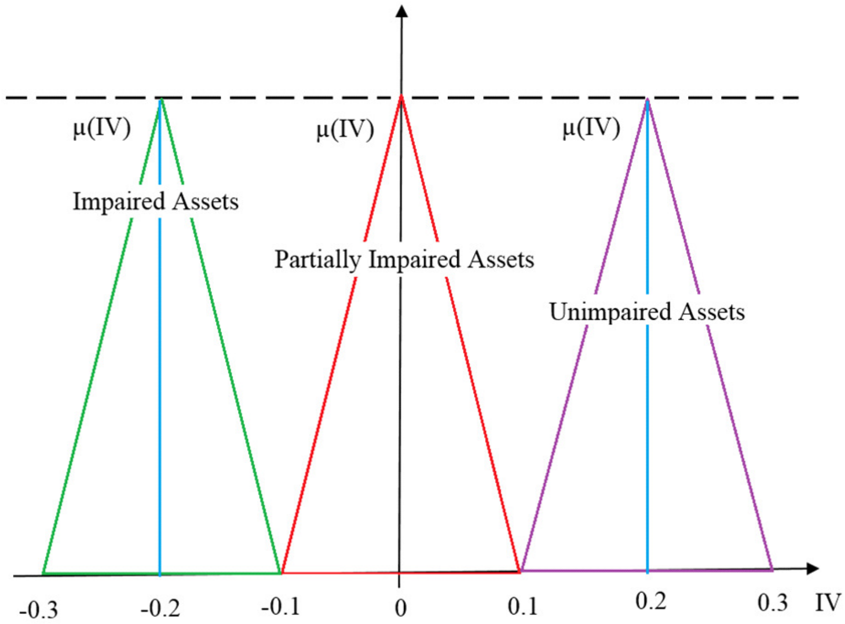

Figure 3: Assets impairment assessment using stochastic fuzzy numbers

6.2 Two assets case

For the two-assets case, were considered the assets $[A_1, A_2]$ for which we know the following:

1. For the (A_1) asset, information on expected future cash flows, actual cash flows and impairment tests are those resulting from calculations for a single asset.
2. For the (A_2), information on the following indicators: the initial investment cost (K_{inv2}), the asset's useful life (Lf_2) 10 years and an investment rate for calculating the 10% impairment, respectively r_2K_{inv2}, as shown in the Figure 4.

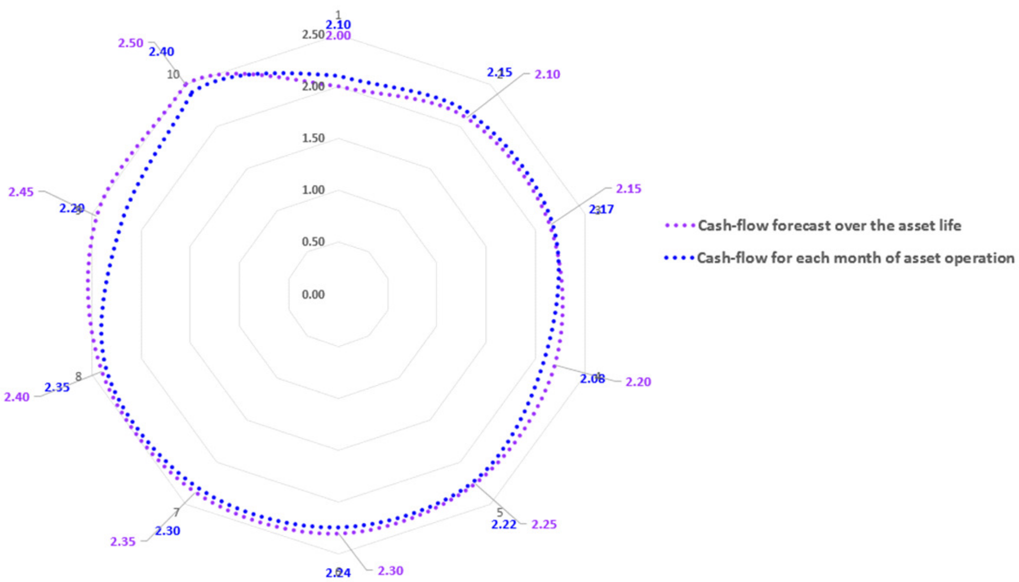

Figure 4: Cash-flow forecast over the asset life and for each month of asset operation

Based on cash flows data, the two statistical indicators are identified; the mean ¯ and the variance œ, respectively μ= 1, 59% and the variance σ= 2, 75%, for which the expected cash flows for the next period of operation are obtained, respectively:$1,8$ *mil. euro* $\leq Cf \leq 2,8$ *mil. euro*.

Within this range, a number based on three triangular fuzzy numbers is also constructed, that characterize the asset A_2 operation, over this range as follows:

$$\begin{cases} \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(1) = [1,8\ 2,13] \\ \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2) = [2,13\ 2,43] \\ \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(3) = [2,43\ 2,80] \end{cases} \tag{66}$$

The fuzzy numbers constructed on the value range $1,8\ mil.\ euro\ \leq Cf \leq 2,8\ mil.\ euro$ characterize the cash flows generated by the tangible asset A_2, as follows:

1. $\tilde{A}^{\Delta}_{Cf_{A2}} ef\,(1) = [1,8\ 2,13]$ characterize the cash flows generated by the tangible asset with *high risk of impairment*;
2. $\tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2) = [2,13\ 2,43]$ characterize the cash flows generated by the tangible asset with *medium risk of impairment*;
3. $\tilde{A}^{\Delta}_{Cf_{A2}} ef\,(3) = [2,43\ 2,80]$ characterize the cash flows generated by the tangible asset with *low risk of impairment*;

According to Theorem 6, the value of the asset impairment will be determined with the formula:

$$V_{impA2} = w_2 \tilde{A}^{\Delta}_{Cf_{A2}} ef - r_2 K_{inv2} \tag{67}$$

Where:

$$w_2 = \frac{r_2 K_{inv2}}{\tilde{A}^{\Delta}_{Cf_{A2}} ef} \tag{68}$$

After calculations, we will have:

$$V_{imp} \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(1) = [1,80\ 2,13] - 2,00 = \ [-0,20 + 0,13]$$

$$V_{imp} \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2) = [2,13\ 2,43] - 2,00 = \ [+0,13 + 0,43]$$

$$V_{imp} \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(3) = [2,43\ 2,80] - 2,00 = \ [+0,43 + 0,60]$$

*Conclusion:*For the (A_2) asset, it follows from the impairment calculations that for the first triangular fuzzy number $\tilde{A}^{\Delta}_{Cf_{A2}} ef\,(1) = [1,80\ 2,13]$ it is necessary to recognize an impairment expense of a maximum value of 0.200 mil. euro, for the second triangular fuzzy number $\tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2) = [2,13\ 2,43]$ as well as for the third triangular fuzzy number $\tilde{A}^{\Delta}_{Cf_{A2}} ef\,(3) = [2,43\ 2,80]$ the asset is not impaired and consequently there is no need to recognize an expense for impairment because the value of impairment is positive respectively $V_{imp} \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(2) = [+0,13 + 0,43]\ \ and\ V_{imp} \tilde{A}^{\Delta}_{Cf_{A2}} ef\,(3) = [+0,43 + 0,60]$.

For the situation where the two assets are taken into account, $(A_1,\ A_2)$, the impairment value will be obtained either by applying Theorem 8 or by summing the values obtained for impairment over the three intervals considered, namely:

$$V_{imp_1} \tilde{A}^{\Delta}_{Cf_{A_1,A_2}} ef\,(1) = [-0,3 - 0,1] + [-0,20 + 0,13] = [-0,50 + 0,03]$$

$$V_{imp_2} \tilde{A}^{\Delta}_{Cf_{A_1,A_2}} ef\,(2) = [-0,1 + 0,1] + \ [+0,13 + 0,43] = [+0,03 + 0,53]$$

$$V_{imp_3} \tilde{A}^{\Delta}_{Cf_{A_1,A_2}} ef\,(3) = [+0,1 + 0,3] + \ [+0,43 + 0,60] = \ [+0,53 + 0,93]$$

There may be C_n^k combinations of intervals that characterize the forecasted cash flows generated by the two assets (A_1, A_2) for the period of operation considered, but the objective is to determine the maximum impairment value that the two assets may register. In our case, the maximum impairment value of the two assets is 0.50 million euro for which the company will record an expense, according to IAS 36. In this case also, full analysis of impairment was made after a certain period of time and generated assets' cash flows were monitored during the operational period.

7 Final conclusions and research limits

The use of stochastic triangular fuzzy numbers in the valuation of impairment of tangible assets enables companies, during the period of using the assets in the current activity, to identify potential impairment risks that may occur when the carrying amount of assets (invested capital) is higher than the discounted cash flows in the value in use of the asset, under the condition that the fair value less costs of disposal of the assets is lower than these cash flows. These asset impairment risks are very important for the company's financial performance for at least two reasons. Firstly, any identified and quantified impairment of assets involves expense recognition, which diminishes the accounting result of the period. Secondly, early identified risks in the operation of tangible assets allow the company to develop strategies and adopt measures to mitigate the impact that asset impairment has on the company's financial performance.

Thus, we consider that there are at least three advantages of using triangular fuzzy numbers:

(1) *first advantage* is that the minimum and maximum value of discounted future cash flows can be estimated on the basis of the cash flows generated by tangible assets in prior periods of operation;

(2) *second advantage* is that the discounted future cash flows generated by the asset's operation can be stratified by means of professional judgments into one or more triangular fuzzy numbers by which the risk of impairment of assets is analysed. The triangular fuzzy numbers thus constructed, in a stochastic environment, allow the identification of those categories of discounted future cash flows of tangible assets with the highest risk of impairment of the assets for which the company has to bear the highest impairment expense;

(3) *third advantage* is that the use of stochastic triangular fuzzy numbers can determine the value of impairment of assets when it exists, depending on the estimated discounted future cash flows. Thus, managers can make realistic estimates of asset's impairment in order to avoid the risk of its financial deterioration in medium and long term.

From the fuzzy modelling of asset's impairment risk, but also from the testing of simulation of this modelling for the situation of a single tangible asset or a group consisting of several tangible assets, it is obvious that value of the impairment for a certain period of time is influenced by three important elements:

(a) *the value of the invested capital* that most often is represented by the carrying amount of the tangible asset which makes the subject to the impairment test;

(b) *the amount of discounted future cash flows* generated by the use of the asset over its useful life;

(c) *the contribution of each tangible asset* for which the impairment test is performed to achieve the company's turnover.

The obtained results allow the asset impairment to be stratified in order to identify those discounted future cash flows ranges for which the company will register impairment costs in line with IFRS's. These impairment values always appear to be negative and are a signal that the financial performance of the company will deteriorate. The positive value of impairment indicates that the company is sheltered from asset impairment risk and its financial performance will not be affected in short and medium term by asset impairment.

Also, the results obtained in this research are the basis of a new chapter in the complex asset impairment field, which allows the quantification of impairment for future periods of time. As far as the limits of this research are concerned, they can be found in scenarios designed to test the risk of impairment of assets and in the construction of the stochastic fuzzy approach. Future research aims to use real data on the assets of listed companies and to build an asset's performance optimization model taking into account all variables from the initial moment of evaluating the cost of assets up to their disposal or derecognition moment.

Author contributions

M.I.B and V.B conceived the study and were responsible for the design and development of the analysis. M.I.B, I.A.B and C.D.S.P were responsible for introduction of stochastic fuzzy numbers.

V.B and D.N.P were responsible for data interpretation. M.I.B wrote the first draft of the article. I.A.B was responsible for the second draft of the study.

Disclosure statement

Authors do not have any competing financial, professional, or personal interests from other parties.

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