Enhanced Interconnection Model in Geographically Interdependent Networks

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Abstract: Interconnection between telecommunication networks and other critical infrastructures is usually established through nodes that are spatially close, generating a geographical interdependency. Previous work has shown that in general, geographically interdependent networks are more robust with respect to cascading failures when the interconnection radius (r) is large. However, to obtain a more realistic model, the allocation of interlinks in geographically interdependent networks should consider other factors. In this paper, an enhanced interconnection model for geographically interdependent networks is presented. The model proposed introduces a new strategy for interconnecting nodes between two geographical networks by limiting the number of interlinks. Results have shown that the model yields promising results to maintain an acceptable level in network robustness under cascading failures with a decrease in the number of interlinks.

Keywords: Cascading failures, interdependent critical infrastructures, robustness, region-based interconnection.

1 Introduction

Interdependent networks depend on a set of Critical Infrastructures (CIs) that function collaboratively to produce and distribute the essential goods and services required for the defence and economic security of nations and the proper functioning of governments and society [11]. Natural disasters (hurricanes, earthquakes, tsunami, tornadoes, floods or forest fires), man-made disasters (Electromagnetic Pulse (EMP) or Weapons of Mass Destruction (WMD) or terrorist attacks), technology-related disasters (power grid blackouts, hardware failures, dam failures or nuclear accidents), or cyber-attacks (viruses, worms or denial of services attacks) are responsible for large-scale disasters in networks [14] [10]. Consequently, failures in critical infrastructures imply service disruptions that affect thousands of people, multiple communities, entire countries, or just one company [9].

Telecommunication networks play a vital role in supporting the control, monitoring, connectivity and data transportation services of a number of critical infrastructures, including banking and finance, emergency and government services, water supply, transportation networks, power grids and oil and gas distribution networks. The interconnection between the nodes of these CIs and telecommunication networks is usually carried out by their spatial proximity. This region-based interconnection model generates a geographically interdependent network in which two

nodes i and j, located in two separate networks are interconnected if the distance (d_{ij}) between them is less than or equal to a given radius (r). Because of such interconnections, failures that occur in one infrastructure can directly or indirectly affect the other and impact large regions with catastrophic consequences [9]. An example of a large-scale failure in interdependent networks is the Italian blackout of 2003, where a single failure in the power grid resulted in failures that propagated over a telecommunications network, ultimately affecting more than 55 million people [2]. Therefore, network topologies, the geographic locations of nodes and their interdependency relationships have a huge impact on how robust interdependent networks are designed and maintained [9].

In contrast to the one-to-one interconnection studied in previous work [2], geographically interdependent networks exhibit a one-to-multiple interdependency model i.e., one node in one network can depend on an arbitrary number of nodes in the other network [16]. In terms of the functional giant component, a geographically interdependent network is more robust with respect to cascading failures when r is large [16]. This is due to the fact that with the increase of r, a node tends to have more interconnection nodes which, in turn, will decrease the probability of that node failing as result of the failures of its interconnection nodes. However, the region-interconnection models proposed in our previous work [16] only consider the geographical distance between nodes to establish the interlink, whereas in most real scenarios, interlink allocation in geographically interdependent networks should be controlled with additional factors in order to mitigate other issues introduced by the large number of interlinks in each r e.g., high deployment cost or exceeding node capabilities.

In the literature, most of the studies have been focused on modifying the interconnection patterns, according to a certain strategy, in order to improve the robustness of interdependent networks against cascading failures. Yagan et al. [17] showed that the regular allocation of bidirectional interlinks always yields stronger robustness than random strategy and unidirectional interlinks do. Li et al. [14] allocated weighted interdependency links under limited budget to obtain a more robust interdependent cyber-physical network. Ji et al. [6] showed that the low Inter Degree-Degree difference addition strategy (IDD) and Random Inter Degree-degree difference addition strategy (RID) are superior to the existing four link addition strategies (random addition, low degree, low betweenness and algebraic connectivity based) in improving the robustness of interdependent networks with high average inter degree-degree difference. However, these studies are focused on interdependent networks where the geographical location of the nodes is not considered to establish an interlink.

J. Martín-Hernández et al. [8] showed the critical number of interlinks beyond which any further inclusion does not enhance the algebraic connectivity (λ_2) of an interdependent network. Therefore, controlling the number of interlinks in geographically interdependent networks is likely a valuable design feature in order to reduce the deployment cost of interdependent networks and not to exceed the capabilities of the nodes to be interconnected. Unlike prior efforts, the major contributions of this paper are: 1) proposing a new strategy for interconnecting nodes between two geographical networks by limiting the number of interlinks and 2) analyzing the impact of limiting the number of interlinks has on the robustness of geographically interdependent networks against cascading failures. As a study case, we focused on interdependent telecommunication networks because they can represent the interconnection of two internet service providers or can refer to multilayer networks. Moreover, in this paper we consider the vulnerability analysis of each network to a certain type of targeted attack to determine the influence the new region-based interconnection model has on the robustness of the resulting interdependent network.

The remainder of this paper is organized as follows: Section 2 describes the proposed interconnection model for region-based interdependent networks and cascading failure process in interdependent networks. Section 3 presents the topologies of the networks to be interconnected
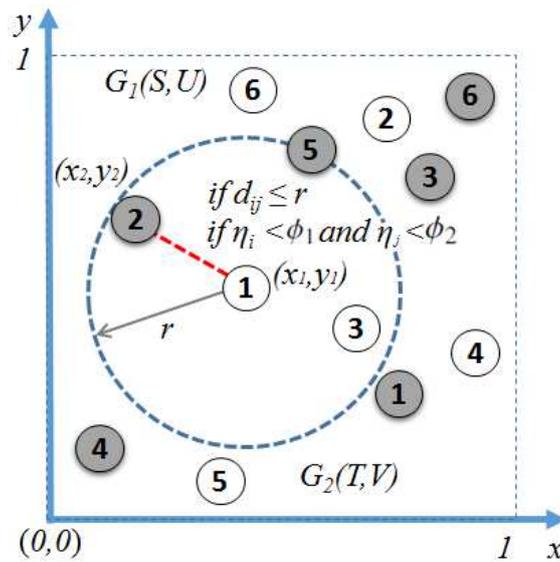
Figure 1: Enhanced interconnection model in geographically interdependent networks

and discusses the impact limiting the number of interlinks has on the robustness of region-based interdependent networks to cascading failures. Finally, Section 4 provides the conclusions and future work.

2   Concepts and models

In addition to the distance between the nodes, interlink allocation in geographic interdependent networks should be controlled by considering factors additional to the geographical constraint. This paper proposes a new region-based interconnection model in which a node i in network G_1 and a node j in network G_2 can be interconnected if 1) the distance d_{ij} between them is less than or equal to a given radius r and 2) the number of interlinks for nodes i and j do not exceed a given percentage for limiting the number of interlinks (ϕ_1 and ϕ_2, respectively). Our new strategy for interlink allocation is based on dividing the nodes in both networks into subsets in accordance with a certain nodal property. Thus, the model prevents ϕ_1 and ϕ_2 being exceeded for any node in G_1 and G_2, respectively.

The model proposed is illustrated in Fig. 1. The nodes in G_1 are represented by filled circles and the nodes in G_2 are represented by unfilled circles. For each node i in G_1, there is a set of nodes in G_2 that can be interconnected if the conditions 1) and 2) are satisfied. Consequently, in contrast to our previous work [16], an enhanced interconnection model for limiting the number of interlinks in geographically interdependent networks is generated. The remainder of this section presents the proposed region-based interconnection model in detail and describes the failure model involving cascading failures.

2.1   Interconnection model for limiting number of interlinks in geographically interdependent networks

Consider two undirected networks $G_1\,(S,U)$ and $G_2\,(T,V)$, each with a set of nodes (S,T) and a set of links (U,V) respectively. Denote N_1 and N_2 as the number of nodes in G_1 and G_2, respectively, and L_1 and L_1 as the number of links in G_1 and G_2, respectively. When G_1 and G_2 interact, a set of bidirectional interlinks I joining the two networks is introduced. Consequently,

an interdependent network is defined as $G(N, L) = (S \cup T, U \cup V \cup I)$ [8]. Let us define the adjacency matrix (A) of G as the $N \times N$ matrix:

$$A_{N \times N} = \begin{pmatrix} A_1 & \alpha B_{12} \\ \alpha B_{12}^T & A_2 \end{pmatrix}, \tag{1}$$

where α represents the coupling strength of the interaction, A_1 is the $N_1 \times N_1$ adjacency matrix of the network G_1, A_2 is the $N_2 \times N_2$ adjacency matrix of the network G_2, and B_{12} is the $N_1 \times N_2$ interconnection matrix representing the interlinks $S_i \leftrightarrow T_j$, between G_1 and G_2. Because we consider bidirectional interlinks, it follows that $B_{21} = B_{12}^T$ [8]. Let b_{ij} denote as the (i, j) entry in the B_{12} matrix, where $b_{ij} = 1$ if the node i and node j are interconnected, and $b_{ij} = 0$ if they are not. The interdependency matrix (B) of the whole system is given by:

$$B_{N \times N} = \begin{pmatrix} 0 & B_{12} \\ B_{12}^T & 0 \end{pmatrix} \tag{2}$$

In the region-based interconnection model previously proposed by us [16], the entry b_{ij} is determined by the geographical location of nodes. Let (x_i, y_i) and (x_j, y_j) denote the spatial coordinates for nodes i and j, then, $b_{ij} = 1$ if the Euclidean distance d_{ij} between node i in G_1 and node j in G_2 is smaller than a given threshold r. This link pattern generates a random geometric graph with a one-to-multiple interdependency model [16]. The Euclidean distances d_{ij} is given by:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{3}$$

In the random geometric graph, a node i in G_1 can depend on an arbitrary number of nodes in G_2 that is no greater than N_2, and vice versa. When the distance between two nodes is considered as the unique interconnection constraint, some issues are evidenced. Specifically, the nodes in one network may have many interlinks from the other network, thus incurring high deployment cost. Note that the cost can be related to the economic investment required to construct an interlink. For instance, in the case of interdependent networks constructed by power grids and telecommunication networks, a new interlink has an associated deployment cost as a function of the cable length. Additionally, nodes in each network have limited capabilities to interconnect to a fixed number of nodes, and so the network's extension requires additional investments. Therefore, limiting the number of interlinks between the nodes in two networks contributes to keeping the deployment cost under control and adjusting to the operator's budget.

Let us define the new factor to be considered in the interconnection of geographically interdependent networks for limiting the number of interlinks in each network. For G_1, this factor is denoted as ϕ_1 and is given by:

$$\phi_1 = \frac{\eta_1}{N_2} \times 100\%, \tag{4}$$

where $\eta_1 \leq N_2$ is the maximum number of nodes from G_2 that each node in G_1 can interconnect to and N_2 is the number of nodes in G_2. Similary, the limit of interlinks (ϕ_2) for nodes in G_2 can be calculated analogous to (4). Therefore, the maximum number of interlinks that each node in G_1 and G_2 can interconnect to is controlled by ϕ_1 and ϕ_2.

As part of our proposal, the nodes in G_1 (G_2) are divided into μ_1 (μ_2) subsets of nodes, each with a maximum of η_1 (η_2) nodes. Subsets of nodes are a key aspect to controlling the allocation of a specific number of interlinks to each node. The number of nodes in a subset is directly related to the capacity of the nodes and the functionality performed by nodes in each network. For instance, in a fixed broadband access architecture, a subset of nodes in the access network can be interconnected to a subset of nodes in the core network. Moreover, a core network can

support the interconnection of a limited number of access nodes. Without loss of generality, a subset of nodes in a network can group nodes with similar properties or randomly. Then, the nodes of a subset in G_1 will be interconnected to the nodes of a subset in G_2 if the distance is less than or equal to a radius (r). As the number of nodes in each subset is limited, the number of interlinks in each node can be kept under control.

Let us consider that the nodes in G_1 are divided into μ_1 subsets of nodes, where μ_1 is given by:

$$\mu_1 = \begin{cases} round(\frac{N_2}{\eta_1}), & \text{if } \phi_1 < 50\% \\ 2, & \text{if } \phi_1 \geq 50\% \end{cases} \tag{5}$$

Similarly, the nodes in G_2 are divided into μ_2 subset of nodes, where μ_2 is given by:

$$\mu_2 = \begin{cases} round(\frac{N_1}{\eta_2}), & \text{if } \phi_2 < 50\% \\ 2, & \text{if } \phi_2 \geq 50\% \end{cases} \tag{6}$$

Let a_i denote the property value of node $i \in G_1$. Then, nodes in G_1 are ordered according to a_i, i.e., $a_1 \geq a_2 \geq ... \geq a_{i-1} \geq a_i \geq a_{i+1} \geq ... \geq a_{N_1-1} \geq a_{N_1}$. Moreover, let Γ_{S_g} denote the ordered set of nodes previously defined in G_1. If $\Gamma_{S_1}, \Gamma_{S_2}, ..., \Gamma_{S_{\mu_1}}$ represent the subsets of Γ_S, then, $\Gamma_S = \bigcup_{g=1}^{\mu_1} \Gamma_{S_g}$, and Γ_{S_g} is given by:

$$\Gamma_{S_g} = \begin{cases} \{i : (g-1) \times \eta_2 < i \leq g \times \eta_2\}, & \text{if } g < \mu_1 \\ \{i : (g-1) \times \eta_2 < i \leq N_1\}, & \text{if } g = \mu_1 \end{cases}, \tag{7}$$

where i represents the $i-th$ element in Γ_{S_g} and $g \in \{1, 2, ..., \mu_1\}$. Similarly, let c_j denote the property value of node $j \in G_2$. Then, nodes $j \in G_2$ are ordered according to c_j, i.e., $c_1 \geq c_2 \geq ... \geq c_{j-1} \geq c_j \geq c_{j+1} \geq ... \geq c_{N_2-1} \geq c_{N_2}$. Additionally, let Γ_{T_h} denote the ordered set of nodes previously defined in G_2. If $\Gamma_{T_1}, \Gamma_{T_2}, ..., \Gamma_{T_{\mu_2}}$ are subsets of Γ_T, then, $\Gamma_T = \bigcup_{H=1}^{\mu_2} \Gamma_{T_h}$, and Γ_{T_h} is given by:

$$\Gamma_{T_h} = \begin{cases} \{j : (h-1) \times \eta_1 < j \leq h \times \eta_1\}, & \text{if } h < \mu_2 \\ \{j : (h-1) \times \eta_1 < j \leq N_2\}, & \text{if } h = \mu_2 \end{cases}, \tag{8}$$

where j represents the $j-th$ element in Γ_{T_h} and $h \in \{1, 2, ..., \mu_2\}$

Let us define B_ϕ as an $N_1 \times N_2$ interconnection matrix, whose entries or elements are $b_{\phi_{ij}} = 1$ if nodes in the subset Γ_{S_g} are connected to nodes in the subset Γ_{T_h} for $g = h$, otherwise $b_{\phi_{ij}} = 0$. Accordingly, the B_ϕ matrix defines which nodes in the networks can be interconnected and establishes the limit for the number of interlinks that each node in the networks can handle. Thus, each node in G_1 or G_2 will have a maximum of η_1 or η_2 interconnected nodes, respectively.

Finally, let us redefine the dependency matrix B_{12}, whose entries are $b_{ji} = 1$ if $d_{ij} \leq r$ and $b_{\phi_{ij}} = 1$, otherwise $b_{ij} = 0$. Note that the new B_{12} matrix captures the interconnection conditions 1) and 2) proposed in this paper and thus the new interdependency matrix B, which is given by the equation (2), can be generated. Therefore, the nodes in each geographical network will interconnect with a limited number of interlinks, consequently improving the model defined in [16].

For simplicity, in this paper we consider that G_1 and G_2 have the same number of nodes ($N_1 = N_2$) and that all the nodes in the interdependent network have the same limit of interlinks ($\phi_1 = \phi_2$). Therefore, each network has $\mu_1 = \mu_2$ subsets of nodes with a maximum number of nodes $\eta_1 = \eta_2$. Figure 2 presents two geographical networks being interconnected by employing the interconnection proposal described in this section. As can be seen in Fig. 2, both networks
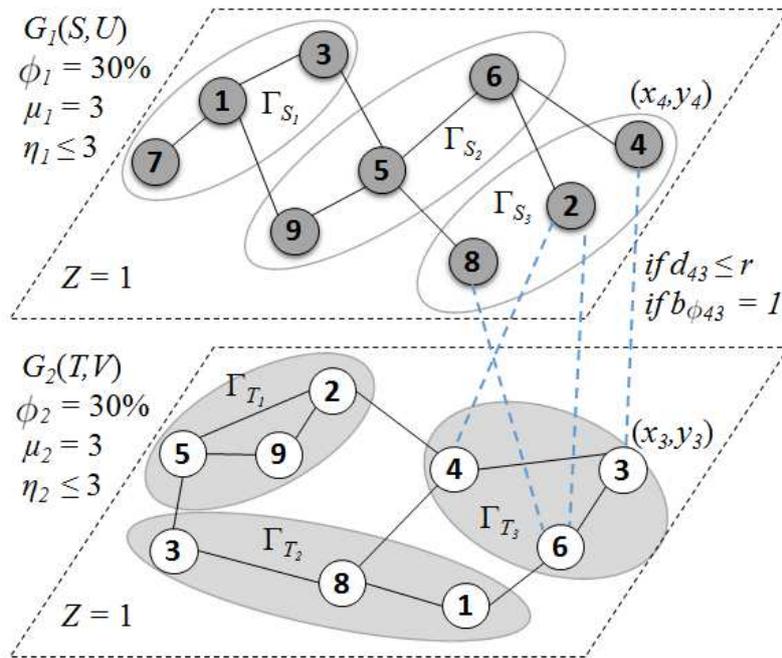
Figure 2: Subsets for limiting the number of interlinks in geographically interdependent networks

have $N_1 = N_2 = 9$ nodes and each node in G_1 and G_2 can support until $\phi_1 = \phi_2 = 30\%$ of nodes from the other. According to what has been described above, nodes in both networks are divided into $\mu_1 = \mu_2 = 3$ subsets, each one with a maximum of $\eta_1 = \eta_2 = 3$ nodes. Then, the B_ϕ matrix is generated with the subsets Γ_{S_g} and Γ_{T_h}. Finally, the interlinks between the nodes from G_1 and G_2 (dashed lines) are established if $d_{ij} \leq r$ and $b_{\phi_{ij}} = 1$.

2.2 Algorithm description

*Algorithm*1 summarizes the interconnection model proposed to limit the number of interlinks in geographically interdependent networks. *Algorithm*1 requires two networks (G_1 and G_2) to be interconnected, the percentage for limiting the number of interlinks (ϕ_1 and ϕ_2) and the radius (r). The output of *Algorithm*1 is a dependency matrix B_{12} with the conditions 1) and 2) previously described. As can be seen, *Algorithm*1 calculates the maximum number of nodes that a node can interconnect to (Lines 1 and 2) and the number of subsets (Lines 3 and 4). Then, the nodes are grouped in subsets according to one property (Lines 5 and 6). The interconnection matrix ($B_{\phi_{12}}$), in which each node in G_1 (G_2) has a maximum of η_1 (η_2) interconnected nodes (Line 7) is generated. Finally, the interdependency matrix B_{12} is generated by considering the distance constraint for a given r and the B_ϕ matrix (Lines 8 to 19). Thus, an enhanced region-based interconnection model is defined for interconnecting the G_1 and G_2 networks and the interdependency matrix B, which is given by the equation (2), can be generated from the resulting B_{12} matrix.

2.3 Cascading failure process in interdependent networks

Consider a geographically interdependent network G generated from the model proposed in this paper. When a random fraction of the nodes in G_1 fails, a cascading failure process is induced. We assume the node i in network G_1 is functional if a) at least one of its interconnected nodes in network G_2 is operative, and b) the node i belongs to the giant component of the

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Algorithm 1 Interconnection model for limiting the number of interlinks in geographically interdependent networks.

Data: two geographical networks (G_1 and G_2), limit for number of interlinks (ϕ_1 and ϕ_2) and radius (r).

Result: dependency matrix B_{12}.

$\eta_1 = round(\phi_1 N_2 / 100)$
$\eta_2 = round(\phi_2 N_1 / 100)$
$\mu_1 = round(N_1 / \eta_2)$
$\mu_2 = round(N_2 / \eta_1)$
$\Gamma_{S_g} \leftarrow getSubsetNodes(S, \mu_1, \eta_2, nodal\_property)$
$\Gamma_{T_h} \leftarrow getSubsetNodes(T, \mu_2, \eta_1, nodal\_property)$
$B_\phi \leftarrow getB_\phi Matrix(\Gamma_{S_g}, \Gamma_{T_h}, \eta_1, \eta_2, \mu_1, \mu_2)$
for all $i \in S$ **do**
    for all $j \in T$ **do**
        $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
        if $d_{ij} \leq r$ and $b_{\phi_{ij}} == 1$ **then**
            $b_{ij} = 1$
        else
            $b_{ij} = 0$
        end if
    end for
end for
    return B_{12}

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functional nodes in network G_1 [4]. Due to interdependency, the failed nodes in G_1 spread failures in G_2. As the assumptions a) and b) are also applied to the nodes j in network G_2, the failed nodes in G_2 spread failures back into G_1, and so on. The cascading failures continue until no more nodes fail. The remaining set of functional nodes is referred to as the *Largest Mutually Connected Component* (*LMCC*):

$$LMCC = \frac{n_1 + n_2}{N_1 + N_2},\tag{9}$$

where n_1 and n_2 are the number of nodes that belong to the giant component of the functional nodes in G_1 and G_2, respectively, when the assumptions a) and b) are satisfied. The cascading failures described in this section can occur in real scenarios such as power grid blackouts [1] and disruptions in economic networks [15]. Note that [16] also considered the case in which a node in G_1 is functional if all of its interconnected nodes in G_2 are operational. Under that condition, in some cases, having more interconnected links makes the geographically interdependent network less robust. However, this case is outside the scope of this paper.

3 Simulation results and discussion

In this section, the topologies for geographically interdependent networks are described. Moreover, the impact limiting the number of interlinks has on the robustness of geographically interdependent network is analyzed.

Table 1: Nodes distribution in G_1 and G_2 according to interlink limits

$\phi_1 = \phi_2$	$\mu_1 = \mu_2$	$\eta_1 = \eta_2$
10%	10	5
25%	4	12
50%	2	25
75%	2	37
100%	1	50

3.1   Topologies for geographically independent networks

The geographically interdependent networks considered as the study case represent two backbone telecommunication networks being interconnected with bidirectional interlinks. The random connection property of a backbone telecommunications network is modeled using an Erdös-Rényi (ER) random graph with a Poisson nodal degree distribution [12]. This indicates that most nodes have approximately the same number of links close to the average nodal degree [3]. Although, scale-free or other graph models can be also used to model telecommunication networks, these are more associated to large networks (such as multi-autonomous systems networks). Moreover, some current backbone topologies are also scaling to other models which are out of the scope of this paper.

In order to analyze the impact the model proposed has on the robustness of interdependent networks against cascading failures, the Largest Mutually Connected Component ($LMCC$) is measured in 100 interdependent telecommunication networks. Each backbone telecommunication network to be interconnected is modeled as an ER random graph with $N_1 = N_2 = 50$ nodes and the average nodal degree ($\langle k \rangle$) equal to 6. The nodes in each network are placed uniformly in a two-dimensional square of the size $Z = 1$ i.e., each node in the G_1 and G_2 networks has as spatial coordinates (x, y), where $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

The interconnection link pattern between the two ER graphs is conditioned by a given radius r. The number of interlinks in each node is limited by a given percentage ϕ. The number of subsets (μ_1, μ_2) and the maximum number of nodes that a node in G_1 and G_2 can interconnect with (η_1, η_2) are presented in Table 1. For instance, when $\phi = 25\%$, this is considered as the design constraint and, as such, the nodes in each network are divided into $\mu_1 = \mu_2 = 4$ subsets. Thus, for a given radius r, it is expected that each node in G_1 and G_2 will have a maximum of 12 interlinks.

As was described in subsection 2.1, a nodal property is also required to define how nodes in each network can be grouped. In the study case considered in this paper, node vulnerability to failures is selected as the property with which to group the nodes into subsets. In most real scenarios, the vulnerability of nodes to failures can be estimated from the historical failure database of their Operation Support Systems (OSS). However, given the difficulty of obtaining access to real data, centrality metrics could be used to measure the importance of nodes for the network connectivity under some failure scenarios [5]. Previous studies have revealed that backbone telecommunication networks modeled as ER are highly vulnerable to a sequential targeted attack based on nodal betweenness centrality (b_c) [13].

Figure 3 depicts a robustness analysis of the backbone telecommunication networks under targeted attacks when networks are not connected to other. The networks' robustness is quantified as a function of the Average Two Terminal Reliability ($ATTR$) metric [9]. As can be seen in Fig. 3, the telecommunication networks considered in this work exhibit high vulnerability to a sequential targeted attack by b_c. Whereas, the networks are more robust to a simultaneous
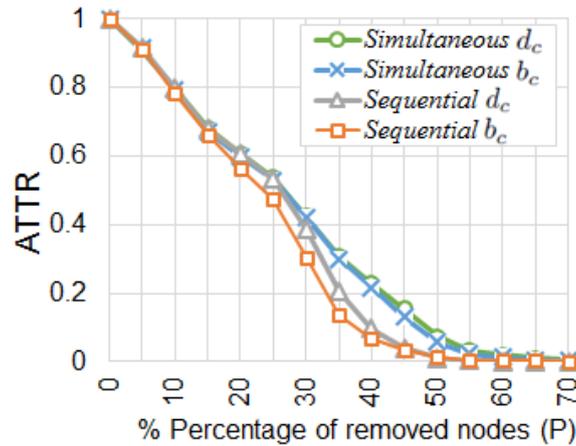
Figure 3: Robustness analysis of backbone telecommunication networks ($N_1 = 50$ and $\langle k \rangle = 6$) in a single scenario

targeted attack by b_c and sequential or simultaneous targeted attacks based on degree centrality (d_c). Consequently, node vulnerability in each ER network could be quantified by their b_c values i.e., the higher the betweenness centrality of node is, the higher the node's vulnerability is.

3.2   Analyzing the impact limiting the number of interlinks has on the robustness of geographically interdependent networks

To investigate the impact the region-based interconnection model has on the robustness of interdependent networks against cascading failures, the Largest Mutually Connected Component ($LMCC$) metric is measured when a fraction of nodes is removed. In the failure scenario considered in this paper, nodes in the network G_1 are removed (according to their vulnerability to a sequential targeted attack by b_c) until the percentage of removed nodes (P) is reached. Removing the nodes in G_1 leads to a cascading failure process as described in Section 2.3.

Although several geographically interdependent networks can be generated by varying the radius and the limit of the number of interlinks, the two scenarios considered as case studies are:

- Scenario 1: The radius (r) is fixed to 0.2 and the limit for the number of interlinks (ϕ) ranges from 25% to 100%. This scenario can represent a real situation in which a telecommunication network operator has a geographical area limited by a radius r and is interested in controlling the number of interlinks to other infrastructures.

- Scenario 2: The number of interlinks is limited to 25% and r is varied from 0.1 to $\sqrt{2}$. This scenario can be used by a telecommunication network operator who has a certain capacity in their network, but wants to restrict its coverage area to a certain radius r to interconnect to fewer number of nodes from other infrastructures.

Both scenarios are replicated in 100 interdependent networks. The robustness analysis presented in this section is the average of the $LMCC$ results measured in these interdependent networks.

Scenario 1: Robustness analysis in geographically interdependent networks against variations in the limit of interlinks (ϕ)

In this scenario, the radius (r) to interconnect the G_1 and G_2 networks is fixed to 0.2. Then, for a given limit in the number of interlinks (ϕ), the $LMCC$ of an interdependent network is
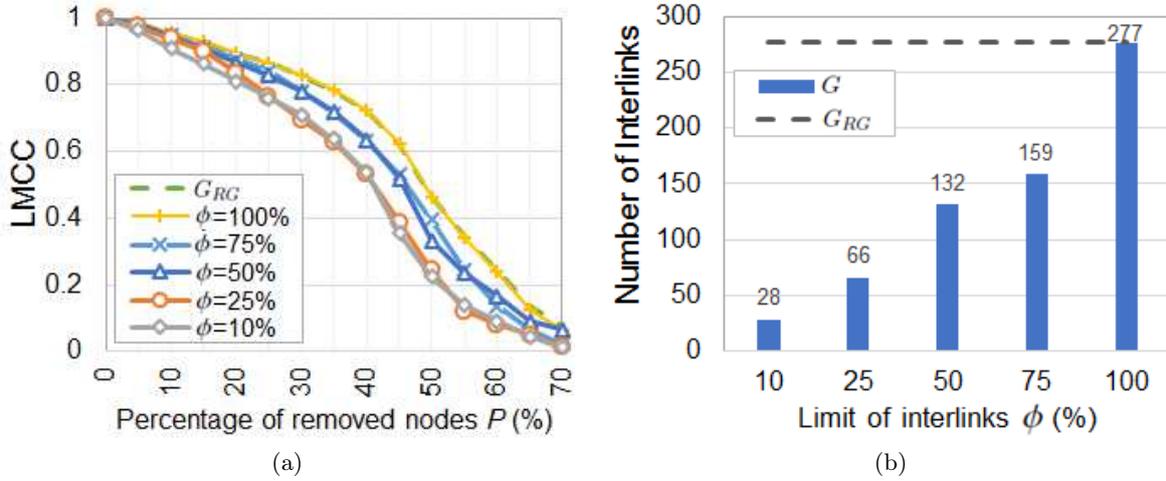
Figure 4: Robustness analysis in geographically interdependent networks ($r = 0.2$) versus variations in the limit of interlinks (ϕ) a) Largest Mutually Connected Component ($LMCC$) as a function of the removed nodes (P) b) Number of interlinks as a function of ϕ

measured when a fraction of nodes (P) is removed in the G_1 network. Figure 4a depicts that for a given ϕ the $LMCC$ first decreases almost linearly with the increase in the fraction of removed nodes ($P \leq 35\%$). Later, the $LMCC$ dramatically decreases until the networks are completely disconnected. Networks with the highest slope in their $LMCC$ curves are those that have less ϕ. This is because with the decrease of ϕ, nodes in the G_1 and G_2 networks are divided into more subsets (μ_1 and μ_2, respectively) which decreases the probability for interconnecting a large number of nodes. Consequently, a node has fewer interconnected nodes and its failure probability is increased thanks to the failures of its interconnected nodes.

Also note that in Fig. 4a there is a zone ($P \leq 20\%$) in which the robustness of interdependent networks for a given ϕ is similar to the robustness reached by a network modeled according to [16] with $r = 0.2$ and without limiting the number of interlinks (G_{RG}). Moreover, in this zone all networks exhibit a high level of robustness against cascading failures ($LMCC > 0.8$). For example, when 20% of the nodes are removed from G_1 and after the cascading failure process, $LMCC = 0.89$ for $\phi = 100\%$ and 0.81 for $\phi = 10\%$. However, for $P > 20\%$, there are more differences between the $LMCC$ values reached by the networks with $\phi \leq 25\%$ and the network G_{RG}. However, in the case of networks with $\phi \geq 50\%$, their robustness remains near to that achieved by G_{RG} until $P \leq 40\%$. Therefore, for some P values, our model is able to maintain the $LMCC$ in values near those achieved by our previous work [16] when the number of interlinks is limited to a certain value of ϕ.

On other hand, as can be seen in Fig. 4b, the number of interlinks is under the maximum number of interlinks reached by the G_{RG} network for $\phi < 100\%$ (compare the dashed line G_{RG} and the blue bars G). This result is due to the strategy proposed in this paper whereby the nodes in the G_1 and G_2 networks are divided into subsets, with a maximum number of nodes η_1 and η_2, respectively. Thus, our new region-based interconnection model guarantees that the number of interlinks in geographically interdependent networks is maintained below the limit ϕ. For instance, when $\phi = 75\%$, the maximum number of interlinks in the interdependent networks is 159. Although this value is not exactly 75% of the maximum number of interlinks, it is below the limit of interlinks considered to be a design constraint.
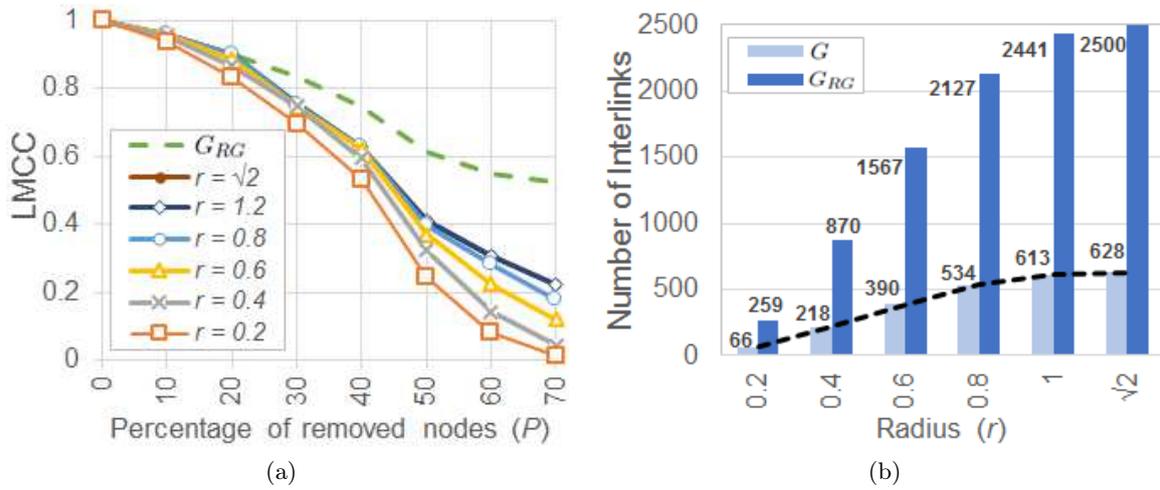
Figure 5: Robustness analysis in geographically interdependent networks ($\phi = 25\%$) versus variations in radius (r) a) Largest Mutually Connected Component ($LMCC$) as a function of removed nodes (P) b) Number of interlinks as a function of r

Scenario 2: Robustness analysis in geographically interdependent networks against variations in radius (r)

In this scenario, the interdependent telecommunication networks are the result of interconnecting the G_1 and G_2 networks by limiting the interlinks (ϕ) to 25% and varying the radius (r). The Largest Mutually Connected Component ($LMCC$) as a function of the fraction of removed nodes from the G_1 network is shown in Fig. 5a. Although the number of interlinks is limited to 25%, Fig.5a shows that geographically interdependent networks better resist cascading failures because of a major number of interlinks when the r is large. This result is to be expected as the nodes in the G_1 and G_2 networks tend to be more probable to interconnect to a greater number of nodes as a wide geographical area is defined by a larger radius r. For example, when 20% of the nodes are removed from G_1 and after the cascading failure process, $LMCC = 0.90$ for $r = 1.2$ and 0.83 for $r = 0.2$.

Additionally, Fig. 5a depicts a zone ($P \leq 20\%$) in which the robustness of geographically interdependent networks for a given radius r remains near to the robustness of a network modeled according to [16] where $r = \sqrt{2}$ and the number of interlinks is not limited (G_{RG}). In this zone, all geographically interdependent networks have the $LMCC > 0.8$. As the percentage of removed nodes in G_1 increases, the networks modeled with our new proposal maintain similar robustness levels until $P \leq 40\%$. Consequently, limiting the number of interlinks to a certain percentage ϕ trends to control interlink allocation against increases in radius r. Thus, our proposal based on subsets is effective in limiting the number of interlinks in geographically interdependent networks.

Regarding the number of interlinks, Fig. 5b shows that for a given radius r our model generates interdependent networks where the interlinks are around 25% of the maximum reached by each network G_{RG} (compare light blue and dark blue bars).The reason is because, independent of the selected radius (r), the model proposed in this paper restricts the number of nodes that a node in the G_1 and G_2 networks can interconnect with to η_1 and η_2, respectively. For example, when $r = 0.6$, the maximum number of interlinks in the interdependent networks is 390, and the number of interlinks per node is 4 on average. Consequently, for some P values our model yields promising results for maintaining network robustness under cascading failures by reducing the number of interlinks.

4   Conclusions

In this paper, an enhanced interconnection model in geographically interdependent networks has been proposed. In contrast to previous work, a new strategy based on subsets of nodes has been proposed to limit the number of interlinks in interdependent networks. The proposed region-based interconnection model has considered the percentage with which to limit the number of interlinks (ϕ) as a new factor in the design of geographically interdependent networks. Moreover, the impact limiting the number of interlinks has on the robustness of geographically interdependent networks against cascading failures has been analyzed.

The interconnection strategy proposed in this paper has proven to be effective in guaranteeing the number of interlinks in geographically interdependent networks is maintained under a certain limit ϕ. This is because for a given ϕ the nodes to be interconnected have been divided into subsets (μ_1, μ_2), each with a maximum number of nodes (η_1, η_2). Results indicate that in some scenarios ($P \leq 20\%$) the robustness for a given ϕ has been maintained at levels close to those reached by [16] ($LMCC \geq 0.80$). This is a relevant outcome because compared to the critical threshold at which $LMCC$ equals zero, quantifying the impact of a small percentage of node failures (P) is essential for network providers to prevent networks from collapsing.

Furthermore, the two scenarios that have been analyzed in this paper represent some situations in which the model proposed can be applied by network providers. Results have shown the robustness behaviour for geographically interdependent networks under cascading failures. In the first case, by limiting the coverage area to a certain radius r and varying the number of interlinks (ϕ), an interdependent networks is more robust against cascading failures when ϕ is increased. Meanwhile, in the second case, by limiting the number of interlinks to a certain ϕ and varying the radius r, the robustness increases for large values of r. In both cases, the results are because with the increase in the number of interlinks, a node tends to be less likely to fail from the failures of its interconnection nodes.

In the future work, the proposed region-based interconnection model can be studied in other interdependent networks and validated with real-world data. Moreover, an in-depth cost-benefit analysis of limiting the number of interlinks in geographically interdependent networks can be carried out.

Acknowledgements

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