ANALYZING THE AHP PRIORITY VECTORS: GOING BEYOND INCONSISTENCY INDEXES

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ABSTRACT

A great concern when utilizing the Analytic Hierarchy Process (AHP) is how the final priority vector, resulting from the inconsistent analysis, behaves when compared with the original priority ranking, resulting from the consistent analysis. The AHP utilizes an inconsistency index to predict rank reversal. In addition to the original inconsistency index of the AHP, several authors have worked on developing alternative inconsistency indexes, with the goal of improving the predictability of rank reversal. However, inconsistency indexes do not help clarify whether a rank reversal is a rejectable outcome or, to some extent, the correct answer. A rank reversal may express the correct priority, particularly when some positions in the original priority rank have small weight differences among them. Therefore, it is very important to develop a method to allow a clear and definitive analysis on how disturbed the weights and ranking of the final priority vectors are when compared to their original consistent rankings. Such a method is developed here and its utilization is demonstrated by analyzing a corporate governance scenario.

Keywords: AHP; MCDA; rank reversal; inconsistency index; corporate governance

1. Introduction

The AHP works to support decisions as different criteria and alternatives are considered. These criteria and alternatives can combine objective and subjective parameters. The AHP, as proposed by Saaty (1977), generates a priority vector from a Pairwise Comparison Matrix (PCM). Such pairwise comparison, made by the Decision Makers (DM) involved in the respective analysis, carries a certain level of inconsistency derived from the inherent subjectivity of human scrutiny.

This ability of the AHP to absorb a certain level of inconsistency is highly valuable. Saaty (1997) emphasizes that the AHP places special focus on integrating human judgment into decision-making process and on evaluating the consistency of such judgments. Saaty (1977) also comments that a certain level of inconsistency in a PCM does not necessarily affect the ranking of the final priority vector.

Grybowski and Starczewski (2020) introduced the *SWFR – significantly-wrong-final-ranking* – as a concept to analyze whether a certain rank reversal should represent the rejection of the final priority vector (FPV). Buede and Maxwell (1995) state that "Through formal and informal discussions about rank reversal, the focus has been on assessing whether the problem exists, what the reasons are, as well as whether the rank reversal is the problem or the desired response". Wang and Triantaphyllou (2008) investigated the rank reversal that occurred in a case of MCDA, not only analyzing the inconsistency index, but analyzing the changes that actually occurred in the ranking positions. Different authors have worked on the development of other inconsistency indexes besides the one originally proposed by Saaty (2009). Saaty's inconsistency index is defined as $IC = (\lambda max - n)/(n - 1)$ where λmax is the maximum eigenvalue of the PCM. Bozóki and Rapcsák (2015) presented alternative inconsistency indexes that have their particular threshold of acceptability, while Grzybowski and Starczewski (2020) introduced the so-called IC ATIA. However, although these are serious contributions to the assessment of the inconsistency in a pairwise comparison, these indexes do not contribute to the assessment of the real impact on final ranking positions. Furthermore, it is important to note that even in the case of no rank reversal, an investigation into the disturbance that occurred in the FPV may be of interest. Cases, for example, that involve the allocation of resources among different alternatives may imply a deeper evaluation on the weight distribution along the ranking positions, even if no reversal has been observed.

Finally, a certain magnitude of rank reversal can be accepted, as it can represent proper prioritization analysis. For example, this occurs in cases where the DM made some pairwise comparisons considering a very similar level of relevance (in the limit, a tie). In these cases, the analysis of inconsistencies can lead to some sorting reversal that, in fact, is adequately clarifying and segregating the criteria judgement performed by the DMs. These considerations support the need to establish a methodology that goes beyond inconsistency indexes to allow an easy, clear, complete and definitive analysis of the FPV generated via the AHP.

To test the method and metrics developed here to evaluate the FPV, an application in a corporate governance scenario is analyzed. A governance maturity analysis has been chosen since, generally, the relevance of the governance criteria do not differ that much. The subjectivity of governance issues is highlighted by IBGC (2015)[1] "In the exercise of corporate governance, the topics dealt with are often subjective and ambiguous, which requires from the governance agents the ability to assess, reason and judge".

The development of the proposed methods and the results regarding the prioritization of the governance theme are presented in this study. The analysis of governance issues, with the support of the methods developed here, also compared the use of the original scale by Saaty (OSS) (1997) and the Generalized Balanced Scale (GBS), as proposed by Goepel (2018).

2. Technical background

2.1 The original AHP methodology

The AHP considers the Perron-Frobenius theory, where the maximum autovector (maximum eigenvector) of a matrix containing positive values forms the priority vector. Furthemore, Saaty (1977) stipulates that a pairwise comparison is the best

[1] IBGC: Brazilian Institute of Corporate Governance.

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method that humans can use to compare different criteria or alternatives. Saaty (1977) refers to the studies by Weber (1846) and Fechner (1860), which focus on the evaluation of human response to stimuli, to define the scale adopted in the original AHP methodology. Saaty (1977) defines a scale from 1 to 9 (Original Saaty Scale – OSS), linked to nine levels of criteria relevance. The pairwise comparison can introduce an inconsistency into the Pairwise Comparison Matrix (PCM). This inconsistency is a result of the fact that DMs may not maintain perfect proportionality among their comparisons in their pairwise comparisons. When the comparison maintains proper proportionality, there will be a consistent PCM and when not, an inconsistent one. In principle, the inconsistency embodies the subjectivity of human judgements.

Due to such inconsistencies, Saaty (1977) defined the Consistency Ratio (CR) with a threshold of 10%[2] as the limit for accepting the AHP autovector. The figures below summarize the main steps and concepts of the AHP methodology.

1	2	3	4	5	6	7	8	9
Equal Importance	Intermediate	Moderate Importance	Intermediate	Strong Importance	Intermediate	Very Strong Importance	Intermediate	Extreme Importance

Figure 1 Original Saaty Scale (OSS)

	Pairwise Comparison Matrix - PCM Parameters (p1 to Pf = weights 1 to 9) of Criteria (C1 to Cn)					
	C1	**C2**	**C3**	**C4**	**.......**	**Cn**
C1	1,0 (P1/p1)	P2/P1	P3/P1	P4/P1	....	Pf/P1
C2	P1/P2	1,0 (P2/p2)	P3/P2	P4/P2	....	Pf/P2
C3	P1/P3	P2/P3	1,0 (P3/p3)	P4/P3	....	Pf/P3
C4	P1/P4	P2/P4	P3/P4	1,0 (P4/p4)	......	Pf/P4
....	....	....	....	....	....	......
Cn	P1/Pn	P2/Pn	P3/Pn	P4/Pn	......	1,0 (Pf/Pn)

	Diagonal = 1,0 once a Criteria is compared with itself
	First Line = Consistent Comparisons (All Criterias compared with first one)
	Region of potential Inconsistencies (comparisons not obligatorily proportional to *Fisrt Line*)
	Region of reciprocal comparisons

Figure 2 Pairwise Comparison Matrix (PCM)

The Final Priority Vector (FPV) $\mathbf{w}_{(i,j)}$ calculated in the AHP method is defined as:

$[\mathbf{w}_{1,j}] = [\mathbf{A}_{i,j}] \times [\mathbf{C}_{1,j}]$, where:
$[\mathbf{w}_{1,j}]$ = Final Priority Vector (FPV);
$[\mathbf{A}_{i,j}]$ = Matrix of normalized priority vectors of the alternatives
$[\mathbf{C}_{1,j}]$ = Criteria eigenvector

and the Saaty inconsistency index is defined as $CI = ((\lambda max - n))/((n-1))$, where $\boldsymbol{\lambda}\mathbf{max}$ = PCM Autovalue and $\mathbf{n}$ = number of criteria.

[2] Consistency Ratio: the relation between CI and RI. RI is the random consistency index as per Saaty (1987)

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2.2 A case study: corporate governance maturity assessment

To test the method and metrics developed here to assess the FPV, an application in a corporate governance scenario was analyzed. To this end, a methodology was structured to assess the maturity of governance based on the proposal by Álvares, Giacometti and Gusso (2008). Botelho (2021) presents more details about the choice of this methodology and the tool that was developed to assess the maturity of corporate governance. The evolution of each element is fictitious (ranging from 30% to 100%) and the pairwise comparison of the criteria involved in this governance maturity was obtained from directors invited from seven different companies**.** The AHP methodology was inserted into the aforementioned governance analysis tool, generating an agenda where, taking into account the pairwise criteria comparison provided by the seven board members, the elements' priorities (the alternatives) were prioritized. Therefore, this exercise allowed a comparison between the original ranking (derived from consistent PCM) and the FPV. In addition, a comparison was also made between the FPV and the priority agenda organized via the AHP. Last but not least, both sets of comparisons were performed using OSS and GBS balances. Figure 3 presents a summary of the mentioned tool, and Figure 4 shows an example of a priority agenda generated by the AHP.

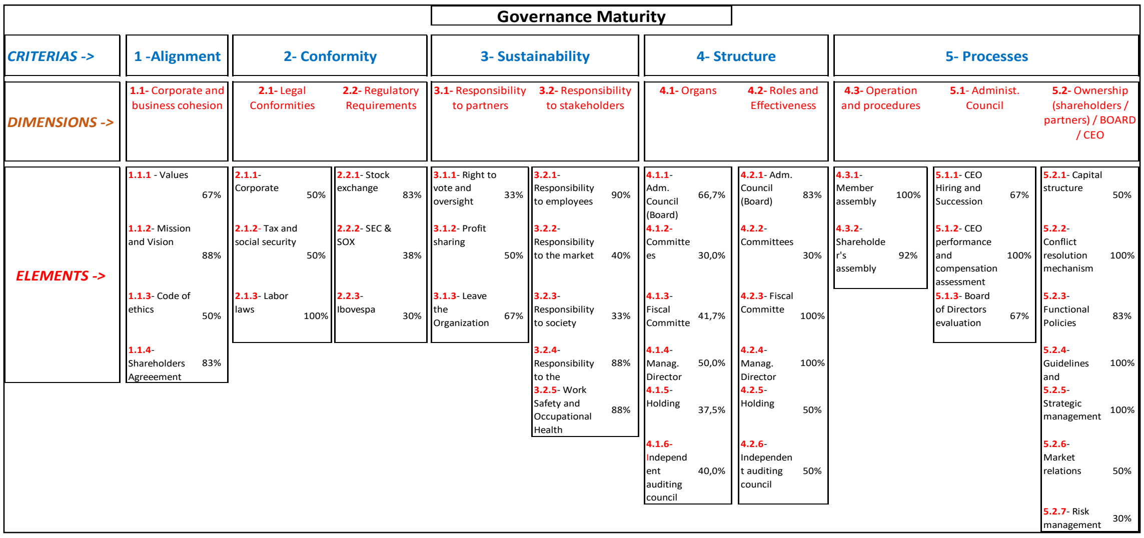

Figure 3 Hierarchy analysis of governance maturity

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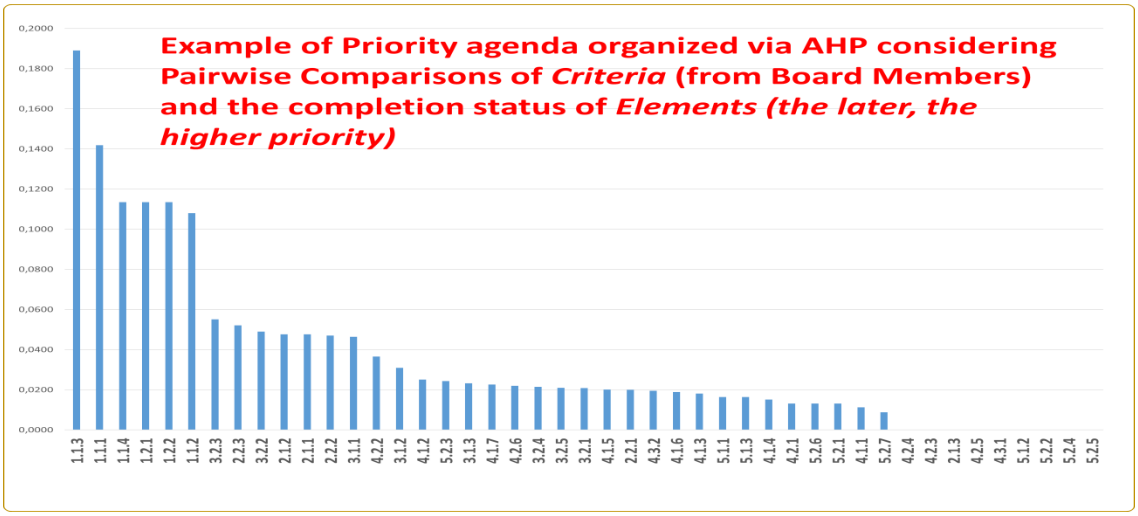

Figure 4 Priority agenda (via AHP over the hierarchy analysis of governance maturity)

2.3 An alternative scale

Several authors have worked on other scales of development besides the OSS. The goal with these alternative scales is to improve the predictability of rank reversal. This study compares the use of the OSS and an alternative scale, an AHP applied process of prioritizing corporate governance criteria, as presented by Botelho (2021).

Goepel (2018) compared different weight scales and introduced the Generalized Balanced Scale (GBS). Goepel (2018) demonstrated that GBS performed better reaching small weight uncertainty and weight dispersion compared to the others. Therefore, Botelho (2021) and this study adopted the GBS as the alternative scale to be applied in the aforementioned AHP governance prioritizing exercise.

The GBS is defined as:

$$\boldsymbol{c} = \frac{[9+(n-1)x]}{9+n-x}$$

where c = weight value in GBS; n = number of criteria and x = weight value in OSS.

Figure 5 shows GBS weights compared with OSS.

OSS	GBS n=3	GBS n=4	GBS n=5	GBS n=6	GB
1	1,0	1,0	1,0	1,0	
2	1,3	1,4	1,4	1,5	
3	1,7	1,8	1,9	2,0	
4	2,1	2,3	2,5	2,6	
5	2,7	3,0	3,2	3,4	
6	3,5	3,9	4,1	4,3	
7	4,6	5,0	5,3	5,5	
8	6,3	6,6	6,8	7,0	
9	9,0	9,0	9,0	9,0	

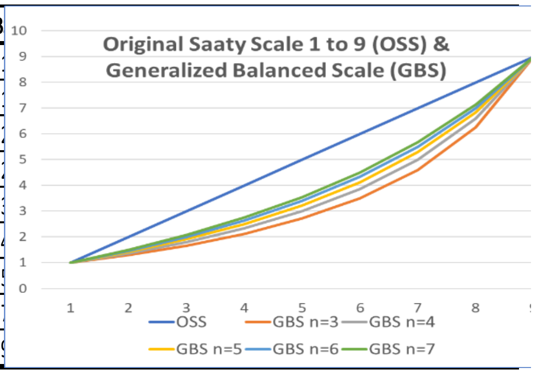

Figure 5 OSS & GBS

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3. Methodology

This study can be classified as applied research, as it aims to generate knowledge for the practical application of scenario analysis. In addition, it uses a practical case study to address the proposal. The methodology developed here, as mentioned in the introduction, presents a complete criterion for evaluating the priority vectors generated by AHP independently and not limited to the consistency indices.

Below, the development of this new approach is demonstrated with respect to a complete analysis of the level of perturbation suffered by the FPVs when compared to the original ordering vectors.

3.1 Analyzing final priority vectors <u>without</u> rank reversal occurrences

Given an Initial Priority Vector (IPV) [$\mathbf{w}_{(i)1}$, $\mathbf{w}_{(i)2}$, $\mathbf{w}_{(i)3}$, ... $\mathbf{w}_{(i)\ [n-1]}$, $\mathbf{w}_{(i)n}$] (the eigenvector of the consistent PCM) and a final one (FPV) [$\mathbf{w}_{(f)1}$, $\mathbf{w}_{(f)2}$, $\mathbf{w}_{(f)3}$,.... $\mathbf{w}_{(f)[n-1]}$, $\mathbf{w}_{(f)n}$] (the eigenvector of the inconsistent PCM), the absolute difference between the weights (**wf - wi**), taken in inverse proportion to their positions in the ranking, weights the intensity of the disturbance, providing a No Reversion Index (NRI), as follows:

$$\text{NRI}= \sum_{i=1}^{n}[wi(i) - wi(f)]/i \qquad (1)$$

See Figure 6 for a NRI definition and Figures 7 and 8 for a NRI histogram and NRI cumulative frequency, respectively.

Furthermore, it established a metric to qualify the level of disturbance without rank reversal. For that, a Monte Carlo analysis was performed evaluating 13.200 random simulations without rank reversal. This Monte Carlo exercise considered a vector with 5 positions in the ranking (n=5). It has defined the “No Reversal Quality Vector” (NRQV), according to Equation 2.

IPV (Consistent)	FPV (Inconsistent)	$NRI = \sum_{i=1}^{n} \lvert[(wi(i) - wi(f))\rvert/i$
$w_{i(1)}$	$w_{f(1)}$	$[w_{i(1)} - w_{f(1)}] / 1$
$w_{i(2)}$	$w_{f(2)}$	$[w_{i(2)} - w_{f(2)}] / 2$
$w_{i(3)}$	$w_{f(3)}$	$[w_{i(3)} - w_{f(3)}] / 3$
.	.	.
.	.	.
.	.	.
$w_{i(n-2)}$	$w_{f(n-2)}$	$[w_{i(n-2)} - w_{f(n-2)}] / (n-2)$
$w_{i(n-1)}$	$w_{f(n-1)}$	$[w_{i(n-1)} - w_{f(n-1)}] /(n-1)$
$w_{i(n)}$	$w_{f(n)}$	$[w_{i(n)} - w_{f(n)}] / n$

Figure 6 Analysis without rank reversal

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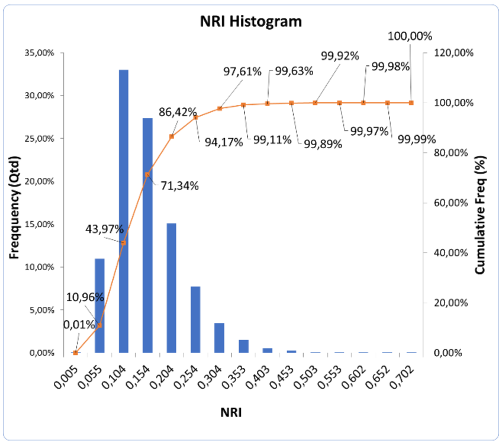

Figure 7 NRI histogram

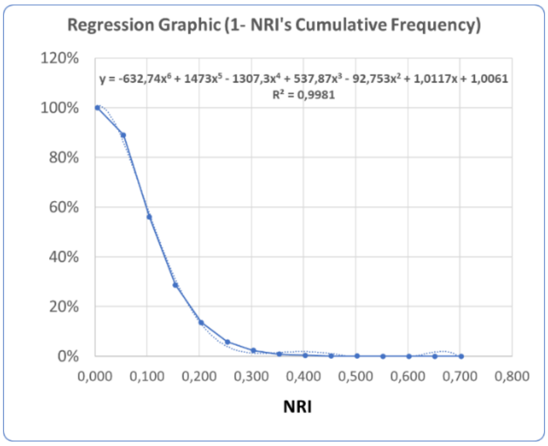

Figure 8 NRI's cumulative frequency

Therefore, the IPV disturbance level is reflected via the NRI index and classified by the NRQV index, represented in Equation 2 below, in line with Figure 8 (for n=5).

$$NRQV(5) = -632.NRI^6 + 1473.NRI^5 - 1307.NRI^4 + 538.NRI^3 - 93.NRI^2 + NRI + 1 \quad (2)$$

3.2 Analyzing final priority vectors with rank reversal occurrences

Similar to the assessment without rank reversal, the impact on the FPV when a rank reversal occurs is more significant the higher in these rankings the positions affected by the reversal are.

The higher the original ranking position impacted by the reversal, the higher the level of relevance of this disturbance. Therefore, an index to represent the magnitude of this particular case of reversal must consider the impact inversely proportional to the original ranking position.

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Considering that, an Index of Reversal (IR) is defined as:

$$\text{IR} = \sum_{i=1}^{n}[Pi(i) - Pi(f)]/i \quad (3)$$

where: *Pi(i)* = Position "i" in original ranking e *Pf(i)*= Position "i" in final ranking

In addition to the IR index, a complementary metric is needed to evaluate how significant the reversal was. To determine this, a minimum and a maximum reversal impact index must be defined.

The minimum reversal that can be observed is between the penultimate (*n-1*) and the last position (*n*) and only a draw between these positions (*n* and *n-1*) represents a lighter impact. This observation is taken to define the so-called Index of Minimum Reversal (IMR) (see Equations 4 and 5).

$$IMR\ (reversal) = [(1/n) + 1/(n-1)] \rightarrow IMR(reversal) = [(2n-1)/(n(n-1))] \quad (4)$$

$$IMR\ (draw) = [(n-(n-1))/(n-1)] \rightarrow IMR\ (draw) = [1/(n-1)] \quad (5)$$

Table 1
IMR x n (reversal and draw on last ranking positions)

n	2	3	4	**5**	6	7	8	9	10
IMR - Reversal of *n and (n-1)*.	1.50	0.83	0.58	**0.45**	0.37	0.31	0.27	0.24	0.21
***IMR* - Draw between (n) and (n-1)**	1.00	0.50	0.33	**0.25**	0.20	0.17	0.14	0.13	0.11

Regarding the definition of a maximum reversal impact index, an extremely relevant ranking reversal that can be conceived is the complete symmetric reversal of the ranking (the first position is reversed with the last, the second with the penultimate, the third with the antepenultimate, successively). It is a fact that significant reversals are undesirable. Therefore, to establish a Maximum Reversal Impact Index (MRII) this must be taken into account and a Monte Carlo simulation carried out to verify the level of confidence of this assumption. Figure 9 presents the above concept, and Figure 10 shows confidence level check.

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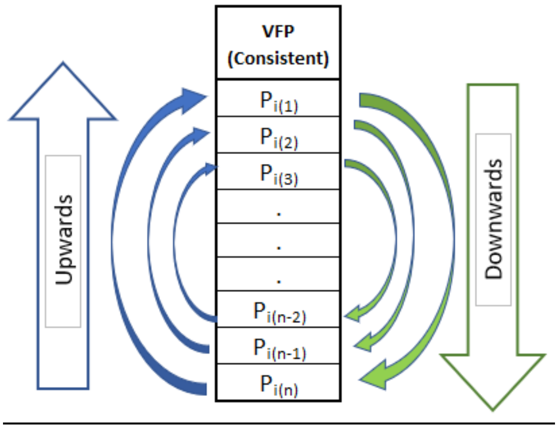

$$Upwards = (n-1)/1 + (n-3)/2 + (n-5)/3 + (n-7)/4 + \cdots$$

Therefore,

$$Upwards = \sum_{i=1}^{int(\frac{n}{2})} (n+1-2i)/i$$

$$Downwards = \frac{n-1}{n} + \frac{n-3}{n-1} + \frac{n-5}{n-2} + \frac{n-7}{n-3} + \cdots$$

Therefore,

$$Downwards = \sum_{i=1}^{int(\frac{n}{2})} (n+1-2i)/(n-i+1)$$

Figure 9 Basis for MRII definition

Therefore, in total:

$$MRII = \sum_{i=1}^{int(\frac{n}{2})} (n+1-2i).\left(\frac{1}{i} + \frac{1}{n-i+1}\right) \qquad (6)$$

Table 2
MRII

MRII	0.00	1.50	2.67	4.58	6.30	8.52	10.59	12.84	15.38	18.01
n	1	2	3	4	5	6	7	8	9	10
1		1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00
2				0.50	1.00	1.50	2.00	2.50	3.00	3.50
3	*Dsc*					0.33	0.67	1.00	1.33	1.67
4								0.25	0.50	0.75
5										0.20
6										0.17
7									0.33	0.43
8	*Asc*					0.25	0.40	0.50	0.57	0.63
9				0.33	0.50	0.60	0.67	0.71	0.75	0.78
10		0.50	0.67	0.75	0.80	0.83	0.86	0.88	0.89	0.90

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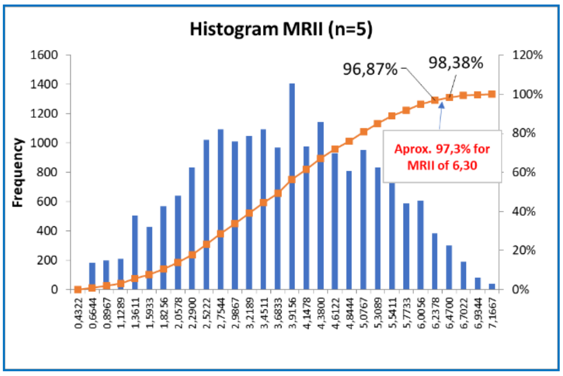

Figure 10 MRII (n=5) level of confidence[3]

Figure 10 shows a confidence level of 97.3% for the adopted MRII concept.

Therefore, based on the above methodologies, an index to qualify the Reversal Quality Vector (RQV), with regard to the relevance of FPV rank reversal, is defined as:

$$RQV = 1 - [\frac{IR - IMR}{MRII - IMR}]^k \qquad (7)$$

Since the ranking reversal of higher magnitudes is not desired, the RVQ was defined considering the exponential characteristic, with the exponent K of ¼ to induce significant reductions in this quality index.

Thus, using this methodology, the governance scenarios were analyzed. These comparisons were considered in a theoretical status of governance maturity. Two different AHP scales (OSS and GBS) were applied. Finally, a comparison was also made between IPV (from consistent PCMs) and FPV (from inconsistent PCMs), as well as between FPV and the priority agenda of elements of governance. The results and discussions are presented in the next sections.

[3] 19,842 occurrences with rank reverse obtained from 20,000 simulations where, randomly, normalized vectors with 5 positions (n=5) had their initial and final ranking compared.

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4. Results

Table 3
Summary of indexes considering 7 pairwise comparison (between consistent and inconsistent PCMs)

(*): *Draw in some positions of IPV (out of Consistents PCMs)*		**PCM Inconsistent x Consistent**				
Board Member	ESCALAS	ISR	IR	QSR	QCR	IC Saaty
C1 (*)	01 ~09	0.057	1.50	0.921	0.326	4.92%
	EGB	0.038	1.50	0.987	0.326	
C2	01 ~09	0.046	0.00	0.935	1.000	16.97%
	EGB	0.031	0.00	0.991	1.000	
C3	01 ~09	0.038	0.00	0.967	1.000	11.73%
	EGB	0.023	0.00	0.977	1.000	
C4	01 ~09	0.0163	0.00	0.9837	1.0000	6.34%
	EGB	0.0111	0.00	0.9889	1.0000	
C5 (*)	01 ~09	0.0383	0.33	0.9726	0.6574	15.92%
	EGB	0.0285	0.87	0.9752	0.4350	
C6	01 ~09	0.0258	0.00	0.9742	1.0000	8.04%
	EGB	0.0047	0.00	0.9953	1.0000	
C7(*)	01 ~09	0.0436	0.25	0.9501	1.0000	0.00%
	EGB	0.0328	0.25	0.9894	1.0000	

4.1 Analysis of FPV x IPV (PCM Consistent x Inconsistent):

Table 4
NRI

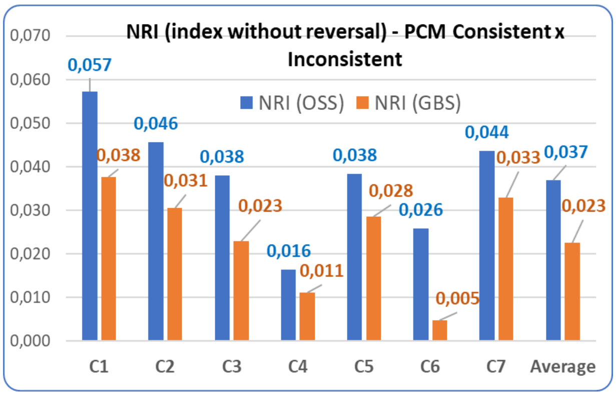


Table 5
NRQV

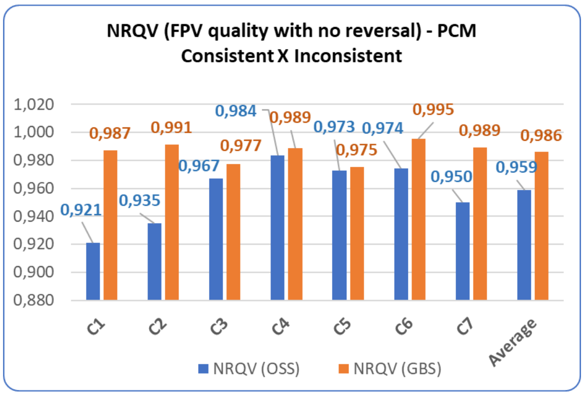


Table 6
IR

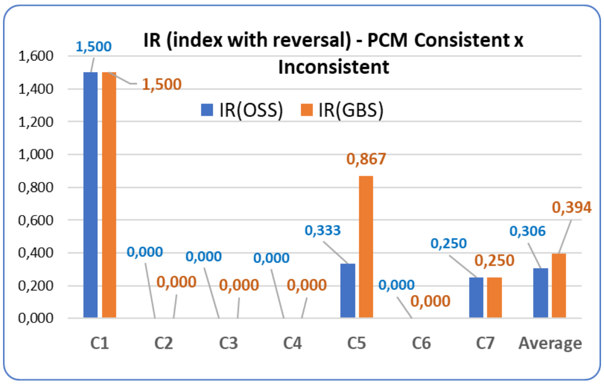
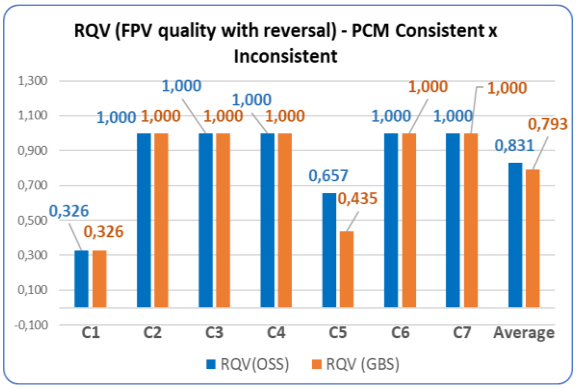


Table 7
RQV



4.2 Analysis of FPV x priority agenda of governance maturity:

Table 8
RQV (Inconsistent PCM)

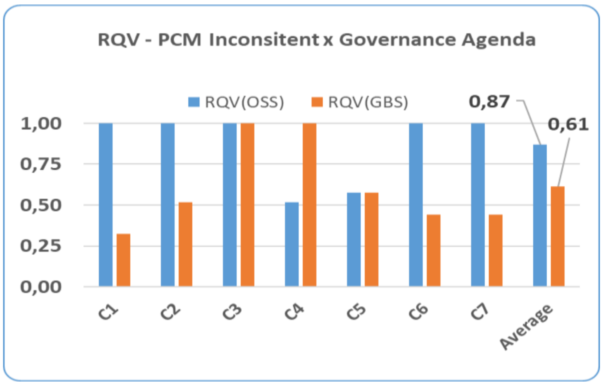
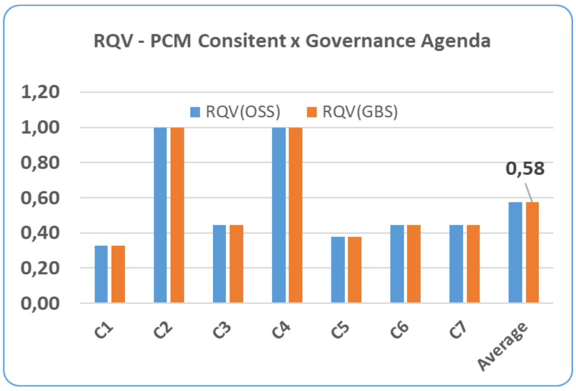


Table 9
RQV (Consistent PCM)



4.3 Interpretation of results

From the above tables, it can be noted that:

a) Concerning the comparisons between inconsistent and consistent PCMs:
   i. The GBS, in all cases, generated a lower level of disturbance in the FPV when no rank reversal has occurred.
   ii. When rank reversal occurred, the GBS generated a higher disturbance in the FPV than OSS.

b) Concerning the comparisons between FPV and the priority governance agenda:
   i. FPV of inconsistent PCMs presented a higher magnitude of rank reversal with GBS than OSS.

ii. No difference was observed regarding the consistent PCMs
iii. The average level of disturbance due to rank reversal was higher with the consistent PCM than with inconsistent ones, for both weight scales (average RQV of 0.61 for GBS and 0.87 for OSS compared to 0.58 for consistent PCMs).

5. Conclusion

Based on these interpretations of the results, it can be concluded that:

I. GBS is a more sensitive scale than OSS, as far as rank reversal trends are concerned.
II. GBS generates a smaller weight difference among ranking positions. Therefore, if the perturbation is not enough to generate rank reversal, GBS provides a FPV with lower weight perturbations.
III. The inconsistent analysis of the invited Board Members generated FPVs with higher adherence to the governance priority agenda than the consistent one. The inconsistency introduced by the pairwise comparison provided by the Board members is an indication that, indeed, such inconsistencies were a refinement/optimization of the process. Therefore, it reinforces the AHP methodology as an adequate process for decision making involving corporate governance issues.
IV. The metrics developed in this study allowed a deeper understanding of the disturbance that occurred with FPV. These disturbances could not be quantified if only the inconsistency index had been used. The indices developed here made it possible to quantify and compare the FPV ranking disorders.

In general, the method and metrics developed in this study went beyond the traditional focus of checking the Consistency Index (CI). A complete and detailed analysis of rank disturbances was feasible due to this new proposed methodology. This proposed methodology focused on an effective evaluation of the disturbances that occurred with the FPV and brings clarity to the final analysis of the outputs provided by the AHP and similar MCDA techniques.

The analysis of the proposed NRI, IR and RQV indices gives a complete view of the ranking disturbance suffered by the FPV when compared to the original ranking. The analysis of these indices allows the DM to interpret whether a given rank reversal should be rejected or, on the contrary, reflects the expected response. A FPV with some rank reversion and a small NRI may represent the appropriate response and, if followed by a high NRI, should be rejected. The magnitude of the RQV also supports this analysis.

Other surveys may carry out further investigations into the limits of acceptance of the FPV based on these proposed new indices.

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