AN ALTERNATIVE FRAMEWORK FOR THE OPTIMIZATION OF SOCIALLY RESPONSIBLE PORTFOLIOS APPLIED TO THE MOROCCAN STOCK EXCHANGE

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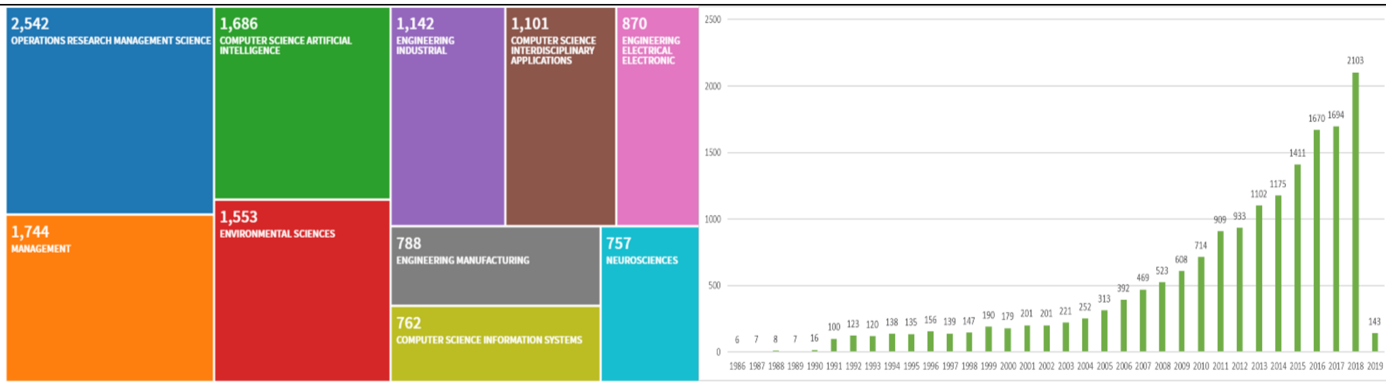
ABSTRACT

The purpose of this article is to propose an alternative approach for portfolio optimization combining financial and ethical constraints as well as objective and subjective preferences of investors. This approach intends to support investors in the selection and optimization of the performance of financial and social portfolios. More precisely, we introduce the Analytic Hierarchy Process (AHP) to measure the ethical performance (EP) score of each asset considering the ethical criteria. Fuzzy multiple criteria decision making (FMCDM) is used to determine the overall financial quality score of the assets with respect to key financial criteria, i.e., short-term return, long-term return, and risk. The interactive fuzzy programming approach is also applied to support the investor's decision, considering his subjective preferences. The robustness of our approach is tested through an empirical study involving the case of the Casablanca Stock Exchange (CSE). The results give evidence that the Socially Responsible (SR) portfolio performed similarly to the conventional one, as no significant differences were found in terms of return. However, the SR portfolio allows the investor to achieve their ethical goals with a slight financial sacrifice.

Keywords: Portfolio optimization; SRI; MCDM; fuzzy set theory; AHP

1. Introduction

Since the founding article by Markowitz (1952), and despite the significance of the literature on portfolio management and Multiple Criteria Decision Making (MCDM) (Figures 1a and 1b), few studies have been dedicated to the optimization of Socially Responsible (SR) portfolios. Moreover, by considering more than one criterion the selection of the optimal portfolio certainly renders it a multi-criteria decision which should be resolved using MCDM techniques.



a) Total Publications by Year   b) MCDM published items by fields

Figure 1 Total publications by year and MCDM published papers by field from January 1, 1986 to February 28, 2019 (Source: Web of Knowledge)

Furthermore, according to EuroSIF (2018), US (2018), and USSIF (2018), Socially Responsible Investing (SRI) has recently experienced a rise in importance among the community of investors. In this context, the aim of this article is to propose a framework that considers an additional constraint, i.e., social responsibility, for determining efficient portfolios.

This paper provides empirical research applied to the case of the Moroccan Stock Exchange. In fact, in addition to the political interest shown by Morocco in the ethical investing movement through hosting globally responsible, ethical and sustainable events (COP7 and the COP22), the Casablanca Stock Exchange is also a member of the United Nations-led Sustainable Stock Exchanges (SSE) initiative. Moreover, the CSE developed an ESG benchmark index including the top ten stocks with respect to their ESG rating in partnership with Vigeo Eiris.

The specific research questions this study seeks to answer are as follows:

- How can asset quality be measured in terms of ethical performance?
- How can an asset be evaluated and measured using financial measures?
- How can hybrid portfolio optimization models be used to obtain well-diversified portfolios that meet investor preferences?

In this article, we attempt to improve upon and complete the significant studies mentioned below, particularly through the proposition of a hybrid framework. Our framework considers both the objective and subjective preferences of the investor in

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order to have a better understanding of the real behaviors of investors. Our approach also combines the advantages of fuzzy logic and constrained multi-objective integer and non-linear programming to reach the preferred compromise solution. This hybrid approach is new because it has not been used in the SRI field.

The main advantages of this approach are:

- It controls the direction of the search by updating the lower (upper) bounds of the objective functions;
- If the investor is not satisfied with the obtained portfolio, more portfolios can be generated by updating the lower (upper) bounds of the objective functions;
- The investor has greater confidence in the solution obtained.

This study is organized as follows: the next section presents a review of the literature related to SRI portfolios. Section 3 presents the methodology of the three-stage multiple criteria decision-making framework for the SR portfolio selection. Section 4 carries out an empirical study applied to the CSE and compares the results of the proposed framework with the conventional approach. Finally, section 5 summarizes the main features and findings of the proposed approach and suggests some improvement tracks.

2. Literature review

The original Markowitz (1952) portfolio selection model assumes that investors are only interested in returns attached to specific levels of risk when choosing their portfolios. Numerous extensions and modifications to Markowitz's theory have been published, all of which consider an alternative measure of risk, namely, semi-variance, absolute deviation and semi-absolute deviation. Despite the widespread use of the Markowitz (1952) framework, this classical approach seems to be necessary but not sufficient to efficiently manage portfolio selection. Indeed, additional criteria could be added to the classical financial criteria (return and risk) including financial and non-financial criteria. In this context, Ballestero et al. (2012), Drut (2010), and Utz et al. (2014) have investigated the portfolio optimization frameworks used in SRI. Although these studies contribute significantly to the SRI literature, they do not go far beyond the Markowitz framework, rather simply extend it by incorporating an ethical goal in the objective function or adding it as an additional constraint.

Furthermore, since the founding article of Markowitz (1952) the literature shows a growing interest in the consideration of SRI within the modern portfolio theory. Specifically, in a universe where optimal portfolio selection is based on a multicriteria approach, the use of MCDM techniques is essential. Moreover, according to Table 1, the GP appears to be the most used technique, followed by fuzzy mathematical programming. These techniques could be used to assess fund performance and set up both an objective function and constraints of the ethical financial decision problem.

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Table 1
MCDM techniques applied in SRI portfolio construction/security analysis
(Source: authors' elaboration)

MCDM based approach	Number of articles	Studies
GP (Goal Programming)	11	Tsai et al. (2009), Ballestero et al. (2012), Bilbao-Terol et al. (2012b), Bilbao-Terol et al. (2012a), Bilbao-Terol et al. (2013), Garcia-Bernabeu et al. (2015), Bilbao-Terol et al. (2016a), Bilbao-Terol et al. (2016c), Masri (2018), Bilbao-Terol et al. (2018)
Compromise Programming	1	Bilbao-Terol et al. (2014)
Reference Point Method	1	Méndez-Rodríguez et al. (2015)
ZOGP (Zero-one goal programming)	1	Tsai et al. (2009)
MAUT (Multi-attribute utility theory)	1	Hallerbach (2004)
ANP (Analytic network process)	1	Tsai et al. (2009)
AHP (Analytic hierarchy process)	3	Gupta et al. (2013), Petrillo et al. (2016), García-Melón et al. (2016)
DEMATEL (Decision making trial and evaluation laboratory)	1	Tsai et al. (2009)
TOPSIS (Technique for the Order of Prioritization by Similarity to Ideal Solution)	1	Bilbao-Terol et al. (2014)
Interactive programming	2	Hallerbach (2004), González et al. (2014)
Chance-constrained	1	Masri (2018)
Fuzzy Mathematical Programming	8	Bilbao-Terol et al. (2012a,b), Hasuike (2012), Gupta et al. (2013), Bilbao-Terol et al. (2016a,b,c), Calvo et al. (2016)
Stochastic programming	2	Dorfleitner and Utz (2012), Masri (2018)

Bilbao-Terol et al. (2014) used the TOPSIS technique to assess the sustainability and sustainability performance of different countries' government bond funds. The proposed model allowed investors to express their preferences regarding the financial and non-financial goals. Additionally, they used the following four indicators of sustainability: Adjusted Net Saving (ANS), the Ecological Footprint (ECF), the Environmental Performance Index (EPI), and the Human Development Index (HDI). Bilbao-Terol et al. (2016b) proposed a mathematical model based on GP to help SR investors that want to manage their investments through a mental accounting structure of portfolios by working jointly with three bounded rationality theories, namely, behavioral portfolio theory with mental accounting, fuzzy logic and GP modeling. García-Melón et al. (2016) proposed a ranking similar to classical financial rankings (e.g. Morningstar ranking of mutual funds) for mutual funds based on the degree of investor social responsibility. This ranking is only intended to complete and update information that can be easily combined with other kinds of financial information, such as the Morningstar classification of funds. Additionally, the authors suggest using the AHP to determine weights. Bilbao-Terol et al. (2016c) suggested a sequential GP with fuzzy hierarchies to solve a portfolio selection problem in two phases. First, they compared criteria of the same nature, and second, they compared the two superior level criteria (the financial and SR objectives). Additionally, the authors used conditional value at risk (CVaR) as a risk measure. Dorfleitner and Utz (2012) introduced stochastic sustainability returns into safety-first models for portfolio choice. They established a general model in three different forms for generalized safety-first portfolio management with probabilistic constraints, namely, the convolution type, the marginal distributions type and joint distribution. Tsai et al. (2009) proposed an innovative model of SRI selection applied to the case of small Taiwanese companies. The model combines the DEMATEL method, ANP and ZOGP to evaluate the SRI selection procedure. DEMATEL helps companies identify the most important criterion or the one that affects other criteria the most. The ANP helps determine the priority weights among alternative stocks, while the ZOGP model helps organizations use resources without exceeding their constraints. Additionally, they used the sustainability-balanced scorecard as a multi-criteria framework for SRI evaluations. Hallerbach (2004) proposed a decision framework to measure the different ethical attributes of an SRI portfolio using the multi-attribute portfolio approach. Bilbao-Terol et al. (2012b) presented an SRI model for selecting portfolios with SRI-funds. They introduced an index called "SRI-Attractiveness" that summarizes the "social, environmental, and ethical performance" of each SRI-fund for a particular investor. More precisely, they combined Fuzzy Set theory techniques and GP to deal with multiple criteria with flexible targets and constraints. Calvo et al. (2016) suggested a fuzzy multi-criteria model for mean-variance portfolio selection by considering SRI as an additional secondary non-financial goal. They introduced a tool to compute the degree of social responsibility of a financial asset. Landi and Sciarelli (2019) tried to identify a direct causal relationship between ESG (environmental, social and corporate governance) rating and financial performances, but no evidence was found.

Moreover, the literature shows that the selection of a SR portfolio is generally based on two methods in order to consider financial and non-financial criteria. The first method relies on subjective preferences of the investor. In this context, Gupta et al. (2013) designed a comprehensive three-stage multiple-criteria decision-making framework for portfolio selection based simultaneously on the investor’s subjective preferences on financial and ethical criteria. The second method is based on quantitative data of return, risk, and social scores. More specifically, Gasser et al. (2017) revisited Markowitz’s portfolio selection theory and proposed a 3-objective model based on return, risk, and ESG scores. Although this study is important, the proposed model relies entirely on the assumption of the investor’s pure and perfect rationality, which has been widely criticized by Cabrerizo et al. (2010), García-Crespo et al. (2012), Rahiminezhad Galankashi et al. (2020) and Zhou et al. (2019) because of the multiple psychological biases it entails for correctly describing the reality of investor decision-making in the real world.

The literature review has been discussed in this section. The following section presents the methodology.

3. Methodology

In order to demonstrate the practical usability of the proposed framework that combines subjective and objective preferences, we provide a detailed flowchart in Figure 2 that summarizes the main steps followed in the framework.

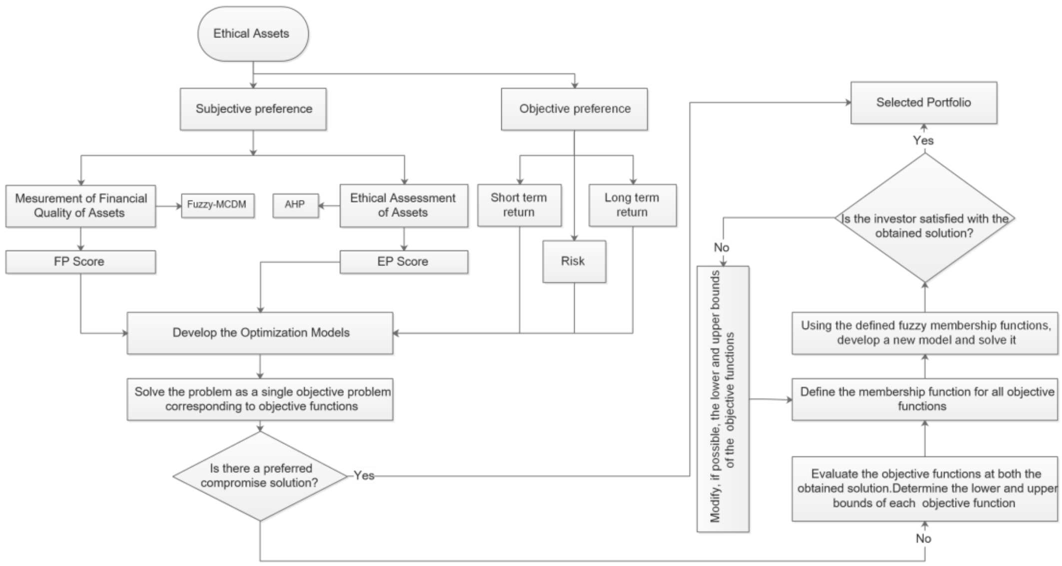

Figure 2 Alternative hybrid framework for portfolio selection
(Source: authors' elaboration)

3.1 EP score using AHP

The AHP, introduced by Saaty (1977), is a multi-criteria theory of measurement that depends on a pairwise comparison of criteria, sub-criteria and assets to be evaluated from the decision maker's preferences. The AHP helps capture both the subjective and objective aspects of a decision. Additionally, the AHP does not consider the internal relationships between criteria; therefore, the AHP is the most suitable method for measuring the EP score of each asset.

3.2 Construction of the model

We selected the main factors for decision-making and arranged them in a hierarchical structure descending from an overall goal to criteria, sub-criteria, and assets in successive levels. Figure 3 shows the structural hierarchy for EP scores proposed by Gupta et al. (2013, 2014).

The first level comprises the overall goal of 'EP score'. The second level contains the three criteria, which contribute to the goal, namely, ES, CSR, and CGBE. The third, level's criteria are broken down into various sub-criteria, i.e., ES is broken down into emissions and waste disposal (EWD), resource conservation (RC), and recycling (RE); CSR is broken down into product safety (PS), occupational safety (OS) and non-discrimination (ND); CGBE is broken down into corruption (CR), disclosure (DI) and code of ethics (CE); and the last level represents the assets that are to be evaluated in terms of the criteria and sub-criteria.

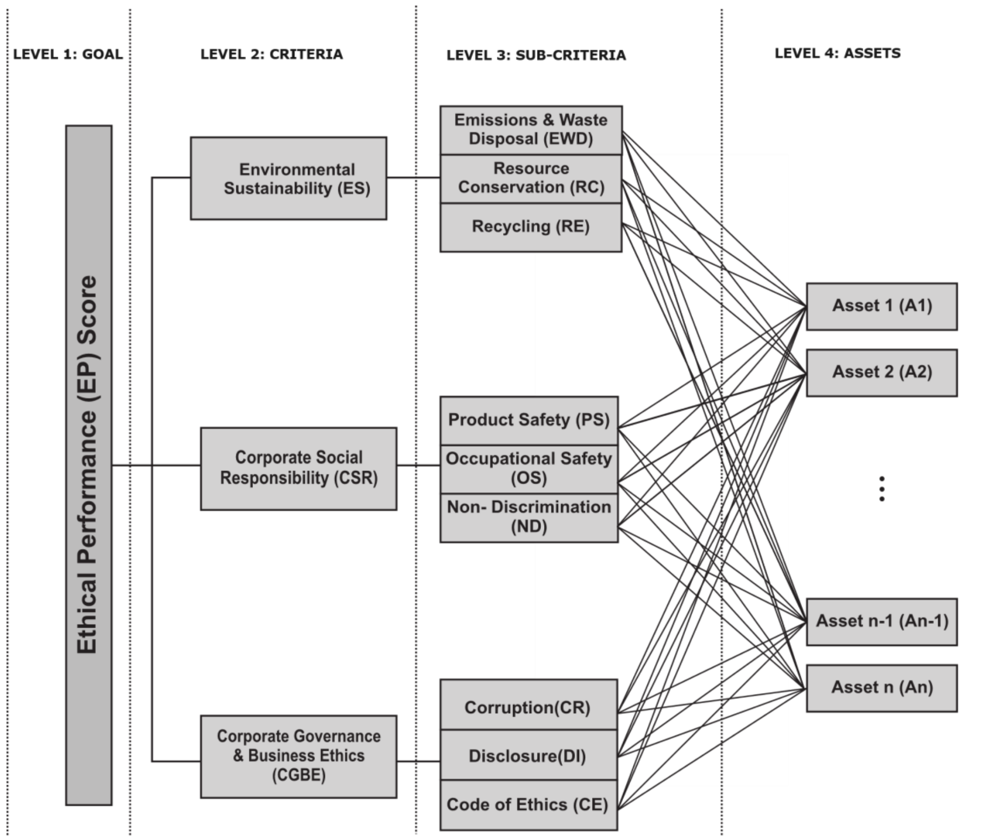

Figure 3 Structural hierarchy for EP score. (Source: Adapted from Gupta et al.)

For the definitions of the criteria, see Gupta et al. (2014). The main steps involved in this methodology are:

- Identification of overall goal.
- Construction of the decision hierarchy of different levels constituting goal, criteria, sub-criteria and assets.
- Comparison of each element at the related level and establishing the normalized matrix.
- Obtaining the overall or final priorities of each element from each normalized matrix, and based on these priorities computing the maximum Eigenvalue and consistency index (CI) of the normalized matrix.
- Checking the consistency of the judgments by computing the CR (the acceptable value to continue the AHP analysis is CR < 0.10).

3.3 Construction of the comparison matrix

The comparisons are made using a scale that indicates the importance of one element over another with respect to a given attribute. Table 2 shows the scale ranging from 1 for 'equally important, likely or preferred' to 9 for 'extremely more important, likely or preferred'.

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Table 2
Saaty's pairwise comparison scale

Verbal scale	Numerical values
Equally important, likely or preferred	1
Moderately more important, likely or preferred	3
Strongly more important, likely or preferred	5
Very strongly more important, likely or preferred	7
Extremely more important, likely or preferred	9
Intermediate values to reflect compromise	2, 4, 6, 8
Reciprocals for inverse comparison	Reciprocals

In Table 2, the intermediate values 2, 4, 6 and 8 are used to address situations of uncertainty. For example, when the decision maker is confused about rating a pairwise comparison as "Strongly more important, likely or preferred (5)" or "Very strongly more important, likely or preferred (7)", a probable option is to rate it as "From Strongly to Very strongly more important, likely or preferred (6)".

3.4 Consistency test

After building the model, the investor then evaluates the elements[1] by making pairwise comparisons. Once all the comparisons are carried out, in order to prove whether the paired comparison matrix is consistent or not, we first calculated the weights of the compared elements, the maximum eigenvalue λ_{max}. Then, the consistency in our judgment for each paired comparison matrix of order n, the consistency index (CI) is calculated as follows:

$$CI = \frac{\lambda_{max} - n}{(n-1)}.$$

While n is the number of compared elements, the consistency ratio (CR) is calculated as follows:

$$CR = \frac{CI}{RI}.$$

where RI is a known random consistency index.

Table 3 shows the values of RI. Saaty and Vargas (2012) demonstrated that a CR of 0.10 or less is acceptable to continue the AHP analysis. If the consistency ratio is greater than 0.10, it is necessary to review the judgments to determine the cause of the inconsistency and modify it.

[1] Each criteria, sub-criteria, assets and the goal are collectively referred to as model elements.

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Table 3
Average random CI (Source: Saaty and Tran (2007))

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49	1.52	1.54	1.56	1.58	1.59

3.5 Evaluation of financial performance using Fuzzy-MCDM

Many factors are considered when measuring the quality of a financial asset, for example, their short and long-term returns, liquidity and risk related characteristics. An estimation of these characteristics by extrapolation of historical data is too risky to measure and judge the quality of an asset. Moreover, the investors are more comfortable expressing their preferences linguistically using terms such as high return and low risk. Under such vagueness, the F-MCDM method is applied to find the financial performance score of each asset with respect to the financial criteria.

Definition 1: Li (1999) A fuzzy preference relation R is a fuzzy subset of $\Re x \Re$ with membership function $\mu_R(A,B)$ representing the degree of preference of fuzzy number A over fuzzy number B.

1. R is reciprocal if $\mu_R(A,B) = 1 - \mu_R(B,A), \forall A,B \subseteq R$.
2. R is transitive if
$$\left.\begin{matrix}\mu_R(A,B) \geq 1/2 \\ \mu_R(B,C) \geq 1/2\end{matrix}\right\} \Rightarrow \mu_R(A,C) \geq 1/2, \forall A,B,C \subseteq R.$$
3. R is a fuzzy total ordering if R is both reciprocal and transitive.

If fuzzy numbers are compared based on fuzzy preference relations, then A is said to be greater than B if $\mu_R(A,B) > 1/2$.

Definition 2: Lee (2005) An extended fuzzy preference relation R is an extended fuzzy subset of $\Re x \Re$ with membership function $-\infty \leq \mu_R(A,B) \leq \infty$ representing the degree of preference of fuzzy number A over fuzzy number B.

1. R is reciprocal if $\mu_R(A,B) = -\mu_R(B,A), \forall A,B \subseteq R$
2. R is transitive if
$$\left.\begin{matrix}\mu_R(A,B) \geq 0 \\ \mu_R(B,C) \geq 0\end{matrix}\right\} \Rightarrow \mu_R(A,C) \geq 0, \forall A,B,C \subseteq R.$$
3. R is additive if $\mu_R(A,C) = \mu_R(A,B) + \mu_R(B,C)$
4. R is a fuzzy total ordering if R is both reciprocal, transitive and additive.

If fuzzy numbers are compared based on extended fuzzy preference relations, then A is said to be greater than B if $\mu_R(A,B) > 0$.

Definition 3: Lee (2005) For any fuzzy number A, B, extended fuzzy preference relation $F(A,B)$ is defined by the membership function:

$$\mu_F(A,B) = \int_0^1 ((A-B)_\alpha^L + (A-B)_\alpha^U) d\alpha$$

Where $(A-B)_\alpha^L = inf_{\mu_{(A-B)} \geq \alpha}(z)$ and $(A-B)_\alpha^U = sup_{\mu_{(A-B)} \geq \alpha}(z)$.

Definition 4: A triangular membership function is specified by three parameters a, b, c defined as $\mu_x = \max\left(min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$

where a and c represent the lower and upper bounds of the fuzzy number, respectively, and b is the median value.

Remark 1 If $A = (a_1, b_1, c_1)$ and $B = (a_2, b_2, c_2)$ are two triangular fuzzy numbers then $\mu_F(A, B) = (a_1 + 2b_1 + c_1 - a_2 - 2b_2 - c_2)/2$.

Definition 5: A linear membership for maximizing and minimizing the objective function $\left(Z^k(x), k = 1,2, \dots, K\right)$, respectively represented in Table 4. While U_k is the worst upper bound and L_k is the best lower bound of the objective function.

Table 4
Linear membership functions of the objective functions (Source: authors' elaboration)

The linear membership for maximizing	$\mu_k(Z^k(x)) = \begin{cases} 1, & if\ \ Z^k(x) \geq U_k, \\ \frac{Z^k(x)-L_k}{U_k-L_k}, & if\ L_k\ < Z^k(x) < U_k, \\ 0, & if\ Z^k(x) \leq L_k. \end{cases}$
The linear membership for minimizing	$\mu_k(Z^k(x)) = \begin{cases} 1, & \text{if } Z^k(x) \leq L_k, \\ \frac{U_k-Z^k(x)}{U_k-L_k}, & \text{if } L_k < Z^k(x) < U_k, \\ 0, & \text{if } Z^k(x) \geq U_k. \end{cases}$

Remark 2 Let $x^k, k = 1,2,\dots,K$ denote the optimal solutions obtained by solving the optimization problem as a single objective problem. We calculate U_k and L_k, respectively by:

$$U_k = max\left\{Z\left(x^k\right), k = 1,2,\dots,K\right\}$$
$$L_k = min\left\{Z\left(x^k\right), k = 1,2,\dots,K\right\}$$

where Z is the objective function.

Multiple Criteria Decision Making (MCDM) contains a set of operational research methods, which help make choices when multiple criteria, goals or objectives exist. In this classical framework, the performance ratings and criteria weights are measured in crisp numbers. Under many circumstances where performance rating and weights cannot be given precisely, the fuzzy set theory is introduced to model the uncertainty of human judgments and problems (Lee, 2005). In Fuzzy MCDM methods, the performance ratings and criteria weights are usually represented by fuzzy numbers.

The preference function of one fuzzy number $\tilde{A}_{ij}$ over another number $\tilde{A}_{kj}$ is written as follows:

$$P\left(\tilde{A}_{ij}, \tilde{A}_{kj}\right) = \begin{cases} \mu_F\left(\tilde{A}_{ij}, \tilde{A}_{kj}\right), & \text{if } \mu_F\left(\tilde{A}_{ij}, \tilde{A}_{kj}\right) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

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Let J be the set of benefit criteria and J' be the set of cost criteria where
$J = \{1 \geq j \geq n \; and \; j \; belongs \; to \; benefit \; criteria\}$
$J' = \{1 \geq j \geq n \; and \; j \; belongs \; to \; cost \; criteria\}$, And $J \cup J' = \{1, \ldots, n\}$

According to Lee (2005), the main steps involved in this methodology are:

- **Step 1**: Identification of the criteria for the selection of distribution selection.
- **Step 2**: Aggregation of the fuzzy decision matrices and fuzzy weight matrices given by decision makers and normalized to the group fuzzy decision matrix. Let $D = \left(\tilde{A}_{ij}\right)$ be the normalized group fuzzy decision matrix and $W = \left(\tilde{W}_j\right)$ be the weight matrix.
- **Step 3**: Calculation of the strength matrix by

$$S_{ij} = \begin{cases} \sum_{k \neq i} P\left(\tilde{A}_{ij}, \tilde{A}_{kj}\right), & \text{if } j \in J \\ \sum_{k \neq i} P\left(\tilde{A}_{kj}, \tilde{A}_{ij}\right), & \text{if } j \in J' \end{cases} \tag{1}$$

- **Step 4**: Calculation of the weakness matrix by

$$I_{ij} = \begin{cases} \sum_{k \neq i} P\left(\tilde{A}_{kj}, \tilde{A}_{ij}\right), & \text{if } j \in J \\ \sum_{k \neq i} P\left(\tilde{A}_{ij}, \tilde{A}_{kj}\right), & \text{if } j \in J' \end{cases} \tag{2}$$

- **Step 5**: Calculation of the fuzzy weighted strength indices by

$$\tilde{S}_i = \sum_j \left(S_{ij} \tilde{W}_j\right) \tag{3}$$

- **Step 6**: Calculation of the fuzzy weighted weakness indices by

$$\tilde{I}_i = \sum_j \left(I_{ij} \tilde{W}_j\right) \tag{4}$$

- **Step 7**: Derivation of the strength index from S_i the fuzzy weighted strength and weakness indices by

$$S_i = \sum_{k \neq i} P\left(\tilde{S}_i, \tilde{S}_k\right) + \sum_{k \neq i} P\left(\tilde{I}_k, \tilde{I}_i\right) \tag{5}$$

- **Step 8**: Derivation of the weakness index I_i from the fuzzy weighted strength and weakness indices by

$$I_i = \sum_{k \neq i} P\left(\tilde{S}_k, \tilde{S}_i\right) + \sum_{k \neq i} P\left(\tilde{I}_i, \tilde{I}_k\right) \tag{6}$$

- **Step 9**: Aggregation of the strength and weakness indices into total performance indices by

$$t_i = \frac{S_i}{S_i + I_i} \qquad (7)$$

- **Step 10**: Ranking assets by total performance indices t_i.

3.6 Asset allocation using hybrid optimization models

We assume that investors allocate their wealth among n assets. We introduced the following assumptions and notations.

3.6.1 Notations and definitions

f_i: The FP score of the i-th asset calculated using the Fuzzy MCDM method
e_i: The EP score of the i-th asset calculated using the AHP
r_i: The expected rate of return of the i-th asset
x_i: The proportion of the total funds invested in the i-th asset
y_i: A binary variable indicating whether the i-th asset is contained in the portfolio, where

$$y_i \begin{cases} 1, & if\ i - th\ asset\ is\ contained\ in\ the\ portfolio \\ 0, & otherwise \end{cases}$$

r_i^{12}: The average performance of the i-th asset during a 12-month period
r_i^{36}: The average performance of the i-th asset during a 36-month period
u_i: The maximum fraction of the capital allocated to the i-th asset
l_i: The minimum fraction of the capital allocated to the i-th asset
h: The number of assets held in a portfolio
β: The desire for an ethical level in the portfolio construction
n: The number of assets in a portfolio.

3.6.2 Objective functions

- **Short-term return**: The short-term return of the portfolio is expressed as:

  $$Z_1(x) = \sum_{i=1}^{n} r_i^{12} x_i \qquad (8)$$

  where $r_i^{12} = \frac{1}{12}\sum_{i=1}^{12} r_{it},\ \ i = 1,2, \dots n;\ r_{it}$ is determined from the historical data.

- **Long-term return**: The long-term return of the portfolio is expressed as

  $$Z_2(x) = \sum_{i=1}^{n} r_i^{36} x_i \qquad (9)$$

  where $r_i^{36} = \frac{1}{36}\sum_{i=1}^{36} r_{it},\ \ i = 1,2, \dots n;\ \ r_{it}$ is determined from the historical data.

- **Financial criteria**: The objective function using FP scores based on the three key financial criteria is expressed as

  $$Z_3(x) = \sum_{i=1}^{n} f_i x_i \qquad (10)$$

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- **Ethicality**: The ethical investing objective function using the EP scores is expressed as

$$Z_4(x) = \sum_{i=1}^{n} e_i x_i \quad (11)$$

- **Risk**: The portfolio risk using semi-absolute deviation measure is expressed as

$$Z_5(x) = \sum_{t=1}^{T} \frac{\left|\sum_{i=1}^{n}(r_{it}-r_i)x_i\right| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2T} = \frac{1}{T}\sum_{t=1}^{T}\theta_t(x) \qquad (12)$$

3.6.3 Constraints

- **Capital budget**: Capital budget constraint on the assets is expressed as
$\sum_{i=1}^{n} x_i = 1 \quad (13)$

- **Minimum fraction**: Minimum fraction of the capital that can be invested in a single asset is expressed as

$$x_i \geq l_i y_i, \mathrm{i} = 1,2,\ldots,n \quad (14)$$

- **Maximum fraction**: Maximum fraction of the capital that can be invested in a single asset is expressed as

$$x_i \leq u_i y_i, \mathrm{i}{=}1,2,\ldots,n \quad (15)$$

- **Cardinality**: Number of assets held in the portfolio is expressed as

$$\sum_{i=1}^{n} y_i = h, \quad y_i \in \{0,1\}, i = 1,2..,n \quad (16)$$

- **No short selling**: No short selling of assets is expressed as

$$x_i \geq 0, \ i = 1,2,\ldots,n \quad (17)$$

3.6.4 Decision problem

In order to reduce the computational burden, we used semi-absolute deviation as a risk measure. The fuzzy multi-objective portfolio optimization model 18 considering the subjective preferences for ethical and financial criteria is shown:

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$$\begin{cases} Max\ Z_1(x) & = \sum_{i=1}^{n} r_i^{12} x_i \\ MaxZ_2(x) & = \sum_{i=1}^{n} r_i^{36} x_i \\ MaxZ_3(x) & = \sum_{i=1}^{n} f_i x_i \\ MaxZ_4(x) & = \sum_{i=1}^{n} e_i x_i \\ MinZ_5(x) & = \frac{1}{T}\sum_{t=1}^{T} \theta_t(x) \\ & = \sum_{t=1}^{T} \frac{|\sum_{i=1}^{n}(r_{it}-r_i)x_i| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2T} \\ & \quad sub\ ject\ to \\ & \quad\quad constraints\ 13 - 17 \end{cases} \tag{18}$$

To eliminate the absolute-valued function in the above model, we transformed the problem into the following form.

$$\begin{cases} Max\ Z_1(x) & = \sum_{i=1}^{n} r_i^{12} x_i \\ MaxZ_2(x) & = \sum_{i=1}^{n} r_i^{36} x_i \\ MaxZ_3(x) & = \sum_{i=1}^{n} f_i x_i \\ MaxZ_4(x) & = \sum_{i=1}^{n} e_i x_i \\ MinZ_5(x) & = \frac{1}{T}\sum_{t=1}^{T} p_t \\ & \quad subject\ to \\ & \quad\quad p_t \geq -\sum_{i=1}^{n}(r_{it} - r_i)x_i,\ \ t = 1,2,\ldots,T, \\ & \quad\quad constraints\ 9 - 17 \\ & \quad\quad p_t \geq 0t = 1,2,\ldots T, \end{cases} \tag{19}$$

The fuzzy multi-objective conventional portfolio optimization model is:

$$\begin{cases} Max\ Z_1(x) & = \sum_{i=1}^{n} r_i^{12} x_i \\ Max\ Z_2(x) & = \sum_{i=1}^{n} r_i^{36} x_i \\ Min\ Z_5(x) & = \frac{1}{T}\sum_{t=1}^{T} p_t \\ sub\ ject\ to & \\ & \quad p_t \geq -\sum_{i=1}^{n}(r_{it} - r_i)x_i, \quad t = 1,2,\ldots,T, \\ & \quad constraints\ 13 - 17 \\ & \quad p_t \geq 0t = 1,2,\ldots T, \end{cases} \tag{20}$$

3.6.5 Solution methodology

First, by using the AHP, we measured the EP score of each asset with respect to the ethical criteria. Next, using Fuzzy MCDM, we determined the overall financial quality score of each asset with respect to the following three financial criteria: short-term return, long-term return, and risk. Then, we constructed model 20 (above) and finally, we used the fuzzy interactive approach to solve the problem. That is identified in the following steps:

- **Step 1**: Solve the problem 20 as a single-objective problem with respect to short-term return 21, long-term return 22 and risk 23 objective functions mathematically:

$$\begin{cases} Max\ Z_1(x) = \sum_{i=1}^{n} r_i^{12} x_i \\ \quad subject\ to \\ \quad p_t \geq -\sum_{i=1}^{n} (r_{it} - r_i) x_i, \quad t = 1,2,\dots,T, \\ \quad constraints\ 9 - 17 \\ \quad p_t \geq 0 t = 1,2, \dots T, \end{cases} \tag{21}$$

$$\begin{cases} Max\ Z_2(x) = \sum_{i=1}^{n} r_i^{36} x_i \\ \quad sub\ ject\ to \\ \quad p_t \geq -\sum_{i=1}^{n} (r_{it} - r_i) x_i, \quad t = 1,2,\dots,T, \\ \quad constraints\ 9 - 17 \\ \quad p_t \geq 0 t = 1,2, \dots T, \end{cases} \tag{22}$$

$$\begin{cases} Min\ Z_5(x) = \frac{1}{T} \sum_{t=1}^{T} p_t \\ \quad subject\ to \\ \quad p_t \geq -\sum_{i=1}^{n} (r_{it} - r_i) x_i, \quad t = 1,2,\dots,T, \\ \quad constraints\ 9 - 17 \\ \quad p_t \geq 0 t = 1,2, \dots T, \end{cases} \tag{23}$$

Let x^1, x^2 and x^3 denote the optimal solutions obtained from solving the single objective problems with respect to each objective function, if all the solutions, i.e., $x^1 = x^2 = x^3 = (x_1, x_2, \dots, x_{14})$ are the same, we obtained the preferred solution and stopped; otherwise, proceed to Step 2.

- **Step 2**: Evaluate the objective functions of all the obtained solutions. Determine the worst lower bound and best upper bound for each objective function.

- **Step 3**: Define the linear membership functions $\mu_{Z_1(x)}$ for short-term return, $\mu_{Z_2(x)}$ for long-term return and $\mu_{Z_5(x)}$ for risk objective functions.

- **Step 4**: Using the obtained fuzzy membership functions, first, develop the fuzzy multi-objective optimization model for the portfolio selection problem as follows:

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$$\begin{cases} Max\ \theta \\ \quad subject\ to \\ \quad\quad \theta \le \mu_{Z_1(x)} \\ \quad\quad \theta \le \mu_{Z_2(x)} \\ \quad\quad \theta \ge \mu_{Z_5(x)} \\ \quad\quad p_t \ge -\sum_{i=1}^{n}(r_{it} - r_i)x_i, \quad t = 1,2,\ldots,T, \\ \quad\quad constraints\ 13 - 17 \\ \quad\quad p_t \ge 0t = 1,2,\ldots T, \\ \quad\quad 0 \le \theta \le 1, \end{cases} \tag{24}$$

where θ is an auxiliary variable representing the grade of membership.

Then, solve problem 24 using the Branch-and-Bound algorithm run by Lingo software (Schrage (2006) to obtain the global optimal solution. Next, we present the solution to the investor. The investor either accepts the decision and the process is over, or the investor re-evaluates all the objective functions. For the benefit objective (i.e., long-term and short-term return), the current worst lower bound is compared with the new objective value. If the new value is higher than the worst lower bound, it is considered a new lower bound; otherwise, the old value should be used. On the other hand, for the risk objective the current worst upper bound is compared with the new objective value. If the new value is lower than the worst upper bound, it should be considered a new upper bound; otherwise, the old value should be used. If there are no changes in the current bounds of all the objective functions then the process is finished; otherwise go to Step 3 and re-iterate the solution process. The same approach discussed above has been followed in order to solve problem 19.

In order to test the robustness and relevance of the proposed framework, an empirical case study was applied to the Moroccan stock exchange.

4. Empirical study and results analysis

In this section, we present the results of an empirical study conducted for a fictitious socially responsible investor. We selected 14 assets (Eiris,2017) listed on the Casablanca Stock Exchange, representing the top performers RSE. The list of selected assets are presented in Table 24 in the Appendix. Based on the historical daily prices of our asset's sample from January 1, 2016 to December 31, 2018, we computed the monthly returns for each asset. When a daily price was missing the average weekly prices were used instead.

4.1 Ethical performance scores

The data of the relative priorities of the fictitious investor were analyzed using the paired comparison matrices (see Tables 5, 6 and 22 to 24). To perform the pairwise comparison, a comparison matrix of the criteria involved in the decision was created as shown in Table 5. The cells in comparison matrices have a value from the numeric scale shown in Table 2 to reflect the relative preference in each of the compared pairs. Since the CR is less than 0.10, the degree of consistency is satisfactory. Now, we know the paired comparison matrix is reasonably consistent and can continue the process of decision-making using the AHP. At level 2, we defined the local priorities of the three main

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criteria with respect to the overall goal (see Table 5). At level 3, we defined the local priorities of the various sub-criteria with respect to their parent criterion[2] in level 2 (see Table 7). At level 4, we defined the local priorities of all the 14 assets with respect to each of the nine sub-criteria of ethical evaluation in level 3 (see Tables 22 to 24).

Table 5
Pair-wise comparisons of the main criteria in relation to the overall goal (Source: authors' elaboration)

Criteria	ES	CSR	CGBE	Local weight
ES	1	5	3	0.6479
CSR	1/5	1	1/2	0.1222
CGBE	1/3	2	1	0.2299
λmax=3.0037	CI=0.0018	CR=0.0036		

Table 6
Pair-wise comparisons of the sub-criteria in relation to the main criteria (Source: authors' elaboration)

ES	RC	RE	EWD	Local weight
RC	1	1/2	2	0.2973
RE	2	1	3	0.5390
EWD	½	1/3	1	0.1637
λmax=3.0092	CI=0.0046			CR=0.0089

CSR	ND	OS	PS	Local weight
ND	1	2	8	0.6380
OS	1/2	1	2	0.2584
PS	1/8	1/2	1	0.1036
λmax=3.0541		CI=0.0271		CR=0.0520

CGBE	CR	CE	DI	Local weight
CR	1	1/4	1/7	0.0824
CE	4	1	1/2	0.3151
DI	7	2	1	0.6025
λmax=3.0020	CI=0.0010		CR=0.0019	

Table 7
EP scores of the assets. (Source: authors' elaboration)

		Global weight													
	Assets	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
Criteria	ES	0.0576	0.0304	0.1036	0.0126	0.0364	0.0358	0.0306	0.0174	0.0218	0.0739	0.0518	0.0374	0.0644	0.0741
	CSR	0.0058	0.0107	0.017	0.0045	0.0062	0.0047	0.0068	0.006	0.0086	0.0113	0.0171	0.0044	0.0022	0.0169
	CGBE	0.0085	0.004	0.0334	0.0148	0.026	0.025	0.0306	0.0079	0.0157	0.0144	0.0159	0.017	0.009	0.0076
	EP scores	0.0718	0.0451	0.154	0.0319	0.0686	0.0655	0.0681	0.0313	0.0462	0.0996	0.0849	0.0588	0.0756	0.0987
	Ranking	**6**	**12**	**1**	**13**	**7**	**9**	**8**	**14**	**11**	**2**	**4**	**10**	**5**	**3**

[2] For example, the sub-criteria, ND, OS and PS are pair-wise compared with respect to the parent criterion CSR.

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4.2 Financial performance (FP) scores

To obtain financial scores of the assets, we used the following three evaluation criteria: short-term return (C1); long-term return (C2); risk (C3); C1 and C2 are benefit criteria, whereas C3 is the cost criterion. The data to evaluate the financial performance of the assets with respect to the three criteria can be obtained from the inputs of the fictitious investor. The preferences of the investor were captured using the linguistic variables employed to represent relative importance and ratings provided in Table 8.

Table 8
Linguistic variables for the importance weights of criteria and the ratings

Importance weights of criteria		Linguistic variables for the ratings	
Linguistic variables	Fuzzy number	Linguistic variables	Fuzzy number
Very low (VL)	(0, 0, 0.1)	Very poor (VP)	(0,0,1)
Low (L)	(0, 0.1, 0.3)	Poor (P)	(0,1,3)
Medium low (ML)	(0.1, 0.3, 0.5)	Medium poor (MP)	(1,3,5)
Medium (M)	(0.3, 0.5, 0.7)	Faire (F)	(3,5,7)
Medium high (MH)	(0.5, 0.7, 0.9)	Medium good (MG)	(5,7,9)
High (H)	(0.7, 0.9, 1.0)	Good (G)	(7,9,10)
Very high (VH)	(0.9, 1.0, 1.0)	Very good (VG)	(9,10,10)

Tables 9 and 10 show respectively, the importance of the financial criteria C1, C2, C3 and the ratings of the assets regarding financial criteria. In order to handle the uncertainty involved in the treatment of the linguistic judgments of the data, we used the scale provided in Table 9 based on triangular fuzzy numbers. The recorded information of the investor preferences with respect to the weights of the criteria and the rating of the assets are found in Tables 11 and 12.

Table 9
Importance weights of the criteria (Source: authors' elaboration)

Criteria	C_1	C_2	C_3
Fictitious investor	M	H	H

Table 10
Ratings of alternatives (assets) given by decision makers (Source: authors' elaboration)

	Fictitious investor														
	Assets	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
Criteria	Short term return	F	F	F	G	G	MG	G	G	MG	F	F	MG	F	F
	Long term return	G	G	G	MP	MP	MG	MP	MP	F	MP	MP	F	G	G
	Risk	MG	MG	MG	VG	VG	MG	VG	VG	MG	VG	VG	G	MG	MG

According to our fictitious investor's preferences for all three criteria (C1, C2, C3) for each asset, the asset A10 is the least preferable in comparison to the other assets. It might also be noted that the FP score of this asset A10 is 0 (see the colored cell of Table 15). In

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addition, its fuzzy strength is the lowest and its fuzzy weakness is the highest when compared to the other assets (see the colored cell of Table 14).

Table 11
Fuzzy weights of the criteria (Source: authors' elaboration)

	C_1	C_2	C_3
Weight	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)

Table 12
Ratings of the assets (Source: authors' elaboration)

	Strength			Weakness		
Assets	C1	C2	C3	C1	C2	C3
A1	(3,5,7)	(7,9,10)	(5,7,9)	(0.3,0.5,0.7)	(0.7,0.9,1)	(0.5,0.7,0.9)
A2	(3,5,7)	(7,9,10)	(5,7,9)	(0.3,0.5,0.7)	(0.7,0.9,1)	(0.5,0.7,0.9)
A3	(3,5,7)	(7,9,10)	(5,7,9)	(0.3,0.5,0.7)	(0.7,0.9,1)	(0.5,0.7,0.9)
A4	(7,9,10)	(1,3,5)	(9,10,10)	(0.7,0.9,1)	(0.1,0.3,0.5)	(0.9,1,1)
A5	(7,9,10)	(1,3,5)	(9,10,10)	(0.7,0.9,1)	(0.1,0.3,0.5)	(0.9,1,1)
A6	(5,7,9)	(5,7,9)	(5,7,9)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.5,0.7,0.9)
A7	(7,9,10)	(1,3,5)	(9,10,10)	(0.7,0.9,1)	(0.1,0.3,0.5)	(0.9,1,1)
A8	(7,9,10)	(1,3,5)	(9,10,10)	(0.7,0.9,1)	(0.1,0.3,0.5)	(0.9,1,1)
A9	(5,7,9)	(3,5,7)	(5,7,9)	(0.5,0.7,0.9)	(0.3,0.5,0.7)	(0.5,0.7,0.9)
A10	(3,5,7)	(1,3,5)	(9,10,10)	(0.3,0.5,0.7)	(0.1,0.3,0.5)	(0.9,1,1)
A11	(3,5,7)	(1,3,5)	(9,10,10)	(0.3,0.5,0.7)	(0.1,0.3,0.5)	(0.9,1,1)
A12	(5,7,9)	(3,5,7)	(7,9,10)	(0.5,0.7,0.9)	(0.3,0.5,0.7)	(0.7,0.9,1)
A13	(3,5,7)	(7,9,10)	(5,7,9)	(0.3,0.5,0.7)	(0.7,0.9,1)	(0.5,0.7,0.9)
A14	(3,5,7)	(7,9,10)	(5,7,9)	(0.3,0.5,0.7)	(0.7,0.9,1)	(0.5,0.7,0.9)

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Table 13
Strength and weakness of the assets (Source: authors' elaboration)

Assets	Strength			Weakness		
	C1	C2	C3	C1	C2	C3
A1	0	8.75	3.65	4.2	0	0
A2	0	8.75	3.65	4.2	0	0
A3	0	8.75	3.65	4.2	0	0
A4	5.95	0	0	0	7.35	4.05
A5	6.3	0	0	0	7.35	4.05
A6	2.8	5.6	3.65	1.4	1.75	0
A7	6.3	0	0	0	7.35	4.05
A8	6.3	0	0	0	7.35	4.05
A9	2.8	2.4	3.65	1.4	4.15	0
A10	0	0	0	4.2	7.35	4.05
A11	0	0	0	3.8	7.35	4.05
A12	2.8	2.4	1.2	1.4	4.15	2.45
A13	0	8.75	3.65	4.2	0	0
A14	0	8.75	3.65	4.2	0	0

Table 14
Indices of the assets (Source: authors' elaboration)

Assets	fuzzy weighted strength index	fuzzy weighted weakness index	strength index	weakness index
A1	(8.68,11.16,12.4)	(1.26,2.1,2.94)	244.95	0
A2	(8.68,11.16,12.4)	(1.26,2.1,2.94)	244.95	0
A3	(8.68,11.16,12.4)	(1.26,2.1,2.94)	244.95	0
A4	(1.785,2.975,4.165)	(7.98,10.26,11.4)	19.9	215.95
A5	(1.89,3.15,4.41)	(7.98,10.26,11.4)	20.95	212.1
A6	(7.315,9.725,11.21)	(1.645,2.275,2.73)	218.175	14.875
A7	(1.89,3.15,4.41)	(7.98,10.26,11.4)	20.95	212.1
A8	(1.89,3.15,4.41)	(7.98,10.26,11.4)	20.95	212.1
A9	(5.075,6.845,8.01)	(3.325,4.435,5.13)	139.775	73.675
A10	(0,0,0)	(9.24,12.36,14.34)	0	338.15
A11	(0,0,0)	(9.12,12.16,14.06)	0.4	332.95
A12	(3.36,4.64,5.56)	(5.04,6.64,7.58)	79.75	133.7
A13	(8.68,11.16,12.4)	(1.26,2.1,2.94)	244.95	0
A14	(8.68,11.16,12.4)	(1.26,2.1,2.94)	244.95	0

Table 15
FP scores of the assets (Source: authors' elaboration)

Assets	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
FP scores	1	1	1	0.0844	0.0899	0.9362	0.0899	0.0899	0.6548	0	0.0012	0.3736	1	1
Normalized scores	0.1366	0.1366	0.1366	0.0115	0.0123	0.1279	0.0123	0.0123	0.0895	0	0.0002	0.051	0.1366	0.1366

4.3 Asset allocation using hybrid optimization models

Let $R_{i,t}$ be a variable representing the rate of return during period t ($t = 1,2,\ldots,T$) of the i-th asset ($i = 1,2,\ldots,14$). In addition, let x_i be the proportion of the total funds invested in the i-th asset. For our purposes, we have historical daily prices of assets, indicating that the fluctuations are quite low for daily returns and sometimes close to zero. Therefore, we are using the logarithmic return (Miskolczi, 2017) to calculate the daily returns. From these data, monthly returns are computed. The logarithmic return at time t of an asset is calculated by:

$$R_{i,t} = \ln\left({C_{i,t}}/{C_{i,t-1}} \right) = \ln\left(C_{i,t}\right) - \ln\left(C_{i,t-1}\right) \qquad (25)$$

where $C_{i,t}$ is the closing price of the i-th asset during the period t, and $C_{i,t-1}$ is the closing price during the period $t-1$.

4.3.1 Fuzzy multi-objective conventional portfolio optimization model

In order to find an optimal asset allocation, we used the solution methodology discussed in the methodology section and the input data from Tables 8, 15, 16 and 20, h = 8. The lower bound and upper bound on allocation in each asset are 0.09 and 0.4, respectively. Finally, the obtained solution for the portfolio selection is provided in Table 17. Next, we needed to check whether the obtained solution $x = (\theta = 0.8222, x_1 = 0.09, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0.2189, x_6 = 0.09, x_7 = 0.09, x_8 = 0.09, x_9 = 0, x_{10} = 0.09, x_{11} = 0.09, x_{12} = 0, x_{13} = 0.2411, x_{14} = 0)$ was a local max point of Equation (24) or not. Then, we verified the Karush-Kuhn-Tucker (KKT) optimality conditions for nonlinear programming quation (24) at the obtained solution.

Table 16
Input data of assets corresponding to short-term return, long-term return, lower bound and upper bounds on allocation in each asset (Source: authors' elaboration)

Assets	A1	A2	A3	A4	A5	A6	A7
Short-term return	0.0010	0.0003	-0.0003	-0.0021	0.0027	0.0011	0.0019
Long-term return	0.0004	-0.0002	0.0001	-0.0004	0	0.0004	0.0002
	A8	A9	A10	A11	A12	A13	A14
	0.0008	0.0010	0.0014	0.0006	0.0003	0.0012	-0.0003
	0.0003	0.0001	0.0003	0.0002	-0.0007	0.0006	-0.0003

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Table 17
Proportions of the assets in the obtained portfolio (Source: authors' elaboration)

	Allocation													
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
Proportions	0.09	0	0	0	0.2189	0.09	0.09	0.09	0	0.09	0.09	0	0.2411	0

Let $y_i \in \{0,1\}, i = 1,2..,14$. We first constructed the lagrangian 26 for problem 24 as follows:

$$L(x, \lambda_1, \lambda_2, \ldots, \lambda_{39}, \gamma_1, \gamma_2, \lambda_{40}, \lambda_{41}, \ldots, \lambda_{67}, \gamma_3, \ldots, \gamma_{16}, \lambda_{68}, \ldots, \lambda_{135}) =$$

$$\theta - \lambda_1(\theta - \mu_{Z_1(x)}) - \lambda_2(\theta - \mu_{Z_2(x)}) - \lambda_3(\mu_{Z_5(x)} - \theta)$$

$$- \sum_{j=4}^{39} \lambda_j \left(-p_t - \sum_{i=1}^{14} (r_{it} - r_i) x_i\right) - \gamma_1 \left(\sum_{i=1}^{14} x_i - 1\right) - \gamma_2 \left(\sum_{i=1}^{14} y_i - 1\right) \tag{26}$$

$$- \sum_{j=40}^{67} \lambda_j (l_i y_i - x_i) - \sum_{j=68}^{95} \lambda_j (x_i - u_i y_i) - \sum_{j=3}^{16} \gamma_j y_i - \sum_{j=96}^{131} \lambda_j (-p_t) - \sum_{j=132}^{145} \lambda_j (-x_i)$$

where $\lambda_i (i = 1,2,\ldots,145)$ and $\gamma_i (i = 1,2,\ldots,16)$ are Lagrange multipliers. We can now write the KKT necessary conditions (Dual Feasibility 27 and Complementary Slackness 28) for this problem as:

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$$Dual\ Feasibility\ \begin{cases} \frac{\partial L}{\partial \theta} = 0 \Rightarrow 1 - 0.0005\lambda_1 - 0.0014\lambda_2 - 0.0608\lambda_3 = 0 \\ \frac{\partial L}{\partial x_i} = 0 \Rightarrow -\lambda_1 r_i^{12} - \lambda_2 r_i^{36} + \sum_{j=4}^{39} \lambda_j (r_{it} - r_i) - \gamma_1 \\ \qquad + \sum_{j=40}^{67} \lambda_j - \sum_{j=68}^{95} \lambda_j + \sum_{j=132}^{145} \lambda_j = 0,\ i = 1,\ldots,14 \\ \frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow \theta - \mu_{Z_1(x)} = 0 \\ \frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow \theta - \mu_{Z_2(x)} = 0 \\ \frac{\partial L}{\partial \lambda_3} = 0 \Rightarrow \mu_{Z_5(x)} - \theta = 0 \\ \frac{\partial L}{\partial \lambda_j} = 0 \Rightarrow p_t + \sum_{i=1}^{14} (r_{it} - r_i) x_i = 0, j = 4,5,\ldots,39; \\ \qquad t = 1,2,\ldots,36 \\ \frac{\partial L}{\partial \gamma_1} = 0 \Rightarrow \sum_{i=1}^{14} x_i - 1 = 0 \\ \frac{\partial L}{\partial \gamma_2} = 0 \Rightarrow \sum_{i=1}^{14} y_i - 1 = 0 \\ \frac{\partial L}{\partial \lambda_j} = 0 \Rightarrow l_i y_i - x_i = 0,\ j = 40,41,\ldots,67;\ i = 1,2,\ldots,14 \\ \frac{\partial L}{\partial \lambda_j} = 0 \Rightarrow x_i - u_i y_i = 0,\ j = 68,69,\ldots,95;\ i = 1,2,\ldots,14 \\ \frac{\partial L}{\partial \gamma_j} = 0 \Rightarrow y_i = 0,\ i = 1,2,\ldots,14 \\ \frac{\partial L}{\partial \lambda_j} = 0 \Rightarrow -p_t = 0,\ t = 1,2,\ldots,36 \\ \frac{\partial L}{\partial \lambda_j} = 0 \Rightarrow -x_i = 0,\ i = 1,2,\ldots,14 \\ \lambda_i \geq 0,\ i = 1,2,\ldots,145 \\ \gamma_i \in \mathbb{R},\ i = 1,2,\ldots,16 \end{cases} \tag{27}$$

$$Complementary\ Slackness \begin{cases} \lambda_1\left(\theta - \mu_{Z_1(x)}\right) = 0 \\ \lambda_2\left(\theta - \mu_{Z_2(x)}\right) = 0 \\ \lambda_3\left(\mu_{Z_5(x)} - \theta\right) = 0 \\ \lambda_j(-p_t - \sum_{i=1}^{14} (r_{it} - r_i) x_i)\ = 0, j = 4,5,\ldots,39; \\ \qquad t = 1,2,\ldots,36 \\ \gamma_1\left(\sum_{i=1}^{14} x_i - 1\right) = 0 \\ \gamma_2\left(\sum_{i=1}^{14} y_i - 1\right) = 0 \\ \lambda_j(l_i y_i - x_i) = 0, j = 40,41,\ldots,67;\ i = 1,2,\ldots,14 \\ \lambda_j(x_i - u_i y_i) = 0, j = 68,69,\ldots,95;\ i = 1,2,\ldots,14 \\ \gamma_j y_i = 0, j = 3,4,\ldots,16; i = 1,2,\ldots,14 \\ \lambda_j(-p_t) = 0,\ j = 96,97,\ldots,131;\ t = 1,2,\ldots,36 \\ \lambda_j(-x_i) = 0, j = 132,133,\ldots,145;\ i = 1,2,\ldots,14 \end{cases} \tag{28}$$

The KKT conditions (Dual Feasibility and Complementary Slackness) are satisfied at the obtained solution:

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$x = (\theta = 0.8222, x_1 = 0.09, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0.2189, x_6 = 0.09, x_7 = 0.09, x_8 = 0.09, x_9 = 0, x_{10} = 0.09, x_{11} = 0.09, x_{12} = 0, x_{13} = 0.2411, x_{14} = 0)$ for $\lambda_1 = 973.4324, \lambda_2 = 380.5106, \lambda_3 = \lambda_4 = \ldots = \lambda_{39} = 0, \gamma_1 = 1.0219, \gamma_i = 0(i = 2,3,\ldots,16), \lambda_{40} = 0.2159, \lambda_i = 0(i = 41,42,43,44), \lambda_{45} = 0.2788, \lambda_{46} = 0.0892, \lambda_{47} = 0.4467, \lambda_{48} = 0, \lambda_{49} = 0.1446, \lambda_{50} = 0.5986, \lambda_i = 0(i = 51,52,\ldots,145). x = (\theta = 0.8222, x_1 = 0.09, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0.2189, x_6 = 0.09, x_7 = 0.09, x_8 = 0.09, x_9 = 0, x_{10} = 0.09, x_{11} = 0.09, x_{12} = 0, x_{13} = 0.2411, x_{14} = 0)$

4.3.2 Fuzzy multi-objective portfolio optimization model considering the subjective preferences

In order to find an optimal asset allocation, we used the solution methodology discussed in the methodlogy section and the input data from Tables 8, 15, 16 and 20, h = 8. The lower bound and upper bound on allocation in each asset are 0.09 and 0.4, respectively. Finally, the obtained solution for the portfolio selection is provided in Table 18. Next, we needed to check whether the obtained solution is a local max point of Equation (19) or not. Then, we verified the KKT optimality conditions for nonlinear programming Equation (19) for the obtained solution following the same approach in the section above. The KKT conditions (Dual Feasibility and Complementary Slackness) are satisfied at the obtained solution:
$x = (\theta = 0.5579, x_1 = 0.09, x_2 = 0, x_3 = 0.1655, x_4 = 0, x_5 = 0.1666, x_6 = 0.09, x_7 = 0.09, x_8 = 0, x_9 = 0, x_{10} = 0.09, x_{11} = 0, x_{12} = 0, x_{13} = 0.2179, x_{14} = 0.09)$ for $\lambda_1 = 3.5177, \lambda_2 = 6.5044, \lambda_3 = 0, \lambda_4 = 328.5171, \lambda_5 = \ldots = \lambda_{41} = 0, \gamma_1 = 0, \gamma_2 = 1.3707, \gamma_i = 0(i = 3,\ldots,16), \lambda_{42} = 0, \lambda_{43} = 0.1037 \lambda_i = 0(i = 44,45,46,47), \lambda_{48} = 0.1477, \lambda_{49} = 0.2727, \lambda_{50} = \lambda_{51} = 0, \lambda_{52} = 0.2588, \lambda_{53} = \lambda_{54} = \lambda_{55} = 0, \lambda_{56} = 0.3457, \lambda_i = 0(i = 57,58,\ldots,147).$

Table 18
Proportions of the assets in the obtained portfolio (Source: authors' elaboration)

	Allocation													
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
Proportions	0.09	0	0.1655	0	0.1666	0.09	0.09	0	0	0.09	0	0	0.2179	0.09

We can notice that the capital is allocated comparatively more to assets A3, A5 and A13 whose FP scores are high and EP scores are within acceptable priority ranking.

5. Conclusions

This paper is conducted in the same manner as other articles discussing the construction of SR portfolios that reflect investor's preferences and incorporating real-world market constraints.

A real-world empirical study based on the 14 top performers according to the RSE score of different assets on CSE was conducted to demonstrate the robustness and practicality of the proposed hybrid framework. In order to reach this objective, we proposed a three-stage approach for the SR portfolio selection. In the first stage, the AHP was used to measure the EP score of each asset with respect to the ethical criteria, and a Fuzzy

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MCDM was used to determine the overall financial quality score of each asset with respect to three key financial criteria namely, short-term return, long-term return, and risk. In the second stage, using real-world market data we calculated the short-term return, long-term return, and risk of each asset. Finally, based on the inputs from each stage, we developed the optimization model to obtain well-diversified portfolios that accomplish financially and ethically satisfying asset allocation. To solve the constrained multi-objective mixed-integer non-linear portfolio optimization problem, we applied an interactive fuzzy programming approach. Furthermore, our proposed approach is innovative within the SRI field.

For the sake of comparison, we demonstrated the the investment proportions differences between the Fuzzy multi-objective conventional portfolio and the proposed approach in the form of a histogram (see Figure 4). It can be seen clearly from Figure 4 that the selected assets differ from one approach to the other

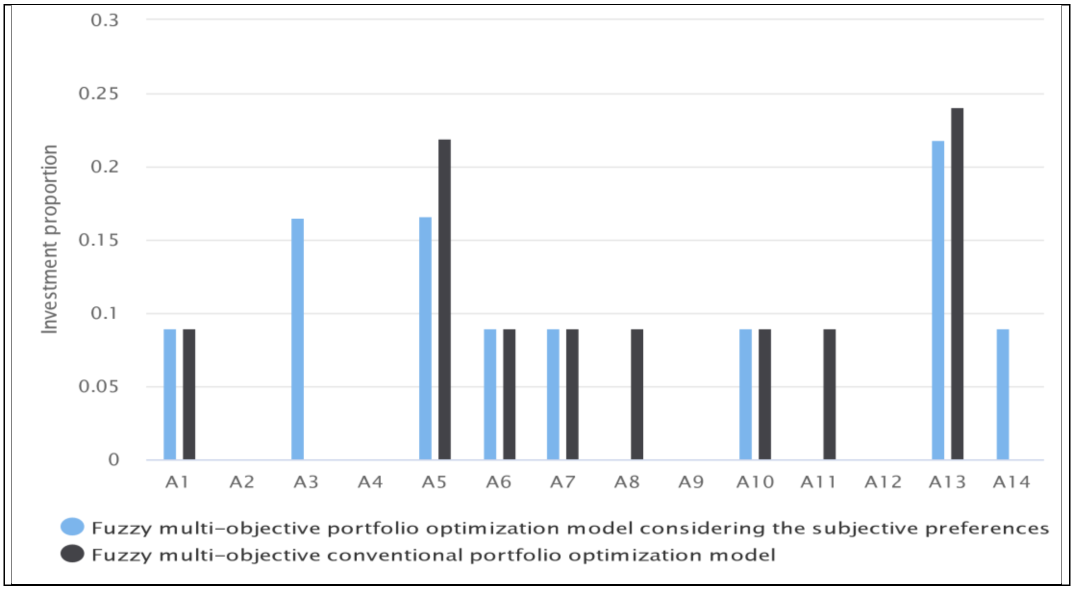

Figure 4 Comparison of the investment proportions on the Fuzzy multi-objective conventional portfolio and the proposed approach (Source: authors' elaboration)

From the comparative results in Table 19 and Figure 5, we found that the proposed approach has a slightly lower risk and short-term return compared to the fuzzy multi-objective conventional portfolio, and both approaches have almost the same long-term return. Therefore, this result supports previous studies that show that SR portfolios tend to perform similarly to conventional portfolios. However, the SR portfolio allows the investor to achieve his ethical goal with only a slight financial sacrifice. Because of this, we strongly believe that these findings could help the ethical investor achieve his ethical goals based on their particular preferences.

Table 19
Results of the Fuzzy multi-objective conventional portfolio and proposed approach (Source: authors' elaboration)

	Objective function value				
	Long-term return	Short-term return	Financial performance	Ethical Performance	Risk
Proposed framework	0.0002	0.0011	0.0916	0.0897	0.0073
Fuzzy multi-objective conventional portfolio	0.0003	0.0015	---	---	0.0086

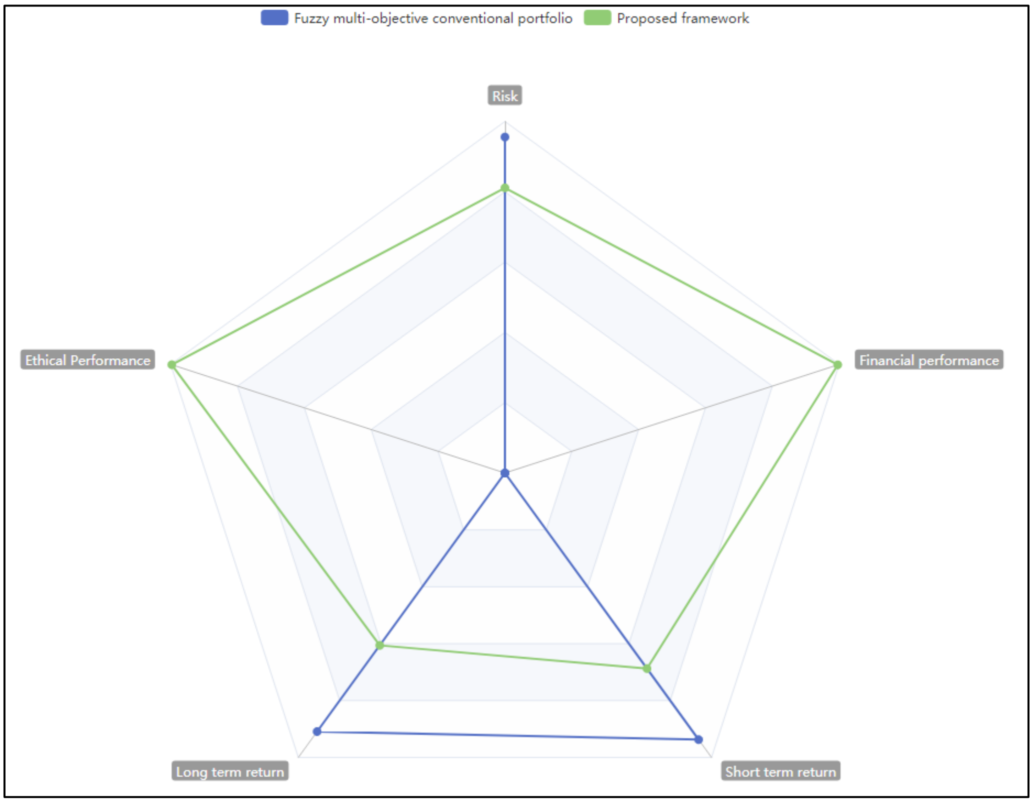

Figure 6 Objective function value

Our proposed approach has several advantages: i) it controls the search direction by updating lower (upper) bounds of the objective functions; ii) if the investor is not satisfied with the obtained portfolio, more portfolios can be generated by updating the lower (upper) bounds of the objective functions; iii) the investor has greater confidence in the obtained solution.

Finally, we believe that another interesting route of research would be an intuitionistic fuzzy portfolio selection because the intuitionistic fuzzy set uses the two indices preference and non-preference (i.e., membership and non-membership degrees or functions) in order to describe fuzziness, and may more flexibly and abundantly represent information compared to the fuzzy set when uncertainty such as hesitancy is present.

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APPENDIX

Table 20
Returns of the assets for the period January 1st, 2016 to December 31st, 2018 (Source: authors' elaboration)

Monthly returns												
Assets	16-Jan	16-Feb	16-Mar	16-Apr	16-May	16-Jun	16-Jul	16-Aug	16-Sep	16-Oct	16-Nov	16-Dec
A1	-0.0006	0.0004	0.0009	0.0027	0.0004	-0.0008	0.0003	0.0003	0.0009	0.0066	-0.0017	0.0023
A2	-0.0005	-0.0025	0.0014	0.0019	0.0006	-0.0013	0.0005	-0.0006	0.0009	-0.0001	0.0004	0.0031
A3	-0.0096	-0.0018	0.0092	-0.0030	0.0041	-0.0001	-0.0021	-0.0007	-0.0008	0	-0.0013	0.0020
A4	0.0052	-0.0029	0	0	-0.0053	-0.0056	-0.0113	-0.0111	0.0032	-0.0001	-0.0040	0.0064
A5	0.0050	-0.0003	-0.0019	0.0023	-0.0008	0.0011	-0.0010	0.0010	0.0029	0.0045	0.0097	0.0097
A6	0.0020	-0.0001	0.0017	0.0046	-0.0024	-0.0016	0.0033	-0.0003	0.0004	0.0022	0	0.0028
A7	-0.0017	0.0002	0.0044	0.0073	-0.0014	-0.0025	0.0090	0.0005	0.0014	0.0031	-0.0005	0.0027
A8	0.0060	-0.0035	0.0004	0.0034	0	0.0002	-0.0005	0.0002	-0.0013	0.0002	0.0009	0.0035
A9	0.0032	-0.0008	0.0031	0.0007	0.0035	0.0005	-0.0028	0.0014	-0.0015	0.0033	0.0013	0.0004
A10	-0.0117	0.0015	0.0066	0.0115	-0.0010	-0.0022	0.0097	-0.0096	0.0021	-0.0006	0.0038	0.0067
A11	0.0004	-0.0016	0.0028	-0.0003	-0.0021	-0.0005	0.0038	-0.0026	0	0.0015	0.0005	0.0057
A12	-0.0074	0.0025	0.0041	-0.0038	0.0105	-0.0059	0.0013	-0.0024	0.0017	0.0034	0.0001	0
A13	0.0025	-0.0006	0.0007	0.0057	-0.0028	0.0005	0.0007	0.0061	-0.0034	0	0.0025	0.0024
A14	-0.0024	0.0016	0.0004	0.0033	0.0036	-0.0024	0.0027	-0.0012	0.0018	-0.0006	0.0030	0.0086
Assets	17-Jan	17-Feb	17-Mar	17-Apr	17-May	17-Jun	17-Jul	17-Aug	17-Sep	17-Oct	17-Nov	17-Dec
A1	0.0008	-0.0006	-0.0016	0.0011	0.0020	0.0012	0.0006	0.0004	0.0016	0.0008	0.0021	-0.0011
A2	-0.0017	-0.0036	0	0	-0.0002	0.004	-0.0029	0	0.0001	0.0001	0.0010	0.0009
A3	0.0007	-0.0016	0.0027	0.0005	-0.0017	0.0021	0.0021	-0.0020	0.0068	0.0073	-0.0002	-0.0015
A4	0.0275	-0.0031	-0.0057	0	-0.0061	0	0.0027	-0.0025	0	0	-0.0028	-0.0031
A5	0.0064	-0.0053	0.0006	0.0040	-0.0003	0.0037	-0.0187	0.0025	-0.0016	0.0012	0.0009	-0.0014
A6	0.0032	-0.0025	-0.0024	0.0005	-0.0023	0.0030	-0.0014	0.0004	-0.0002	-0.0006	0.0007	-0.0012
A7	0.0016	0.00024	-0.0072	0.0002	-0.0018	0.0002	0.0018	-0.0008	-0.0083	0.0031	0.0004	0.0011
A8	0.0039	-0.0014	-0.0009	0.0017	0	0.0012	0.0034	0	0.0014	0.0003	0.0011	0.0013
A9	0.0017	0	0.0001	0.0001	0.0028	0.0015	0.0008	-0.0010	0.0034	-0.0024	-0.0019	0.0016
A10	0.0085	-0.0016	-0.0013	0.0069	0.0022	0.0010	0.0075	0.0029	-0.0004	-0.0013	0.0020	-0.0014
A11	0.0015	-0.0027	0.0025	0.0089	-0.0037	0.0025	0.0003	-0.0012	0.0028	0.0025	-0.0055	0
A12	0.0013	-0.0021	0.0033	-0.0010	0.0012	0.0003	0.0027	0.0026	0.0041	-0.0036	-0.0009	-0.0053
A13	0.0012	0	-0.0014	0.0020	0.0021	0.0010	-0.0010	0.0003	-0.0012	0.0021	0.0030	-0.0005
A14	0.0027	-0.0020	-0.0010	0.0003	-0.0011	0.0005	0.0020	0.0018	-0.0015	0.0004	0.0008	-0.0003
Assets	18-Jan	18-Feb	18-Mar	18-Apr	18-May	18-Jun	18-Jul	18-Aug	18-Sep	18-Oct	18-Nov	18-Dec
A1	0.0014	0.0008	-0.0007	0.0009	-0.0022	-0.0015	0.0003	-0.0009	-0.0014	-0.0034	0.0032	0.0006
A2	0.0022	0.0011	0	-0.0042	-0.0016	0.0009	-0.0016	0	-0.0026	-0.0015	0.0004	-0.0005
A3	0.0079	-0.0010	-0.0045	0.0026	-0.0002	-0.0045	-0.0096	0.0029	0.0019	-0.0023	-0.0045	0.0033
A4	0	-0.0028	0.0001	-0.0028	0.010	0	0	0	0	0	0	0
A5	0.0013	-0.0010	-0.0002	-0.0019	-0.0016	-0.0009	-0.0205	0.0005	-0.0012	-0.0008	0.0003	0.0018
A6	0.0042	0.0003	0.0001	0.0028	-0.0043	-0.0016	0.0012	-0.0002	0.0003	-0.0014	0.0008	0.0003
A7	0.0012	-0.0002	0.0003	-0.0017	-0.0038	-0.0027	0.0005	-0.0008	-0.0018	-0.0016	0.0036	0.0025
A8	0.0011	-0.0004	-0.0023	0.0007	-0.0042	0.0011	-0.0018	-0.0008	-0.0010	0.0004	-0.0019	-0.0026

A9	-0.0007	0.0008	-0.0015	-0.0003	0.0009	0.0012	-0.0058	0.0019	-0.0060	-0.0008	0.0017	-0.0023
A10	0.0039	-0.0015	-0.0021	-0.0026	-0.0003	-0.0007	-0.0085	-0.0002	-0.0061	-0.0011	-0.0038	-0.0063
A11	0.0048	-0.0031	0.0003	0.0003	-0.0021	-0.0121	-0.0025	0.0014	-0.0039	0.0001	0.0081	-0.0003
A12	0.0049	-0.0001	0.0020	0	-0.0027	-0.0014	-0.0270	0.0016	-0.0098	-0.0015	-0.0006	0.0034
A13	0.0053	-0.0020	-0.0019	0.0007	-0.0054	0.0010	-0.0016	0.0005	0.0017	-0.0024	0.0017	0.0008
A14	0.0002	0.0012	-0.0009	-0.0002	-0.0029	-0.0042	-0.0001	-0.0040	0.0009	0.0011	-0.0005	0

Table 21
Pair-wise comparisons of the alternatives in relation to the sub-criteria RC, RE and EWD (Source: authors' elaboration)

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	Local weight
RC															
A1	1	6	1/3	9	5	1	5	7	9	1/2	1	6	5	8	0.1478
A2	1/6	1	1/2	3	3	1	3	3	2	1/3	1/2	4	1/2	3	0.0618
A3	3	2	1	9	5	3	7	8	7	3	1/2	4	4	8	0.1758
A4	1/9	1/3	1/9	1	1/3	1/7	1	1/3	1/4	1/9	1/9	1	1/5	1/3	0.0137
A5	1/5	1/3	1/5	3	1	1/3	1	5	1	1/2	1/3	1	1/3	3	0.0372
A6	1	1	1/3	7	3	1	5	7	5	1	1	4	3	6	0.1063
A7	1/5	1/3	1/6	1	1	1/5	1	1	1	1/7	1/3	1	1/6	3	0.0245
A8	1/7	1/3	1/8	3	1/5	1/7	1	1	1	1/7	1/3	2	1/4	2	0.0247
A9	1/9	1/2	1/7	4	1	1/5	1	1	1	1/2	1/9	2	1/2	6	0.0335
A10	2	3	1/3	9	2	1	7	7	2	1	1/3	4	3	8	0.1132
A11	1	2	2	9	3	1	3	3	9	3	1	7	3	9	0.1518
A12	1/5	1/4	1/4	1	1	1/4	1	1/2	1/2	1/4	1/6	1	1/2	6	0.0268
A13	1/5	2	1/4	6	3	1/3	7	4	2	1/3	1/3	2	1	6	0.0682
A14	1/8	1/3	1/8	3	1/3	1/6	1/3	1/2	1/5	1/8	1/9	1/5	1/5	1	0.0145
$\lambda_{\max}$ =15.6900 CI=0.1300 CR=0.0823															
RE															
A1	1	2	1/3	4	3	6	1	3	1	1/3	1	1	1	1/2	0.0649
A2	1/2	1	1/8	2	1	1	1/2	1	1/2	1/2	1/2	1/4	1/5	1/5	0.0252
A3	3	8	1	8	5	8	7	8	3	2	3	3	2	1/3	0.1613
A4	1/4	1/2	1/8	1	1/4	1/2	1/4	1/3	1/2	1/8	1/3	1/4	1/5	1/5	0.0151
A5	1/3	1	1/5	4	1	1	1/3	1	1/3	1/7	1/3	1/5	1/5	1/8	0.0229
A6	1/5	1	1/8	2	1	1	1/5	1	1/2	1/5	1/3	1/6	1/5	1/5	0.0197
A7	1	2	1/7	4	3	6	1	7	1	1/3	1/2	1/4	1/5	1/6	0.0499
A8	1/3	1	1/8	3	1	1	1/6	1	1/3	1/8	1/3	1/8	1/8	1/9	0.0192
A9	1	2	1/3	2	3	2	1	3	1	1/2	1/3	1/3	1/4	1/5	0.0409
A10	3	2	1/2	8	7	6	3	8	2	1	2	2	1/2	1/2	0.1072
A11	1	2	1/3	3	3	3	2	3	3	1/2	1	1	1/3	1/5	0.0570
A12	1	4	1/3	4	5	6	4	8	3	1/2	1	1	1/5	1/4	0.0794
A13	1	5	1/2	6	5	5	5	8	4	2	3	5	1	1	0.1411
A14	2	6	3	6	8	6	7	9	6	2	6	4	1	1	0.1965
$\lambda_{\max}$ =15.2684 CI=0.0976 CR=0.0618															
EWD															

A1	1	1/2	1/3	1	1/5	1	1	1	9	1/2	3	1	5	4	0.0606
A2	2	1	1/2	2	1/2	2	2	2	9	1/2	3	2	5	4	0.0916
A3	3	2	1	4	1/3	3	3	3	9	1/2	3	3	5	6	0.1268
A4	1	1/2	1/4	1	1/4	1/2	1/4	1	4	1/4	3	1	4	2	0.0444
A5	5	2	3	4	1	3	5	5	9	2	5	5	5	6	0.1999
A6	1	1/2	1/3	2	1/3	1	1	2	9	1/2	3	4	5	6	0.0798
A7	1	1/2	1/3	4	1/5	1	1	2	9	1/2	3	4	5	4	0.0801
A8	1	1/2	1/3	1	1/5	1/2	1/2	1	8	1/2	3	2	5	3	0.0563
A9	1/9	1/9	1/9	1/4	1/9	1/9	1/9	1/8	1	1/9	1/3	1/4	1/5	1/2	0.0101
A10	2	2	2	4	1/2	2	2	2	9	1	8	4	6	6	0.1387
A11	1/3	1/3	1/3	1/3	1/5	1/3	1/3	1/3	3	1/8	1	1/3	3	1/2	0.0251
A12	1	1/2	1/3	1	1/5	1/4	1/4	1/2	4	1/4	3	1	4	3	0.0428
A13	1/5	1/5	1/5	1/4	1/5	1/5	1/5	1/5	5	1/5	1/3	1/4	1	1/4	0.0187
A14	1/4	1/4	1/5	1/2	1/5	1/5	1/4	1/3	2	1/5	2	1/3	4	1	0.0252
$\lambda_{\mathbf{max}}$ =15.1678 CI=0.0898 CR= 0.0569															

Table 22
Pair-wise comparisons of the alternatives in relation to the sub-criteria ND, OS and PS (Source: authors' elaboration)

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	Local weight
ND															
A1	1	1/7	1/8	1	1/3	1	1	1	1	1/2	1/3	4	4	1/8	0.0342
A2	7	1	1/2	8	1	3	1	2	2	2	1	9	7	1/2	0.1052
A3	8	2	1	4	3	5	2	2	2	2	1/3	9	5	1/3	0.1128
A4	1	1/8	1/4	1	1/2	2	1/6	1/4	1/2	1/2	1/6	2	5	1/6	0.0272
A5	3	1	1/3	2	1	5	1	1	1	1/2	1/5	5	5	1/6	0.0563
A6	1	1/3	1/5	1/2	1/5	1	1/5	1/3	1/2	1/4	1/9	2	2	1/6	0.0209
A7	1	1	1/2	7	1	6	1	1	1	1/2	1/5	7	5	1/2	0.0702
A8	1	1/2	1/2	4	1	3	1	1	1	1/3	1/4	4	5	1/5	0.0489
A9	1	1/2	1/2	2	1	2	1	1	1	1	1/5	9	5	1/9	0.0513
A10	2	1/2	1/2	2	2	4	2	3	1	1	1	6	5	1/3	0.0786
A11	3	1	3	7	5	9	5	4	5	1	1	8	7	1/2	0.1587
A12	1/4	1/9	1/9	1/2	1/5	1/2	1/6	1/4	1/9	1/5	1/8	1	1/2	1/6	0.0119
A13	1/4	1/6	1/5	1/5	1/5	1/2	1/5	1/5	1/5	1/5	1/6	2	1	1/9	0.0139
A14	8	2	3	7	6	6	2	6	9	3	2	7	9	1	0.2100
$\lambda_{\mathbf{max}}$ =15.3585 CI=0.1045 CR=0.0661															
OS															
A1	1	1	1/3	1	5	1	7	1	1	1/2	1/3	1	3	6	0.0690
A2	1	1	1/2	1/2	2	1/2	2	1/2	1/2	1/2	1/3	1/2	2	3	0.0444
A3	3	2	1	3	5	3	7	3	3	2	3	3	5	6	0.1750
A4	1	2	1/3	1	5	1	3	1	1/2	1/2	1/3	1	4	6	0.0648
A5	1/5	1/2	1/5	1/5	1	1/5	1	1/5	1/9	1/5	1/5	1/4	1/5	1	0.0168
A6	1	2	1/3	1	5	1	3	1	1/2	1/2	1/3	1/4	5	6	0.0627
A7	1/6	1/2	1/7	1/3	1	1/3	1	1/3	1/6	1/6	1/6	1/4	2	6	0.0243

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A8	1	2	1/3	1	5	1	3	1	1/3	1/2	1/3	1/4	3	6	0.0584
A9	1	2	1/3	2	9	2	7	3	1	2	1	2	5	6	0.1161
A10	2	2	1/2	2	5	2	7	2	1/2	1	1/2	2	5	8	0.1017
A11	3	3	1/3	3	5	3	7	3	1	2	1	3	6	6	0.1380
A12	1	2	1/3	1	4	4	4	4	1/2	1/2	1/3	1	5	6	0.0879
A13	1/3	1/2	1/5	1/4	5	1/5	1/2	1/3	1/5	1/5	1/5	1/5	1	5	0.0276
A14	1/5	1/3	1/5	1/6	1	1/5	1/5	1/5	1/6	1/8	1/5	1/5	1/5	1	0.0133
$\lambda_{\max}$ =15.2942 CI=0.0996 CR=0.0630															
PS															
A1	1	1/2	1/7	4	1	1	1	5	1	1/3	7	1	5	6	0.0721
A2	2	1	1/3	5	1	1	3	4	2	1/2	3	2	5	5	0.0901
A3	7	3	1	8	3	6	6	8	3	1	6	4	4	7	0.2086
A4	1/4	1/5	1/8	1	1/3	1/7	1/2	1	1/5	1/5	1	1/3	2	4	0.0233
A5	1	1	1/3	3	1	1	5	5	1	1/2	9	4	5	6	0.1011
A6	1	1	1/5	7	1	1	3	4	1	1/3	6	2	6	9	0.0892
A7	1	1/3	1/5	2	1/5	1/3	1	2	1/2	1/5	2	1/2	7	4	0.0421
A8	1/5	1/4	1/8	1	1/5	1/4	1/2	1	1/4	1/8	1/3	1/4	5	4	0.0254
A9	1	1/2	1/3	5	1	1	2	4	1	1/2	4	2	4	6	0.0766
A10	3	2	1	6	2	3	6	8	2	1	2	4	4	9	0.1541
A11	1/6	1/3	1/5	1	1/9	1/5	1/2	3	1/4	1/2	1	1/2	2	3	0.0301
A12	1	1/2	1/4	3	1/4	1/2	2	4	1/2	1/4	2	1	5	6	0.0543
A13	1/5	1/5	1/4	1/2	1/5	1/5	1/7	1/5	1/4	1/4	1/2	1/5	1	5	0.0214
A14	1/5	1/5	1/6	1/4	1/5	1/9	1/4	1/4	1/5	1/9	1/3	1/5	1/5	1	0.0116
$\lambda_{\max}$ =15.6748 CI=0.1288 CR= 0.0815															

Table 23
Pair-wise comparisons of the alternatives with respect to the sub-criteria CR, CE and DI (Source: authors' elaboration)

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	Local weight
CR															
A1	1	1	1	1	5	6	1	1	3	1	3	1	1/5	1/5	0.0630
A2	1	1	1/2	2	5	2	2	2	2	1/2	2	1/2	1/3	1/2	0.0600
A3	1	2	1	3	5	3	3	3	3	2	3	1/3	1/3	1/3	0.0853
A4	1	1/2	1/3	1	4	4	1	1	4	1/2	2	1/4	1/4	1/3	0.0487
A5	1/5	1/5	1/5	1/4	1	1/2	1/5	1/5	1/2	1/5	1/3	1/5	1/5	1/5	0.0152
A6	1/5	1/2	1/3	1/4	2	1	1	1/5	2	1/3	1/3	1/5	1/5	1/5	0.0231
A7	1	1/2	1/3	1	5	1	1	1	2	1/2	3	1/4	1/5	1/7	0.0415
A8	1	1/2	1/3	1	5	6	1	1	4	1/2	3	1/4	1/5	1/5	0.0526
A9	1/3	1/2	1/3	1/4	2	1/2	1/2	1/4	1	1/2	1/3	1/4	1/9	1/9	0.0202
A10	1	2	1/2	2	5	3	2	2	2	1	2	1/2	1/2	1/2	0.0712
A11	1/3	1/2	1/3	1/2	3	3	1/3	1/3	3	1/2	1	1/3	1/5	1/5	0.0327
A12	1	2	3	4	5	6	4	4	4	2	3	1	1/4	1/4	0.1109
A13	5	3	3	4	5	6	5	5	9	2	5	4	1	2	0.1956

A14	6	2	3	3	6	6	7	5	9	2	6	4	1/2	1	0.1800
$\lambda_{\max}$ =15.3019	CI=0.1001	CR=0.0634													
CE															
A1	1	6	1	1	1/4	1/4	1/6	1	1	2	1/3	4	3	8	0.0580
A2	1/6	1	1/9	1/8	1/9	1/9	1/7	1/5	1/9	1/2	1/5	1/2	1/5	1/2	0.0120
A3	1	9	1	4	1/2	1/2	1/3	4	2	2	1/2	3	5	6	0.0864
A4	1	8	1/4	1	1/4	1/3	1/5	3	2	1	1/4	4	3	4	0.0556
A5	4	9	2	4	1	5	1/2	5	1	2	1/5	8	8	6	0.1320
A6	4	9	2	3	1/5	1	1/2	5	2	2	1/3	6	6	6	0.1046
A7	7	7	3	5	2	2	1	8	2	2	1/2	7	5	9	0.1554
A8	1	5	1/4	1/3	1/5	1/5	1/8	1	1/3	1/2	1/8	6	2	5	0.0363
A9	1	9	1/2	1/2	1	1/2	1/2	3	1	2	1/3	3	5	5	0.0686
A10	1/2	2	1/2	1	1/2	1/2	1/2	2	1/2	1	1/3	2	2	6	0.0467
A11	3	6	2	4	5	3	2	8	3	3	1	9	4	9	0.1868
A12	1/4	2	1/3	1/4	1/8	1/5	1/6	1/5	1/3	1/2	1/9	1	1/3	2	0.0175
A13	1/3	5	1/5	1/3	1/8	1/6	1/5	1/2	1/5	1/2	1/4	3	1	1/2	0.0247
A14	1/8	2	1/5	1/4	1/5	1/5	1/9	1/5	1/5	1/5	1/9	1/2	2	1	0.0153
$\lambda_{\max}$ =15.8600	CI=0.1431	CR=0.0906													
DI															
A1	1	1	1/7	1/2	1/5	1/4	1/7	1	1/3	1/3	2	1/5	1	1	0.0221
A2	1	1	1/7	1/8	1/9	1/9	1/7	1/5	1/9	1/9	3	1/7	1/3	1/3	0.0143
A3	7	7	1	3	2	3	3	7	3	3	5	3	5	7	0.1846
A4	2	8	1/3	1	1/2	1/3	1/5	4	2	2	5	1/3	5	3	0.0714
A5	5	9	1/2	2	1	1	1/2	5	1	2	9	5	5	3	0.1167
A6	4	9	1/3	3	1	1	1/2	5	2	2	9	4	6	6	0.1223
A7	7	7	1/3	5	2	2	1	5	2	2	7	1	4	6	0.1342
A8	1	5	1/7	1/4	1/5	1/5	1/5	1	1/3	1/3	3	1/4	4	1	0.0306
A9	3	9	1/3	1/2	1	1/2	1/2	3	1	1	7	1/2	4	6	0.0749
A10	3	9	1/3	1/2	1/2	1/2	1/2	3	1	1	4	1/2	5	6	0.0695
A11	1/2	1/3	1/5	1/5	1/9	1/9	1/7	1/3	1/7	1/4	1	1/9	1/3	1/3	0.0129
A12	5	7	1/3	3	1/5	1/4	1	4	2	2	9	1	7	4	0.0984
A13	1	3	1/5	1/5	1/5	1/6	1/4	1/4	1/4	1/5	3	1/7	1	3	0.0255
A14	1	3	1/7	1/3	1/3	1/6	1/6	1	1/6	1/6	3	1/4	1/3	1	0.0226
$\lambda_{\max}$ =15.6239	CI=0.1249	CR= 0.0791													

Table 24
List of selected assets (Source: authors' elaboration).

	Assets
A1	ATTIJARIWAFA BANK
A2	BMCE BANK
A3	BMCI
A4	CENTRALE DANONE
A5	COSUMAR
A6	ITISSALAT AL-MAGHRIB
A7	LAFARGEHOLCIM MAR
A8	LESIEUR CRISTAL
A9	LYDEC
A10	MANAGEM
A11	OULMES
A12	SMI
A13	TAQA MOROCCO
A14	WAFA ASSURANCE