DECISION-MAKING WITH MODELING OF PROBLEM SITUATIONS USING THE ANALYTIC NETWORK HIERARCHY PROCESS

Tatiana Kravchenko
State University – Higher School of Economics
Moscow, Russia
E-mail: tkravchenko@hse.ru

Natalia Seredenko
State University – Higher School of Economics
Moscow, Russia
E-mail: alia_nata@mail.ru

ABSTRACT

The Modeling of problem situations is a very important issue in decision-making theory. Actually, there are no decision support systems which include decision making methods under risk and uncertainty. The main advantage of a proposed approach is the ability to process dependences and feedbacks which may exist between conditions, sub-conditions and their realizations.

Keywords: modeling of economic decision-making problem situations, analytic hierarchy process (AHP), analytic network process (ANP), relevant significance estimates of problem situations.

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1. Introduction

The decision maker (DM) has to consider interrelation of his particular case with other areas of concern during the decision process. Thereto DM needs a comprehensive model of an external environment, which is called a scenario. In practice the DM constructs a more limited model focused on his specific problem – the local scenario (Figure 1).

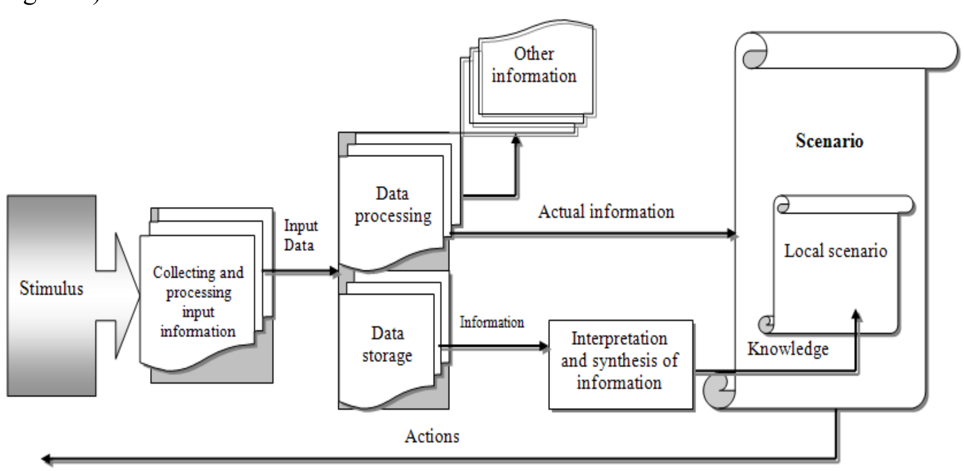

Figure 1 Model of the local scenario

When DM doesn't model even a local scenario, then, in fact, the decision is accomplished in "ideal conditions" without considering an external environment.

Quite often the local scenario is represented in the form of variant problem situations, which DM should take into account. In this case the choosing of an efficient alternative becomes more complicated because experts should evaluate alternatives considering all possible problem situations and DM should compare all the estimates obtained. Nevertheless, only the modeling of alternative problem situations can increase the efficiency of decision-making.

There are two approaches which are mainly used for the modeling of problem situations: simulation modeling and expert forecasting.

It is necessary to assign a set of conditions and dependences between them to deal with a simulation model. Then we may model a problem situation by combining different results of implementation.

If it is difficult to specify all dependences in the formal way, the modeling of problem situations are often based on expert forecast. There are questions which have to be solved for successful forecast:

- Organizational support of forecasting;
- Stating demands for forecasting;
- Organizing an analytical work group;
- Setting up an expert committee; and
- Preparing all needed background materials, software and data.

Precise expert forecast can be stated only if well prepared and competent experts engaged. Information source must be reliable and evaluations must be correctly collected and processed. Only experienced specialists are invited to join the expert committee.

The main advantage of problem situations modeling approach is ability to process dependences and feedbacks which may exist between conditions, sub-conditions and their realizations (most probable results of checking sub-conditions).

The methodological basis of proposed approach to the modeling of problem situations and evaluating judgments are the Analytical hierarchy process (AHP) and Analytical network process (ANP) (Saaty, 1993; Saaty, 2003; Saaty, 2008).

2. Theoretical background of the decision-making conditions modeling

We use AHP and ANP for building hierarchy of conditions, which are decomposed into sub-conditions and realizations (probable results of checking sub-conditions). All these items are considered in the problem to be solved.

There is a main decision goal "to find priorities of conditions, sub-conditions and their realizations" on the top of the hierarchy structure. The number of levels of conditions depends on the statement of the problem.

We assume that the main goal and levels of conditions, which are located under the main goal, are composed in the *Control Hierarchy of Conditions*.

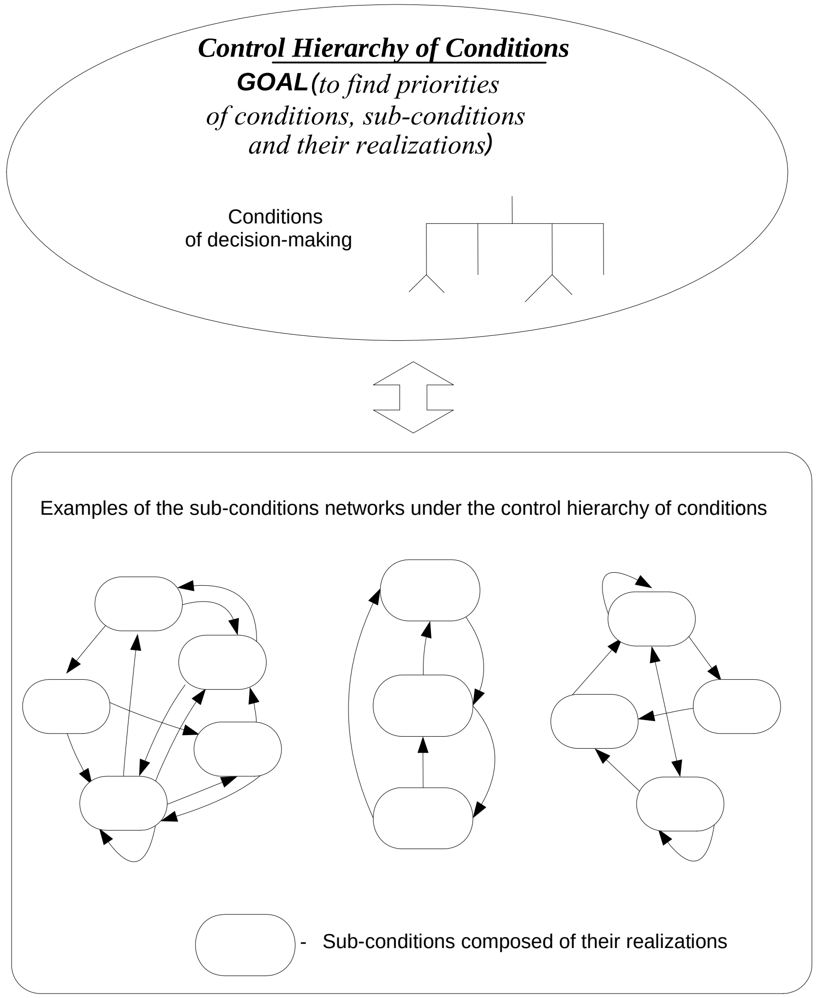

Figure 2 Examples of the sub-conditions networks

We use Analytical hierarchy process for modeling sub-conditions and their realizations. Under each condition of the control hierarchy the *sub-conditions network* should be modeled. Type of networks under the control hierarchy depends on the structural complexity of the problem. Figure 2 shows possible variants of the sub-conditions networks.

To process the control hierarchy of conditions we may use the Analytical hierarchy process (Saaty, 1993).

To get results (to calculate the relevant significance estimates of sub-conditions and their realizations) we may use the Analytical network process (Saaty, 2008).

3. Calculating the relevant significance estimates of decision-making conditions, sub-conditions and their realizations

3.1 Calculating the relevant significance estimates of decision-making conditions

Notation

- $U = (U_1, ..., U_m, ..., U_M),\ m = 1 \dots M$ — decision-making conditions;
- P_{mn} — elements of the pairwise comparison matrix for conditions, formed by DM, $m, n = 1 \dots M$;
- $p_{eig.M} = (p_{eig.1}, ..., p_{eig.M})$ — principal eigenvector of the pairwise comparison matrix for conditions;
- $\lambda_{\max M}$ — principal eigenvalue of the pairwise comparison matrix for conditions; and
- $P_M = (p_1, ..., p_M)$ — vector of the relevant significance estimates (weights) of conditions.

Input data

- U_m — decision-making conditions, $m = 1 \dots M$;
- P_{mn} — elements of the pairwise comparison matrix for conditions, $m = 1 \dots M$.

All ratios should be estimates as numbers using the Fundamental Scale of the AHP, consisting of: $\{1/9, 1/8, 1/7, 1/6, 1/5, 1/4, 1/3, 1/2, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. There is a correlation between evaluations P_{mn} and P_{nm}: $P_{nm} = 1/P_{mn}$. It means that if $P_{nm} = 7$, then the relevant significance estimates of *m*- condition is very strong dominate *n*- condition. Note that in this case P_{nm} = 1/7, so it means that the relevant significance estimates of *n*- condition is very strongly dominated by *m*- condition.

Solution algorithm

1. DM forms input data.
2. DM fills up the pairwise comparison matrix with elements P_{mn}, where $m, n = 1 \dots M$, to evaluate the relevant significance estimates of *m*- and *n*- conditions.
3. Then the principal eigenvector of the pairwise comparison matrix for conditions $p_{eig.M}$ has to be obtained (Saaty, 2003). There is a general equation for obtaining the principal eigenvector (in compliance with the principal eigenvector definition):

$$(1)\quad P_{mn} \times p_{eig.M} = \lambda_{\max M} \times p_{eig.M}$$

4. The elements of obtained eigenvector have to be normalized:

$$(2)\quad p_m = \frac{p_{eig.m}}{\sum_m p_{eig.m}}$$

5. The vector $P_M = (p_1, ..., p_M)$ is the required vector relevant significance estimates of conditions, which will be used further for calculating the relevant significance estimates of sub-conditions and their realizations.

The example of forming and processing the control hierarchy of conditions is described in chapter 5.2.

Table 1
The Fundamental Scale of absolute numbers

Rating	*Interpretation of the evaluations*
1	relevant significance estimate is equal importance
2	relevant significance estimate is weak
3	relevant significance estimate is moderate importance
4	relevant significance estimate is moderate plus
5	relevant significance estimate is strong importance
6	relevant significance estimate is strong plus
7	relevant significance estimate is demonstrated importance
8	relevant significance estimate is very, very strong
9	relevant significance estimate is extreme importance

3.2 Calculating relevant significance estimates of decision-making sub-conditions and their realizations

After processing the control hierarchy of conditions, DM should form and make calculations over all sub-networks, presenting the sub-conditions structure for each condition.

As stated above, the structure of sub-networks under the control hierarchy of conditions depends on the decision-making problem. One of the possible views of decision sub-conditions and their realizations sub-networks is shown in Figure 3.

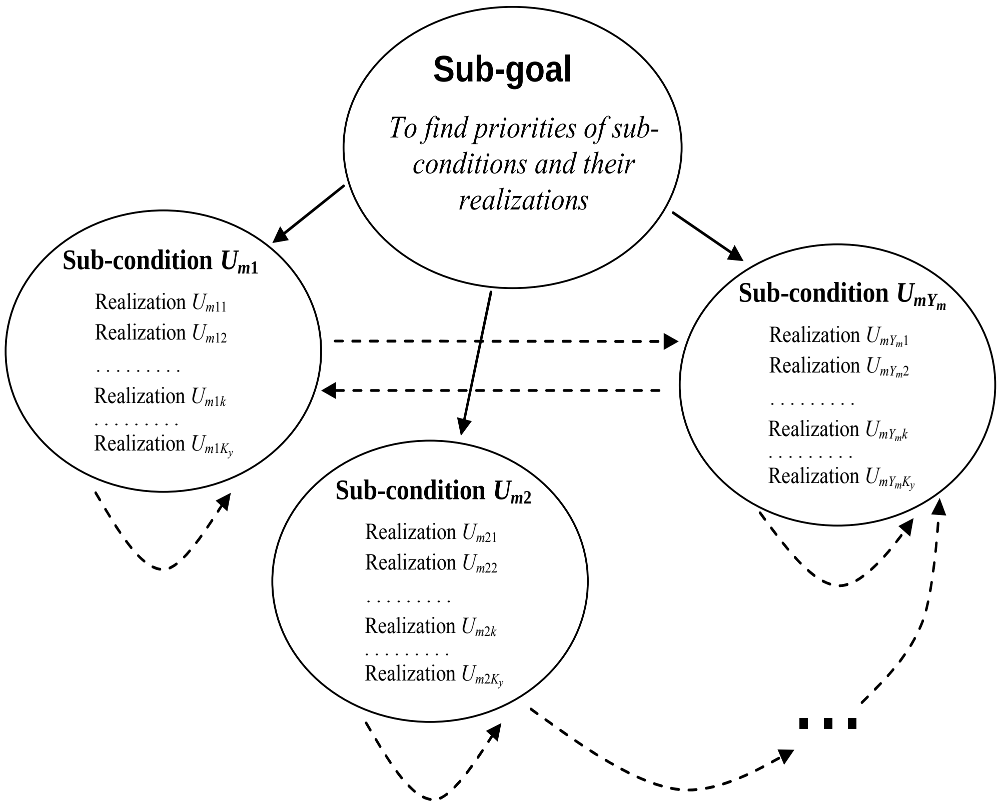

Figure 3 Example of the sub-networks under the control hierarchy of conditions

Such sub-network structures can have the following dependences between the sub-goal, sub-conditions and their realizations:

- Connections between the sub-goal and sub-conditions, influencing the sub-goal. This type of outer dependence is shown by the solid arrows outgoing the sub-goal in Figure 3 (dependences of *type I*).
- Connections between the sub-conditions and their realizations:
  - Connections between two different sub-conditions and their realizations. This type of outer dependence is shown in Figure 3 by the dotted arrows, drawn from one sub-condition to another sub-condition whose elements influence it (dependences of *type II.a*);
  - Interconnections between two different sub-conditions and their realizations. This type of outer dependence is shown in Figure 3 by the two dotted multidirectional arrows between two different sub-conditions (dependences of *type II.b*).
- Connections between realizations within some sub-condition. This type of inner dependence is shown in Figure 3 by the dotted arrows, drawn from one sub-condition to the same sub-condition (dependences of *type III*).

As stated above, it is applicable to use ANP for processing such types of sub-networks. In this paper we give the interpretation of ANP for modeling sub-conditions and their realizations.

<u>Notation</u>

<u>Dependences of *type I*</u>

- U_{my} — decision-making sub-conditions of *m*- condition, where $y = 1 \ldots Y_m$, for all $m = 1 \ldots M$;
- U_{myk} — *k*- realizations, formed within *y*- sub-condition, within *m*- condition, where $k = 1 \ldots K_y$, $y = 1 \ldots Y_m$, $m = 1 \ldots M$.
- $P^I_{m(y,t)}$ — elements of the pairwise comparison matrix for *y*- and *t*- sub-conditions influencing *m*- condition.

These matrixes reflect the dependences of *type I*. The number of such type of matrixes equals to the number of conditions, which are considered in the decision-making problem. The general view of the pairwise comparison matrix of *type I* is illustrated in Table 2.

Table 2
General view of the pairwise comparison matrix for the sub-conditions of *m*- condition

	U_{m1}	…	U_{mt}	…	U_{mY_m}
U_{m1}			…		
…			…		
U_{my}	…	…	$P^I_{m(y,t)}$		
…					
U_{mY_m}					

- $p^I_{eig.\,m}$ — principal eigenvector of the pairwise comparison matrix for U_{my} sub-conditions influencing *m*- condition; $m = 1 \ldots M$.
- $\lambda^I_{\max m}$ — principal eigenvalue of the pairwise comparison matrix for sub-conditions influencing *m*- condition; $m = 1 \ldots M$.
- P^I_{my} — vector of the relevant significance estimates (weights) of sub-conditions influencing *m*- condition; $y = 1 \ldots Y_m$, $m = 1 \ldots M$.

<u>Dependences of *type IIa*</u>

- *t* — sub-conditions of *m*-condition, influencing on the other sub-conditions;
- *y* — sub-conditions of *m*-condition, being influenced by the other sub-conditions;
- U_{myk} — *k*- realizations of *y*- sub-condition (of *m*-condition), being influenced by the other sub-conditions, $k = 1 \ldots K_y$;
- U_{mth} — *h*- realization of *t*- sub-condition (of *m*-condition), influencing the realizations of other sub-conditions, $h = 1 \ldots K_t$; and

- $P^{IIa}_{U_{myk}}(U_{mth}, U_{mtl})$ — elements of the pairwise comparison matrix for *h*- and *l*-realizations of *t*- sub-condition of *m*- condition, influencing *k*- realizations of *y*- sub-condition.

Note that in compliance with the notation conventions, as shown in Figure 3, *y*- sub-conditions are the sub-conditions from which the dotted arrows come and *t*- sub-conditions are the sub-conditions in which the dotted arrows are.

Each generated dependence (dotted arrow) would be defined by such number of pairwise comparison matrixes for *h*- and *l*- realizations of *t*- sub-conditions, what number of *k*- realizations is contained in *y*- sub-condition, being influenced by *t*- sub-condition. The general view of the pairwise comparison matrixes of *type IIa* is illustrated in Table 3.

Table 3
General view of the pairwise comparison matrix for the realizations of *t*- sub-condition, influencing the realizations of *y*- sub-condition

	U_{mt1}	…	U_{mtl}	…	U_{mtK_t}
U_{mt1}			…		
…			…		
U_{mth}	…	…	$P^{IIa}_{U_{myk}}(U_{mth},$		
…					
U_{mtK_t}					

- $p^{IIa}_{eig.m(y,t)}$ — principal eigenvectors of the pairwise comparison matrixes in which the realizations of *t*- sub-condition, influencing the realizations of *y*- sub-conditions, are estimated;
- $\lambda^{IIa}_{\max.m(y,t)}$ — principal eigenvalues of the pairwise comparison matrixes; and
- $P^{IIa}_{m(y,t)}$ — vectors of the relevant significance estimates (weights) of decision-making realizations.

Dependences of *type IIb*

In case of interdependence between two different sub-conditions and their realizations (dependences *type IIb*, as shown in Figure 3 by the two dotted multidirectional arrows), besides matrixes of *type IIa*, additional matrixes should be completed.

$P^{IIb}_{U_{mth}}(U_{myk}, U_{myl})$ — elements of the pairwise comparison matrix for *k*- and *l*- realizations of *y*- sub-condition of *m*- condition, being influenced by the *h*- realizations of *t*- sub-condition.

Note that in compliance with the notation conventions, as shown in Figure 3, *t*- sub-conditions are the sub-conditions, from which the dotted arrows come and *y*- sub-conditions are the sub-conditions in which the dotted arrows are.

Each generated dependence (dotted arrow) would be defined by such number of pairwise comparison matrixes for *k*- and *l*- realizations of *y*- sub-conditions, what number of *h*- realizations is contained in *t*- sub-condition, being influenced by *y*- sub-condition. The general view of the pairwise comparison matrixes of *type IIb* is illustrated in Table 4.

Table 4
General view of the pairwise comparison matrix for the realizations of *t*- sub-condition, being influenced by the realizations of *y*- sub-condition

	U_{my1}	…	U_{myl}	…	U_{myK_y}
U_{my1}			…		
…			…		
U_{myk}	…	…	$P^{IIb}{}_{U_{mth}}(U$		
…					
U_{myK_y}					

Thus, Tables 3 and 4 show the *type II* dependences.

- $p^{IIb}{}_{eig.m(t,y)}$ — principal eigenvectors of the pairwise comparison matrixes, in which the realizations of *t*- sub-condition, being influenced by the realizations of *y*- sub-conditions of *m*- condition, are estimated;
- $\lambda^{IIb}{}_{\max.m(t,y)}$ — principal eigenvalues of the pairwise comparison matrixes; and
- $P^{IIb}{}_{m(t,y)}$ — vector of the relevant significance estimates (weights) of decision-making realizations.

<u>Dependences of *type III*</u>

- *t* — sub-conditions of *m*-condition with internal dependences between realizations;
- U_{mtk} — *k*- realizations of *t*- sub-condition of *m*- condition, being influenced by the other realizations of the same sub-condition, where $k = 1 \ldots K_t$;
- U_{mth} , U_{mtl} – *h*- and *l*- realizations of *t*- sub-condition of *m*- condition, influencing the other realizations of the same sub-condition, where $k = 1 \ldots K_t$; and
- $P^{III}{}_{U_{mtk}}(U_{mth}, U_{mtl})$ – elements of the pairwise comparison matrix for *h*- and *l*- realizations of *t*- sub-condition of *m*- condition, influencing *k*- realizations of the same *t*- sub-condition.

Note that each generated dependence (shown as dotted arrow drawn from one sub-condition to the same sub-condition) would be defined by such number of pairwise comparison matrixes for *k*- and *l*- realizations, what number of realizations is contained in *t*- sub-condition. The general view of the pairwise comparison matrixes of *type III* is illustrated in Table 5.

Table 5
General view of the pairwise comparison matrix for the realizations of sub-condition with internal dependences and feedback

	U_{mt1}^{III}	…	U_{mtl}^{III}	…	$U_{mtK_t}^{III}$
U_{mt1}^{III}			…		
…			…		
U_{mth}^{III}	…	…	$P_{U_{mtk}(U_{mth}}^{III}$		
…					
$U_{mtK_t}^{III}$					

- $P_{eig.U_{mt}}^{III}$ — principal eigenvectors of the pairwise comparison matrixes for the realizations of each *t*- sub-condition of *m*- condition;
- $\lambda_{\max U_{mt}}^{III}$ — principal eigenvalues of the pairwise comparison matrixes for the realizations of each *t*- sub-condition of *m*- condition;
- $P_{U_{mt}}^{III}$ — vector of the relevant significance estimates (weights) of decision-making realizations of sub-conditions with internal dependences of *m*- condition;
- $P_{mSuperMatr}$ – weighted supermatrix composed of the eigenvectors derived from the pairwise comparison matrixes for the realizations of each sub-condition of *m*- condition with elements: $P_{U_{myk}(U_{mth},U_{mtl})}^{IIa}$, $P_{U_{mth}(U_{myk},U_{myl})}^{IIb}$, $P^{III}{}_{U_{mtk}(U_{mth},U_{mtl})}$;
- $P_{mSuperMatr}^{\lim}$ — weighted supermatrix being raised to powers (the limit supermatrix); and
- P_{myk} — required vectors of the global relevant significance estimates (weights) of decision-making realizations.

Input data

- U_{my} — decision-making sub-conditions of *m*- condition, where $m = 1 ... M$, $y = 1 ... Y_m$;
- U_{myk} — *k*- realizations of *t*- sub-condition of *m*- condition; and
- $P^I{}_{m(y,t)}$, $P_{U_{myk}(U_{mth},U_{mtl})}^{IIa}$, $P_{U_{mth}(U_{myk},U_{myl})}^{IIb}$, $P^{III}{}_{U_{mtk}(U_{mth},U_{mtl})}$ — elements of the pairwise comparison matrixes formed by DM.

Solution algorithm

1. DM forms input data.

2. DM fills up the pairwise comparison matrixes with elements $P^{I}_{m(y,t)}$ to evaluate the relevant significance estimates of sub-conditions. All ratios should be estimates as numbers using the Fundamental Scale of the AHP. Interpretation of the evaluations is illustrated in Table 1.
3. Then the principal eigenvector of the pairwise comparison matrix for sub-conditions $p^{I}_{eig.m}$ has to be obtained (Saaty, 2003). There is a general equation for obtaining the principal eigenvector (in compliance with the principal eigenvector definition):

$$(3)\ P^{I}_{m(y,t)} \times p^{I}_{eig.m} = \lambda^{I}_{\max m} \times p^{I}_{eig.m}\ .$$

4. The elements of obtained eigenvector have to be normalized:

$$(4)\ p^{I}{}_{my} = \frac{p^{I}_{eig.my}}{\sum_{y=1}^{Y_m} p^{I}_{eig.my}}.$$

5. DM forms the pairwise comparison matrixes with elements $P^{IIa}{}_{U_{myk}(U_{mth},U_{mtl})}$, $P^{IIb}{}_{U_{mth}(U_{myk},U_{myl})}$ and $P^{III}{}_{U_{mtk}(U_{mth},U_{mtl})}$. Filling the matrixes, the DM has to answer the basic question in all pairwise comparisons: «How many times more dominant is one element than the other with respect to a certain element (sub-condition or condition)? ». All ratios should be estimates as numbers using the Fundamental Scale of the AHP. This allows to take into account the mutual influences and inner dependences of sub-conditions. Interpretation of the evaluations is illustrated in Table 1.
6. Then the principal eigenvectors of the pairwise comparison matrixes for realizations $p^{IIa}_{\text{eig.}\,m(y,t)}$, $p^{IIb}_{\text{eig.}\,m(t,y)}$ and $p^{III}_{\text{eig.}\,mt}$ have to be obtained (Saaty, 2003). There are general equations for obtaining the principal eigenvectors:

$$(5.1)\ P^{IIa}{}_{U_{myk}(U_{mth},U_{mtl})} * p^{IIa}{}_{eig.m(y,t)} = \lambda^{IIa}{}_{\max m(y,t)} * p^{IIa}{}_{eig.m(y,t)}\ ;$$

$$(5.2)\ P^{IIb}{}_{U_{mth}(U_{myk},U_{myl})} * p^{IIb}{}_{eig.m(t,y)} = \lambda^{IIb}{}_{\max m(t,y)} * p^{IIb}{}_{eig.m(t,y)}\ ;$$

$$(6)\ P^{III}{}_{U_{mtk}(U_{mth},U_{mtl})} * p^{III}{}_{eig.mt} = \lambda^{III}{}_{\max mt} * p^{III}{}_{eig.mt}\ .$$

7. The elements of obtained eigenvectors have to be normalized:

$$(7.1)\ p^{IIa}{}_{m(y,t)k} = \frac{p^{IIa}{}_{eig.m(y,t)k}}{\sum_{k=1-K_t} p^{IIa}{}_{eig.m(y,t)k}}\ ;$$

$$(7.2)\ p^{IIb}{}_{m(t,y)k} = \frac{p^{IIb}{}_{eig.m(t,y)k}}{\sum_{k=1-K_t} p^{IIb}{}_{eig.m(t,y)k}}\ ;$$

$$(8)\ p^{III}{}_{mtk} = \frac{p^{III}{}_{eig.mtk}}{\sum_{k=1-K_t} p^{III}{}_{eig.mtk}}.$$

8. All derived vectors of the relevant significance estimates (weights) of decision-making realizations and sub-conditions (pt. 4 and pt. 7) should be placed in the col-

umns of the supermatrix $P_{mSuperMatr}$ (Table 6). The blocks of the supermatrix are to be filled according to the following rules:

- The vectors P^{I}_{m} presenting dependences of *type I* are used to weigh the elements of the corresponding blocks of the supermatrix.
- The vectors $P^{IIa}_{m(y,t)}$, $P^{IIb}_{m(t,y)}$ presenting dependences of *type IIa* and *type IIb* are placed within the outside of the main diagonal blocks according to the following rules:
    - In the case of outer dependence between two different sub-conditions and their realizations (dependence of *type IIa*), only the one block of the supermatrix has to be filled. The symmetry with respect to the main diagonal block is all zero. This case is illustrated in Table 6 by blocks 2.1 and 2.2; and
    - In the case of interdependence between two different sub-conditions and their realizations (dependence of *type IIb*), both blocks symmetry with respect to the main diagonal of the supermatrix have to be filled. This case is illustrated in Table 6 by blocks 4.1 and 4.2.
- The vectors P^{III}_{mt} presenting dependences of *type III* should be placed on the main diagonal blocks. This case is illustrated in Table 6 by block 3. In case there are no inner dependences within the sub-condition then the corresponding block is all zero. This case is illustrated in Table 6 by block 1.

9. Then, to obtain the limit supermatrix, it is necessary to raise the supermatrix to powers until it is orderly cyclic (Saaty, 2008):

$$(9)\ P^{\lim}_{SuperMatr} = \lim_{k\to\infty}\frac{1}{N}\sum_{k=1}^{N} P^{k}_{SuperMatr}.$$

10. In compliance with ANP the required vectors of the global relevant significance estimates (weights) of decision-making realizations P_{myk} may be obtained from the limit supermatrix $P^{\lim}_{mSuperMatr}$ (Saaty, 2008). Note that all inner and outer dependences between sub-conditions and their realizations are taken into account. The similar calculations should be implemented for each *m*- condition, where *m=1...M*.

Table 6
General view of the Weighted Supermatrix

		Sub-conditions				
		Realizations	…	Realizations	…	Realizations
Sub-conditions	Realiza-tions	(1) Zero entries		(2.2) Zero entries		
	Realiza-tions	(2.1) Zero entries		(3) Nonzero entries		(4.2) Nonzero entries
	Realiza-tions			(4.1) Nonzero entries		

The example of processing the sub-conditions and their realizations network structure is described in chapter 5.3.

4. Modeling of decision-making problem situations

In fact, the problem situation is a set of different realizations of all considered sub-conditions of all considered decision-making conditions, between which there is a logical conjunction “AND”. Besides, one problem situation should contain only one realization of each sub-condition. Because of the logical connective “and” it is reasonable to add the relevant significance estimates (weights) of decision-making realizations which make a problem situation in aggregate.

Thus, the indicator of the relevant significance estimate (weight) of decision-making problem situation may be calculated as a sum of weights of all realizations which are made in aggregate this problem situation.

If there is a considerable quantity of different sub-conditions and their realizations, then the quantity of decision-making problem situations will be very great. In addition, after forming a set of problem situations every expert should provide all corresponding judgments (evaluating alternatives with respect to the criteria etc.) considering all problem situations. So the labour-intensiveness of decision-making can be unreasonably high. Therefore only six-to-eight problem situations with the highest weights are usually considered.

The example of forming a set of problem situations is described in chapter 6.4.

5. The example of forming a set of problem situations and their relevant significance estimates

To illustrate the stated theoretical basis consider IT outsourcing decision making. This decision-making problem has been described by the scientific group of Youxu Tjader, Jennifer Shang, Luis Vargas and Jerry May. These researchers made a report at the international symposium ISAHP2009. In that paper (Tjader, Shang, Vargas and May, 2009) the ANP and the Balanced Scorecard (BSC) are used for IT outsourcing decision making.

According to the current article, it is offered to use the BSC for revealing a set of decision-making conditions and their sub-conditions. Thus the concept of “perspective” is interpreted as a decision-making condition and concept of “key indicator” is interpreted as a decision-making sub-condition. The AHP/ANP are used for the modeling of decision-making problem situations and calculating their relevant significance estimates.

5.1 Target setting

IT outsourcing decision making is a really topical problem for any large-scale enterprise. It is necessary to analyze all circumstances around the company which may occur in the future to make an effective decision. So, as shown in Figure 4, we consider perspectives (financial, customer, company learning and growth, internal operations) as decision-making conditions and their key indicators as decision-making sub-conditions (Tjader, Shang, Vargas and May, 2009).

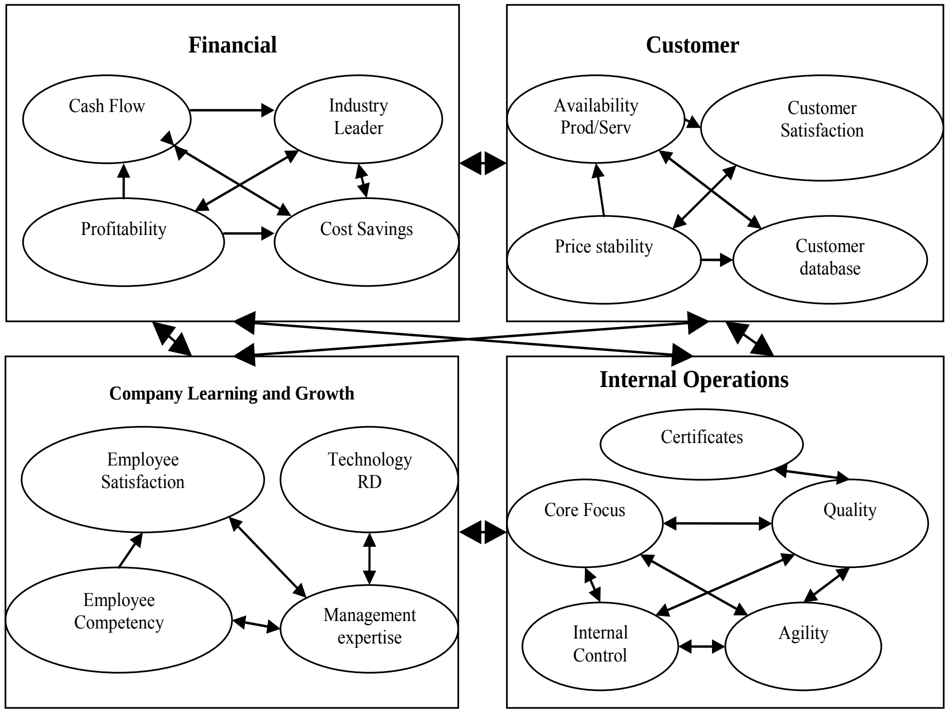

Figure 4 Framework of perspectives and key indicators

5.2 Forming the Control Hierarchy of Conditions

Figure 5 gives the Control Hierarchy of Conditions for the IT outsourcing decision making.

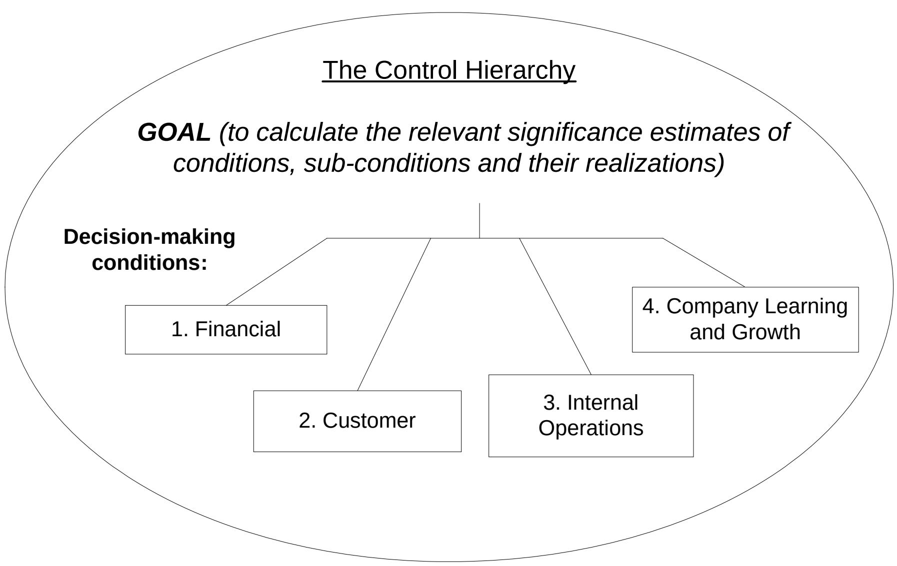

Figure 5 The Control Hierarchy for the IT outsourcing decision making

After forming the set of conditions it is necessary to model network sub-conditions structure for each of the conditions.

5.3 The example of the sub-conditions network structure for the Financial condition

Figure 6 gives the sub-conditions network structure for the *Financial* condition for the IT outsourcing decision making.

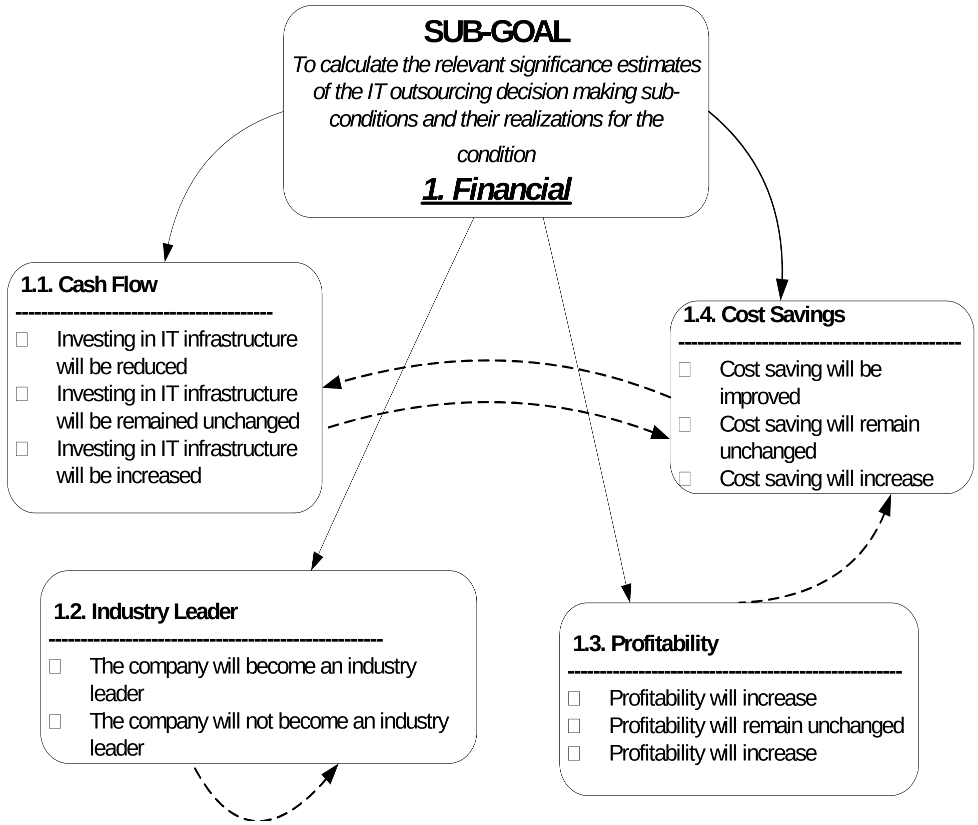

Figure 6 The sub-conditions network structure for the *Financial* condition

As shown in Figure 6, the *Financial* condition consists of the following sub-conditions:

- 1.1. Cash flow.
- 1.2. Industry Leader.
- 1.3. Profitability.
- 1.4. Cost Savings.

In turn, each of the sub-conditions consists of the appropriate set of realizations. For example, the sub-condition "*Cash flow*" may subdivide into the following realizations:

- Investing in IT infrastructure will be reduced.
- Investing in IT infrastructure will remain unchanged.
- Investing in IT infrastructure will be increased.

5.4 The example of modeling the problem situations

According to the rules stated above, we may form the required set of problem situations for the IT outsourcing decision making. The elements in this set are as follows:

- *S1*: {Investing in IT infrastructure will be reduced; the company will become an industry leader; profitability will increase; …}.
- *S2*: { Investing in IT infrastructure will remain unchanged; the company will become an industry leader; profitability will increase; …}.
- *S3*: { Investing in IT infrastructure will be increased; the company will become an industry leader; profitability will increase; …}.
- *S4*: { Investing in IT infrastructure will be reduced; the company will not become an industry leader; profitability will increase; …}.
- …

By this way all possible combinations of realizations must be searched. Then all the relevant significance estimates (weights) of decision-making realizations of sub-conditions for each condition have to be calculated. As an example the relevant significance estimates of decision-making realizations of sub-conditions for the *Financial* condition are resulted in Table 7. All calculations are made by the decision support system *SuperDecisions* (Saaty, 2002).

Table 7
The relevant significance estimates of decision-making realizations of sub-conditions for the *Financial* condition

Realizations	**Estimates of sub-conditions weighted by the estimates of conditions**	**Local estimates of realizations (within sub-conditions)**	**Global estimates of realizations**
1.1. Cash flow:	**0,04**		
▪ Investing in IT infrastructure will be reduced		0,6	0,024
▪ Investing in IT infrastructure will remain unchanged		0,2	0,008
▪ Investing in IT infrastructure will be increased		0,2	0,008
1.2. Industry leader:	**0,09**		
▪ The company will become an industry leader		0,3	0,027
▪ The company will not become an industry leader		0,7	0,063
1.3. Profitability:	**0,19**		
▪ Profitability will increase		0,5	0,095
▪ Profitability will remain unchanged		0,4	0,076
▪ Profitability will increase		0,1	0,019
…	…	…	…

After calculating such values for every condition, the relevant significant estimates of problem situations may be obtained. As stated above, these coefficients may be calculated as a sum of the relevant significant estimates of realizations of which the problem situations are composed.

We may cite as an example the problem situation S_1: investing in IT infrastructure will be reduced; the company will become an industry leader; profitability will increase; cost saving will be improved; availability of product will decrease; customer satisfaction will decrease; price stability will remain unchanged; customer database will be reduced by 10 percent; employee satisfaction will remain unchanged; technology RD will be developed; employee competency will remain unchanged; management knowhow will increase; certifications will remain unchanged; core focus will remain unchanged; quality will increase; internal control will be strengthened; agility will reduce.

The relevant significant estimate of P_{S_1} may be calculated this way:

$$\text{(10)} \quad \mathrm{P}_{S_1} = 0{,}024 + 0{,}027 + 0{,}095 + 0{,}054 + 0{,}077 + 0{,}033 + 0{,}055 + 0{,}016 + \\ + 0{,}035 + 0{,}009 + 0{,}012 + 0{,}014 + 0{,}087 + 0{,}022 + 0{,}008 + 0{,}016 + 0{,}014 = 0{,}742$$

In this sum (10) the first four terms represent the relevant significant estimates of realizations of sub-conditions for the *Financial* condition, the next four are the estimates of realizations of sub-conditions for the condition *Customer*, the next five are the estimates of realizations of sub-conditions for the condition *Internal Operations* and the last four are the estimates of realizations of sub-conditions for the condition *Company Learning and Growth*.

By the same way the relevant significant estimates of all the other problem situations should be calculated. Then the problem situations have to be sorted by the coefficients and the most significant of them are to be included in the task of the IT outsourcing decision making.

6. Conclusion

This paper provides an approach to the modeling of economic decision-making problem situations using the Analytical hierarchy process and the Analytical network process.

The calculation algorithm of the relevant significant estimates of decision-making conditions, sub-conditions and their realizations is described. The decision support system *SuperDecisions* has been used for the auxiliary calculations.

The development of an algorithm and software for modeling of problem situations and finding among them the most significant are the directions for future researches.

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