AN EVALUATION OF METHODS TO MAKE TRANSITIVE THE MULTIPLE COMPARISONS MATRICES

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ABSTRACT

In this paper we analyse the response of several methods to construct indices for consistent multiple comparison. We consider also the close formal connection between the comparison of preference judgements and the comparison of economic aggregates. To evaluate the various scaling methods, we have used official data furnished by Eurostat. Consequently our analysis is based on real-life data and not on simulations as is usually the case in study in this kind. The most important results that we have achieved are the close concordance of the weights obtained with the various methods and the robustness of the evaluations performed.

Keywords: Ratio-scale matrices, Multiple comparisons, Index numbers.

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1. Introduction

The objective of this paper is to examine some procedures of economic entity comparisons. As is well known, comparisons may be bilateral (or binary) when individual pairs of entities are compared, or they may be multiple (or multilateral) when the intention is instead to grade the entities compared.

The binary comparisons are simpler to set up and easier to implement. However, the information obtained in this way and arranged in appropriate ratio-scale matrices is normally inconsistent when used for multilateral comparisons. We shall illustrate this aspect in the second section.

In the third section we discuss methods which enable consistent matrices to be constructed even independently of matrices for binary relationships. This is a line of inquiry which has been pursued and developed especially by economic statisticians.

The results obtained from application of the methods discussed in the second and third sections, results which enable the entities compared to be ranked consistently, are analyzed in the fourth section.

Our procedure is to consider official data furnished by Eurostat. The analysis is consequently based on real-life data, and not on simulations as is usually the case in

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studies of this kind. Although on the one hand this might be considered a limitation, in that only a limited number of cases are examined, on the other it has the advantage that situations of little or no practical importance are eliminated, thereby yielding, we believe, a clearer and more concrete picture. The fifth section makes some concluding remarks.

2. Methods to Give Consistency to Reciprocal Positive Matrices Constructed by Binary Comparisons

Binary comparisons between alternatives in order to rank preferences for each of them or, in other words, to determine an ordered set of weights to be associated with them, have for some time been the subject of a broad strand of studies (Saaty, 1980; Saaty and Vargas, 1984; Crowford and Williams, 1985) falling under the general heading of AHP (Analytic Hierarchy Process) and which is still developing in various directions.

Of particular interest among these various lines of development are those involving the more systematic use of statistical tools (Haines, 1998; Carriere and Finster, 1992; Basak, 2002) and those that combine AHP with various methods of multicriterion analysis (Lootsma, 1997; Lootsma, Ramanathan and Schuijt, 1998; Guitouni and Martel, 1998).

To restrict the discussion to the matters treated empirically in this study, we consider the set $A = \{A_1, \dots, A_n\}$ of the n alternatives to compare, and we use **C** to denote the n order square matrix whose generic element c_{ij} expresses the extent to which alternative A_i is more important than A_j, assuming that this extent is expressible on a ratio scale measurement.

At most n(n-1)/2 comparisons are required because a minimum consistency is imposed whereby:

$$c_{ji} = 1/c_{ij} \qquad i,j = 1, \dots, n \tag{1}$$

so that if, for example $c_{ij} = 3$ that is, if A_i is deemed to be three times more important than A_j, the importance of A_j is one third that of A_i hence $c_{ji} = 1/3$. It straightforwardly follows from (1) that $c_{ii} = 1$ for i = 1, …, n.

Matrices with positive elements possessing property (1) are known as “positive reciprocal matrices”.

It should be pointed out, however, that constraint (1) does not ensure complete consistency when multiple comparisons are made. In fact, this consistency only comes about if the elements of the positive reciprocal matrix **C** satisfy the relations:

$$c_{ij} = c_{ik} \bullet c_{kj} \qquad \forall\ i, j, k \tag{2}$$

If the weights (importance) w_i to associate with the individual alternatives A_i were known, and if matrix **C** were consistent, one would have:

$$c_{ij} = w_i/w_j \qquad i, j = 1, \dots, n \tag{3}$$

Note that the weights w_i, which we shall consider in what follows as components of the vector **w**, are determined up to a multiplication by a constant, which is arbitrarily determinable.

As is evident from (3), in the case of perfect consistency, the vector **w** of the weights would be immediately deducible from any row or column of matrix **C**. In practice, however, only rarely does this matrix display the complete consistency defined by (2) when it is actually constructed.

This feature has prompted the development of various methods to determine the weights by minimizing the divergence, opportunely defined, between the values c_{ij} and the theoretical ones w_i/w_j.

Various studies (Golany and Kress, 1993; Dodd, Donegan and McMaster, 1995) have used stochastic simulation techniques to assess the performance of the various adjustment methods. In our empirical analysis, we applied the best known and most widely used of these methods, which are now briefly surveyed.

Before beginning the survey, however, we would point out that the multiplier constant up to which the weights are defined can be chosen in various ways: by giving unitary value to one component of vector **w** or imposing the unit-sum constraint or the unitary product constraint on the components of vector **w**. Obviously, this is an arbitrary choice made purely for reasons of computational convenience.

2.1 Dominant eigenvalue method (henceforth DE)

This is the method originally proposed by Saaty (1980) and based on the consideration that if the relations in (3) hold, then matrix **C** has one single eigenvalue different from zero whose value corresponds to the order of the matrix. Consequently the vector of the weights **w** coincides with the corresponding eigenvector.

$$\mathbf{Cw} = \lambda \mathbf{w} \tag{4}$$

In the practice, the eingenvector **w** associated with the dominant eigenvalue λ of the **C** is considered to be a reasonable evaluation of the vector of the weights. Frobenius's theorem ensures the positivity of the components of **w** and therefore the latter's acceptability in the context considered.

Moreover, it is possible to measure the degree of approximation associated with this evaluation by means of the index:

$$I(\mathbf{C}) = (\lambda - n)/(n-1) \tag{5}$$

known as the "consistency ratio".

2.2 Modified dominant eigenvalue method (henceforth MDE)

Introduced by Cogger and Yu (1985), this method is a variant of the previous one. Bearing in mind that matrix **C** is reciprocal, this technique considers only the upper triangle. Using **T** to denote the matrix such that:

$$t_{ij} = \begin{cases} c_{ij} & \text{if } i \geq j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

and **G** to denote the diagonal matrix with the elements:

$$g_{ii} = n\text{-}i+1 \qquad i=1,\dots,n \tag{7}$$

the weights correspond to the solution of the system:

$$(\mathbf{G}^{-1}\mathbf{T} - \mathbf{U})\underline{w} = 0 \tag{8}$$

where **U** is a unitary matrix. The solution is obtained recursively by means of the relations:

$$w_i = \sum_{j=i+1}^{n} c_{ij} w_j / (n-1) \qquad i = 1, \dots, n-1 \tag{9}$$

2.3 Direct least squares method (henceforth DLS)

This is the classic least squares method by which the weights are determined so as to minimize the objective functions:

$$\varphi_1(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij} - \frac{w_i}{w_j})^2 \tag{10}$$

In this case the solution **w** does not have an analytical definable expression but must be calculated with iterative methods.

2.4 Weighted least squares method (henceforth WLS)

This is a method similar to the previous one but differing from it in the weighting given to deviations from the square. In this case the objective function to minimize takes the following form:

$$\varphi_2(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij} - \frac{w_i}{w_j})^2 w_j^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij} w_j - w_i)^2 \tag{11}$$

It has been shown (Blankmeyer, 1987) that unlike the previous DLS method, which may have multiple solutions, (11) admits to only one and strictly positive solution. This can be straightforwardly obtained by solving the following system of linear equations:

$$(\sum_{i \neq k}^{n} c_{ik}^2 + n - 1) w_k - \sum_{j \neq k}^{n} (c_{jk} + c_{kj}) w_j + \lambda = 0 \qquad k=1,\dots,n \tag{12}$$

generated by the first-order conditions for the Lagrange auxiliary function constructed by taking (11) as the objective function and constraining the weights to unitary sum.

2.5 Logarithmic least squares method (henceforth LLS)

Given the multiplicative nature of the positive reciprocal matrix **C**, it is natural to measure the divergences between observed values c_{ij} and the theoretical values w_i/w_j considering the ratio between them. That is, by calculating the relative error given by the deviation of this ratio from unity.

Considering, for obvious reasons of simplicity, the logarithm of these ratios and the least squares method, the objective function becomes:

$$\varphi_3(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\ln\frac{c_{ij}}{w_i/w_j}\right)^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(\ln c_{ij} - \ln w_i - \ln w_j)^2 \qquad (13)$$

Selecting the arbitrary multiplying constant of the weights so that their product is unitary, the components of vector **w** that minimize (13) correspond to the geometric mean of the elements in the corresponding row of matrix **C**:

$$w_i = \left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}} \qquad i=1, \ldots, n \qquad (14)$$

3. Methods to Construct Consistent Matrices Even Without Binary Comparisons

The methodology for performing accurate comparisons among economic phenomena observed on different occasions, namely index number theory, is a key area of research for economic statisticians and to which they have made a number of crucial contributions.

With index numbers, comparison is made among the relative differences of a phenomenon by examining their corresponding intensities on different occasions. For example, if we consider the unit price of a generic good on two occasions A and B, which may be two regions, two periods, and so on, the ratios:

$$\frac{p_A}{p_B} \text{ and } \frac{p_B}{p_A} \qquad (15)$$

measure the relative difference in the price on occasion A with respect to occasion B considered as the base situation, and vice versa.

Simple index numbers obviously satisfy the consistency conditions mentioned in the second section. However, when the phenomenon is analysed across a number of entities, or, to refer to the example again, when the unit prices of n goods are considered, the statistical relationships that simultaneously and succinctly measure the relative diversity of prices in the situations compared are known as “complex index numbers” or simply as “index numbers”.

There are various methods with which to define complex index numbers. However, many of the complex indexes proposed and widely used do not permit consistent comparisons.

This is a problem with particular practical implications for the calculation of purchasing power parities.

The theoretical and practical importance of comparison among complex index numbers is certainly not restricted to price index numbers, for there is an immediate logical symmetry between the latter and the index numbers of quantities.

Nevertheless, in what follows, we shall restrict our treatment to the former, both because it was on these that we conducted our empirical analysis and because they suffice to illustrate the issue analysed here.

In the following sections we briefly survey the best known methods for the construction of complex and consistent index numbers. We assume that n situations are to be compared, each of them characterized by m goods described by the following vectors of prices and quantities:

$$\mathbf{p}_i = [p_{i1}, p_{i2}, \ldots, p_{im}]; \quad \mathbf{q}_i = [q_{i1}, q_{i2}, \ldots, q_{im}]\ ; \quad i=1,\ldots,n \tag{16}$$

3.1 The Gini-Elteto-Köves-Szulc method (henceforth GEKS)

This method was originally proposed by Gini (1924) and then taken up by Elteto and Köves (1964) and Szulc (1964). It is based on the matrix **C** of binary comparisons whose elements are Fisher price index numbers:

$$c_{ij} = \left(\frac{\sum_{k=1}^{m} p_{ik} q_{jk}}{\sum_{k=1}^{m} p_{ik} q_{ik}} . \frac{\sum_{k=1}^{m} p_{jk} q_{jk}}{\sum_{k=1}^{m} p_{ik} q_{jk}} \right)^{1/2} \qquad i, j = 1, \ldots, n \tag{17}$$

Independently of the authors cited above, the logarithmic least squares method mentioned earlier has been proposed in order to give consistency to the comparisons expressed by the **C** matrix which is evidently positive reciprocal.

3.2 The Theil method (henceforth T)

In this case too, consideration is made of a **C** matrix of binary comparisons, which are then made consistent by means of the logarithmic least squares method. Unlike in the previous case, however, the elements that make up the matrix are Törnquist bilateral indices:

$$c_{ij} = \prod_{k=1}^{m} \left(\frac{p_{ik}}{p_{jk}} \right)^{\alpha_k} \qquad i, j=1, \ldots,n \tag{18}$$

where:

$$\alpha_k = \frac{1}{2} \left(\frac{p_{ik} q_{ik}}{\sum_{k=1}^{m} p_{ik} q_{ik}} + \frac{p_{jk} q_{jk}}{\sum_{k=1}^{m} p_{jk} q_{jk}} \right) \tag{19}$$

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3.3 The Economic Commission for Latin America method (henceforth ECLA)

This method too can be traced back to the work of Gini, and it has been used by the Economic Commission for Latin America. Its distinctive feature is that it directly and consistently constructs the **C** matrix of comparisons, whose elements are defined in the following manner:

$$c_{ij} = \frac{\sum_{k=1}^{m} p_{ik}\bar{q}_k}{\sum_{k=1}^{m} p_{jk}\bar{q}_k} \qquad i, j=1, \ldots, n \tag{20}$$

where $\bar{q}_k = \frac{1}{n}\sum_{i=1}^{n} q_{ik}$ represents the mean of the quantities of the k-th good treated in the set of statistical units considered.

3.4 The Geary-Khamis method (henceforth GK)

Nor does this method require the construction of a matrix of binary comparisons to be then rendered consistent. Rather, it is constructed directly, by means of (3), from the weights w_i, which in the context treated by Gear and Khamis act as conversion factors.

This technique proposed by Geary (1958) and Khamis (1969), is of iterative type and is divided into two phases. In the first, the conversion factors (weights) w_i are used to determine the mean price (π_k) of each good in the basket of the statistical collective:

$$\pi_k = \frac{\sum_{i=1}^{n} w_i p_{ik} q_{ik}}{\sum_{i=1}^{n} q_{ik}}; \qquad k=1, \ldots, m \tag{21}$$

In the second phase, (22) below is used to determine the conversion factors:

$$w_i = \frac{\sum_{k=1}^{m} \pi_k q_{ik}}{\sum_{k=1}^{m} p_{ik} q_{ik}}; \qquad i=1, \ldots, n \tag{22}$$

Each iteration involves two phases: in the first, after giving w_i an arbitrary initial value, (21) is used to determine the mean prices; in the second, the latter is inserted in (22) to determine the conversion factors (weights). The procedure is reiterated until the solutions of two successive iterations are judged equal to each other.

3.5 The Gerardi method (henceforth G)

This technique does not substantially differ from the GK method. As regards the evaluation procedure it is entirely analogous to it, the only difference being that in this case the mean prices of the individual goods are calculated using a geometric mean of the prices observed on the n occasions:

$$\pi_k = \left(\prod_{i=1}^{n} p_{ik} \right)^{1/n} \qquad k=1, \ldots, m \tag{23}$$

4. Empirical Analysis of The Methods to Construct Indices For Multiple Consistent Comparisons

In this section we describe a series of experiments conducted on empirical data from official sources in order to examine the responses of the various methods surveyed above.

The analysis was carried out using information on prices and real spending volumes of goods and services purchased in the countries belonging to the EUROSTAT-OECD Purchasing Power Parity Programme.

The programme was started in the 1980s to compare the price and volume levels of gross domestic product (GDP) of the member state of the European Union and the member countries of the OECD. See for details Eurostat (1985) and Eurostat (2000).

The GDP of each country is divided in 36 comparable items of goods and services. For each of them the prices and the real spending volumes are known. The data available were used to construct two distinct binary comparison matrices: one with Fisher price index numbers, and one with Törnqvist price index numbers.

Application of the LLS method to give them consistency led, respectively, to the GEKS and the T method. As regards the other methods (DE, MDE, DLS and WLS) which can be used to give consistency to binary comparison matrixes, we applied these only to the matrix of Fisher index numbers, because we had ascertained that, in the various situations examined, this matrix differs to a negligible extent from the corresponding matrix of Törnqvist index numbers.

Taking account of the multiplicative nature and the asymmetry of these matrices, comparison was made using the following similarity index (Mean Logarithmic Variation):

$$MVL = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \ln \frac{F_{ij}}{To_{ij}} \right| ; \tag{24}$$

where F_{ij} and To_{ij} are respectively the Fisher and Törnqvist index numbers calculated for the purpose of comparison between the i-th and the j-th country.

The values assumed by index (24) in the four sets of data on which we made our subsequent calculations are given in Table 1, which shows that the divergence between the two types of matrix never exceeded the threshold of 0.38%.

It may also be of interest to point out that the behaviour of the index matched our *a priori* expectations regarding the heterogeneity of the four situations analysed.

Table 1
Mean of the logarithmic variations between the Fisher and Törnqvist indices.

DATA SET	MVL
Europe 15, Eurostat comparable data year 1998	0.0014
Europe 12+3, Eurostat comparable data year 1985	0.0022
All country with Eurostat comparable data, year 1998	0.0022
All country with Eurostat comparable data, year 1985	0.0038

The first set of calculations was performed on the 1998 price indexes of the 15 countries of the European Union. The results are given in the following Table 2.

Table 2
Normalized weights (*) – Countries of the European Union – 1998.

	DE	MEV	DLS	WLS	LLS GEKS	GK	G	T	ECLA	Mean
EURO15	100	100	100	100	100	100	100	100	100	100
Austria	6.81	6.79	6.8	6.79	6.81	6.85	6.83	6.81	6.78	6.81
Belgium	2.49	2.49	2.49	2.49	2.49	2.5	2.5	2.49	2.49	2.49
Denmark	10.83	10.85	10.85	10.84	10.83	10.93	10.91	10.83	4.48	10.15
Finland	15.11	15.09	15.14	15.08	15.11	15.24	15.2	15.1	14.9	15.11
France	13.95	13.95	13.85	13.94	13.95	14.06	14.07	13.96	13.88	13.96
Germany	46.36	46.36	46.41	46.38	46.36	46.6	46.68	46.34	46.06	46.39
Greece	0.38	0.38	0.38	0.38	0.38	0.39	0.39	0.38	0.38	0.38
England	143.1	143.4	143.0	143.1	143.1	143.3	143.2	143.2	143.0	143.1
Ireland	130.1	130.1	130.2	130.1	130.1	131.9	131.6	130.1	128.6	130.3
Italy(**)	57.95	57.92	57.97	57.99	57.95	58.62	58.61	57.92	57.46	58.04
Luxemburg	2.19	2.19	2.21	2.21	2.19	2.22	2.23	2.18	2.16	2.2
Holland	46.69	46.65	46.52	46.65	46.69	46.96	46.98	46.71	46.51	46.71
Portugal	0.71	0.71	0.71	0.71	0.71	0.73	0.72	0.71	0.7	0.71
Spain	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
Sweden	9.4	9.45	9.35	9.38	9.4	9.48	9.48	9.4	9.28	9.4
MSD	0.18	0.19	0.18	0.18	0.18	0.48	0.41	0.18	1.49	

(*) These weights are interpretable as purchasing power parities. (**) Expressed as thousands of lire.
All value are multiplied by 100

The most striking aspect to emerge from Table 2 is the close concordance indeed, often the perfect coincidence of the results obtained when applying the various methods. As a synthesis measure we considered only the following consistency index (Mean Squared Deviation) of the weights with respect to those calculated as a mean of the nine methods applied:

$$MSD_h = \left[\frac{1}{n} \sum_{i=1}^{n} (w_{ih} - \overline{w}_i)^2 \right]^{1/2} \qquad h=1, \ldots, 9 \qquad (25)$$

where:

$$\overline{w}_i = \frac{1}{9}\sum_{h=1}^{9} w_{ih} \qquad i=1, \ldots, n \tag{26}$$

The values of this index highlight not only the negligible amount of deviation, directly deducible from the close concordance already mentioned, but also a slightly different behaviour from the others, in order, of the G, GK and ECLA methods, namely, precisely those methods which do not presuppose a matrix of binary comparisons to be adjusted.

In order to verify the reliability of these results as regards both their stability in time and their robustness, we conducted three further tests. The first consisted in replication of the analysis using comparable data from further back in the past, specifically those relative to 1985, and including in the analysis the three countries (Austria, Finland, and Sweden) which did not yet belong to the Union at the time.

The results are given in Table 3, and they are a perfect match with the previous ones, displaying considerable stability in the degree of the concordance over time. It must be pointed out that stability is referred to concordance in the methods' responses and not to the weight calculated in 1988 and 1995, which depict different situation.

As regards the consistency index in the three methods differing most from the others, we find, as well as GK and G, also the DLS method, but the difference with the ECLA method (0.0040 as opposed to 0.0039) is so slight that it can be taken to be entirely insignificant.

Table 3
Normalized weights(*) – Countries of Europe of 12 plus the three countries (Austria, Finland, and Sweden) that joined later – 1985.

	DE	MEV	DLS	WLS	LLS GEKS	GK	G	T	ECLA	Mean
EURO12+3	100	100	100	100	100	100	100	100	100	100
Austria	7.17	7.12	7.16	7.18	7.17	7.21	7.23	7.17	7.14	7.17
Belgium	2.65	2.65	2.64	2.65	2.65	2.68	2.67	2.66	2.65	2.66
Denmark	12.13	12.09	12.06	12.12	12.13	12.27	12.16	12.12	12.07	12.13
Finland	19.68	19.71	19.49	19.66	19.68	20.18	20.05	19.68	19.31	19.72
France	16.38	16.38	16.33	16.38	16.38	16.44	16.44	16.39	16.36	16.39
Germany	47.84	47.84	47.89	47.84	47.84	48.16	48.41	47.87	47.46	47.91
Greece	1.44	1.45	1.44	1.45	1.44	1.49	1.5	1.44	1.37	1.45
England	209.6	209.2	208.5	209.6	209.6	212.0	211.3	209.8	209.1	209.9
Ireland	167.0	167.9	166.1	167.0	167.0	166.4	166.7	166.8	166.3	166.8
Italy (**)	91.67	91.78	91.7	91.59	91.67	92.39	92.15	91.69	90.61	91.7
Luxemburg	2.73	2.74	2.74	2.75	2.73	2.78	2.79	2.72	2.68	2.74
Holland	46.99	46.96	46.99	47.01	46.99	47.25	47.21	46.94	46.91	47.03
Portugal	1.66	1.67	1.63	1.64	1.66	1.81	1.79	1.66	1.52	1.67
Spain	1.23	1.23	1.22	1.23	1.23	1.28	1.26	1.23	1.19	1.23
Sweden	14.3	14.15	14.11	14.27	14.3	14.84	14.66	14.25	14.04	14.32
MSD	0.07	0.33	0.4	0.08	0.07	0.6	0.41	0.03	0.39	

(*) These weights are interpretable as purchasing power parities. (**) Expressed as thousands of lire.

All value are multiplied by 100

In order to conduct thorough verification of not only the stability in the concordance of the responses but also the robustness of the responses obtained with the various methods, we again replicated the calculations, at the two times already considered, including in the two occasions the countries not belonging to the European Union in its present form but for which comparable data, again furnished by Eurostat, were available.

The results are set out in Tables 4 and 5. Once again they confirm the close concordance of the responses and the slight divergence of the GK, G and ECLA methods.

To these results should now be added the considerable robustness of the evaluations of the weights inferable from comparison of Table 2 with Table 4, and Table 3 with Table 5.

Leaving further comparisons to the reader, here we merely point out that, as regards the weight of Italy, the mean value in 1985 changes only from 0.5504 to 0.5806 because of the inclusion of Cyprus, Iceland, Norway, Poland and Switzerland.

Likewise with the 1985 data, because of the inclusion of Australia, Canada, Japan, New Zealand, Norway, United States and Turkey, the mean value shifts from 0.9170 to 0.9171, and therefore to only a very slight extent bearing in mind the significance and the order of magnitude of those weights.

Table 4
Normalized weights(*) – Countries for which standardized Eurostat data are available – 1998.

	DE	MEV	DLS	ELS	LLS GEKS	GK	G	T	ECLA	Mean
EURO15	100	100	100	100	100	100	100	100	100	100
Austria	6.81	6.78	6.81	6.78	6.81	6.84	6.81	6.8	6.78	6.8
Belgium	2.49	2.49	2.51	2.49	2.49	2.5	2.5	2.49	2.49	2.5
Denmark	10.84	10.86	10.87	10.86	10.84	10.93	10.91	10.84	10.74	10.85
Finland	15.11	15.08	15.16	15.14	15.1	15.24	15.15	15.1	14.9	15.11
France	13.93	13.93	13.87	13.93	13.93	14.06	14.08	13.95	13.87	13.95
Germany	46.38	46.38	46.42	46.57	46.38	46.63	46.71	46.36	46.02	46.43
Greece	0.38	0.38	0.38	0.38	0.38	0.39	0.39	0.38	0.38	0.38
England	143.2	143.5	143.0	143.2	143.2	143.3	143.4	143.4	143.0	143.3
Ireland	129.9	129.9	130.3	129.8	129.9	131.9	131.4	130.0	128.5	130.2
Italy (**)	57.94	57.92	57.94	57.94	57.94	58.54	58.51	57.9	57.5	58.01
Luxemburg	2.19	2.2	2.23	2.22	2.19	2.23	2.23	2.19	2.15	2.2
Holland	46.67	46.64	46.5	46.65	46.67	46.96	46.97	46.69	46.51	46.7
Portugal	0.71	0.71	0.72	0.71	0.71	0.73	0.72	0.71	0.7	0.71
Spain	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
Sweden	9.38	9.35	9.39	9.39	9.38	9.48	9.46	9.38	9.28	9.39
Cyprus	220.9	224.2	222.9	220.7	220.9	225.5	225.2	221.0	215.2	221.8
Iceland	1.12	1.12	1.11	1.1	1.12	1.13	1.12	1.12	1.11	1.11
Norway	9.74	9.69	9.69	9.63	9.74	9.97	9.87	9.74	9.6	9.74
Poland	53.36	53	52.88	53.09	53.35	58.22	57.56	52.9	49.28	53.74
Switzerland	47.28	47.33	47.44	47.46	47.28	47.8	47.79	47.29	46.43	47.34
MSD	0.23	0.54	0.31	0.3	0.23	1.33	1.16	0.26	1.8	

(*) These weights are interpretable as purchasing power parities. (**) Expressed as thousands of lire.
All value are multiplied by 100

Table 5
Normalized weights(*) – Countries for which standardized Eurostat data are available – 1985.

	DE	MEV	DLS	ELS	LLS GEKS	GK	G	T	ECLA	Mean
EURO12	100	100	100	100	100	100	100	100	100	100
Austria	7.17	7.16	7.17	7.17	7.17	7.16	7.21	7.18	7.12	7.17
Belgium	2.65	2.65	2.65	2.65	2.65	2.68	2.67	2.65	2.65	2.66
Denmark	12.15	12.14	12.13	12.15	12.15	12.28	12.17	12.15	12.13	12.16
Finland	19.7	19.74	19.73	19.69	19.7	20.05	19.98	19.71	19.49	19.75
France	16.39	16.39	16.4	16.39	16.39	16.46	16.45	16.4	16.34	16.4
Germany	47.92	47.92	48.15	47.88	47.92	47.79	48.36	47.92	47.73	47.96
Greece	1.44	1.44	1.44	1.44	1.44	1.51	1.49	1.44	1.36	1.44
England	209.5	209.2	208.3	209.5	209.5	212.8	211.3	209.7	208.5	209.8
Ireland	167.4	168.1	169.4	167.3	167.4	166.7	167.4	167.5	166.2	167.5
Italy (**)	91.63	91.68	91.91	91.57	91.63	92.4	92.28	91.69	90.58	91.71
Luxemburg	2.73	2.73	2.73	2.73	2.73	2.72	2.78	2.72	2.69	2.73
Holland	47.02	47.01	47.04	47.04	47.02	47.54	47.25	46.98	46.78	47.08
Portugal	1.66	1.66	1.65	1.65	1.66	1.84	1.8	1.66	1.51	1.68
Spain	1.23	1.23	1.23	1.23	1.23	1.29	1.27	1.23	1.19	1.24
Sweden	14.3	14.27	14.12	14.33	14.3	14.79	14.6	14.27	14.25	14.36
Australia	97.36	97.93	97.35	97.3	97.36	96.89	97.28	97.21	97.64	97.37
Canada	97.96	97.97	97.3	97.92	97.96	98.07	100	97.52	98.48	98.13
Japan	0.51	0.51	0.51	0.51	0.51	0.54	0.52	0.51	0.51	0.51
N. Zealand	86.57	85.97	86.89	86.49	86.56	88.41	88.4	86.59	83.31	86.58
Norway	13.34	13.34	13.37	13.34	13.34	13.74	13.72	13.34	13.12	13.4
United St.	121.1	121.8	119.6	121.2	121.1	121.0	124.4	120.5	121.1	121.3
Turkey	0.7	0.7	0.7	0.7	0.7	0.76	0.76	0.71	0.64	0.71
MSD	0.09	0.27	0.65	0.1	0.09	0.79	0.91	0.21	0.82	

(*) These weights are interpretable as purchasing power parities. (**) Expressed as thousands of lire.
All value are multiplied by 100

5. Conclusions

There is a large body of literature on the methodology with which to perform consistent comparisons. These methods have been analysed in a relatively independent manner by scholars of decision-making processes in business and by economic statisticians with regard to the comparison of economic aggregates. In this paper we have first emphasised the close connection between the two approaches: the one that uses subjective preference judgements, and the one based on objective data regarding prices and quantities. Using data of the latter type, which are more incontrovertible, we have conducted a series of calculations using both cross-section data and time series data.

The results that we believe to be most significant are the following:

- the close concordance, sometimes almost the coincidence, of the weights obtained with the various methods; and
- the notable robustness of the evaluations performed.

The most immediate conclusion to be drawn from our analysis is that these methods should be used more widely. Which method in particular should be selected is of little importance from the practical point of view, given the extremely close concordance obtained in the responses. More specifically as regards index numbers, we have provided clear evidence that those methods which prove incoherent in multiple comparisons should be discarded. These were methods used in the past because of a computational simplicity which is now wholly irrelevant, and they are still too widely used.

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