AN INVESTMENT CHOICE PROBLEM AND CALENDAR ANOMALIES: A GROUP AHP MODEL FOR INVESTORS

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ABSTRACT

An investment choice can be influenced by numerous qualitative and quantitative factors that often conflict with one other. Therefore, portfolio management choice is a multi-criteria decision problem that requires flexible and analytic decision tools for investors. For this task, the Analytic Hierarchy Process (AHP) is suitable. We propose an AHP group-based model to analyze an investment choice problem looking at two financial markets including Spain and France. The evaluation criteria that we used in our model are the return of the stock market, performance of government bonds and calendar effects in the financial markets. The 2017 French and Spanish equity market returns and the government bond performances for each country are available in public databases. Mean tests were performed in order to analyze calendar anomalies for both of the markets from 2007-2017. The aim of our study is to propose a model that allows simultaneous evaluation of the impact of the previously mentioned factors on investment choice. Our analysis involves 69 students from the Department DEMM of the University of Sannio (Italy) who have worked on financial market simulators. The data were obtained using questionnaires. The common priority vector procedure (CPVP) was used to determine the individual priorities (derived by individual judgments matrices) and aggregate the individual priorities (derived by individual judgments matrices) to obtain the group preferences. The results show that the decision makers prefer to invest in diversified portfolios.

Keywords: AHP; calendar anomalies; government bonds; stock market; investment choice; consistency

1. Introduction

In recent years, the financial markets have changed because of globalization. Today, a wide range of investment opportunities is available to investors. Investors can select different products (stocks, bonds, currencies, options) in the diverse financial markets. In this way, the financial markets are related but each one of them has specific characteristics with particular opportunities for investors. Financial decision makers differ in their aims and restrictions, which makes portfolio management choice more complex and dynamic (Khaksari et al., 1989). In addition, a financial problem could be constructed because it is classified as a strategic choice (Zopounidis, 1999). The selection of an investment depends on several qualitative and quantitative criteria that can be conflicting. Therefore, an investment decision is a multi-criteria problem that requires some flexible and analytic tools for the financial agents. The Analytic Hierarchy Process (AHP) can be applied in this type of problem (Saaty, 1980; Ülengin & Ülengin, 1994). In fact, the AHP method can assist investors throughout the entire process, from the formulation of the problem to the evaluation and all of the choices in between (Zopounidis, 1999).

Based on this, we analyzed financial decisions by applying the AHP to investment choice problems. We prefer the AHP model to other multicriteria methods (such as ANP) because the analysis is simplified, assuming that criteria are independent (Marcarelli, 2018).

To define the ranking of investment choices, our model considers the following evaluation criteria: return of the stock market, performance of government bonds, and the presence of calendar effects (CE). Roy, Kolte, Sangvikar, and Pawar (2019) introduced two models to study the stock market volatility. Jang and Park (2019) suggested the return of the stock market and the performance of government bonds as the main criteria to consider. They affirmed that the previous research has shown that the bond and stock markets influence investments by decision makers (Christiano et al., 1996; Kontonikas & Zekaite, 2018). In addition to these two criteria that are connected to market efficiency, we propose considering a factor that detects the irrationality of financial agents, that is, the presence of calendar anomalies.

The aim of our study is to propose a model that allows simultaneous evaluation of the impact of the previously mentioned mixed factors for the investment choice. CE refer to the cycling irregularity in a financial market (Latif et al., 2011), and in order to analyze these anomalies we perform some statistical tests. We constructed a hierarchical model that is composed of the following three levels. On the top is the selection of the investment choice, the intermediate level is represented by the return of the stock market, the performance of government bonds, and the presence of calendar effects, and on the bottom level, there are four financial investments, two of which are diversified in two different markets and two are entirely in one market. We considered these four choices in order to avoid the neutral option (50% - 50%). The neutral option could lead investors to choose a response that did not reflect their true behavior (Krosnick, 1991).

The structure of the paper is as follows: Section 2 offers literature reviews on the AHP method and calendar anomalies; section 3 presents some statistical tests used to check for the presence of calendar anomalies and the AHP method; section 4 discusses the results of our procedure; finally, section 5 provides some conclusions and future developments.

2. Literature review

Saaty introduced the AHP as a multicriteria method in the 1970s. This method represents a problem by using a hierarchical structure and derives relative and global weights for the hierarchical elements that are based on expert judgements (Saaty, 1980). The AHP analyzes complex decision problems with both qualitative and quantitative elements and provides priorities using pairwise comparison matrices.

The AHP has been applied in several fields, such as portfolio selection, resource allocation, environmental impact evaluation, risk and performance management (Naji, Mousrij, Cillo & Chierici, 2019; Mital, Del Giudice & Papa, 2018). Saaty and Vargas (1982) illustrated some applications in business, energy, health and transportation. In finance, AHP has been used for portfolio comparisons (Martel et al., 1988), the evaluation of the exchange rate (Ülengin & Ülengin, 1994), and financial decision problems (Zopounidis, 1999). Spronk et al. (2005) analyzed the contributions of multi-criteria decision methods in finance, and Zouponidis et al. (2015) published a bibliographic survey of multicriteria analysis contributions in financial decision making. Furthermore, the AHP method has been used to select target markets and distribution channels, and direct resource allocation among portfolio elements (Saaty & Vargas, 1982). Calendar effects are among the evaluation criteria used to reach the goal of the study, and represent an important topic in the financial field.

Calendar effects describe when a change in stock prices is influenced by specific periods of the calendar year. Various studies have documented unexpected and abnormal regularities in relation to certain moments of a day, days of the week, periods of a month or months of the year (Wachtel, 1942; Rozeff & Kinney, 1976; French, 1980; Barone, 1990; Agrawal & Tandon, 1994). Essentially, the academic world and professional operators have, in various ways, analyzed stock returns by looking for a link between price changes and what times these changes occur. Therefore, this study explores the principal calendar effects.

Weekend effect

The weekend calendar effect describes how stock prices tend to fall on Mondays. The first two studies that verify the speed of the generative process of stock prices are Fama (1965) and Granger and Morgenstern (1970). These studies show that when the market is closed, the stochastic process followed by the share price (random walk) continues to operate, but at a lower speed. This means that the closing price on a Monday is less than the closing price on the previous Friday (Latif et al., 2011). These results have been substantiated by other studies (French, 1980; Gibbons & Hess, 1981; Jaffe & Westerfield, 1985; Schwert, 2003; Chen & Singal, 2003; Miller et al., 2006).

January effect

"As goes January, so goes the year" is a famous law in the stock market. This is also called the "turn of the year" effect. In other words, there are abnormal returns in January in most countries (Gultekin & Gultekin, 1983). The first evidence of abnormal stock returns in January for the U.S. stock markets was observed by Wachtel (1942). This effect was confirmed by many other scholars (Rozeff & Kinney 1976; Barone, 1990; Wong et al., 2006; Rossi & Fattoruso, 2017).

Holiday effect

The holiday effect shows a significant return on days before public holidays (Pettengill, 1989; Ariel, 1990). This effect influences the performance of daily share returns. Ariel (1990) verifies a significant growth on Christmas Eve and May Day Eve when compared with other holidays. This abnormality is present in different markets. Different observations (Lakonishok & Smidt, 1988; Barone, 1990; Kim & Park, 1994; Meneu & Pardo, 2004; Cao et al., 2009) confirm the presence of abnormal post-holiday returns.

Turn of the Month (ToM) anomaly

In 1987, Ariel first identified the ToM effect for the U.S. stock market. He discovered that mean returns are higher at the end of a month and at the beginning of the next month. This result is confirmed by many scholars (Pettengill & Jordan, 1988; Agrawal & Tandon, 1994). Some recent research has confirmed that this effect is also present in other stock markets (Hensel & Ziemba, 1996; McConnell & Xu, 2008).

3. Methodology

3.1 Analysis of calendar effects

This paper proposes an application of the group-AHP model to an investment choice problem regarding the French and Spanish financial markets. For this reason, we considered government bond performance and the stock market return as evaluation criteria. These data were obtained from the Sella SGR[1] report (2018) and Yahoo Finance. The third criterion was the presence of calendar anomalies (CA) for both markets. To analyze CA, we introduced some statistical tests to evaluate if the difference between the number of changes between 2007-2017 were significant.

First, we analyzed the calendar effects described above. We computed the averages of the rates of change for both indices and used some figures to evaluate the behavior as the first descriptive analysis of each calendar effect. Then, the following statistical test on the proportion of differences was performed to verify if there was a significant effect on the markets:

$$H_0: q_1 - q_2 = 0$$
$$H_1: q_1 - q_2 > 0 \quad \text{or} \quad H_1: q_1 - q_2 < 0$$

indicating with q_1 the population ratio of positive (or negative) changes in the analyzed period (for example, during the weekend), whereas q_2 is the proportion of the population in the other periods.

To verify if the null hypothesis (H_0) can be rejected or not:

[1] Sella SGR is the Asset Management Firm of Sella Group which operates since 1983.

$$Z = \frac{p_1 - p_2}{\sqrt{p_c(1-p_c)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim N(0,1)$$

where:

- p_1 is the relative frequency in the sample obtained from the first population
- p_2 is the relative frequency in the sample obtained from the second population
- $p_c = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$
- n_1 and n_2 are the sizes of the two samples

If $Z > z_\alpha$ (or $Z < -z_\alpha$) then H_0 has to be rejected. z_α is the quantile of the normal standardized distribution and α is the selected significance level (Piccolo, 2010).

Weekend effect (WE)
To verify the presence of the WE, we considered the correlation between changes in stock values that occurred on Monday and Friday. We checked to see if the percentage of positive changes on Monday was lower than the percentage of positive changes on the weekends.

January effect (JE)
To test if the JE was present in our data, we analyzed if the ratio of positive changes in stock values seen in January was greater than the percentage of positive changes in stock values in the other months of the year.

Holiday effect (HE)
To verify the existence of the HE, we examined if the ratio of positive changes that occurred during the days before a holiday was greater than the percentage of positive changes on the other days of the year.

Turn-of-the-Month effect (ToME)
To test for the presence of the ToME, we verified if the ratio of the positive changes in the stock prices during the last day of the month and the first three days of the following month was greater than the ratio of the positive changes on the other days of the month.

3.2 Group AHP to prioritize criteria and evaluate alternatives

The AHP procedure involves the following steps (Saaty, 1980; Saaty & Vargas, 1982; Saaty, 1994):

1. Decomposition of the problem into a hierarchy;
2. Data collection using pairwise comparisons;
3. Deriving relative priorities;
4. Synthesizing relative priorities to obtain the global ranking.

Step 1
The problem was decomposed into three levels. On the bottom level are the alternatives; the intermediate level contains the criteria used to compare the investment choices; on the top level is the objective of the problem.

Step 2

This step involves the use of pairwise comparisons to establish the relative priorities, which represent the importance of the components of a level in relation to the components in the level immediately above. We assigned a judgment, a_{ij}, to couples of elements (x_i,x_j) in a level as they related to a given component in an upper level. This judgment is a value greater than 1 if the component x_i is preferred to the component x_j, whereas the opposite preference relationship is indicated by a value lower than 1, and the difference between the two components is indicated by *1*. A positive reciprocal square matrix is obtained when comparing n components of a level. All of the values on the main diagonal are equal to 1. We used a nine-point scale proposed by Saaty to assign the judgements (Saaty & Vargas, 1982).

The determination of a priority vector is always accompanied by a check of logical consistency in the allocation of judgements. The matrix is perfectly consistent if the following condition exists:

$$a_{ij} \cdot a_{jk} = a_{ik} \qquad \text{for each } i, j, k = 1, 2, \ldots\ldots, n \qquad (1)$$

Unfortunately, this matrix may not be consistent. This can happen because of inaccuracies, errors or simply a violation of transitivity and/or proportionality. A preference relation is transitive, assuming that x_i is preferred to x_j and x_j is preferred to x_k, then x_i is preferred to x_k. Furthermore, the inconsistency may be caused by a violation of the proportionality between the elements even if the transitive property is satisfied. The preference relationship is proportional if $a_{ij} = 4$ and $a_{jk} = 2$, then $a_{ik} = 8$.

Consistency indices to check the inconsistency level in a set of pairwise judgements have been proposed in the literature (Saaty, 1980; Koczkodaj, 1993; Salo-Hamalainen, 1997; Crawford & Williams, 1985). Each index represents the degree of inconsistency in the judgements expressed as a real number. To check the inconsistency of judgments, Saaty suggested the consistency ratio (CR):

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \qquad (2)$$

where

$$CI = (\lambda_{max} - n) \,/\, (n - 1) \qquad (3)$$

is the consistency index and RI is the average value of CI derived from a sample of 50,000 randomly generated reciprocal matrices (Saaty, 1980). The CR value increases as inconsistency increases. The matrix has a tolerable inconsistency if CR < 0.1. The consistency of judgements is important because it is strictly linked to the accuracy of the preferences. When the judgements are inconsistent, the priority estimates are not reliable because each prioritization method may provide a different priority vector (Grzybowski, 2016). Instead, if the matrix is fully consistent, then all of the prioritization methods should give the same result.

The consistency indices and thresholds that are proposed in the literature may be useful to address cardinal consistency, but they do not take into account ordinal consistency or transitivity (Siraj et al., 2015). To overcome this kind of a problem, Amenta et al. (2018,

2020) proposed some approximated transitivity thresholds for some consistency indices. These thresholds are useful because they allow judgement revision to be avoided if a qualitative ranking of preferences is the goal. If the consistency index ranges between the consistency and transitivity threshold values, then we may be sure of the accuracy of the preferences.

Step 3
The relative relevance of the components can be obtained as the eigenvector associated with the maximum eigenvalue of A:

$$A \cdot \underline{w} = \lambda_{max} \cdot \underline{w} \qquad \text{for } i, j = 1, 2, \dots, n \qquad (4)$$

by using the eigenvector method (EM) proposed by Saaty.

Other methods can be used to estimate the priority vector (Saaty, 1980, Aguaron & Moreno-Jimenez, 2003; Pelaez & Lamata, 2003, Gass & Rapsak, 2004) and include the arithmetic mean method (AMM), the row geometric mean method (RGMM), the logarithmic least squares (LLS) method, the singular value decomposition (SVD).

Step 4
In the final AHP step, the global priorities are derived, which express the relevance of the investment choices. The relative weights of the components are aggregated by the principle of hierarchical composition. The global weights provide the global ranking of the alternatives. Once the final ranking is obtained, a sensitivity analysis is used to verify the stability of the resulting solution.

4. Analysis of results

4.1 Test to check for the presence of anomalies

Before applying the AHP method, the calendar anomalies in the two markets were analyzed. An empirical analysis was performed to verify the presence of the anomalies taking into account the price data from two different indices as follows: CAC 40 (French stock market) and IBEX 35 (Spanish stock market). This research studied the CE for ten years between 2007-2017. The data were obtained from "Yahoo finance". The number of observations for the two indices are 2811 and 2807 from the CAC and IBEX 35, respectively. A significance level of α =0.05 (z_α = 1.645) was used to determine the presence of CE in the two different markets.

Weekend effect
To evaluate the weekend effect, the proportion of positive changes on Monday and Friday was considered. The data are described in the following tables and figures.

Table 2
Weekend effect

Days	**CAC 40**		**IBEX 35**	
	# of positive changes	# of changes	# of positive changes	# of changes
Monday	264	556	235	555
Friday	291	553	301	553

Source: Authors' calculations

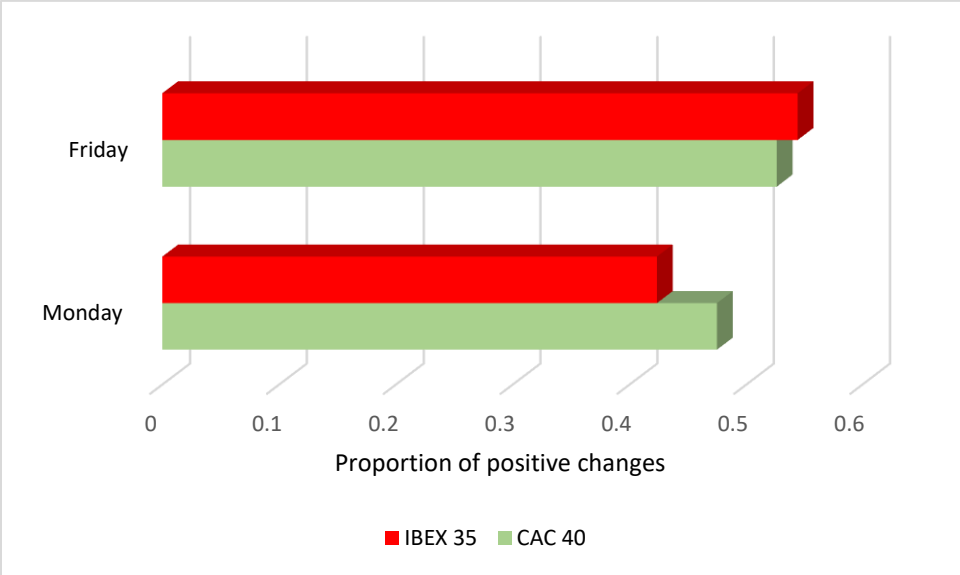


Figure 2 Weekend effect for the CAC 40 and IBEX 35 indices
Source: Authors' calculations

From Figure 2, we note that the proportion of positive changes in both markets is higher on Friday. Therefore, it is necessary to verify if the differences are statistically significant using the following test:

$$H_0: q_1 - q_2 = 0 \text{ and } H_1: q_1 - q_2 < 0$$

Considering the data in the previous tables, we have $Z_{\text{CAC }40} = -1.7117$ and $Z_{\text{IBEX }35} = -4.0258$. In both markets, the difference between the positive changes on Monday and Friday is significant; therefore, there is a weekend effect.

January effect

Table 3
January effect

Months	**CAC 40**		**IBEX 35**	
	# of positive changes	# of changes	# of positive changes	# of changes
January	109	234	106	233
Other months	1325	2576	1334	2573

Source: Authors' calculations

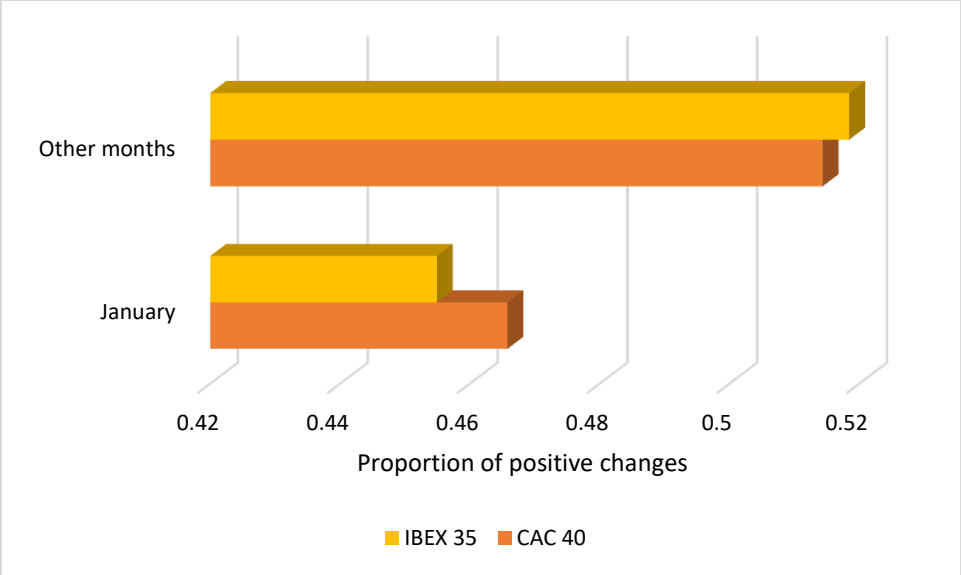


Figure 3 January effect for the CAC 40 and IBEX 35 indices
Source: Authors' calculations

It is clear from the data that in both France and Spain the proportion of positive changes in January is smaller than in other months. For this reason, there is no January effect, as confirmed by the following test:

$H_0: q_1 - q_2 = 0$ and $H_1: q_1 - q_2 > 0$

$Z_{CAC\,40} = -1.4225$ and $Z_{IBEX\,35} = -1.8577$. It is not possible to reject H_0.

Holiday effect

Table 4
Holiday effect

	CAC 40		**IBEX 35**	
	# of positive changes	# of changes	# of positive changes	# of changes
Holiday	24	44	28	44
No Holiday	1410	2766	1395	2762

Source: Authors' calculations

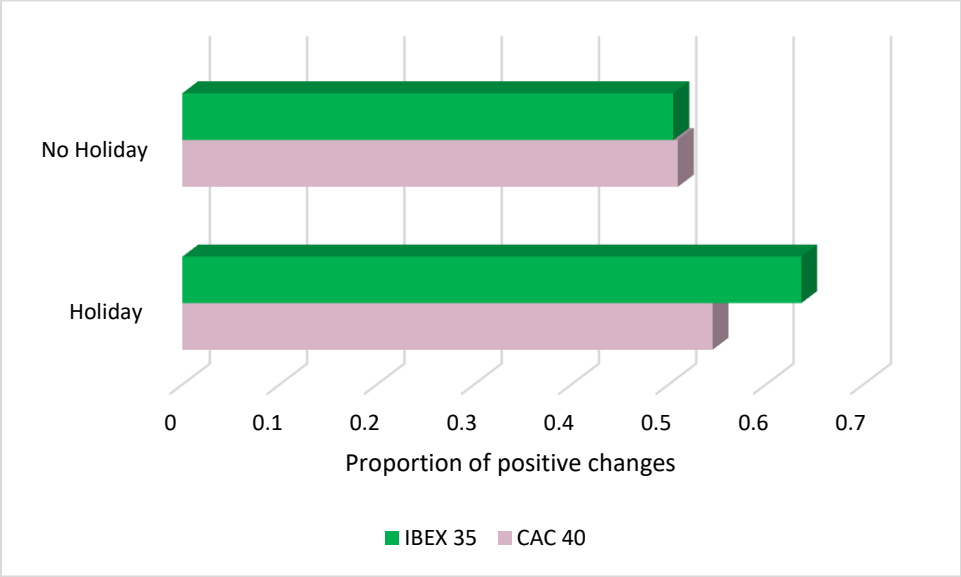


Figure 4 Holiday effect for the CAC 40 and IBEX 35 indices
Source: Authors' calculations

The relative frequency of the positive changes seems to be higher for both indices in the days before a holiday when compared with other days. To evaluate if the differences are statistically significant, we consider the classical test:

$H_0: q_1 - q_2 = 0$ and $H_1: q_1 - q_2 > 0$

$Z_{CAC\,40} = 0.4699$ and $Z_{IBEX\,35} = 1.7283$ allows us to affirm that the HE only happens in the Spanish market.

Turn-of-the-month effect

Table 5
Turn-of-the-month effect

Days	**CAC 40**		**IBEX 35**	
	# of positive changes	# of changes	# of positive changes	# of changes
Days (–1, +3)	271	526	285	527
Other days	1163	2284	1140	2279

Source: Authors' calculations

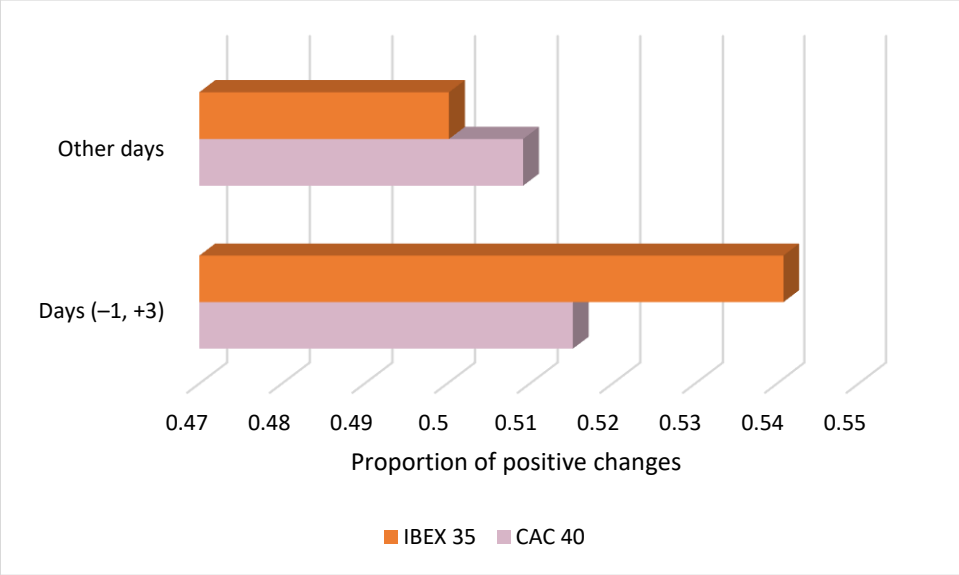


Figure 5 ToME for the CAC 40 and IBEX 35 indices
Source: Authors' calculations

The number of positive changes is greater at the ToME when compared with other days. $Z_{CAC\,40} = 0.2488$ and $Z_{IBEX\,35} = 1.6792$ lead us to reject H_0 for the Spanish market, whereas in the French market, there are no significant differences.

$$H_0: q_1 - q_2 = 0 \text{ and } H_1: q_1 - q_2 > 0$$

Table 6 highlights the existence of the CEs.

Table 6
Synthesis of effects

	FRANCE	**SPAIN**
Weekend effect	Yes	Yes
January effect	No	No
Holiday effect	No	Yes
Turn-of-the month effect	No	Yes

Source: Authors' calculations

In conclusion, our study has shown that the French stock market is only affected by the weekend effect, while in Spain all three effects (weekend, holiday and turn-of-the-month) are present.

4.2 Analysis of the AHP model results

After the analysis of the CE, we structured the financial choice problem (Fig. 9) and implemented the AHP using three levels that included the objective, the three criteria, and the four financial choices.

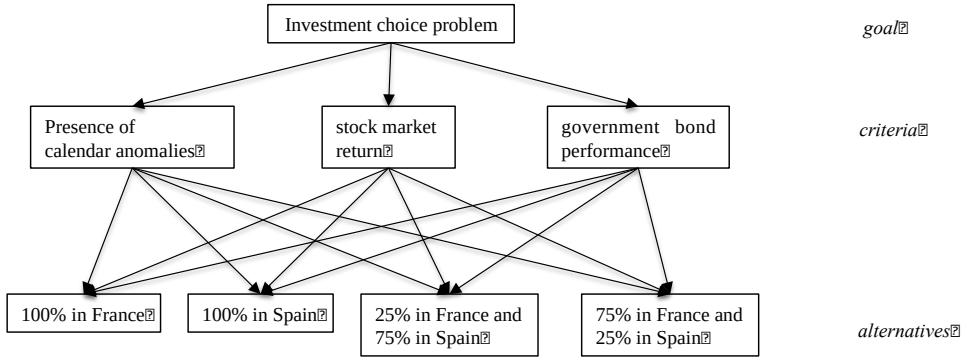


Figure 6 Hierarchical structure of the problem

Source: Authors' calculations

Our analysis took place between April-June 2019, and included 69 students from the Department DEMM of the University of Sannio (Italy) who have worked on financial market simulators. To help the experts express their judgements using pairwise comparisons, a simple questionnaire was constructed with guidelines for the completion (see Appendix 1). Many of the matrices have CR > 0.1. In-depth analysis of the these matrices shows that they are transitive because their CR is lower than the transitivity threshold (Amenta et al., 2018, 2019). We know that transitivity ensures the reliability of

the priority vector (Sjrai et al., 2015). If the consistency ratio ranges between the consistency and transitivity threshold values, then we can be confident of the accuracy of the preferences.

We may consider the following matrix:

$$\begin{pmatrix} 1 & 1/4 & 1/7 & 3 \\ 4 & 1 & 1/4 & 1/5 \\ 7 & 4 & 1 & 7 \\ 1/3 & 5 & 1/7 & 1 \end{pmatrix}$$

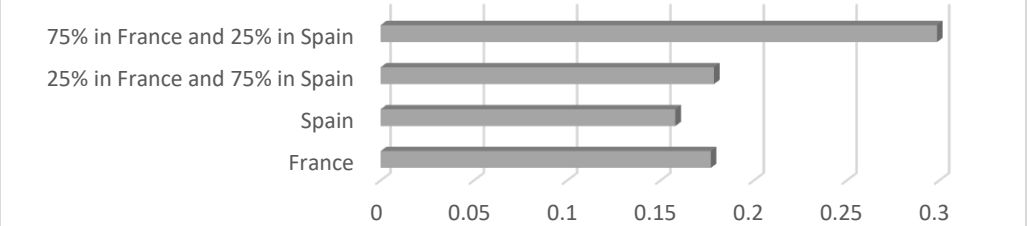
The CR is higher than the consistency threshold suggested by Saaty (0.1), but lower than the transitivity threshold suggested by Amenta et al. (2019), so we may consider the judgements acceptable. The ranking of preferences is the same when calculating the priority vector using different methods.

Table 7
Ranking of the alternatives for a transitive matrix

Investment choice	Aggregation methods			Ranking
	AMM	GMM	EM	
France	0.124376	0. 100707	0.130	4
Spain	0.154308	0. 117714	0.143	3
25% in France and 75% in Spain	0.537953	0. 658619	0.554	1
75% in France and 25% in Spain	0.183363	0. 122958	0.172	2

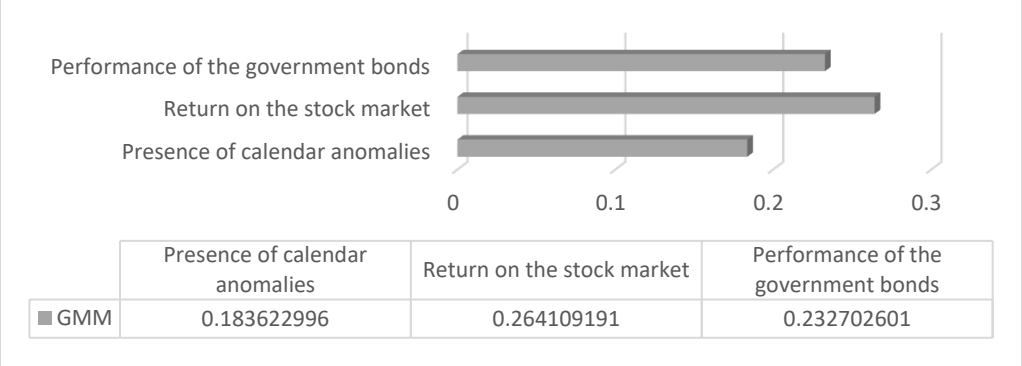
Source: Authors' calculations

Some matrices have a consistency index that is greater than the transitivity threshold; in these cases, since it is not possible to revise the judgements (the experts are anonymous), the corresponding questionnaires were removed from our analysis. The analysis of cardinal and ordinal consistency eliminated 24 questionnaires. The aggregation of the relative weights provides a global priority vector that represents the classification of the investment choices. The AHP was applied for each decision maker, and in this way we obtained 45 preference vectors. To aggregate the overall priorities that were calculated for each of the experts, we considered the common priority vector procedure (CPVP) proposed by Amenta et al. 2019 (see Appendix 2). This procedure considers the majority group preference and diminishes the influence of extreme individual opinions when deriving the common vector. Figure 10 exhibits the global order of the investment choices.



	France	Spain	25% in France and 75% in Spain	75% in France and 25% in Spain
GMM	0.176938844	0.157945227	0.17863068	0.298792379

Figure 7 Global ranking of the investment choices
Source: Authors' calculations



	Presence of calendar anomalies	Return on the stock market	Performance of the government bonds
GMM	0.183622996	0.264109191	0.232702601

Figure 8 Global ranking of criteria
Source: Authors' calculations

The analysis of 45 students' preferences shows that they prefer a diversified investment. In particular, the best choice is a diversified investment with three-quarters in France and a quarter in Spain (preferred by approximately 40% of the investors); the worst choice is investing only in Spain (approximately 19%). Regarding the importance of the criteria, Figure 11 shows that the presence of CE (0.27) is less important than the equity market return (0.39) and government bond performance (0.34). Tables 8 and 9 show normalized priority vectors.

Table 8
Normalized priority vectors for alternatives

Investment choice	**Normalized priority vector**	**Ranking**
Total in France: 100% France	0,217822591	3
Total in Spain: 100% Spain	0,194440281	4
Diversified favoring Spain: 25% in France and 75% in Spain	0,219905345	2
Diversified favoring France: 75% in France and 25% in Spain	0,367831782	1

Source: Authors' calculations

Table 9
Normalized priority vectors for criteria

Criteria	**Normalized priority vector**	**Ranking**
Presence of calendar anomalies	0,26986127	3
Stock market return	0,38814769	1
Government bond performance	0,34199104	2

Source: Authors' calculations

5. Concluding remarks

This paper applied the AHP multi-criteria method to investigate an investment choice problem in two European countries. The AHP model required an in-depth analysis of calendar anomalies. To check for the presence of anomalies, we performed some statistical tests. The analysis showed that the Spanish stock market exhibits three effects, while the French market exhibits only one anomaly. Our model shows that the investors chose a diversified portfolio. In particular, a "diversified favoring France" was preferred to a "diversified favoring Spain" portfolio. The presence of calendar anomalies was the least used criterion as determined by the weights. The procedure has strengths and weaknesses. First, the AHP methodology allows the phenomenon to be broken down into a hierarchical structure. Then, it assigns personal judgements using pairwise matrices. In addition, the AHP is adaptable and makes it possible to analyze changes in the ranking based on varying the weights of criteria. Humans are involved in the decision-making process, and for this reason, it is important to consider both individual preferences and knowledge (Zopounidis & Doumpos, 2002). From this point of view, the model is obviously correct according to the calculations and the procedure, but could not always be entirely predictable and controllable, which affects its reliability and authenticity.

Our proposal has theoretical and practical implications. The model allows us to analyze the influence of rational and irrational criteria, and highlights the poor financial knowledge of investors. The model considers a combination of different elements that have never been considered together in the existing literature. The investor's poor

financial knowledge is demonstrated by the fact that the calendar anomalies are considered the least important criterion, although it is the only criterion that detects the irrationality of the financial agents. Furthermore, the choice of a diversified investment had the highest priority. Therefore, we think that if we had considered a fifth alternative (the 50% - 50% investment), this would have most likely been the top choice. This leads us to believe that there is a lack of financial literacy. Financial literacy is more than just knowledge or information, but the ability to use the information and resources to achieve and maintain financial wellbeing (Huston, 2010). According to West (2012), our study confirms that financial literacy activities should seek to educate investors about the stock market, government bonds, other financial products and individual behavior. In fact, psychological biases and the limitations of the decision maker can influence financial behavior and, as a result, the investment choice (Figure 9).

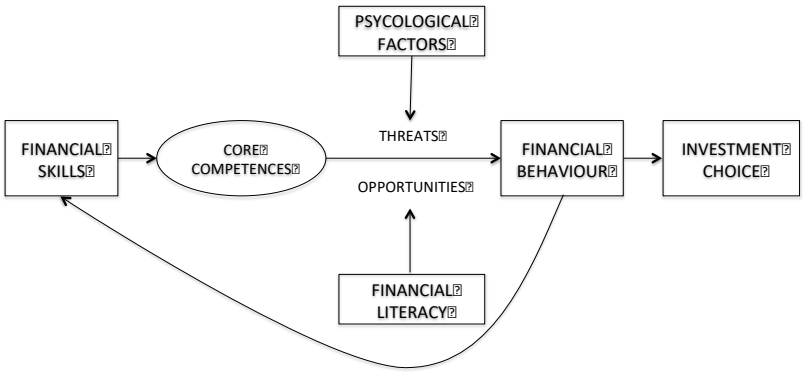


Figure 9 Impact of psychological factors and financial literacy on investment choice
Source: Authors' calculations

In this paper, we presented preliminary results from our study. Our aim was to extend the analysis in three different ways in the future:

- to involve a greater number of investors,
- to consider investors from different countries,
- to apply some other methods and compare the results,
- to study the impact of financial literacy on the investment choice.

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APPENDIX 1

Questionnaire

For each pair of criteria, indicate which is better than the "Investment choice" objective with a cross. (In case of indifference do not indicate any choice)	Indicate the degree of preference for the selected criterion on the left. (In the case of indifference, indicate 1.) The values from 2 to 9 indicate progressively increasing degrees of importance, from weak to absolute.
☐ Criterion 1: "Presence of calendar anomalies" [2] ☐ Criterion 2: "Stock market return in 2017"	1 2 3 4 5 6 7 8 9
☐ Criterion 1: "Presence of calendar anomalies" ☐ Criterion 3: "Government bond performance in 2017"	1 2 3 4 5 6 7 8 9
☐ Criterion 2: "Stock market return in 2017" ☐ Criterion 3: "Government bond performance in 2017"	1 2 3 4 5 6 7 8 9

[2] **"Presence of calendar anomalies":** We speak of calendar anomalies when precise dates or periods of the calendar year affect the variation in share prices. Four calendar effects were considered:

1. Weekend effect: negative equity returns on Monday and positive on Friday;
2. Holiday effect: higher equity returns in the days preceding the holidays;
3. Turn of the month effect: higher bond yields on the last trading day of the month and the first three of the next;
4. January effect: higher yields in January compared to other months.

For each pair of alternatives, indicate which is better than the criterion "Presence of calendar anomalies" with a cross, considering that there were 1 out of 4 anomalies in France and 3 out of 4 anomalies in Spain. (In the case of indifference, do not indicate any choice)	Indicate the degree of preference for the alternative selected on the left. (In the case of indifference. indicate 1.) The values from 2 to 9 indicate progressively increasing degrees of importance, from weak to absolute.
☐ Alternative 1: "Investment in France" ☐ Alternative 2: "Investment in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 1: "Investment in France" ☐ Alternative 3: "Investment 25% in France and 75% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 1: "Investment in France" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 2: "Investment in Spain" ☐ Alternative 3: "Investment 25% in France and 75% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 2: "Investment in Spain" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 3: "Investment 25% in France and 75% in Spain" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9

For each pair of alternatives, indicate which is better than the criterion "Stock market return in 2017" with a cross, considering that the French CAC 40 gained 9.3%, and the Spanish IBEX 35 gained 7.4%. (In the case of indifference, do not indicate any choice)	Indicate the degree of preference for the alternative selected on the left. (In the case of indifference, indicate 1.) The values from 2 to 9 indicate progressively increasing degrees of importance, from weak to absolute.
☐ Alternative 1: "Investment in France" ☐ Alternative 2: "Investment in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 1: "Investment in France" ☐ Alternative 3: "Investment 25% in France and 75% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 1: "Investment in France" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 2: "Investment in Spain" ☐ Alternative 3: "Investment 25% in France and 75% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 2: "Investment in Spain" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 3: "Investment 25% in France and 75% in Spain" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9

For each pair of alternatives, indicate which is better than the criterion "Government bond performance in 2017" with a cross, considering that there has been a growth in the ten-year yields with levels of 0.78% for the French OAT and 1.57% for Spanish Bonos. (In the case of indifference, do not indicate any choice.)	Indicate the degree of preference for the alternative selected on the left. (In the case of indifference, indicate 1.) The values from 2 to 9 indicate progressively increasing degrees of importance, from weak to absolute.
☐ Alternative 1: "Investment in France" ☐ Alternative 2: "Investment in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 1: "Investment in France" ☐ Alternative 3: "Investment 25% in France and 75% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 1: "Investment in France" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 2: "Investment in Spain" ☐ Alternative 3: "Investment 25% in France and 75% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 2: "Investment in Spain" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9
☐ Alternative 3: "Investment 25% in France and 75% in Spain" ☐ Alternative 4: "Investment 75% in France and 25% in Spain"	1 2 3 4 5 6 7 8 9

APPENDIX II

Common Priority Vector Procedure

The Common Priority Vector Procedure is an aggregation method recently introduced by Amenta et al. (2019). The aim of this procedure is to aggregate the DMs judgments with the hypothesis that they can have a common priority vector. Some of them agree in a higher way with this common solution, whereas others have a smaller concordance with it. The goal is to find this common vector, indicated by q, which synthesizes the values of the alternatives as seen by the majority of DMs. The solution is given by:

$$W_k = \widetilde{W}_k + E_k = \tilde{\lambda}_k q q^T + E_k$$

where $W_k = \lambda_k q q^T$ is a positive matrix of unitary rank providing the same information of judgment matrix X_k; $\widetilde{W}_k = \tilde{\lambda}_k q q^T$; $\tilde{\lambda}_k$ is the salience of order k associated to common vector q, E_k is the residual matrix, with $k = 1, \dots, K$ (K is the number of DMs) and q^T is the transpose of q vector. The final common vector and the saliences can be found by minimizing the following loss function:

$$L(q, \tilde{\lambda}_k, \alpha_k) = \sum_{k=1}^{K} \alpha_k \left\| W_k - \tilde{\lambda}_k q q^T \right\|_F^2$$

$$= \sum_{k=1}^{K} \alpha_k \left\| \lambda_k q_k q_k^T - \tilde{\lambda}_k q q^T \right\|_F^2$$

$$= \sum_{k=1}^{K} \alpha_k \left\| W_k - \widetilde{W}_k \right\|_F^2$$

The weights α_k are generally fixed a priori according to previous information. The common vector and the saliences then minimize the distance between the individual priority vectors and the common vector q. The solution of the previous function can be obtained by implementing an iterative algorithm, based on an alternating least squares procedure. All of the salience $\tilde{\lambda}_k$ are initialized to fixed values and the other quantities are computed iteratively until a convergence criterion is reached.