INTUITIONISTIC FUZZY ORIGINATED INTERVAL TYPE-2 FUZZY AHP: AN APPLICATION TO DAMLESS HYDROELECTRIC POWER PLANTS

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ABSTRACT

Intuitionistic fuzzy sets (IFS) proposed by Atanassov (1983, 1986) are a generalization of ordinary fuzzy sets. They incorporate the degree of hesitation which is defined as 1 minus the sum of membership and non-membership degrees. Type-2 fuzzy sets were first introduced by Zadeh (1975) as an extension of the concept of an ordinary fuzzy set. Type-2 fuzzy sets have grades of membership that are themselves fuzzy. The membership function of a type-2 fuzzy set is three-dimensional, and it is the new third dimension that provides additional degrees of freedom for handling uncertainties. An intuitionistic fuzzy set can be converted to a Type-2 fuzzy set by subtracting its non-membership function from 1. Thus, an intuitionistic fuzzy multi-criteria decision making problem can be solved by using type-2 fuzzy multi-criteria decision making techniques. In this paper, an intuitionistic fuzzy originated interval type-2 fuzzy AHP method is developed and applied to the technology selection problem of a damless hydroelectric power plant. Damless hydroelectric power plants are environmentally friendly and sustainable energy production systems. Several criteria and damless technology alternatives along the

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Sakarya River in Turkey are considered. Linguistic evaluations are considered in this multi-criteria damless technology selection problem.

Keywords: Intuitionistic fuzzy sets; type-2 fuzzy sets; AHP; multi-criteria decision making; damless hydroelectric power

1. Introduction

Multi-criteria decision making approaches model the decision problem by considering all related characteristics, perspectives and views of the stakeholders. In the traditional multi-criteria decision making approaches (MCDM) the evaluations are done with exact numbers. Fuzzy MCDM approaches are helpful tools in order to deal with the problems that involve uncertainty and imprecision. Type-1 fuzzy sets introduced by Zadeh (1965) where each element is defined with a membership degree in the interval [0, 1] are important for dealing with vagueness. Yet, type-1 fuzzy sets have limitations in defining the uncertainties. Therefore, there are several extensions of fuzzy sets that deal with the shortcomings. Intuitionistic fuzzy sets introduced by Atanassov (1983) enable defining both the membership and non-membership degrees. Type-2 fuzzy sets as fuzzy sets define a three-dimensional fuzzy set where the new third dimension represents the degrees of freedom (Zadeh, 1975). Both of these two extensions have various advantages and transitivity among these sets is possible.

Energy consumption has been rising due to population growth and exponential technology improvements. Carbon based energy resources such as petroleum or gas are not enough to fulfill the demand by the population on earth. Carbon based energy resources release greenhouse gases and increase carbon emissions which create severe environmental and health problems. Renewable energy can be the major solution to the increased energy need. Renewable resources like solar, wind, geothermal, and hydro energy are clean and limitless. Renewable energy investments are crucial for supplying the energy need. Hydroelectric power is the leading renewable energy resource that accounts for approximately 16% of total electricity production and 85% of renewable electricity production (Özcan et al., 2017). It is one of the most convenient energy resources that can balance the fluctuations between energy demand and supply. Damless hydroelectric power plants are eco-friendly and sustainable energy systems. Selecting the appropriate damless hydroelectric power is important for the success of renewable energy investments. Yet, the criteria in damless hydroelectric power selection includes uncertainties and imprecision.

Classical AHP as a selection method uses a linguistic scale involving some degree of vagueness. In this linguistic scale, every linguistic term has a corresponding numerical value. However, an expert may want to use a continuous interval rather than an exact discrete number such as “between 2 and 3” or “larger than 7”. In this case, “between 2 and 3” and “larger than 7” can be represented by triangular fuzzy numbers. There are several ordinary fuzzy AHP methods developed by some researchers in the literature which include Buckley (1985), Laarhoven and Pedrycz (1983), Chang (1996), etc. Emerging extensions of ordinary fuzzy sets caused new extensions of the fuzzy AHP method such as intuitionistic fuzzy AHP, hesitant fuzzy AHP, type-2 fuzzy AHP and Pythagorean fuzzy AHP (Otay et al., 2017; Büyüközkan et al., 2017; Zhu & Xu, 2014; Cevik Onar et al., 2014; Kahraman et al., 2014; Ilbahar et al., 2018).

The aim of this study is to show the potential applicability of type-2 fuzzy conversion of intuitionistic fuzzy sets in multi-criteria selection problems. An intuitionistic fuzzy set can be converted to a Type-2 fuzzy set by subtracting its non-membership function from 1. Thus, an intuitionistic fuzzy multi-criteria decision making problem can be solved by using type-2 fuzzy multi-criteria decision making techniques. This is the advantage of our approach since type-2 fuzzy AHP methods have been already well-developed in the literature. The originality of the paper is the development of an intuitionistic fuzzy originated interval type-2 fuzzy AHP and its application to the technology selection problem of a damless hydroelectric power plant. The rest of the study is organized as follows: Section 2 explains the preliminaries on intuitionistic and type-2 fuzzy sets. The proposed intuitionistic fuzzy originated interval type-2 fuzzy AHP is given in Section 3. Application of the proposed methodology and the sensitivity analysis for a damless hydroelectric power plant selection problem is given in Section 4 and in the final section, conclusions are given.

2. Intuitionistic and type-2 fuzzy sets

2.1 Triangular intuitionistic fuzzy sets

The intuitionistic fuzzy sets introduced by Atanassov (1986, 1999) are expressed by a membership value and a non-membership value for any *x* in *X* so that their sum is at most 1.

Definition 1:

Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set in X is an object A given by

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle; x\epsilon X\}, \tag{1}$$

where $\mu_{\tilde{A}}: X \to [0,1]$ and $v_{\tilde{A}}: X \to [0,1]$ satisfy the condition

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \tag{2}$$

for every $x\epsilon X$. Hesitancy is equal to "1-$\left(\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x)\right)$"

Definition 2:

A Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{A}$ is an intuitionistic fuzzy subset in $\mathbb{R}$ with the following membership function and non-membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, for\ a_2 \leq x \leq a_3 \\ 0\ ,\ otherwise \end{cases} \tag{3}$$

And

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, for\ a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2}, for\ a_2 \leq x \leq a_3' \\ 1\ ,\ otherwise \end{cases} \tag{4}$$

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where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3', 0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$ and TIFN is denoted by $\widetilde{A}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$.

Definition 3:
Arithmetic operations for TIFNs are as follows:
Let $\tilde{A}_{TrIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{B}_{TrIFN} = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two TIFNs.

Then,
Addition: $\tilde{C} = \tilde{A} + \tilde{B}$ is also a TIFN:

$$\tilde{C} = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, \; a_3' + b_3'). \tag{5}$$

Multiplication: $\tilde{C} = \tilde{A} \otimes \tilde{B}$ is also a TIFN:

$$\tilde{C} \cong (a_1 b_1, a_2 b_2, a_3 b_3; a_1' b_1', a_2 b_2, a_3' b_3'). \tag{6}$$

Subtruction: $\tilde{C} = \tilde{A} \ominus \tilde{B}$ is also a TIFN:

$$\tilde{C} = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 - b_2, a_3' - b_1'). \tag{7}$$

Division: $\tilde{C} = \tilde{A} \oslash \tilde{B}$ is also a TIFN:

$$\tilde{C} \cong (a_1/b_3, a_2/b_2, a_3/b_1; a_1'/b_3', a_2/b_2, a_3'/b_1'). \tag{8}$$

Definition 4:
Let $I_i = \left(a_i^L, a_i^M, a_i^U; a_i'^L, a_i^M, a_i'^U\right)$ is a TIFN. Then, the defuzzification is realized by using function defined in Eq. (9).

$$d_f = \frac{a_i^L + a_i^M + a_i^U}{3} + \frac{a_i'^L + a_i^M + a_i'^U}{\tau} \tag{9}$$

where τ is a very large number.
Aggregation operator for triangular intuitionistic fuzzy sets is defined as in Definition 5.

Definition 5:
Let $\tilde{A}_j = \left(\left(a_j^L, a_j^M, a_j^U\right), \left(a_j'^L, a_j^M, a_j'^U\right)\right)$, *j=1, 2, ..., n* be a set of Triangular Fuzzy Number Intuitionistic Fuzzy Numbers ($TFNIFN$) where $a_j^L, a_j^M, a_j^U, a_j'^L, a_j^M, a_j'^U \geq 1$.

The aggregation operator in Equation 10 for these numbers is proposed by us.

$$f_m(I_1, I_2, \ldots, I_n) = \left(\left[\prod_{j=1}^{n}\left(a_j^L\right)^{w_j}, \prod_{j=1}^{n}\left(a_j^M\right)^{w_j}, \prod_{j=1}^{n}\left(a_j^U\right)^{w_j}\right], \left[\prod_{j=1}^{n}\left(a_j'^L\right)^{w_j}, \prod_{j=1}^{n}\left(a_j^M\right)^{w_j}, \prod_{j=1}^{n}\left(a_j'^U\right)^{w_j},\right]\right) \tag{10}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of

$$I_j (j = 1, 2, \ldots, n), \; w_j \in [0,1], \sum_{j=1}^{n} w_j = 1.$$

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2.2 Preliminaries: type-2 fuzzy sets

In type-1 (ordinary) fuzzy sets each element has a degree of membership which is given by a membership function valued in the interval [0, 1] (Zadeh, 1965). The concept of a type-2 fuzzy set was introduced by Zadeh (1975) as an extension of the concept of an ordinary fuzzy set called a type-1 fuzzy set. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets. They are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set; hence, they are useful for incorporating linguistic uncertainties, e.g., the words that are used in linguistic knowledge can mean different things to different people (Karnik & Mendel, 2001). While the membership functions of ordinary fuzzy sets are two-dimensional, the membership functions of type-2 fuzzy sets are three-dimensional. It is the new third-dimension that provides additional degrees of freedom that make it possible to directly model uncertainties. Figure 1 illustrates an interval type-2 fuzzy set.

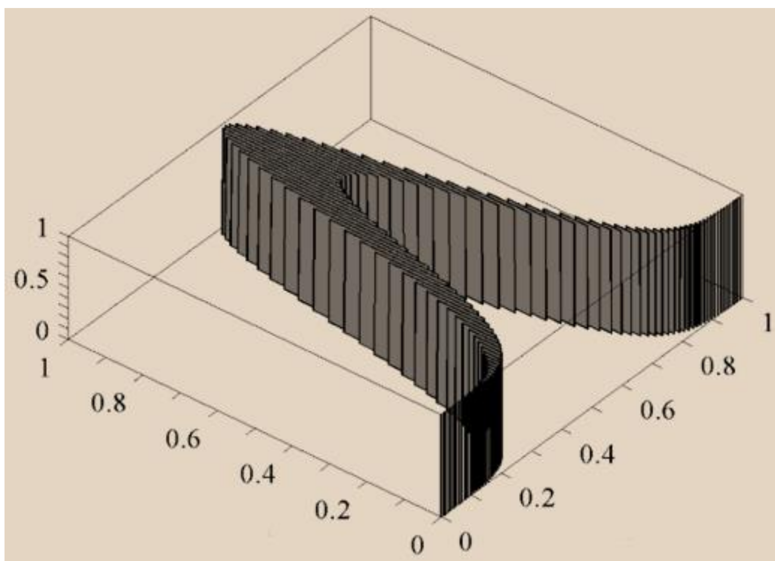

Figure 1 Interval type-2 fuzzy set

where J_x denotes an interval [0,1]. The type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented by Equation 11 (Mendel et al., 2006):

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x, u) / (x, u) \quad (11)$$

where $J_x \subseteq [0,1]$ and $\iint$ denote union over all admissible x and u.

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x, u) = 1$, then $\tilde{\tilde{A}}$ is called an interval type-2 fuzzy set (Buckley, 1985). An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of a type-2 fuzzy set, represented by Eq. (12) (Mendel et al., 2006):

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u) \quad (12)$$

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where $J_x \subseteq [0,1]$.

Arithmetic operations with triangular interval type-2 fuzzy sets are given in the following.

Definition 6:
The upper and lower membership functions of an interval type-2 fuzzy set are type-1 membership functions.
A triangular interval type-2 fuzzy set is illustrated as $\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U; \tilde{A}_i^L\right) = \left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U\,; H(\tilde{A}_i^U)\right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L,; H(\tilde{A}_i^L)\right)\right)$ where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets; a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i1}^L, a_{i2}^L and a_{i3}^L are the reference points of the interval type-2 fuzzy set $\tilde{\tilde{A}}_i$. $H\left(\tilde{A}_i^U\right)$ denotes the membership value of the element a_{i2}^U in the upper triangular membership function. $H\left(\tilde{A}_i^L\right)$ denotes the membership value of the element a_{a2}^L in the lower triangular membership function. $H\left(\tilde{A}_i^U\right) \in [0,1]$, $H\left(\tilde{A}_i^L\right) \in [0,1]$ and $1 \leq i \leq n$ (Chen & Lee, 2010).

Definition 7:
The addition operation between the triangular interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U\,; H(\tilde{A}_1^U)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L,; H(\tilde{A}_1^L)\right)\right)$ and $\tilde{\tilde{A}}_2 = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U\,; H(\tilde{A}_2^U)\right), \left(a_{21}^L, a_{22}^L, a_{23}^L,; H(\tilde{A}_2^L)\right)\right)$ is defined by Equation 13 (Chen & Lee, 2010):

$$\begin{gathered}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = \Big((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U; \\ min\left(H\left(\tilde{A}_1^U\right); H\left(\tilde{A}_2^U\right)\right), min\left(H\left(\tilde{A}_1^U\right); H\left(\tilde{A}_2^U\right)\right)\Big), \\ (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L,; \\ min\left(H\left(\tilde{A}_1^L\right); H\left(\tilde{A}_2^L\right)\right), min\left(H\left(\tilde{A}_1^L\right); H\left(\tilde{A}_2^L\right)\right)\Big)\Big)\end{gathered} \tag{13}$$

Definition 8:
The subtraction operation between the triangular interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U\,; H(\tilde{A}_1^U)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L,; H(\tilde{A}_1^L)\right)\right)$ and $\tilde{\tilde{A}}_2 = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U\,; H(\tilde{A}_2^U)\right), \left(a_{21}^L, a_{22}^L, a_{23}^L,; H(\tilde{A}_2^L)\right)\right)$ is defined by Equation 14 (Chen & Lee, 2010):

$$\begin{gathered}\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = \Big((a_{11}^U - a_{23}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{21}^U; \\ min\left(H\left(\tilde{A}_1^U\right); H\left(\tilde{A}_2^U\right)\right), min\left(H\left(\tilde{A}_1^U\right); H\left(\tilde{A}_2^U\right)\right)\Big), \\ \Big((a_{11}^L - a_{23}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{21}^L; \\ min\left(H\left(\tilde{A}_1^U\right); H\left(\tilde{A}_2^U\right)\right), min\left(H\left(\tilde{A}_1^U\right); H\left(\tilde{A}_2^U\right)\right)\Big)\end{gathered} \tag{14}$$

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Definition 9:

The multiplication operation between the triangular interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left((a_{11}^U, a_{12}^U, a_{13}^U\,; H(\tilde{A}_1^U)),\ (a_{11}^L, a_{12}^L, a_{13}^L,; H(\tilde{A}_1^L))\right)$ and $\tilde{\tilde{A}}_2 = \left((a_{21}^U, a_{22}^U, a_{23}^U\,; H(\tilde{A}_2^U)),\ (a_{21}^L, a_{22}^L, a_{23}^L,; H(\tilde{A}_2^L))\right)$is defined by Eq. (15) (Chen & Lee, 2010):

$$\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 \cong \Big((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U;$$
$$min\left(H(\tilde{A}_1^U); H(\tilde{A}_2^U)\right), min\left(H(\tilde{A}_1^U); H(\tilde{A}_2^U)\right)\Big),$$
$$(a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L;$$
$$min\left(H(\tilde{A}_1^U); H(\tilde{A}_2^U)\right), min\left(H(\tilde{A}_1^U); H(\tilde{A}_2^U)\right)\Big) \quad (15)$$

Definition 10:

The arithmetic operations between the triangular interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left((a_{11}^U, a_{12}^U, a_{13}^U\,; H(\tilde{A}_1^U)),\ (a_{11}^L, a_{12}^L, a_{13}^L,; H(\tilde{A}_1^L))\right)$ and $\tilde{\tilde{A}}_2 = \left((a_{21}^U, a_{22}^U, a_{23}^U\,; H(\tilde{A}_2^U)),\ (a_{21}^L, a_{22}^L, a_{23}^L,; H(\tilde{A}_2^L))\right)$ and the crisp value k is defined by Equations 16 and 17 (Chen & Lee, 2010):

$$k\tilde{\tilde{A}}_1 = \Big((k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U); H(\tilde{A}_1^U),$$
$$\left(k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L; H(\tilde{A}_1^L)\right)\Big) \quad (16)$$

$$\frac{\tilde{\tilde{A}}_1}{k} = \left(\left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U\right); H(\tilde{A}_1^U),\right.$$
$$\left.\left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L; H(\tilde{A}_1^L)\right)\right) \quad (17)$$

where k > 0.

3. Intuitionistic fuzzy originated interval type-2 fuzzy AHP

The steps of the methodology are given in the following:

Step 1: Determine the evaluation criteria and alternatives and define the hierarchy of the problem.

Step 2: Construct the pairwise comparison matrices using a triangular intuitionistic fuzzy linguistic evaluation scale for each of the experts based on our proposed scale given in Table 1.

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Table 1
Triangular intuitionistic fuzzy scale

Linguistic Terms	Intuitionistic Fuzzy Numbers
Absolutely Strong (AS)	{(7.5, 9, 10.5), (7, 9, 11)}
Very Strong (VS)	{(5.5, 7, 8.5), (5, 7, 9)}
Fairly Strong (FS)	{(3.5, 5, 6.5), (3, 5, 7)}
Slightly Strong (SS)	{(1.5, 3, 4.5), (1, 3, 5)}
Exactly Equal (E)	{(1, 1, 1), (1, 1, 1)}
If factor *i* has one of the above linguistic variables assigned to it when compared with factor j, then j has the reciprocal value when compared with *i*: $1/\tilde{A}_j = \left((1/a_j^U, 1/a_j^M, 1/\ a_j^L), (1/a_j'^U, 1/a_j^M, 1/\right.$	Reciprocals of above

Step 3: Check the consistency of each fuzzy pairwise comparison matrix. Assume $\tilde{\tilde{A}} = \left[\tilde{\tilde{a}}_{ij}\right]$ is a fuzzy positive reciprocal matrix and $A = \left[a_{ij}\right]$ is its defuzzified positive reciprocal matrix. If the result of the comparisons of $A = \left[a_{ij}\right]$ is consistent, then it can imply that the result of the comparisons of $\tilde{\tilde{A}} = \left[\tilde{\tilde{a}}_{ij}\right]$ is also consistent. In order to check the consistencies of the fuzzy pairwise comparison matrices, the proposed defuzzification method given in Equation 9 is used.

Step 4: Aggregate the intuitionistic fuzzy pairwise comparison matrices using Equation 10.

Step5: Transform the aggregated triangular intuitionistic fuzzy pairwise comparison matrix into triangular type-2 fuzzy (TT2F) pairwise comparison matrix. An example of this transformation is illustrated in Figure 2.

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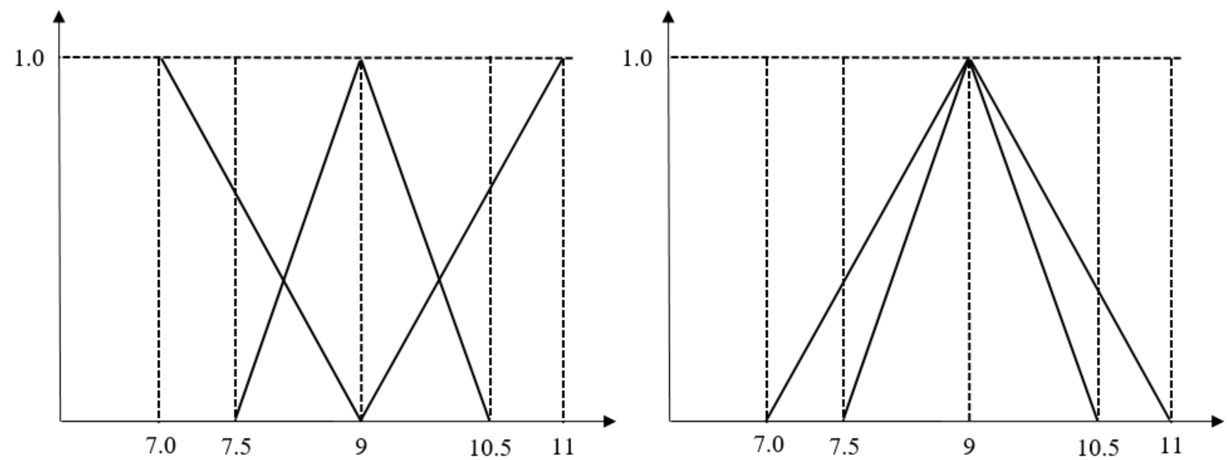

(a) (b)

Figure 2 Transformation from TIFN to TT2F number

A type-2 fuzzy pairwise comparison matrix is represented by Equation 18.

$$\tilde{\tilde{A}} = \begin{bmatrix} 1 & \tilde{\tilde{a}}_{12} & \cdots & \tilde{\tilde{a}}_{1n} \\ \tilde{\tilde{a}}_{21} & 1 & \cdots & \tilde{\tilde{a}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{a}}_{n1} & \tilde{\tilde{a}}_{n2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{\tilde{a}}_{12} & \cdots & \tilde{\tilde{a}}_{1n} \\ 1/\tilde{\tilde{a}}_{12} & 1 & \cdots & \tilde{\tilde{a}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{\tilde{a}}_{1n} & 1/\tilde{\tilde{a}}_{2n} & \cdots & 1 \end{bmatrix} \tag{18}$$

where

$$1/\tilde{\tilde{a}} = \left\{\left(\frac{1}{a_{13}^U},\frac{1}{a_{12}^U},\frac{1}{a_{11}^U};1\right),\left(\frac{1}{a_{23}^L},\frac{1}{a_{22}^L},\frac{1}{a_{21}^L};1\right)\right\}$$

Step 6: Calculate the geometric mean of each row and then compute the fuzzy weights by normalization.

The geometric mean of each row $\tilde{\tilde{r}}_j$ is calculated by Equation 19.

$$\tilde{\tilde{r}}_j = \left[\tilde{\tilde{a}}_{j1} \otimes \ldots \otimes \tilde{\tilde{a}}_{jn}\right]^{1/n} \tag{19}$$

where

$$\sqrt[n]{\tilde{\tilde{a}}_{jn}} = \left\{\left(\left(\sqrt[n]{\prod_{j=1}^{n} a_{j1}^U}, \sqrt[n]{\prod_{j=1}^{n} a_{j2}^U}, \sqrt[n]{\prod_{j=1}^{n} a_{j3}^U};1\right)\right),\left(\left(\sqrt[n]{\prod_{j=1}^{n} a_{j1}^L}, \sqrt[n]{\prod_{j=1}^{n} a_{j2}^L}, \sqrt[n]{\prod_{j=1}^{n} a_{j3}^L};1\right)\right)\right\}$$

The fuzzy weight $\tilde{\tilde{w}}_i$ of the i^{th} criterion is calculated by Equation 20.

$$\tilde{\tilde{w}}_i = \tilde{\tilde{r}}_i \otimes [\tilde{\tilde{r}}_1 \oplus \dots \oplus \tilde{\tilde{r}}_i \oplus \dots \oplus \tilde{\tilde{r}}_n]^{-1} \tag{20}$$

where

$$\frac{\tilde{\tilde{a}}_{ij}}{\tilde{\tilde{b}}_{ij}} = \left\{\left(\frac{a_1^u}{b_3^u}, \frac{a_2^u}{b_2^u}, \frac{a_3^u}{b_1^u}, 1\right), \left(\frac{a_1^L}{b_3^L}, \frac{a_2^L}{b_2^L}, \frac{a_3^L}{b_1^L}, 1\right)\right\}$$

Step 7: Aggregate the fuzzy weights and fuzzy performance scores by Equation 21:

$$\tilde{\tilde{U}}_i = \sum_{j=1}^{n} \tilde{\tilde{w}}_j \tilde{\tilde{r}}_{ij}, \forall i. \tag{21}$$

where $\tilde{\tilde{U}}_i$ is the fuzzy utility of alternative *i*; $\tilde{\tilde{w}}_j$ is the weight of the criterion *j*, and $\tilde{\tilde{r}}_{ij}$ is the performance score of alternative *i* with respect to criterion *j*.

Step 8: Apply the classical AHP method's procedure to determine the best alternative. In order to find the crisp weights, the defuzzification formula given in Equation 22 is applied. For an interval type-2 triangular fuzzy number

$$\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U; H(\tilde{A}_1^U)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L; H(\tilde{A}_1^L)\right)\right)$$

the defuzzification formula is as follows (Kahraman et al., 2014):

$$DTtrT = \frac{\frac{\left(a_{13}^U - a_{11}^U\right) + \left(a_{12}^U - a_{11}^U\right)}{3} + a_{11}^U + H(\tilde{A}_1^L)\left[\frac{\left(a_{13}^L - a_{11}^L\right) + \left(a_{12}^L - a_{11}^L\right)}{3} + a_{11}^L\right]}{2} \tag{22}$$

4. An application to damless hydroelectric power plants

Existing energy structures that seriously pollute the environment because of the use of fossil fuels must be gradually replaced by clean and renewable energy structures (Li et al., 2018). All over the world, efforts to reduce carbon emissions have increased interest in renewable energy production. With the global acceptance of the effort to reduce the use of fossil fuels, renewable energy will gain importance in the next three to four decades (Sarasúa et al., 2014; Sternberg, 2010). Hydroelectric power generation is of special interest to generate clean energy (Ioannidou & O'Hanley, 2018). One of the hydroelectric power plant types is damless hydroelectric power. It can be referred to as flow-of-the-river power plants. The transition of the flow energy of a river into electricity by a damless or free-flow hydroelectric power plant is called damless hydroelectric transition (Volshanik, 1999). Damless hydropower plants are characterized by the storage capacity of the energy (Sarasúa et al., 2014). The energy thus produced varies greatly depending on the current flow in the stream.

Damless hydroelectric plants have two main advantages which are high social sustainability and low environmental cost in rural areas (Jager et al., 2015). Damless plants have shown promise in rural areas of the U.S., Europe, Africa and Asia with the potential of generating hydroelectricity (Tilmant et al., 2012; Kuby et al., 2005; Szabó et al., 2013; Reddy et al., 2006). Irrigation systems and waste water streams can provide opportunities for harmless hydropower generation (Adhau et al., 2012). Although

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damless hydroelectric plants have advantages, there are also some limitations. Generating energy with damless technologies is water dependent and depends on the speed of streamflow (Sternberg, 2010). However, new damless technologies try to overcome these limitations to generate energy permanently.

There are two alternatives that will be compared in this study. These alternatives are the cascaded damless hydroelectric turbine (A1) and the helical turbine (A2). The cascades of hydro turbines are installed on pontoons or fixed in the water stream by means of cantilever suspension. They work without building a dam on the river. Figure 3 presents an illustration of cascaded hydro turbines.

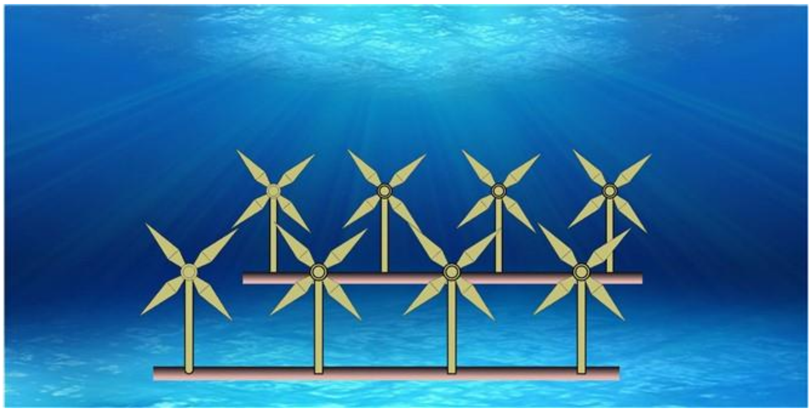

Figure 3 Cascades of hydro turbines

Helical Unique Generation (HUG) is a new alternate source of hydroelectric energy without a dam (Figure 4). The helical turbine is a cross flow unidirectional rotation machine that makes it valuable for underwater currents generated by wave fluctuations.

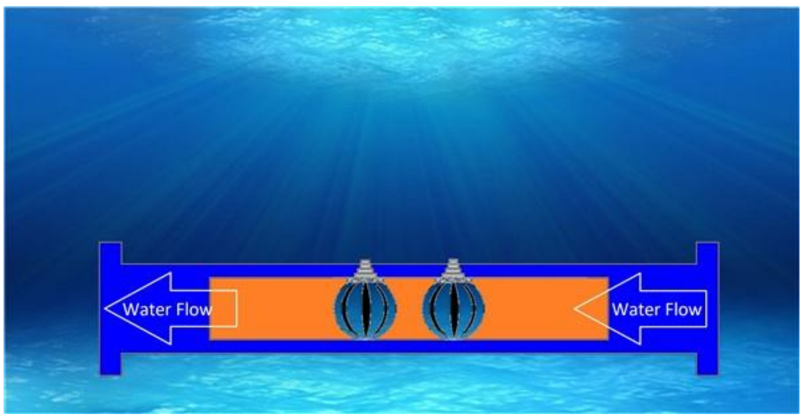

Figure 4 Helical turbine

The following evaluation criteria have been considered for this multi-criteria decision making problem (https://www.turbulent.be/damless-hydro-power-plants-and-its-implications/).

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Cost (C1): In the damless hydroelectric power plant projects, several factors affect the cost such as plant size, local economy, resources to be used, site-specific conditions (river flow, water amount etc.). The costs of the turbine technology are also taken into account as cost criterion, which refers to the sum of equipment, electrical connection, infrastructure, planning and other installation costs.

Maintenance frequency (C2): Operating and maintenance frequency refers to the frequency of activities required to maximize the energy produced and to avoid any costly breakdowns.

Corrosion resistance (C3): Submerged metals corrode in time. The duration of corrosion exposure is an important factor in evaluating turbine alternatives as it directly affects cost.

Production capacity (C4): The amount of electricity that a turbine can generate is another consideration in damless hydroelectric power plant projects.

Technological upgrade (C5): At a particular point, it will often be economical to upgrade the current technology in order to increase production capacity and reduce increased operating and maintenance costs. The refurbishment may require additional construction work depending on the degree of corrosion and wear.

4.1 Application of the method

***Step 1**:* The hierarchy of the problem is presented in Figure 5.

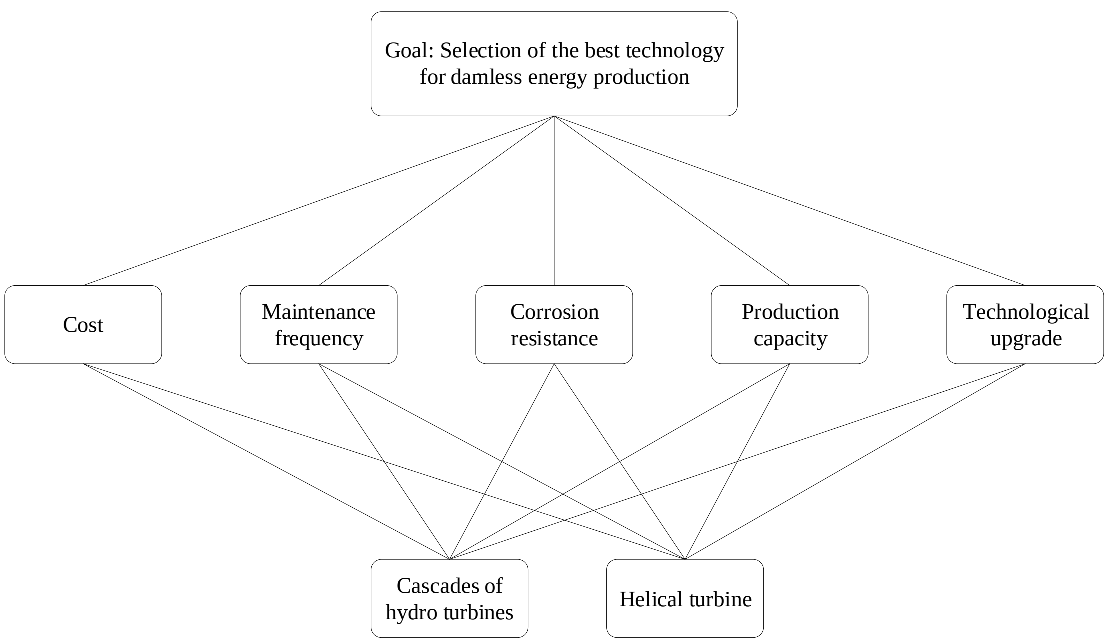

Figure 5 Hierarchy of the damless technology selection problem

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Step 2: The weights of three experts are determined as 0.25, 0.45 and 0.30 based on their experiences. Their pairwise comparison matrices are given in Table 2.

Table 2
Pairwise comparison matrices of the main criteria with respect to the goal

Expert 1

	C1	**C2**	**C3**	**C4**	**C5**
C1	E	FS	VS	1/SS	SS
C2	1/FS	E	SS	1/VS	1/SS
C3	1/VS	1/SS	E	1/AS	1/VS
C4	SS	VS	AS	E	VS
C5	1/SS	SS	VS	1/VS	E

Expert 2

	C1	**C2**	**C3**	**C4**	**C5**
C1	E	SS	FS	1/FS	E
C2	1/SS	E	E	1/AS	1/FS
C3	1/FS	E	E	1/AS	1/FS
C4	FS	AS	AS	E	AS
C5	E	FS	FS	1/AS	E

Expert 3

	C1	**C2**	**C3**	**C4**	**C5**
C1	E	FS	SS	1/VS	E
C2	1/VS	E	E	1/FS	1/SS
C3	1/SS	E	E	1/AS	1/VS
C4	VS	FS	VS	E	VS
C5	E	SS	VS	1/AS	E

The pairwise comparisons of the alternatives with respect to the criteria have been made by three experts by using the linguistic scale given in Table 1. These comparisons are presented in Table 3.

Table 3
Pairwise comparison matrices of the alternatives with respect to the criteria

Expert 1

C1

	A1	**A2**
A1	E	VS
A2	1/VS	E

C2

	A1	**A2**
A1	E	SS
A2	1/SS	E

C3

	A1	**A2**
A1	E	1/SS
A2	SS	E

C4

	A1	**A2**
A1	E	FS
A2	1/FS	E

C5

	A1	**A2**
A1	E	FS
A2	1/FS	E

Expert 2

C1

	A1	**A2**
A1	E	VS
A2	1/VS	E

C2

	A1	**A2**
A1	E	FS
A2	1/FS	E

C3

	A1	**A2**
A1	E	E
A2	E	E

C4

	A1	**A2**
A1	E	SS
A2	1/SS	E

C5

	A1	**A2**
A1	E	SS
A2	1/SS	E

Expert 3

C1

	A1	**A2**
A1	E	FS
A2	1/FS	E

C2

	A1	**A2**
A1	E	SS
A2	1/SS	E

C3

	A1	**A2**
A1	E	1/FS
A2	FS	E

C4

	A1	**A2**
A1	E	FS
A2	1/FS	E

C5

	A1	**A2**
A1	E	E
A2	E	E

Step 3: In this step, the consistencies of the intuitionistic fuzzy pairwise comparison matrices are checked. First of all, the values in the pairwise comparison matrices of the main criteria are defuzzified by using Equation 9 and then the crisp consistency measurement method is applied. The consistency of each pairwise comparison matrix has been provided to be under 0.1. The value of τ has been assigned as 100 in our

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calculations. The defuzzified pairwise comparison matrices for the main criteria are given in Table 4.

Table 4
Defuzzified pairwise comparison matrices

Expert 1

	C1	**C2**	**C3**	**C4**	**C5**
C1	1.0	5.2	7.2	0.3	3.1
C2	0.2	1.0	3.1	0.1	0.3
C3	0.1	0.3	1.0	0.1	0.1
C4	3.1	7.2	9.3	1.0	7.2
C5	0.3	3.1	7.2	0.1	1.0

CR = 0.085

Expert 2

	C1	**C2**	**C3**	**C4**	**C5**
C1	1.0	3.1	5.2	0.2	1.0
C2	0.3	1.0	1.0	0.1	0.2
C3	0.2	1.0	1.0	0.1	0.2
C4	5.2	9.3	9.3	1.0	9.3
C5	1.0	5.2	5.2	0.1	1.0

CR = 0.073

Expert 3

	C1	**C2**	**C3**	**C4**	**C5**
C1	1.0	3.1	3.1	0.1	1.0
C2	0.3	1.0	1.0	0.2	0.3
C3	0.3	1.0	1.0	0.1	0.1
C4	7.2	5.2	9.3	1.0	7.2
C5	1.0	3.1	7.2	0.1	1.0

CR = 0.097

Since the pairwise comparisons of alternatives are based on 2×2 matrices, it is not necessary to check the consistencies of these matrices.

Step 4: The aggregated triangular intuitionistic fuzzy pairwise comparison matrices for the main criteria and alternatives are given in Table 5 and Table 6, respectively.

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Table 5
Aggregated triangular intuitionistic fuzzy pairwise comparison matrix for the criteria

	C1	**C2**	**C3**
C1	{(1, 1, 1), (1, 1, 1)}	{(2.39, 3.97, 5.51), (1.83, 3.97, 6.02)}	{(3.04, 4.67, 6.22), (2.45, 4.67, 6.74)}
C2	{(0.17, 0.23, 0.37), (0.15, 0.23, 0.47)}	{(1, 1, 1), (1, 1, 1)}	{(1.11, 1.32, 1.46), (1, 1.32, 1.5)}
C3	{(0.16, 0.21, 0.33), (0.15, 0.21, 0.41)}	{(0.69, 0.76, 0.9), (0.67, 0.76, 1)}	{(1, 1, 1), (1, 1, 1)}
C4	{(3.24, 4.87, 6.43), (2.66, 4.87, 6.94)}	{(5.52, 7.09, 8.63), (4.99, 7.09, 9.14)}	{(6.83, 8.35, 9.86), (6.33, 8.35, 10.36)}
C5	{(0.69, 0.76, 0.9), (0.67, 0.76, 1)}	{(2.2, 3.78, 5.31), (1.64, 3.78, 5.82)}	{(4.49, 6.02, 7.53), (3.97, 6.02, 8.04)}

	C4	**C5**
C1	{(0.16, 0.21, 0.31), (0.14, 0.21, 0.38)}	{(1.11, 1.32, 1.46), (1, 1.32, 1.5)}
C2	{(0.12, 0.14, 0.18), (0.11, 0.14, 0.2)}	{(0.19, 0.26, 0.46), (0.17, 0.26, 0.61)}
C3	{(0.1, 0.11, 0.13), (0.09, 0.11, 0.14)}	{(0.13, 0.17, 0.22), (0.12, 0.17, 0.25)}
C4	{(1, 1, 1), (1, 1, 1)}	{(6.32, 7.84, 9.35), (5.82, 7.84, 9.85)}
C5	{(0.1, 0.12, 0.14), (0.1, 0.12, 0.16)}	{(1, 1, 1), (1, 1, 1)}

Table 6

Aggregated triangular intuitionistic fuzzy pairwise comparison matrices for the alternatives

		A1	A2
C1	A1	{(1, 1, 1), (1, 1, 1)}	{(4.8, 6.33, 7.84), (4.29, 6.33, 8.35)}
	A2	{(0.13, 0.16, 0.21), (0.12, 0.16, 0.23)}	{(1, 1, 1), (1, 1, 1)}
C2	A1	{(1, 1, 1), (1, 1, 1)}	{(2.2, 3.78, 5.31), (1.64, 3.78, 5.82)}
	A2	{(0.19, 0.26, 0.46), (0.17, 0.26, 0.61)}	{(1, 1, 1), (1, 1, 1)}
C3	A1	{(1, 1, 1), (1, 1, 1)}	{(0.39, 0.47, 0.62), (0.37, 0.47, 0.72)}
	A2	{(1.61, 2.13, 2.55), (1.39, 2.13, 2.68)}	{(1, 1, 1), (1, 1, 1)}
C4	A1	{(1, 1, 1), (1, 1, 1)}	{(2.39, 3.97, 5.51), (1.83, 3.97, 6.02)}
	A2	{(0.18, 0.25, 0.42), (0.17, 0.25, 0.55)}	{(1, 1, 1), (1, 1, 1)}
C5	A1	{(1, 1, 1), (1, 1, 1)}	{(1.64, 2.45, 3.14), (1.32, 2.45, 3.36)}
	A2	{(0.32, 0.41, 0.61), (0.3, 0.41, 0.76)}	{(1, 1, 1), (1, 1, 1)}

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Step 5: Now, the aggregated triangular intuitionistic fuzzy pairwise comparison matrix is transformed into TT2F pairwise comparison matrix for the main criteria (Table 7) and the alternatives (Table 8).

Table 7
Transformation to TT2F pairwise comparison matrix for the main criteria

	C1	**C2**
C1	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(1.83, 3.97, 6.02; 1), (2.39, 3.97, 5.51 ;1)}
C2	{(0.15, 0.23, 0.47; 1), (0.17, 0.23, 0.37 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}
C3	{(0.15, 0.21, 0.41; 1), (0.16, 0.21, 0.33 ;1)}	{(0.67, 0.76, 1; 1), (0.69, 0.76, 0.9 ;1)}
C4	{(2.66, 4.87, 6.94; 1), (3.24, 4.87, 6.43 ;1)}	{(4.99, 7.09, 9.14; 1), (5.52, 7.09, 8.63 ;1)}
C5	{(0.67, 0.76, 1; 1), (0.69, 0.76, 0.9 ;1)}	{(1.64, 3.78, 5.82; 1), (2.2, 3.78, 5.31 ;1)}
	C3	**C4**
C1	{(2.45, 4.67, 6.74; 1), (3.04, 4.67, 6.22 ;1)}	{(0.14, 0.21, 0.38; 1), (0.16, 0.21, 0.31 ;1)}
C2	{(1, 1.32, 1.5; 1), (1.11, 1.32, 1.46 ;1)}	{(0.11, 0.14, 0.2; 1), (0.12, 0.14, 0.18 ;1)}
C3	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(0.09, 0.11, 0.14; 1), (0.1, 0.11, 0.13 ;1)}
C4	{(6.33, 8.35, 10.36; 1), (6.83, 8.35, 9.86 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}
C5	{(3.97, 6.02, 8.04; 1), (4.49, 6.02, 7.53 ;1)}	{(0.1, 0.12, 0.16; 1), (0.1, 0.12, 0.14 ;1)}
	C5	
C1	{(1, 1.32, 1.5; 1), (1.11, 1.32, 1.46 ;1)}	
C2	{(0.17, 0.26, 0.61; 1), (0.19, 0.26, 0.46 ;1)}	
C3	{(0.12, 0.17, 0.25; 1), (0.13, 0.17, 0.22 ;1)}	
C4	{(5.82, 7.84, 9.85; 1), (6.32, 7.84, 9.35 ;1)}	
C5	{(1, 1, 1; 1), (1, 1, 1 ;1)}	

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Table 8
Transformation to TT2F pairwise comparison matrices for the alternatives

		A1	**A2**
C1	**A1**	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(4.29, 6.33, 8.35; 1), (4.8, 6.33, 7.84 ;1)}
	A2	{(0.12, 0.16, 0.23; 1), (0.13, 0.16, 0.21 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}
C2	**A1**	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(1.64, 3.78, 5.82; 1), (2.2, 3.78, 5.31 ;1)}
	A2	{(0.17, 0.26, 0.61; 1), (0.19, 0.26, 0.46 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}
C3	**A1**	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(0.37, 0.47, 0.72; 1), (0.39, 0.47, 0.62 ;1)}
	A2	{(1.39, 2.13, 2.68; 1), (1.61, 2.13, 2.55 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}
C4	**A1**	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(1.83, 3.97, 6.02; 1), (2.39, 3.97, 5.51 ;1)}
	A2	{(0.17, 0.25, 0.55; 1), (0.18, 0.25, 0.42 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}
C5	**A1**	{(1, 1, 1; 1), (1, 1, 1 ;1)}	{(1.32, 2.45, 3.36; 1), (1.64, 2.45, 3.14 ;1)}
	A2	{(0.3, 0.41, 0.76; 1), (0.32, 0.41, 0.61 ;1)}	{(1, 1, 1; 1), (1, 1, 1 ;1)}

Step 6: The geometric means of each row for alternatives are calculated and given in Table 9 and Table 10, respectively.

Table 9
Geometric means of TT2F sets for the main criteria the main criteria and

C1	{(0.92, 1.38, 1.87; 1), (1.05, 1.38, 1.73 ;1)}
C2	{(0.31, 0.41, 0.61; 1), (0.33, 0.41, 0.54 ;1)}
C3	{(0.26, 0.31, 0.43; 1), (0.27, 0.31, 0.39 ;1)}
C4	{(3.45, 4.68, 5.78; 1), (3.78, 4.68, 5.52 ;1)}
C5	{(0.84, 1.15, 1.49; 1), (0.93, 1.15, 1.39 ;1)}

Table 10
Geometric means of TT2F sets for the alternatives

C1	**A1**	{(2.07, 2.52, 2.89; 1), (2.19, 2.52, 2.8 ;1)}
	A2	{(0.35, 0.4, 0.48; 1), (0.36, 0.4, 0.46 ;1)}
C2	**A1**	{(1.28, 1.94, 2.41; 1), (1.48, 1.94, 2.3 ;1)}
	A2	{(0.41, 0.51, 0.78; 1), (0.43, 0.51, 0.67 ;1)}
C3	**A1**	{(0.61, 0.68, 0.85; 1), (0.63, 0.68, 0.79 ;1)}
	A2	{(1.18, 1.46, 1.64; 1), (1.27, 1.46, 1.6 ;1)}
C4	**A1**	{(1.35, 1.99, 2.45; 1), (1.55, 1.99, 2.35 ;1)}
	A2	{(0.41, 0.5, 0.74; 1), (0.43, 0.5, 0.65 ;1)}
C5	**A1**	{(1.15, 1.57, 1.83; 1), (1.28, 1.57, 1.77 ;1)}
	A2	{(0.55, 0.64, 0.87; 1), (0.56, 0.64, 0.78 ;1)}

Tables 11 and 12 give the normalized type-2 fuzzy weights of each criterion and alternative, respectively.

Table 11
Normalized type-2 fuzzy weights of the main criteria with respect to goal

C1	{(0.09, 0.17, 0.32; 1), (0.11, 0.17, 0.27 ;1)}
C2	{(0.03, 0.05, 0.11; 1), (0.03, 0.05, 0.08 ;1)}
C3	{(0.03, 0.04, 0.07; 1), (0.03, 0.04, 0.06 ;1)}
C4	{(0.34, 0.59, 1; 1), (0.4, 0.59, 0.87 ;1)}
C5	{(0.08, 0.15, 0.26; 1), (0.1, 0.15, 0.22 ;1)}

Table 12
Normalized type-2 fuzzy weights of the alternatives with respect to criteria

C1	**A1**	{(0.14, 0.21, 0.31; 1), (0.15, 0.21, 0.28 ;1)}
	A2	{(0.02, 0.03, 0.05; 1), (0.03, 0.03, 0.04 ;1)}
C2	**A1**	{(0.09, 0.16, 0.26; 1), (0.1, 0.16, 0.23 ;1)}
	A2	{(0.03, 0.04, 0.08; 1), (0.03, 0.04, 0.07 ;1)}
C3	**A1**	{(0.04, 0.06, 0.09; 1), (0.04, 0.06, 0.08 ;1)}
	A2	{(0.08, 0.12, 0.18; 1), (0.09, 0.12, 0.16 ;1)}
C4	**A1**	{(0.09, 0.16, 0.26; 1), (0.11, 0.16, 0.23 ;1)}
	A2	{(0.03, 0.04, 0.08; 1), (0.03, 0.04, 0.06 ;1)}
C5	**A1**	{(0.08, 0.13, 0.2; 1), (0.09, 0.13, 0.17 ;1)}
	A2	{(0.04, 0.05, 0.09; 1), (0.04, 0.05, 0.08 ;1)}

Step 7: The fuzzy performance scores of alternatives with respect to the main criteria are calculated using Equation 21 and given in Table 13.

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Table 13
Fuzzy performance scores of the alternatives

A1	{(0.05, 0.16, 0.45; 1), (0.07, 0.16, 0.34 ;1)}
A2	{(0.02, 0.04, 0.14; 1), (0.02, 0.04, 0.1 ;1)}

In Figure 6, the TT2F performance scores of the alternatives are illustrated.

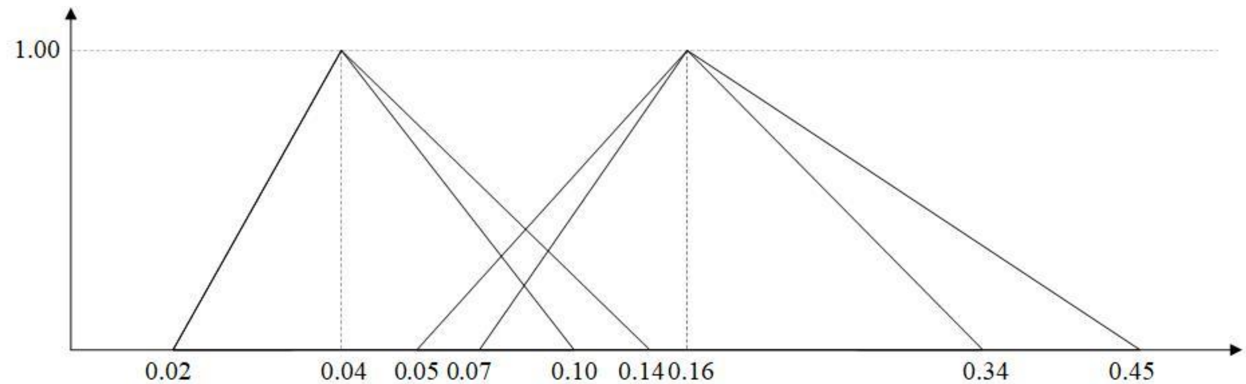

Figure 6 Type-2 fuzzy performance scores

Step 8: Applying the defuzzification formula in Equation 22, the following results are obtained:

$$\mathrm{DTtrT}(A_1) = 0.21$$
$$\mathrm{DTtrT}(A_2) = 0.06$$

Based on these results, Alternative A_1 is by far superior to Alternative A_2.

4.2 Validity and sensitivity

To check the validity of the proposed methodology, the classical AHP is applied. Table 14 shows the crisp numerical evaluations which are the mid-points in the linguistic scale for each expert.

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Table 14
Crisp numerical evaluations for criteria by three experts

Expert 1						Expert 2						Expert 3					
	C1	**C2**	**C3**	**C4**	**C5**		**C1**	**C2**	**C3**	**C4**	**C5**		**C1**	**C2**	**C3**	**C4**	**C5**
C1	1	5	7	0.3	3	**C1**	1	3	5	0.2	1	**C1**	1	5	3	0.1	1
C2	0.2	1	3	0.1	0.3	**C2**	0.3	1	1	0.1	0.2	**C2**	0.1	1	1	0.2	0.3
C3	0.1	0.3	1	0.1	0.1	**C3**	0.2	1	1	0.1	0.2	**C3**	0.3	1	1	0.1	0.1
C4	3	7	9	1	7	**C4**	5	9	9	1	9	**C4**	7	5	7	1	7
C5	0.3	3	7	0.1	1	**C5**	1	5	5	0.1	1	**C5**	1	3	7	0.1	1
CR = 0.007						CR = 0.049						CR = 0.074					

Table 15 presents the crisp numerical evaluations for alternatives by three experts.

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Table 15
Crisp numerical evaluations for alternatives by three experts

Expert 1

C1

	A1	A2
A1	1.00	7.00
A2	0.14	1.00

C2

	A1	A2
A1	1.00	3.00
A2	0.33	1.00

C3

	A1	A2
A1	1.00	0.33
A2	3.00	1.00

C4

	A1	A2
A1	1.00	5.00
A2	0.20	1.00

C5

	A1	A2
A1	1.00	5.00
A2	0.20	1.00

Expert 2

C1

	A1	A2
A1	1.00	7.00
A2	0.14	1.00

C2

	A1	A2
A1	1.00	5.00
A2	0.20	1.00

C3

	A1	A2
A1	1.00	1.00
A2	1.00	1.00

C4

	A1	A2
A1	1.00	3.00
A2	0.33	1.00

C5

	A1	A2
A1	1.00	3.00
A2	0.33	1.00

Expert 3

C1

	A1	A2
A1	1.00	5.00
A2	0.20	1.00

C2

	A1	A2
A1	1.00	3.00
A2	0.33	1.00

C3

	A1	A2
A1	1.00	0.20
A2	5.00	1.00

C4

	A1	A2
A1	1.00	5.00
A2	0.20	1.00

C5

	A1	A2
A1	1.00	1.00
A2	1.00	1.00

The aggregated pairwise comparison matrix is shown in Table 16.

Table 16
Aggregated pairwise comparison matrix

with respect to the goal	C1	C2	C3	C4	C5
C1	1.00	4.22	4.72	0.21	1.44
C2	0.24	1.00	1.44	0.15	0.28
C3	0.21	0.69	1.00	0.11	0.16
C4	4.72	6.80	9.00	1.00	7.61
C5	0.69	3.56	6.26	0.13	1.00

From the matrix in Table 17, the weights are obtained as $w_{C1} = 0.17$, $w_{C2} = 0.05$, $w_{C3} = 0.04$, $w_{C4} = 0.58$, $w_{C5} = 0.16$.

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Table 17 gives the scores of alternatives with respect to the criteria together with their weights.

Table 17
Alternative scores and criteria weights

	C1	C2	C3	C4	C5
Weights	0.17	0.05	0.04	0.58	0.16
A1	0.86	0.79	0.32	0.80	0.71
A2	0.14	0.21	0.68	0.20	0.29

From Table 17, the overall scores of alternatives are obtained as $A_1 = 0.8$, $A_2 = 0.2$. This shows that crisp and fuzzy approaches yield the same ranking result. However, they may produce different ranking results when asymmetric fuzzy numbers are used with large supports in pairwise comparison matrices.

In order to check the robustness of the given decisions one-at-a-time, a sensitivity analysis is applied in the following. In this sensitivity analysis, the weight of each criterion between 0-1 is changed and the ranking of the alternatives is observed. When the weight of a criterion is increased or decreased, the weights of other criteria are allocated proportional to their initial weights. The sum of the weights equals 1 in each case. The sensitivity analysis results are given in Figure 7.

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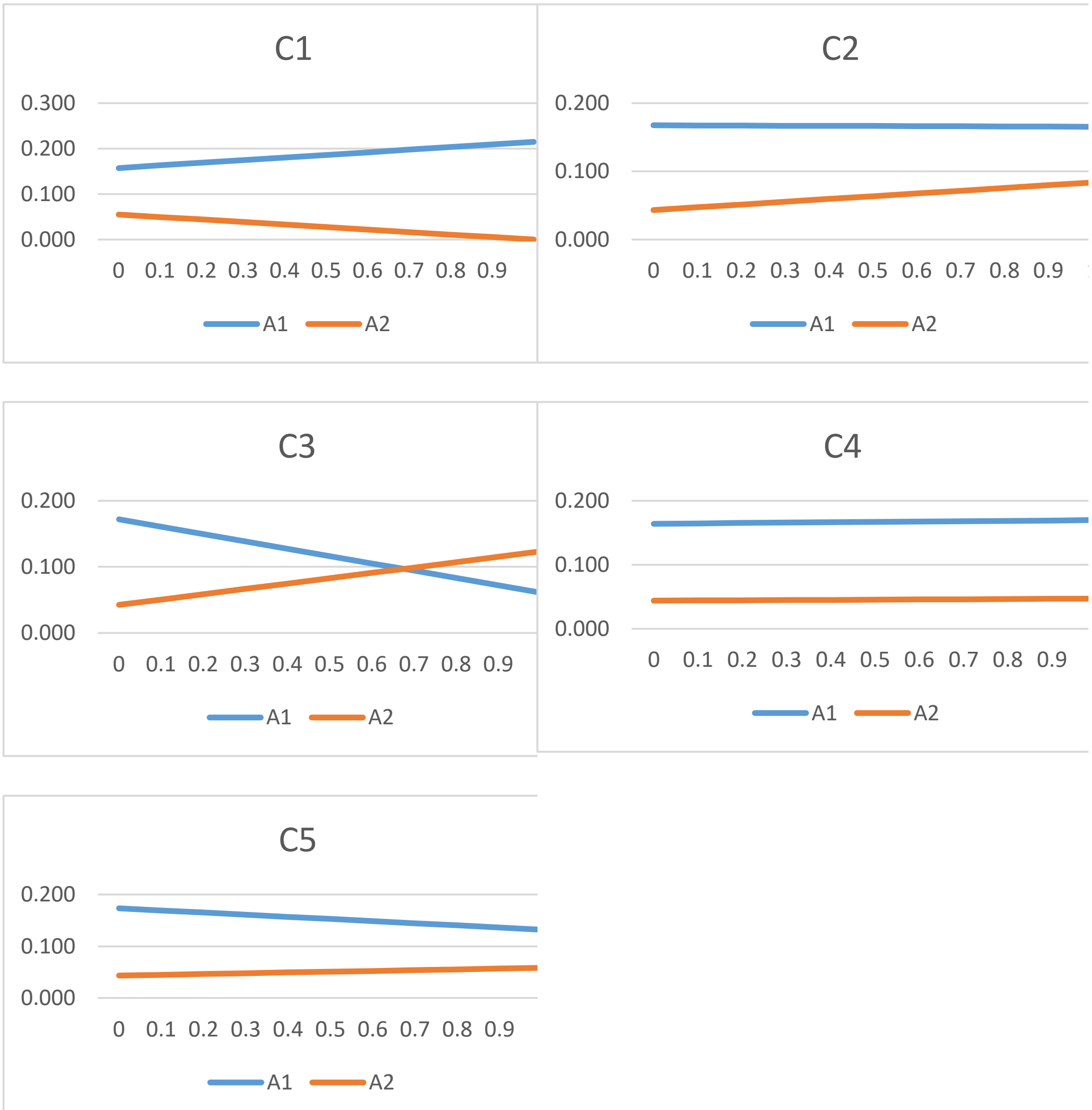

Figure 7 Sensitivity analysis results

The selection of A1 is a robust decision since in almost all sensitivity analyses A1 is superior to A2. For the changes in the weights of criteria cost (C1), maintenance frequency (C2), production capacity (C4) and technological upgrade (C5), A1 is never overtaken by A2. However, for changes in the weights of the criterion corrosion resistance (C3), A2 is selected when its weight is greater than approximately 0.7. Unless the criterion corrosion resistance (C3) is highly important, A1 is certainly better than A2.

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5. Conclusion

AHP is the most used multi-criteria decision making method all over the world. It has been extended to several fuzzy versions such as hesitant fuzzy AHP and intuitionistic fuzzy AHP. We have proposed a fuzzy AHP method which is originally formulated by intuitionistic fuzzy numbers and then converted to type-2 fuzzy AHP. Although modeling with intuitionistic fuzzy sets is popular and easier with respect to the other fuzzy extensions, the solution of intuitionistic fuzzy models is somewhat harder than type-2 fuzzy modeling. Hence, a transformation from intuitionistic fuzzy sets to type-2 fuzzy sets has been preferred. The pairwise comparative evaluations of multiple experts has also been considered and these evaluations through intuitionistic fuzzy aggregation operators have been aggregated. Another important advantage of our study is the usage of fuzzy sets along the whole process and the usage of defuzzification in the final step. The damless energy production technology selection problem proved the validity and consistency of our proposed methodology. The sensitivity analysis showed that the decision given by the methodology is quite robust. For further research, other possible transformations between extensions of fuzzy sets such as from Pythagorean fuzzy sets to type-2 fuzzy sets are suggested. In addition, interval valued intuitionistic fuzzy numbers or trapezoidal intuitionistic fuzzy numbers can be used instead of TIFNs in the proposed methodology.

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