HOW TO WRITE A CONTRACT WITH THE AHP

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ABSTRACT

In this paper, we show how the Analytic Hierarchy Process could be used to develop a legal contract in the process of a negotiation. We illustrate the process with a well-known case used routinely in negotiation courses to illustrate that the AHP is particularly well suited for this type of application where most of the dimensions and criteria are intangibles, and the scales used to measure the gains and costs of parties involved in the negotiation do not always exist.

Keywords: Negotiation; gain and loss ratios; value claim; value creation

1. Introduction

The dictionary definition of "contract" is "a binding agreement between two or more persons or parties" or "a document describing the terms of a contract." This implies that a contract has multiple dimensions and the parties must agree on each of the dimensions. For example, in the case of a recruiter trying to hire a candidate for a position in a company, the dimensions could be the signing bonus, salary, job assignment, company car, starting date, number of vacation days, percentage of moving expenses covered, the type of insurance coverage offered, and so on. Each dimension has a different impact on each of the parties.

There are two types of outcome at work when two parties negotiate: Value claim, and Value creation (see Figure 1).

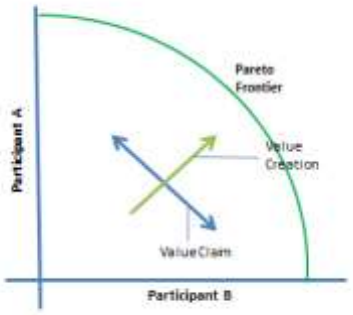

Figure 1. Value Claims, Value Creation and the Pareto Frontier

Value claim occurs when one party can capture value from the other party during the negotiation process. This is most prevalent among those dimensions of the negotiation that are distributive (i.e., what one party gains, the other party experiences as a comparable loss). However, it can also manifest itself for integrative elements (i.e., when multiple factors are negotiated – some of which are more important to one of the parties, and some of which are more important to the other party). However, for both integrative and compatible dimensions (i.e., factors where the same element is perceived as a gain for both parties), there are also opportunities for exchange that leads to value creation. Thus, value creation takes place when both parties are made better off during the negotiation. When value creation occurs, the parties move closer towards the Pareto frontier – the point at which neither party can be made better off without the counterparty being made worse off.

2. A simple example

We mentioned above that a contract has multiple dimensions and the parties must agree on each of the dimensions for the contract to be accepted by both parties. Thus, for a negotiation to arrive at a mutually agreed contract it needs to consider the gains and losses of the parties in each of the dimensions. For example, a recruiter is negotiating with a prospective employee for a position. They need to agree on the conditions of employment. The negotiation involves agreement on several dimensions. Each dimension can be considered a benefit or a cost. Table 1 shows an example of dimensions of a negotiation and their type.

Table 1
Dimensions and their type

Dimensions	Type
SIGNING BONUS (SB)	Benefit
SALARY (S)	Cost
JOB ASSIGNMENT (JA)	Cost
COMPANY CAR (CC)	Benefit
STARTING DATE (SD)	Benefit
VACATION DAYS (VD)	Benefit
MOVING EXPENSES REIMB (MER)	Benefit
INSURANCE COVERAGE (IC)	Benefit

In addition, within each type, the dimensions are not equally important. Table 2 shows the importance of the dimensions from both, the recruiter's and the employee's perspective.

Table 2
Recruiter/Employee Priorities for Benefits and Costs

	Priorities	
Benefits	Recruiter	Employee
SIGNING BONUS (SB)	0.270	0.270
COMPANY CAR (CC)	0.081	0.081
STARTING DATE (SD)	0.108	0.270
VACATION DAYS (VD)	0.270	0.108
MOVING EXPENSES (MER)	0.054	0.216
INSURANCE COVERAGE (IC)	0.216	0.054
	Priorities	
Costs	Recruiter	Employee
SALARY (S)	0.75	0.75
JOB ASSIGNMENT (JA)	0.25	0.25

In many real-life contract negotiations, neither the dimensions of the contract nor the intensity scales may be known. To make tradeoffs we need to identify the dimensions and the intensity scales. Consider the dimensions of the recruiter-candidate example with scales as given in Table 3.

Table 3
Intensities and the benefits/costs accrued by the recruiter and the candidate

	INTENSITY	RECRUITER	CANDIDATE
SIGNING BONUS (SB)	**10%**	0	4000
	8%	1000	3000
	6%	2000	2000
	4%	3000	1000
	2%	4000	0
SALARY (S)	**$ 60,000.00**	-6000	0
	$ 58,000.00	-4500	-1500
	$ 56,000.00	-3000	-3000
	$ 54,000.00	-1500	-4500
	$ 52,000.00	0	-6000
JOB ASSIGNMENT (JA)	**Division A**	0	0
	Division B	-600	-600
	Division C	-1200	-1200
	Division D	-1800	-1800
	Division E	-2400	-2400
COMPANY CAR (CC)	**LUX EX2**	1200	1200
	MOD 250	900	900
	RAND XTR	600	600
	DE PAS 450	300	300
	PALO LSR	0	0
STARTING DATE (SD)	**1-Jun**	1600	0
	15-Jun	1200	1000
	1-Jul	800	2000
	15-Jul	400	3000
	1-Aug	0	4000
VACATION DAYS (VD)	**30 days**	0	1600
	25 days	1000	1200
	20 days	2000	800
	15 days	3000	400
	10 days	4000	0
MOVING EXPENSES	**100%**	0	3200
REIMBURSEMENT (MER)	**90%**	200	2400
	80%	400	1600
	70%	600	800
	60%	800	0
INSURANCE COVERAGE (IC)	**Allen Insurance**	0	800
	ABC Insurance	800	600
	Good Health Insurance	1600	400
	Best Insurance Co.	2400	200
	Insure Alba	3200	0

These scales are not usually known and need to be constructed using relative measurement. For the moment, consider the intensity scales in Table 3 expressed in ideal terms (i.e., the elements are divided by the largest value) in Table 4. In this example, the scale values are all equispaced, i.e., they form a linear scale. However, in practice these values would be obtained through prioritization and they do not need to be linear.The negotiation process consists in finding out what value each dimension should take for the recruiter and the candidate so that the total amount they get (benefit/cost ratio) is maximized and satisfies the constraint that neither party gets more than the other, i.e., the contract is fair and equitable (Fisher & Ury, 1981).

Table 4
Priorities of dimensions and relative scales for the recruiter – candidate case

RECRUITER PRIORITIES		CANDIDATE PRIORITIES						
Benefits' Priorities	Costs' Priorities	Benefits' Priorities	Costs' Priorities					
					RECRUITER		CANDIDATE	
0.2702701		0.27027		SIGNING BONUS	Scale	Ideal Scale	Scale	Ideal Scale
				10%	0.01	0.00	4,000	1.00
				8%	1,000	0.25	3,000	0.75
				6%	2,000	0.50	2,000	0.50
				4%	3,000	0.75	1000	0.25
				2%	4,000	1.00	0.01	0.00
	0.714285918		0.714285918	SALARY	**10000.01**		**10000.01**	
				$60,000	-6,000	1	0.01	0.00
				$58,000	-4,500	0.75	-1,500	0.25
				$56,000	-3,000	0.50	-3,000	0.50
				$54,000	-1,500	0.25	-4,500	0.75
				$52,000	0.01	0.00	-6,000	1.00
	0.285714082		0.285714082	JOB ASSIGNMENT	**-14999.99**		**-14999.99**	
				Division A	0.01	0.00	0.01	0.00
				Division B	-600	0.25	-600	0.25
				Division C	-1,200	0.50	-1,200	0.50
				Division D	-1,800	0.75	-1,800	0.75
				Division E	-2,400	1.00	-2,400	1.00
0.0810812		0.081081		COMPANY CAR	**-5999.99**		**-5999.99**	
				LUX EX2	1200	1.00	1200	1.00
				MOD 250	900	0.75	900	0.75
				RAND XTR	600	0.50	600	0.50
				DE PAS 450	300	0.25	300	0.25
				PALO LSR	0.01	0.00	0.01	0.00
0.1081082		0.27027		STARTING DATE	**3000.01**		**3000.01**	
				1-Jun	1,600	1.00	0.01	0.00
				15-Jun	1,200	0.75	1,000	0.25
				1-Jul	800	0.50	2,000	0.50
				15-Jul	400	0.25	3,000	0.75
				1-Aug	0.01	0.00	4,000	1.00
0.2702701		0.108108		VACATION DAYS	**4000.01**		**10000.01**	
				30 days	0.01	0.00	1,600	1.00
				25 days	1,000	0.25	1,200	0.75
				20 days	2,000	0.50	800	0.50
				15 days	3,000	0.75	400	0.25
				10 days	4,000	1.00	0.01	0.00
				MOVING EXPENSES	**10000.01**		**4000.01**	
0.0540542		0.216216		REIMBURSEMENT				
				100%	0.01	0.00	3,200	1.00
				90%	200	0.25	2,400	0.75
				80%	400	0.50	1,600	0.50
				70%	600	0.75	800	0.25
				60%	800	1.00	0.01	0.00
0.2162161		0.054054		INSURANCE COVERAGE	**2000.01**		**8000.01**	
				Allen Insurance	0.01	0.00	800	1.00
				ABC Insurance	800	0.25	600	0.75
				Good Health Insurance	1,600	0.50	400	0.50
				Best Insurance Co.	2,400	0.75	200	0.25
				Insure Alba	3,200	1.00	0.01	0.00
					8000.01		**2000.01**	

3. The trading model

To find the solution of this problem we model it with integer programming. A solution is represented by an 8-by-5 matrix (x_{ij}) of 0's and 1's. Each row corresponds to a dimension and each column corresponds to an intensity of the scale corresponding to that dimension (see Table 5).

Table 5
A solution with the Benefit/Cost Ratios

A Solution			Intensities			Recruiter		Candidate	
	1	2	3	4	5	Benefits	Costs	Benefits	Costs
SB	0	0	1	0	0	0.135135	0	0.135135	0
S	0	1	0	0	0	0	0.535714	0	0.178571
JA	0	1	0	0	0	0	0.071429	0	0.071429
CC	1	0	0	0	0	0.081081	0	0.081081	0
SD	0	0	1	0	0	0.054054	0	0.135135	0
VD	0	1	0	0	0	0.067568	0	0.081081	0
MER	0	0	1	0	0	0.027027	0	0.108108	0
IC	0	0	1	0	0	0.108108	0	0.027027	0
					B/C Ratio	0.7790		2.2703	

The benefits and costs are obtained from Table 4. For example, the benefit for the recruiter of selecting a 6% Signing Bonus (0.135135) is obtained by multiplying the weight of Signing Bonus (0.27027) by the scale intensity 3 (0.50).

Thus, $x_{ij} = 1$ if the i[th] dimensions takes the j[th] intensity value. Let b_{ij}^R (b_{ij}^C) and c_{ij}^R (c_{ij}^C) the benefit and cost corresponding to the j[th] intensity of the i[th] dimension for the recruiter (candidate).

The benefits/costs ratios of the recruiter and the candidate are given by

$$r_R(x) = \frac{\text{benefits}}{\text{costs}} = \frac{\sum_i w_i^R \sum_j x_{ij} b_{ij}^R}{\sum_i v_i^R \sum_j x_{ij} c_{ij}^R} \text{ and } r_C(x) = \frac{\text{benefits}}{\text{costs}} = \frac{\sum_i w_i^C \sum_j x_{ij} b_{ij}^C}{\sum_i v_i^C \sum_j x_{ij} c_{ij}^C},$$

respectively.

The objective is to find a solution x^* such that the parties gain as much as possible,

$$r_R(x^*) = r_C(x^*) = \underset{x \in X_S}{Max}\{\text{Min}\{r_A(x), r_B(x)\}\},$$

where X_S is the solution space defined as the set of matrices (x_{ij}) that satisfy the conditions $\sum_{j=1}^{5} x_{ij} = 1$, for all i, $x_{ij} = 0,1$, for all i and j, and the two parties gain the same, i.e., their ratios are equal.

A given solution has benefits/costs ratios that are different for the parties. For example, in Table 5 we give a solution. In this solution, the recruiter has a lower benefits/costs ratio than the candidate, so the recruiter will try to change to another solution where he will get a greater benefits/costs ratio. Table 6 shows the solution in matrix form.

Table 6
Optimal solution

Optimal Solution			Intensities			Recruiter		Candidate	
	1	2	3	4	5	Benefits	Costs	Benefits	Costs
SB	0	0	1	0	0	0.135135	0	0.135135	0
S	0		1	0	0	0	0.357143	0	0.357143
JA	1	0	0	0	0	0	0	0	0
CC	1	0	0	0	0	0.081081	0	0.081081	0
SD	0	0	0	0	1	0	0	0.27027	0
VD	0	0	0	0	1	0.27027	0	0	0
MER	1	0	0	0	0	0	0	0.216216	0
IC	0	0	0	0	1	0.216216	0	0	0
					B/C Ratio	1.9676		1.9676	

Translated into the original scale values of the dimensions we have Table 7. Note that now both the recruiter and the candidate gain the same.

Table 7
Terms of the contract

	SB	S	JA	CC	SD	VD	MER	IC	Total
	6%	$56,000.00	Division A	LUX EX2	1-Aug	10 days	100%	Insure Alba	Points
Recruiter	2000	-3000	0	1200	0	4000	0	3200	7400
Candidate	2000	-3000	0	1200	4000	0	3200	0	7400

Obviously, the scales within each dimension do not have to be linear. For example, if the recruiter and the candidate have relative intensities as given in Table 8, the solution (Table 9) would not be the same as the one in Table 7. The solutions in Table 9 are within 3.125% of each other. No other closer solutions exist.

4. General contract model

In many contract negotiations, the parties do not always act in good faith or share information with the other party. In this case, one should also consider the perceptions of the parties about the benefits and costs of the tradeoffs. For example, in a merger transaction, the buyer (A) and the seller (B) may not always agree as to the terms of the merger, and hence the transaction may fail. The steps to make tradeoffs in this more general situation are as follows:

1. Identify the dimensions of the problem
2. Identify the tradeoffs of each party within the dimensions
3. Identify the benefits accrued by a party from the other party's tradeoffs
4. Identify the costs incurred by a party from its own tradeoffs
5. Identify the perceived benefits that the other party received from your tradeoffs
6. Identify the perceived costs incurred by the other party from their tradeoffs
7. Find out what tradeoff each party must make to maximize the total minimum gain they obtain, ensuring that the gains of a party are as close as possible to the other party gains. This is what makes the final contract fair, equitable and balanced.

Table 8
Intensities with Non-Linear Relative Scales

			Relative Scales	
Dimensions	RECRUITER	CANDIDATE	RECRUITER	CANDIDATE
SIGNING BONUS				
10%	0.01	4,000	1E-06	1
8%	1,000	3,000	0.1	0.75
6%	2,000	2,000	0.5	0.5
4%	3,000	1000	0.9	0.1
2%	4,000	0.01	1	1E-06
SALARY	**10000.01**	**10000.01**		
$60,000	-6,000	0.01	1	1E-06
$58,000	-4,500	-1,500	0.75	0.1
$56,000	-3,000	-3,000	0.5	0.5
$54,000	-1,500	-4,500	0.1	0.9
$52,000	0.01	-6,000	1E-06	1
JOB ASSIGNMENT	**-14999.99**	**-14999.99**		
Division A	0.01	0.01	1E-06	1E-06
Division B	-600	-600	0.1	0.1
Division C	-1,200	-1,200	0.5	0.5
Division D	-1,800	-1,800	0.9	0.9
Division E	-2,400	-2,400	1	1
COMPANY CAR	**-5999.99**	**-5999.99**		
LUX EX2	1200	1200	1	1
MOD 250	900	900	0.75	0.75
RAND XTR	600	600	0.5	0.5
DE PAS 450	300	300	0.1	0.1
PALO LSR	0.01	0.01	1E-06	1E-06
STARTING DATE	**3000.01**	**3000.01**		
1-Jun	1,600	0.01	1	1E-06
15-Jun	1,200	1,000	0.75	0.1
1-Jul	800	2,000	0.5	0.5
15-Jul	400	3,000	0.1	0.9
1-Aug	0.01	4,000	1E-06	1
VACATION DAYS	**4000.01**	**10000.01**		
30 days	0	1,600	0	1
25 days	1,000	1,200	0.1	0.75
20 days	2,000	800	0.5	0.5
15 days	3,000	400	0.9	0.1
10 days	4,000	0.01	1	1E-06
MOVING EXPENSES	**10000**	**4000.01**		
REIMBURSEMENT				
100%	0.01	3,200	1E-06	1
90%	200	2,400	0.1	0.75
80%	400	1,600	0.5	0.5
70%	600	800	0.9	0.1
60%	800	0.01	1	1E-06
INSURANCE COVERAGE	**2000.01**	**8000.01**		
Allen Insurance	0.01	800	1E-06	1
ABC Insurance	800	600	0.1	0.75
Good Health Insurance	1,600	400	0.5	0.5
Best Insurance Co.	2,400	200	0.9	0.1
Insure Alba	3,200	0.01	1	1E-06
	8000.01	**2000.01**		

Table 9
Terms of the contract for the Non-Linear Intensity case

	SB	S	JA	CC	SD	VD	MER	IC	Total
	10%	$ 56,000.00	Division A	LUX EX2	1-Jul	10 days	90%	Insure Alba	Points
Recruiter	0	-3000	0	1200	800	4000	200	3200	6400
Candidate	4000	-3000	0	1200	2000	0	2400	0	6600

The mathematical model that helps identify the proper contract is given below.

Let $X_k(x)$ the scale of the *k*th dimension. The parties will negotiate on the value of that scale according to their preferences. The realized value of the scale is determined by the benefit, the cost, the perceived benefits and the perceived cost that the value has for each party.

Let $B_i(x_k)$ be the benefits accrued by party *i* from the other party tradeoffs in dimension *k*. Let $C_i(x_k)$ be the costs incurred by party *i* from its own tradeoffs in dimension *k*. Let $PB_i(x_k)$ be the benefits party *i* perceives the other party receives from its tradeoffs in dimension *k*, and let $PC_i(x_k)$ be the costs the other party perceives that party *i* incurs from its tradeoffs in dimension *k*. Thus, for a given dimension *k*, the gain of party *i* is given by the benefits it accrues from the tradeoffs of the other party in that dimension times the costs it perceives the other party incurs in that dimension, i.e., $B_i(x_k)PC_i(x_k)$. Similarly, the loss in each dimension *k* is given by $C_i(x_k)PB_i(x_k)$. Thus, the gain to loss ratio for a party for a given dimension *k* is given by:

$$\frac{B_i(x_k)PC_i(x_k)}{C_i(x_k)PB_i(x_k)}$$

and the total gain-to-loss ratio for a party is given by

$$r_i \equiv \sum_{all\ k} \frac{B_i(x_k)PC_i(x_k)}{C_i(x_k)PB_i(x_k)}.$$

This formulation was suggested by Saaty (1988) to handle retributive conflicts.

Let $x_k(s)$ be a binary variable, where $x_k(s)=1$ if the parties agree on selecting the intensity *s* of the *k*th dimension as the best decision for both. The problem now consists in finding values of *s* for each dimension that maximizes the smallest gain-to-loss ratio of both parties, i.e.,

$$\underset{s}{Max}\left\{Min\left\{r_i(s) \equiv \sum_{all\ k} \frac{B_i[x_k(s)]PC_i[x_k(s)]}{C_i[x_k(s)]PB_i[x_k(s)]}, r_j(s) \equiv \frac{B_j[x_k(s)]PC_j[x_k(s)]}{C_j[x_k(s)]PB_j[x_k(s)]}\right\}\right\}$$

Subject to $\sum_s x_k(s) = 1$ and $\left|1 - \frac{r_i(s)}{r_j(s)}\right| \leq \varepsilon$, where ε is the tolerance that measures how far the two parties are in terms of their total gain-to-loss ratio.

5. Conclusions

In this paper, we assume that a multidimensional contract between two parties is an agreement among the parties about the values of the dimensions that gives each party a fair and equitable gain. The values of the dimensions can be estimated through relative measurement when no scales exist. This approach was first used by Saaty (1988) to address the conflict in South Africa.

The main difference between the approach in this paper and that used in the analysis of the Palestinian-Israeli conflict is that in the later the tradeoffs were analyzed in pairs (Saaty & Zoffer, 2011; 2013). In the case of a contract, the scales in which the dimensions are measured makes it impossible to analyze all possible pairs of tradeoffs. For example, in the simple case given above, the number of tradeoffs is $2^{8\times4} = 4,294,967,296$. Here we used a non-linear integer optimization formulation to derive the solutions shown using a genetic algorithm.

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