PARALLEL BETWEEN SPACE GEOMETRY & DECISION MAKING SPACE

(Modelling the Growing Complexity of Measurement)

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Introduction:

The aim of this essay is to show some interesting analogies between space geometry and what we call "decision making (DM) space", as well as, proposing a possible way of measuring how close (or far) two or more priority vectors are. For this purpose, we suggest applying the compatibility index *G* as a new way to measure the distance of priority vectors in the DM space.

The aim for this essay is that it should be easy to read, (made in a kind of "divertimento" format), without complex formulae or hard math.

First of all, we posit that the set of priorities (i.e. P1, P2,…,Pn) obtained from an AHP/ANP synthesis process can be considered as a vector in an n-dimensional space and each of the priorities (e.g. Pn) constitutes a vector component in the specific n-dimension. The n-dimensional space in which the priority vector can be plotted is what we define as "decision-making (DM) space" and will constitute the focus of our discussion here.

The main idea proposed here is to establish a parallel between the classic Cartesian vision of space[1] and its way of measurement through the Pythagorean Theorem, with the decision making space using the compatibility index *G* as a sort of "distance measure". By the way, compatibility is intended as the degree of closeness between 2 decision vectors (like the distance between 2 points in a Cartesian system). The essay was written using a Physics analogy in three incremental stages of complexity, in terms of measurement, as follows: first, looking at the DM space from the classic Cartesian point of reference (homogenous flat space); second, looking at the decision-making space from a non-Cartesian (homogeneous curved space)[2] to consider a situation in which different dimensions may have different weights; and finally, from the point of view of a flexible bended space framework (see Figure 4) to incorporate the decision-making feedback.

[1] A Cartesian coordinate system is a coordinate system that specifies each point uniquely in a plane by a set of 3 numerical coordinates, which are the signed distances from the point to three fixed perpendicular directed lines, measured in the same unit of length. Each reference line is called a *coordinate axis* or just *axis* of the system, and the point where they meet is its *origin*, usually at ordered pair (0, 0,0). The coordinates can also be defined as the positions of the perpendicular projections of the point onto the three axes, expressed as signed distances from the origin (see Figure 2).
[2] In this approach, coordinate hypersurfaces of different coordinates are also orthogonal, although curved (see Figure 3).

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The Three Stages of Complexity:

As previously indicated, we can establish three different stages of increasing complexity to explore the analogy between measurement in physical space (metric topology) and measurement in the decision making space (order topology[3]).

Before starting with the analogy, we need to establish three basic conditions for the DM space coordinate systems used in this essay:

First:

Every point (x, y, z) of a DM space, can be interpreted as a point with the coordinates of one possible decision vector for a given decision maker (DM), in other words the coordinates (x, y, z) may represent the field of potential priorities for a given DM (similar to the Faraday's interpretation of magnetic fields in physics).

Second:

Every point can be represented by a positive normalized vector. The set of coordinates of any point of the space has to add to one ($x_i+y_i+z_i=1$), no one coordinate may present negative values and all axes range from 0 to 1.

Third:

The decision making structure represents the space decision making which represents the way that the DM makes decisions. In other words, the decision structure synthesizes the rule of measurement in that specific decision making space (an AHP or ANP structure for instance).

In Figure 2, the coordinate system may represent the space of points of possible decisions for a given DM in a (0 - 1) range for every axe. Each component of the decision vector may correspond, for example, to the priority given to each criterion (assuming the decision constitutes prioritization of criteria) and the three axial components (x1, y1 and z1) corresponds to the prioritization given by the first decision maker while (x2, y2 and z2) corresponds to the criteria prioritization given by the second decision maker.

[3] Order topology is the topology used for measuring the intensity of preferences of the DMs.

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First Stage of Complexity: The Space of Non Weighted Axes (homogenous flat space)

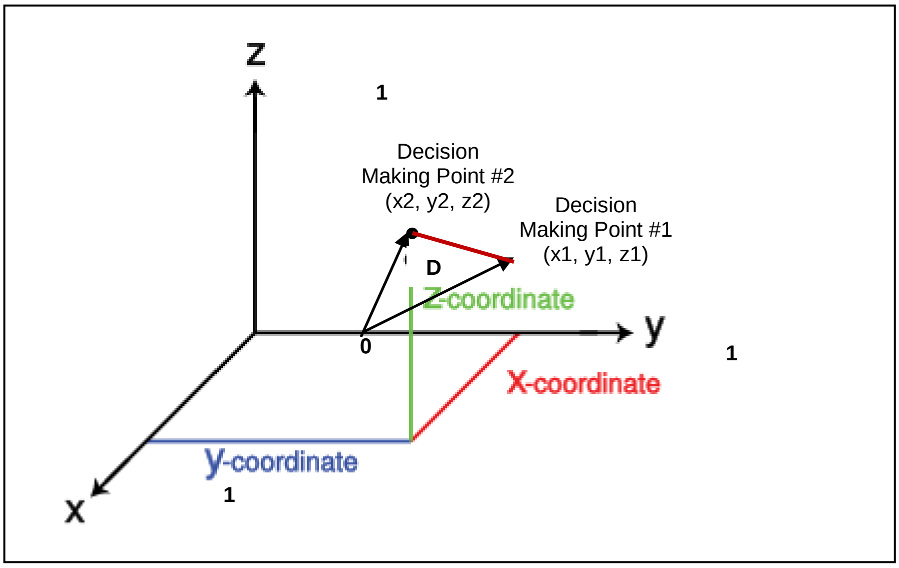

Figure 2. Coordinates system in a homogenous flat space *(Figure from Wikipedia)*

Figure 2, represents the first stage of the analogy between space geometry & geometry of the decision making space; its shows a 3D Cartesian coordinate system representing a homogenous or "flat" space (Euclidean space). In this stage, the concept of distance or closeness between points is the same that one has learned at school, and applies the classic formula of distance: $D(x,y,z)= \sqrt{}(\square x^2+\square y^2+\square z^2)$ *(Eq.1)*. Equation 1 represents the situation where every axe is equally important, meaning that one unit of axe X has the same impact (or influence) in the assessment of distance "D" than a unit in axe Y or axe Z. This stage for the decision making space is equivalent to the situation where every criterion has the same importance or weight ($w_x=w_y=w_z$=constant). Of course, in the decision making domain such a situation is not likely but it is useful as a reference case for a distance calculation between two different points in a homogenous space.

The Second Stage of Complexity: Weighting the Axes

The second stage of complexity corresponds to a space where the axes have weights, i.e. each axis may have a different importance from the others, this space can be represented as a bended or curved space (Fig.3). In this second stage, the initial weights are fixed, they cannot vary under any circumstance and may be represented as a bended space within a rigid frame.

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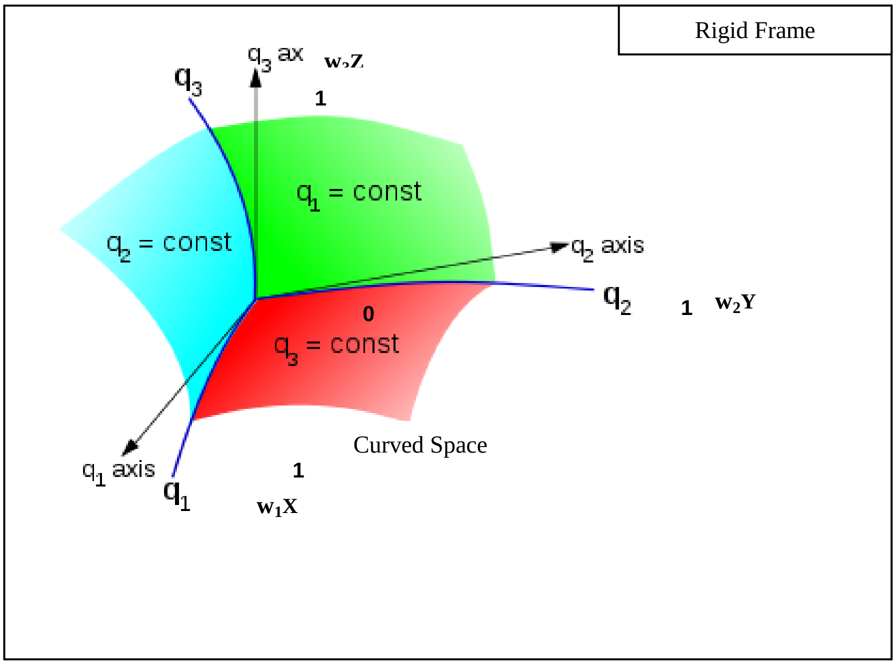

Figure 3. Coordinate System for curved space in a rigid frame. *(Figure from Creative Common License, Wikipedia)*

The 3 planes (q_1, q_2, q_3) are constant. These bended or curved surfaces cannot be altered by the presence or absence of other objects (alternatives for instance). This decision making geometry corresponds to the standard AHP decision making structure (representing the AHP rule of measure), where each axe that represents each terminal criteria has its own global weight that produces a deflection or a change in the slope of the surface, that weight cannot be changed (unless we change the structure). Of course, bigger weights represent bigger slopes. We realize that with weighed axes a variation in one axe (□*x* for instance), may produce different results for distant calculation than a variation in any other for distance calculations. The fact that the importance of a change in X (□*x*) may differ in importance to a change in Y or Z (□*y or* □*z*), even if the size of the change is the same, can represent a big difference for the closeness calculation index. The classic formula of distance calculation (Eq.1) will not be adequate anymore. We need to project the coordinates of the straight axes over the bended axes and renormalize the coordinates to correctly represent the new priority points P1 and P2 (as an initial approximation), which means to weight the initial coordinates by the importance of the axes. *(Most of the time reality in decision making environment is better represented within a heterogeneous space than a homogenous one).*

Continuing with the analogy with the space and structures built on AHP, we may say that in this stage once the structure is built, there is no feedback interaction between criteria or criteria with alternatives; the alternatives must follows the rule of measure established initially in the AHP structure. The space under AHP might be bended only once (at the beginning). However, this frame is still too rigid to represent the full picture due to this artificial restriction to the geometry over the decision making frame. Also, this frame shows that this geometry cannot be altered by adding or deleting alternatives. But, we know that this action (adding or deleting alternatives) may change the actual decision making geometry.

Observation: When modelling with AHP one has to be careful when adding or deleting elements (even alternatives), since one could be modelling another problem and creating another rule of measure. This is especially true in the relative measurement of AHP, where a change in the set of alternatives (adding or deleting) may produce a change in the priority values of the alternatives including its own rank. The alternatives do not follow all the rules of the rigid frame created in stage 2. Thus, if all the alternatives are settled at the same time rank reversal may happen, because the alternatives are connected between them in a relative measurement form. This connection is not fully captured by this rigid frame of decision making. But, if the alternatives are settled one at a time, we break down that connection (for good and for bad) and no rank reversal is produced. That is what we are doing when measuring alternatives with absolute measurement scales specially created for this mode (the absolute measurement mode).

The Third Stage of Complexity: Getting into a Flexible Frame (the feedback process)

The third stage of this analogy corresponds to an ANP model representing a weighted space in a flexible frame (see Fig.4):

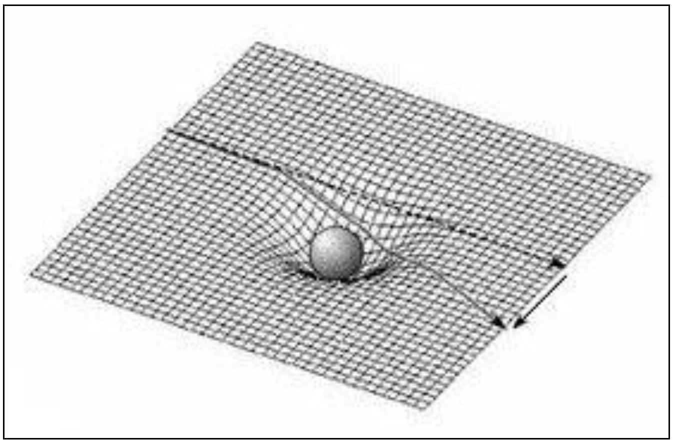

Figure 4. Coordinate System in a flexible bended space *(Figure from Wikipedia)*

This decision structure (the ANP model), represents a flexible frame of the creating space. In this stage, the alternative can be physically interpreted as a mass of a body that alters (curve) the space (see Fig.4). This represents the way the alternative tells the space structure (the ANP structure) how much the weights of the criteria have to change due to its presence, changing the curvature of the space where the alternatives belong and so altering its pattern behaviour (the initial line/path is altered/deflected to another position, see Fig.4). In this manner, we are creating an interaction network between the frame and the elements that belong to the frame that better recreate the system behaviour *(and that's why we need a flexible frame).*

We have to keep the ANP model interacting with the set of alternatives until reaching a steady point (the final supermatrix's eigenvector) They conform an interrelated unity that responds for the functionality of the whole system. This is what happens in a relativistic frame between mass and space. This is like the old Einstein quote: *"The space tells to the mass how to move and the mass tells to the space how to curve"*.
Now we may say, the ANP model (the space structure with its own rule of measurement) tells the alternatives how to behave and the alternatives tell the ANP

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model how to re-shape the space structure (re-building its own rule of measurement). In synthesis, the ANP represents a new flexible and interdependent rule of measurement, which is very close to what happens in the real decision making world.

A Final Observation of Measuring in the Growing Complexity:

The better way to measure closeness (distance) between points, in the second and third stage of complexity (the decision making space in weigthed environment), is using the general compatibility index *G*.
($G= \frac{1}{2}\sum_i(a+b)*Min(a,b)/Max(a,b)$, with: A, B n-dimensional normalized vectors).

Other conditions that are important to remember:

1. Each axes of this special Cartesian system, is bounded between 0 – 1.
2. The curvature of axes also belong to a 0 – 1 scale, where 0 represents no deflection or flat space (0° degree) and 1 represent maximum deflection (90° degree).
3. Each node (criterion) will produce a deflection proportional to its own weight.
4. The interaction between nodes and between nodes and alternatives should be seen as the degree of connectivity of the space geometry we are creating. The greater the connectivity, greater the complexity ("*rugosity")* of the space.
5. The high degree of connectivity of the system is the main reason that the classic Cartesian axes representation is not a good representation anymore.
6. Each point in the deflected mesh represents one possible or potential solution for the priority vector before reaching the equilibrium or steady point.
7. This final equilibrium point in the decision making space (using an ANP model) may have more than one steady solution. This behaviour has it's "mirror" in the gravitational strange attractor (a fractal attractor) where a final equilibrium point of a system may have more than one simple attractor. For example, the system Earth-Sun presents this kind of strange attractor points, called Henon attractor in honour of Michel Henon, an astronomer that first noticed a disturbing behaviour in a simple model of stars orbiting within a galaxy at the Observatory of Nice (France) in the 1960s. At first glance it was thought that this was some error in calculation, but later, this phenomenon was noticed in almost every orbit (stars and planets). The shape of Henon's orbits is not the classical ellipses that astronomers have used for centuries. If one planet orbits one star in an otherwise empty universe, the linear curves of Newton and Kepler would accurately model its trajectory. But, in the real universe gravitational attractions from other bodies make the planet´s orbit much less predictable. (Henon discovered that this sort of "chaotic behaviour" was an essential part of the dynamics of stellar and planet orbits). In the same analogous situation, if a DM has just one criterion of decision and nothing else (just one body mass), then the multiple possible paths to reach the equilibrium (as Graph Theory and Cesaro´s Sum shown) will collapse to just one path and the steady point will be reached immediately (just like a 100% consistent matrix of judgments). In this case, the equilibrium point will be easily represented trough a clean linear function.

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Finally, it is possible to show the whole parallelism in a simple short table as below:

More Idealistic	
Flat space	No weights (The DM space is built on an ordinal kind of scale)
Curved space in a rigid frame	AHP model. The DM space is built on a cardinal scale without including feedback, unable to change if any change happens within the frame.
Curved space in a flexible frame	ANP model. The DM space is built on a cardinal scale including feedback, enables changes in the frame itself.
Strange attractor that best represents the motion of the bodies	More complex ANP model (more connected) with more than one steady point.
More Realistic	

Figure 5. Parallel between space geometry and decision making space

It is also interesting to note that when working in a relativistic space the notion of distance change from the classic 3D formula $SQRT(\square x^2+\square y^2+\square z^2)$ to the more complex one $SQRT(\square x^2{}_x+\square y^2{}_y+\square z^2{}_z+(C\square ti)^2{}_i)$[4], makes clear that time is simply another expression of space (the fourth dimension if you like). Also, the importance of this fourth axe when relevant (within a relativistic space for instance) is far greater than the other three, distorting the surrounding space (just like a big weight will make it in a DM environment). So, if we consider that distortion as a result of the presence of a big weight (a star for instance), we can say that weights do matter when making measurements, whether we are talking of physical or a decision making environment.

Notes:
It has been mathematically proven that in a strong curved space one would be able to move without applying any external force or changing one's total mass (contradicting the classic Newton's law of action-reaction). This shows that space is something more complex than just a set of absolute (and independent) points in a Cartesian graph. Space might be something that one can interact with, moving oneself like laying in a flexible and touchable frame. This is similar to what happens when a decision maker interacts long term with an ANP model, seeing how his/her own values are transformed into priorities and these priorities (when seen it in a structured and explicit way) may eventually change his/her own initial values.

By the way, the movement in this curved space might be done just like swimming in a pool, where the water of the pool corresponds to this flexible and curved space, the more curved the space the easier it is to move in it by "swimming". Of course, in a nearly flat space, if we try to move ourselves using this swimming process it would take millions of years just to move our body a couple of centimeters[5].

[4] **"***i"=sqrt(-1), direction perpendicular to the axes x, y, z. C= speed of light.*
[5] *Scientific American*, August 2009

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