A PRACTICAL IMPLEMENTATION OF INTERVAL ARITHMETIC IN AHP

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ABSTRACT

One of the well-established methods that help the decision makers deal with multiple criteria is the Analytical Hierarchy Process (AHP) which utilizes a weighting approach. The process of decision making with multiple criteria is faster if all the weights of the factors related to a particular problem are clearly stated. However, if the weights of said factors are not well defined, or only their lower and upper weight limits are known, then the decision makers face considerable uncertainty because the standard AHP numerical procedure operates with deterministic values. As a result, the corresponding assessment preferences cannot be expressed in the form of a sequence of numerical values and implemented in the AHP evaluation. A practical approach is presented in this work to deal with the data uncertainty by implementing interval arithmetic in the AHP calculations so that the assessment preferences are presented in the form of interval numbers.

Keywords: AHP; interval arithmetic; interval numbers; decision making

1. Introduction

Decision making can be considered as a human brain process that occurs in almost every human activity. It can be categorized as one of several brain activities which are called cognitive processes which are known as brain activity associated with attention, memorization, language production and understanding, learning, and problem solving which includes decision making.

Decision making can be described as a process which is activated after having some initial information, and then analyzing this information using one's judgment in order to decide which option is the best from all the available options. The process of one's

[1] The author gratefully acknowledges the Directorate General of Higher Education, Ministry of Education, Republic of Indonesia, which provides the BPPLN scholarships including the support of the Stikubank University Semarang.

judgment typically uses several criteria for every available option. The weighting of each criterion for an option will cause each option to have a set of different weights. One can perform subsequent calculations on the basis of the weights that are available and obtain possible solutions. After further comparison, the best solution can be determined by applying some known criteria. The obtained result can be seen as the best solution from all available options.

The decision makers can make proper decisions quickly only if the factor weights are clearly stated. However, if the upper and lower limits of these weights are known instead, then the decision making process faces obvious difficulties. Human intuition can be used with relative success as even for a small number of options the resultant decisions could be unreliable.

The upper and lower limits of the factor weights can be interpreted as an interval number. By definition, an interval number is a set of real numbers in which the members of the set lie between two limiting numbers. The concept of an interval number is the cornerstone of interval arithmetic. By implementing interval arithmetic into the decision making process, especially in the AHP technique, the uncertainty that occurs is expected to be taken into consideration. This raises a few adjustments in the weight calculation of decision making scenarios.

The aim of this paper is to demonstrate the implementation of interval arithmetic in the decision making process, especially in the AHP technique. For that purpose, the structure of the paper is organized accordingly. The second section provides a literature review which covers recent developments in the theory of interval arithmetic in relation to the decision making process. The third section gives a description of the methodology for the implementation of interval arithmetic in the AHP technique and the obtained results are discussed in the last section.

2. Literature Review

The decision making methods should be used to solve complex problems when the number of existing criteria or options are beyond the natural human processing ability. At present, there are numerous published studies presenting different aspects of decision making methods used for dealing with a wide range of such complex problems. Some methods rely on giving weights for all criteria and options and subsequently processing the weights with a specific technique. One of the well-established techniques for decision making which is known as the Analytical Hierarchy Process (AHP) was proposed by Thomas Lorie Saaty (Lane & Verdini, 1989; Saaty, 1990).

The aim of the operation of a decision making technique is to reduce the uncertainty in the decision outcomes. The uncertainty results in hesitation among the decision makers when giving preferences during problem evaluation. There are two types of uncertainties related to decision making. The first is the uncertainty about the occurrence of events which cannot be controlled. The second is the set of judgment values for expressing the preferences that can be used by the decision makers which is related to the available information (Saaty & Vargas, 1987). Obviously, as more reliable information is gathered, the better the assessment of the existing preferences will be.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

An overview of the decision making process using the Fuzzy AHP technique, the theory of interval arithmetic, and the link between the AHP and interval arithmetic as a new proposed method is provided below.

2.1 Fuzzy Analytic Hierarchical Process

The Fuzzy Analytic Hierarchy Process (Fuzzy AHP) is another method that deals with uncertainty faced by the decision makers. The conventional AHP method is ineffective when applied to the uncertain nature or significance problems. As in the real world the uncertainty problems are many, some researcher extend the capability of conventional AHP with the fuzzy set theory to handle and overcome the limitation of the conventional AHP method (Javanbarg, Scawthorn, Kiyono, & Shahbodaghkhan, 2012). The benefit of combining fuzzy set theory with the conventional AHP is the capability of fuzzy set theory to represent uncertain or vague data in a natural form with the purpose to reduce the uncertainty for decision makers.

Although the fuzzy set theory is combined with AHP in order to expand the capability of conventional AHP, the main ideas of AHP are not changed. The fuzzy AHP still needs to derive the weight of the criterion and according to Wang & Chin (2011) the fuzzy AHP has two approaches to derive the weight. The first approach derives the weight of the criterion from a set of fuzzy weight pair-wise comparison matrices and the other approach uses a set of crisp weights from a fuzzy pair-wise comparison matrix. The first approach involves more than one method to get the weight of the criterion such as the geometric mean method, fuzzy logarithmic least-squares methods (LLSM), lambda-max method, and the linear programming goal (LGP) method. The second method to get the weight of the criterion uses extent analysis and the fuzzy preference programming (FPP) which is based on nonlinear methods.

Wang & Chin (2011) reported that it is not simple to calculate the weight of the criterion using the second approach which causes many researches to adopt the first approach or the simple extent analysis method for deriving fuzzy AHP weights. One example of the simple extent analysis method implementation for fuzzy AHP is by Koul & Verma (2012). Though using the first approach or adopting the simple extent analysis, the fuzzy AHP still has some problems of inconsistency (Wang & Chin, 2011; Koul & Verma, 2012).

To overcome the inconsistency, Javanbarg et al. (2012) reported that the consistency check process will be incorporated in one step of the fuzzy AHP sequence. The fuzzy AHP method steps are like the conventional AHP method and those steps are: first, structuring the decision hierarchy; second, developing the pair-wise fuzzy comparison matrix. In this step, the triangular fuzzy numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ are used to develop the fuzzy reciprocal comparison matrix. The judgment scores for the fuzzy AHP that convert the decision maker’s judgments into scores are displayed in Table 1.

Vol. 8 Issue 2 2016
ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

Table 1
The Judgment scores in fuzzy AHP (Javanbarg *et al.* (2012))

Fuzzy Judgements	Fuzzy Score
About equal	(1/2, 1, 2)
About *x* times more important	(*x* - 1, *x*, *x* + 1)
About *x* times less important	(1/(*x* + 1), 1/*x*, 1/(*x* - 1))
Between *y* and *z* times more important	(*y*, (*y* + *z*)/2, z)
Between *y* and *z* times less important	(1/*z*, 2/(*y* + *z*), 1/*y*)

Note: x=2, 3, 9; y and *z* =1, 2, …, 9; *y* < *z*

The third step conducts a consistency check and derives priorities. In the consistency check process, a fuzzy comparison matrix $\tilde{A} = \{\tilde{a}_{ij}\}$ is called consistent if $\tilde{a}_{ik} \otimes \tilde{a}_{kj} \approx \tilde{a}_{ij}$ where i, j, and k is 1, 2, …, n ($\otimes$ is fuzzy multiplication and $\approx$ denotes fuzzy equal to). When the fuzzy comparison matrix passes the consistency check then fuzzy priorities $\tilde{W}_i$ can be calculated. The last step of fuzzy AHP is priorities aggregations and alternatives ranking. In this step, if there are i alternatives and j criteria, then the final priority of alternative i will be as $A_i = \sum_{j=1}^{n} w_j a_{ij}$ where w_j is the j th criterion weight and a_{ij} is the alternative evaluation of A_i against j criterion. The higher the value of A_i, the more preferred the alternative. Still, if the priorities result in fuzzy form, an appropriate ranking procedure should be applied to de-fuzzify the rank of alternatives.

2.2 Interval arithmetic

The interval arithmetic becomes increasingly popular at present in the virtual battle against uncertainties in numerical computations and is widely used in scientific and engineering applications which deal with incomplete data. The concept of interval arithmetic used in the context of uncertainty mitigation is not quite new as it has been studied since the invention of the digital electronic computer and its immediate utilization in control science.

The development of the modern theory of interval arithmetic began in 1959 with the technical reports of Ramon Edgar Moore (Dawood, 2011). He developed a number system and arithmetic for real intervals by introducing the so-called range numbers and range arithmetic. Those terms are the first popular synonyms of modern interval numbers and interval arithmetic. The subsequently developed theory of interval arithmetic has provided a better solution for the problems occurring between computation and applied mathematics when utilizing floating-point arithmetic or traditional numerical approximation methods.

The interval arithmetic can be broadly defined as a field of study dealing with real intervals and is also referred to as interval mathematics, interval analysis or interval computations. It provides specific rules for doing arithmetic operations with closed intervals. The interval arithmetic is centered on interval numbers at its foundation.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

An interval number is a set of real numbers in which any number that lies between two limiting numbers is a member of the set. Dawood (2011) stated that the concept of an interval number system is straightforward: “Each interval number represents some fixed real number between the lower and upper limits of the closed interval”. The exact definition of an interval number is provided below:

Let $\underline{x},\overline{x} \in R$ such $\underline{x} \leq \overline{x}$, then an interval number [$\underline{x}$, $\overline{x}$] is a closed and bounded non-empty real interval that is [$\underline{x}$, $\overline{x}$] ={x∈ R | $\underline{x}$≤ x ≤ $\overline{x}$} where $\underline{x}$=min([$\underline{x},\overline{x}$]) and $\overline{x}$=max([$\underline{x}$, $\overline{x}$]) are called, respectively, the lower and upper bounds (endpoints) of [$\underline{x}$, $\overline{x}$]. (Dawood, 2011).

The basic algebraic operations such as addition, multiplication, subtraction, and division for real numbers can be extended to interval numbers. The following are the basic arithmetic operations for two intervals [$\underline{x}$, $\overline{x}$] and [$\underline{y}$, $\overline{y}$] that can be performed with interval numbers (Hickey, Ju, & Emden, 2001).

(1) Addition is given by Equation (7),

$$[\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}] \tag{7}$$

(2) Subtraction is given by Equation (8),

$$[\underline{x}, \overline{x}] - [\underline{y}, \overline{y}] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}] \tag{8}$$

(3) Multiplication is given by Equation (9),

$$[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] = [\min(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}), \max(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y})] \tag{9}$$

(4) Division is given by Equation (10),

$$\left[\frac{\underline{x},\overline{x}}{\underline{y},\overline{y}}\right] = \left[\min\left(\frac{\underline{x}}{\underline{y}}, \frac{\underline{x}}{\overline{y}}, \frac{\overline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}\right), \max\left(\frac{\underline{x}}{\underline{y}}, \frac{\underline{x}}{\overline{y}}, \frac{\overline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}\right)\right], \text{ where } \underline{y} \text{ and } \overline{y} \text{ is not 0.} \tag{10}$$

2.3 Interval arithmetic and AHP

As an extension of the standard AHP technique, Saaty and Vargas (1987) investigated the possibility of decision makers being able to express their judgment by using an interval judgment. They proposed the term interval judgment to be used when dealing with the uncertainty during pair-wise comparisons. The interval judgment was implemented by using the Monte Carlo simulation approach to derive the resultant weights because the conventional AHP technique derives said weights without considering the uncertainty of human intuition (Saaty & Vargas, 1987; Yu, Hsiao, & Sheu, 2011).

Although the interval judgment can be used to facilitate one’s intuition during the decision making process, the problem with the uncertainty does not stop here. It raises another issue, a difficulty in measuring the inconsistency of the generated weights from the interval judgment. An additional study attempted to address this issue by proposing the Fuzzy Preference Programming (FPP) technique. The FPP technique initially defines the values of all tolerance parameters for all judgments.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

The Lexicographic Goal Programming for AHP (LGAHP) technique was also proposed to address the inconsistency in the interval AHP technique. The LGAHP technique generates the weights of interval pair-wise comparison matrices by using deviation variables. By using LGAHP, it is not required to initially define tolerance parameters and it is also possible to minimize the summation of the deviation variables through a positive objective value (Yu et al., 2011).

The rank reversal is another issue of the interval AHP technique which arises due to a misuse of the aggregation methods. When dealing with rank reversal problems, the multiplicative AHP (MAHP) technique was proposed in order to maintain the ratio scaling properties. Several studies developed and extensively used the MAHP technique because of the absence of rank reversal problems. In addition, there are several proposed approaches intended to make the interval AHP technique more efficient such as the logarithmic goal programming (LGP), the integrated logarithmic goal programming, the logarithmic least-squares method (LLS), the logarithmic least absolute value method (LLAV), etc. (Yu et al., 2011)

When the interval arithmetic is integrated into the AHP technique, the main AHP algorithmic stages remain intact. Each element in the standard AHP technique is modified to have two values, namely, a lower limit (L) and an upper limit (U). The basic algebraic operations for interval numbers are used in all computations, thus replacing the standard algebraic operations.

The pair-wise matrix of AHP with interval numbers is formed in accordance with Equation (11) because each element of said matrix must be presented by a pair of lower and upper values,

$$\left[A^L, A^U\right] = \begin{bmatrix} \left[x_{11}^L, x_{11}^U\right] & \left[x_{12}^L, x_{12}^U\right] & \cdots & \left[x_{1j}^L, x_{1j}^U\right] \\ \left[x_{21}^L, x_{21}^U\right] & \left[x_{22}^L, x_{22}^U\right] & \cdots & \left[x_{2j}^L, x_{2j}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[x_{i1}^L, x_{i1}^U\right] & \left[x_{i2}^L, x_{i2}^U\right] & \cdots & \left[x_{ij}^L, x_{ij}^U\right] \end{bmatrix} \tag{11}$$

The relative weighs of the pair-wise matrix components are obtained with Equation 12,

$$\left[A_{norm}^L, A_{norm}^U\right] = \begin{bmatrix} \left[a_{11}^L, a_{11}^U\right] & \left[a_{12}^L, a_{12}^U\right] & \cdots & \left[a_{1j}^L, a_{1j}^U\right] \\ \left[a_{21}^L, a_{21}^U\right] & \left[a_{22}^L, a_{22}^U\right] & \cdots & \left[a_{2j}^L, a_{2j}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[a_{i1}^L, a_{i1}^U\right] & \left[a_{i2}^L, a_{i2}^U\right] & \cdots & \left[a_{ij}^L, a_{ij}^U\right] \end{bmatrix}, \text{ where } \left[a_{ij}^L, a_{ij}^U\right] = \frac{\left[x_{ij}^L, x_{ij}^U\right]}{\sum_{i=1}^{n}\left[x_{ij}^L, x_{ij}^U\right]} \tag{12}$$

The weights of the criteria are given by Equation (13),

$$\left[W_i^L, W_i^U\right] = \frac{\sum_{j=1}^{n}\left[a_{ij}^L, a_{ij}^U\right]}{n}, \text{ where } \sum_{i=1}^{n}\left[W_i^L, W_i^U\right] = \left[W_i^L, W_i^U\right] \tag{13}$$

The consistency index (CI) equation of the pair-wise matrix is given by Equation (14) which requires the prior calculation of the λ_{max} interval,

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

$$[\lambda_{\max}^L, \lambda_{\max}^U] = \frac{\sum_{i=1}^{n}\left[b_i^L, b_i^U\right]}{n}, \text{ where } \left[b_i^L, b_i^U\right] = \frac{\sum_{j=1}^{n}\left[W_i^L, W_i^U\right]\times\left[a_{ij}^L, a_{ij}^U\right]}{\left[W_i^L, W_i^U\right]}$$

so that

$$\left[CI^L, CI^U\right] = \frac{\left[\lambda_{\max}^L, \lambda_{\max}^U\right] - n}{n-1} \tag{14}$$

Finally, the coherence ratio (CR) is obtained with Equation (15) and the value of random index (RI) is determined in accordance with Table 2,

$$\left[CR^L, CR^U\right] = \frac{\left[CI^L, CI^U\right]}{\left[RI^L, RI^U\right]} \tag{15}$$

Table 2
Random consistency index table with interval numbers

N	1	2	3	4	5	6	7	8	9	10
RI	[0, 0]	[0, 0]	[0.58, 0.58]	[0.90, 0.90]	[1.12, 1.12]	[1.24, 1.24]	[1.32, 1.32]	[1.41, 1.41]	[1.45, 1.45]	[1.49, 1.49]

3. Results and Discussion

This section is divided into two parts; the first part summarizes the AHP research results of Jamali, Samadi, and Marthandan (2014) which are used as an example of the standard AHP process. The second part discusses the implementation of interval arithmetic in AHP by using the basic algebraic operations with intervals for AHP computations. The reference data which is used for the second part is taken from Jamali et al. (2014) and all calculations are performed by utilizing Wolfram Mathematica® version 9 software.

3.1 Standard AHP process

It is commonly known that the adoption of electronic commerce (e-commerce) technology in small and medium-sized enterprises (SMEs) has significant effects on their competitiveness and performance. Therefore, the technology selection becomes important and must meet the SMEs requirements and uniqueness. Jamali et al. (2014) investigated the most relevant spectrum of e-commerce technology which is suitable for Iranian family SMEs.

The objective of the case study from Jamali et al. (2014) was extracted from the related literature, refined with unstructured interviews and discussed with the expert member team. The obtained results indicated that the e-commerce technology to be selected must meet the main objectives of the Iranian family SMEs such as: (1) sales growth; (2) maximization of profit; (3) increased market share; (4) minimization of family conflicts; and (5) preservation of family independence. The relevant technologies that meet their needs were identified as e-mail, website technology, Intranet, Extranet, electronic data

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

exchange (EDI), electronic funds transfer (EFT), and barcode technology. The data sample was collected from the Iranian SME database (ISIPO) using non-probabilistic sampling technique. The selection of the respondents for the conducted study was based on the following criteria: must be the CEO of a family SME; must have at least 5-year managerial work experience; or at least a Bachelor's education in management or in a related field of study (Jamali et al., 2014).

After the AHP computations, the analysis of the weighted criteria indicated that the minimization of family conflicts had the most significant weight among other criteria, and the final weights of the alternatives against the criteria led to the conclusion that the website technology is the most fitted e-commerce technology for Iranian family SMEs (Jamali et al., 2014).

3.2 AHP with interval arithmetic

The practical implementation of interval arithmetic in AHP is shown below. Uniform increments of 0.5 were used for creating the differences between the minimum and maximum values of the interval numbers. The intervals range from 0 to 4 which results in 9 different initial interval ranges for AHP calculations.

The implementation of the modified technique does not exceed the AHP rating scale used in the pair-wise matrix. As an example, the base number starts from 8.0 and increases gradually by 0.5. The number increment will stop at 9.0 and then there is a gradual decrease by 0.5 until stopping at 1.0.

The criterion pair-wise matrix (Table 3) starts from range 0; there is no difference between the minimum and maximum values. Equation (11) is used for the pair-wise matrix with the interval numbers (Table 3).

Table 3
Pair-wise criteria matrix with interval numbers

	Criteria 1	Criteria 2	Criteria 3	Criteria 4	Criteria 5
Criteria 1	[1, 1]	[1.78, 1.78]	[2.31, 2.31]	[2.49, 2.49]	[1.34, 1.34]
Criteria 2	[0.56, 0.56]	[1, 1]	[2.16, 2.16]	[2.70, 2.70]	[1.74, 1.74]
Criteria 3	[0.43, 0.43]	[0.46, 0.46]	[1, 1]	[1.14, 1.14]	[2.35, 2.35]
Criteria 4	[0.40, 0.40]	[0.37, 0.37]	[0.88, 0.88]	[1, 1]	[1.31, 1.31]
Criteria 5	[0.75, 0.75]	[0.57, 0.57]	[0.43, 0.43]	[0.76, 0.76]	[1, 1]

The result of the normalized pair-wise matrix of the criterion table can be seen in Table 4. All calculations use basic algebra operations with interval numbers. The interval of λ_{max} of the criterion matrix is [5.255, 5.263] and the interval of the consistency index (CI) is [0.064, 0.066]. The interval of the coherence ratio (CR) of the criterion table is [0.057, 0.059]. Here Equations (14) and (15) are used for the calculation of λ_{max} , CI, and CR for the criterion matrix with interval numbers.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

Table 4
Pair-wise criterion matrix normalization with interval numbers

	Criteria 1	Criteria 2	Criteria 3	Criteria 4	Criteria 5
Criteria 1	[0.318, 0.318]	[0.427, 0.427]	[0.340, 0.340]	[0.308, 0.308]	[0.173, 0.173]
Criteria 2	[0.178, 0.178]	[0.240, 0.240]	[0.319, 0.319]	[0.334, 0.334]	[0.225, 0.225]
Criteria 3	[0.134, 0.134]	[0.110, 0.110]	[0.147, 0.147]	[0.141, 0.141]	[0.304, 0.304]
Criteria 4	[0.127, 0.127]	[0.089, 0.089]	[0.130, 0.130]	[0.124, 0.124]	[0.169, 0.169]
Criteria 5	[0.239, 0.239]	[0.134, 0.137]	[0.063, 0.063]	[0.094, 0.094]	[0.129, 0.129]

The final weights of the criteria matrix can be seen in Figure 1. Figure 1 has non-linear behavior and the highest input interval ranges is the prominent point. For the input interval ranges from 1.5 to 4.0, the initial linear growth is replaced with a smooth decrease of the final intervals.

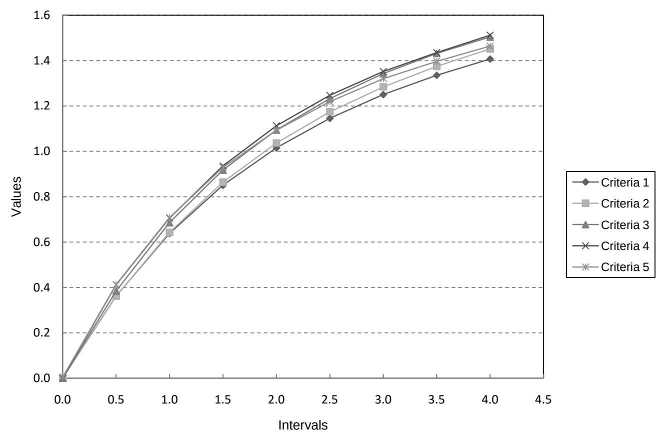

Figure 1. Priority vectors of criteria matrix

The third alternative is the final solution in the Jamali et al. (2014) research which gives the dominant weight. After the implementation of the interval arithmetic in the AHP, the third alternative is still the final solution that gives the dominant weight among the other alternatives. The weights of the third alternative for the input range interval 0.0 is [0.362, 0.363] and sequentially the weight values of the input range interval from 0.5 to 4.0 are [0.274, 0.483], [0.218, 0.616], [0.178, 0.759], [0.150, 0.913], [0.122, 1.059], [0.099, 1.219], [0.083, 1.400], and [0.067, 1.623].

The other graphical illustrations of the interval arithmetic implementation in the AHP technique are shown in Figures 2-7. Figures 2-6 show the graphs of the priority vectors from alternative 1 to alternative 7 for criteria 1-5 and Figure 7 shows the graphs of the final priority vectors of all alternatives. The graph tendency of the priority vectors from alternative 1 to alternative 7 for criteria 1-5 is linear but for the final priority vectors is saturated.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

The graphs of the priority vectors for criteria 1-5 in Figures 2-6 increase steadily. In Figures 2-6, some of the linear graphs have similar values which results in the graph lines being close to each other and sometimes they are overlapping. As an example, in Figure 4 the differences between the interval values are smaller than in the remaining graphs so that the graph lines between the alternatives slowly fluctuate in a narrow range. In Figure 5, the curve of the fifth alternative follows a noticeably different path as compared to the linear paths of the other alternatives. In some intermediate points it is plotted noticeably lower than the other graphs but it is stabilized for the highest input ranges similarly to the graphs of the other alternatives.

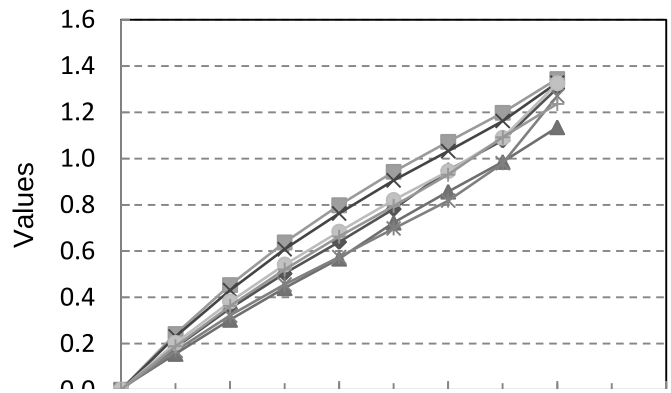

Figure 2. Priority vectors of criterion 1

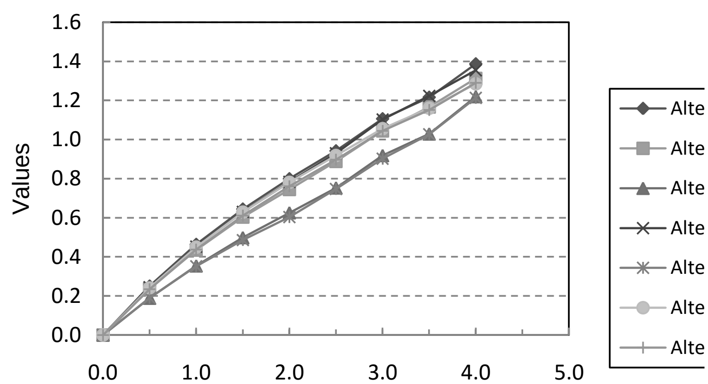

Figure 3. Priority vectors of criterion 2

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

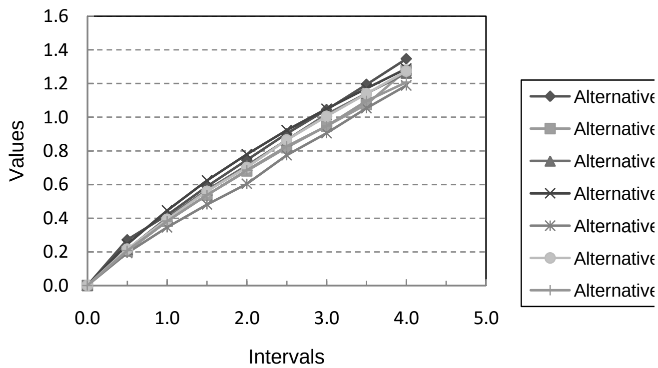

Figure 4. Priority vectors of criterion 3

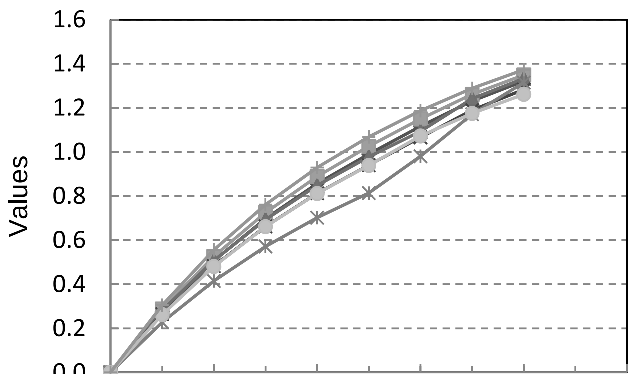

Figure 5. Priority vectors of criterion 4

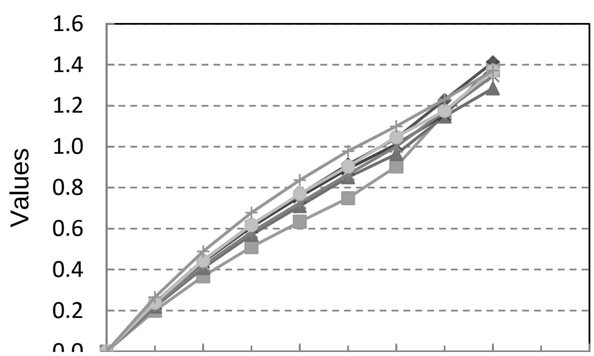

Figure 6. Priority vectors of criterion 5

The graphs of the final priority vectors (Figure 7) in the last stage of computation are quite different when compared with the intermediate graphs. The graphs in Figure 7 have a non-linear behavior which becomes increasingly noticeable for the highest input ranges. For input interval ranges from 1.5 to 2.5, the initial linear growth is replaced with a smooth decrease of the final intervals. Interestingly, the smallest graph increments are observed for the largest input intervals in the ranges from 2.5 to 4.0. The observed non-linearity occurs in the graphs of the final priority vectors for all alternatives.

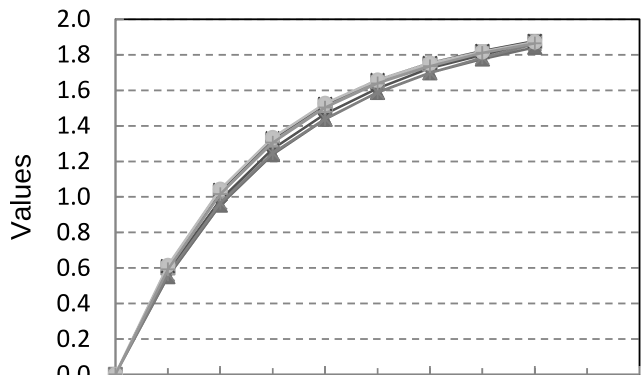

Figure 7. Final priority vectors

4. Conclusion

The battle with uncertainty has been addressed in many ways especially in the decision making field and has resulted in more than one method. Those methods are useful and help decision makers when they are faced with uncertainty. AHP as one popular method expands its method to overcome uncertainty by adopting fuzzy set theory which results in fuzzy AHP. The implementation of the fuzzy set theory into AHP is not the final and perfect answer to dealing with uncertainty. The fuzzy AHP still has imperfections concerned with inconsistency, but that does not mean that this inconsistency cannot be resolved. When decision makers use fuzzy AHP, they have to convert judgments into triangular fuzzy numbers to get the weights of the factors. The triangular fuzzy numbers represent numerical crisp data which are used as a foundation when developing the AHP comparison matrix. The process of fuzzy AHP is like conventional AHP except that steps for consistency checking and de-fuzzifying have been added in order to arrive at the final result.

Another method to deal with uncertainty is to introduce interval numbers into the AHP method. Interval numbers have a special form with minimum and maximum limits not represented by conventional numbers and they can be used to represent uncertainty. Single numbers used in the AHP represent exact conditions, but by using interval arithmetic the decision makers have the possibility of representing uncertainty in a formal way.

The differences between the fuzzy AHP and the method proposed here can be described as follows. When using the fuzzy AHP, the decision makers have to first choose which method they will use to derive the factors weights. Once they decide which method they will use, then the judgment conversion process that uses the triangular fuzzy numbers is started. The next step is building the comparison matrix. The decision makers also have to implement the consistency check in a specific step and must de-fuzzify if the priority result is still in the fuzzy form. The final result of the fuzzy AHP is still in the single number form which can ignore the uncertain value of preferences.

The use of interval numbers in AHP will not change the process sequence of AHP. In the conventional AHP that uses a single number, the calculation process uses normal algebraic operations. However, normal algebraic operations cannot be used with the

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

interval numbers. The calculations with interval numbers have to use interval numbers algebraic operations.

With the implementation of interval numbers in AHP, the decision makers do not have to choose any method as in fuzzy AHP from the beginning of the process. They only need to assign values for the upper and the lower limits of their judgments. The form of the pair-wise comparison matrix for AHP with interval numbers is illustrated in Table 3 or Table 4 and during the calculation process the interval number algebraic operations are used. The final result of AHP with interval numbers will be in the form of interval numbers and uncertainty is still present but it is in an optimum state.

In summary, the development and the implementation of interval arithmetic has the advantage of obtaining reliable information about the uncertainty at the final stage of decision making. In this study, the implementation of interval arithmetic is carried out for the AHP technique, one of the well-established decision making techniques.

The input interval numbers in all pair-wise table comparisons used in this study have a uniform increment of 0.5 from 0 to 4 in order to systematically investigate the behavior of the output interval values at the end of the computation process. When introducing interval numbers in the pair-wise comparison tables, it must be noted that the values of the reciprocal interval numbers must be properly ordered according to their minimum and maximum values and cannot exceed the AHP rating scale limit. Also, the basic algebraic operations for interval numbers must be thoroughly applied with no exceptions for the computation of all equations within the AHP algorithmic sequence.

After comparing the computational results between the AHP techniques with and without interval numbers, it can be seen that the result of the AHP technique without interval numbers is within the range of the AHP technique with interval numbers. The implementation of interval numbers in the judgment process provides a more reliable way for the representation of the corresponding judgment values. The judgment process with the use of interval numbers can estimate the uncertainty which is a common issue during decision making.

The interval numbers which are applied in the judgment process can be regarded as uncertainty estimators which have distinct minimum and maximum values. The increased distance between the minimum and maximum values is an indicator of the accumulated uncertainty due to the sequential use of basic algebraic operations with interval numbers.

The resultant graphs of the AHP technique with interval numbers show the tendency of interval growth during the computational process. In Figures 2-6, the trend is linear with slight fluctuations in every increment of the intervals. However, the final priority vector graphs (Figure 7) have a tendency of stabilization. The linearity in Figure 7 for input ranges from 0.0 to 1.5 is changed to a non-linear reduction of interval growth for input ranges from 1.5 to 2.5 and the interval growth for input ranges from 2.5 until 4.0 is on an even smaller scale.

The main observation is that for bigger ranges of input intervals, the final intervals become saturated as can be seen in the final tendency obtained for the sample input ranges from 1.5 to 4.0. This can be explained with the principle of operation of the AHP

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

technique which results in a stabilization of weight differences in the final algorithmic stage.

This study is limited to the use of a uniform distribution of weights within an interval. Further extensions on the use of interval arithmetic may include non-uniform distributions of weights for specific applications.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v8i2.325

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