GROUP INTERVAL WEIGHTS BASED ON CONJUNCTION APPROXIMATION OF INDIVIDUAL INTERVAL WEIGHTS

Tomoe Entani
Graduate School of Applied Informatics
University of Hyogo
Kobe, JAPAN
E-mail:entani@ai.u-hyogo.ac.jp

ABSTRACT

The individual and group decisions in this study are denoted as the normalized interval weights of alternatives as in Interval AHP. It assumes that a decision maker uses crisp values in the interval weights in giving comparisons. The interval weights reflect uncertainty in a decision maker's mind. Then, the group interval weight is obtained as a conjunction approximation of the individual interval weights. For a consensus, the group interval weight is obtained so as to intersect with all the individual interval weights. In other words, the group interval weight has something in common with each individual interval weight. The group decision depends on how much the decision makers are satisfied or dissatisfied with it. The satisfaction of a decision maker is measured by the ranges of the group interval weights which s/he can support. Similarly, the decision maker's dissatisfaction is defined by the ranges which are out of his/her decision. It is better to maximize the satisfaction and simultaneously to minimize the dissatisfaction. However, there is a trade-off between these two objectives. In the proposed model, the importance of the satisfaction or dissatisfaction is given. Then, the decision makers find not only the group decision but also their satisfaction and dissatisfaction with it.

Keywords: Group decision making; interval analysis; Analytic Hierarchy Process

1. Introduction

In the Analytic Hierarchy Process (AHP), the crisp priority weights of alternatives are obtained from the pairwise comparison matrix given by a decision maker (Saaty, 1980). The elements of a matrix are crisp values such as 1/3, and 5. The well-known techniques to obtain the weights from the given crisp comparisons are geometric mean and eigenvector methods. The obtained weights are also crisp. The other technique is Interval AHP, where the weights are obtained as an interval to reflect the inconsistency among the given crisp comparisons (Sugihara & Tanaka, 2001; Sugihara, Ishii, & Tanaka, 2004). The interval weights are obtained so as to include the given comparisons as close as possible. The crisp comparisons are extended into interval ones to reflect our uncertain judgments (Saaty & Vargas, 1987; Arbel, 1989). In group AHP, some works handle the interval or fuzzy comparisons, instead of crisp comparisons (Dopazo, Chouinard & Guisse, 2014; Xu, 2013). However, this study handles the crisp comparisons and the proposed approach with crisp comparisons can be extended into the interval comparisons on the same principle.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

Group decision making is discussed from the viewpoint of AHP (Dyer & Forman, 1992; Basak & Saaty, 1993). There are two approaches to aggregate the individuals into a group by geometric mean and so on (Aczel & Saaty, 1983; Altuzarra, Moreno-Jimenez, & Salvador, 2007; Entani & Inuiguchi, 2010; Forman, Peniwati, 1998; Yeh & Chang, 2009). One approach is to aggregate the individual judgments first and then the group decision is obtained from the aggregated judgments. The other approach is to aggregate the individual decisions which are independently obtained from the individually given judgments. Since the latter aggregation can show a decision maker his/her decision, it helps him/her to understand the relationship between his/her decision and the group decision. This study also assumes that the group decision is obtained as the aggregation of individual decisions and discusses how to aggregate them. For a consensus, the group weight is obtained so as to have something in common with each individual weight. The quality of the group decision is measured by the satisfaction or dissatisfaction of the decision makers. Some parts of the group interval weights are supported by a decision maker but the other parts are not. Therefore, the satisfaction and dissatisfaction of all decision makers are maximized and minimized, respectively. However, they have a trade-off relationship so that the importance of the satisfaction or dissatisfaction is introduced and the group decision depends on it.

2. Individual decisions by Interval AHP

In AHP, decision maker k gives the following pairwise comparison matrix

$$A_k = \begin{bmatrix} 1 & \cdots & a_{k1n} \\ \vdots & a_{kij} & \vdots \\ a_{kn1} & \cdots & 1 \end{bmatrix}, \forall k, \tag{1}$$

where a_{kij} is his/her intuitive judgment on the importance ratio of alternative i to that of alternative j. The comparisons satisfy $a_{kii} = 1$ and $a_{kij} = 1/a_{kji}$ for all k, i and j. The comparisons are consistent if and only if

$$a_{kij} = a_{kil}a_{klj}, \forall i, j, l. \tag{2}$$

By the eigenvector method, the weights of alternatives are obtained as the eigenvector corresponding to principal eigenvalue, $A_k\boldsymbol{w}_k = \lambda\boldsymbol{w}_k$, where $\boldsymbol{w}_k = (w_{k1}, \dots, w_{kn})^T$ and $\sum_i w_{ki} = 1$. It is noted that the weights, $w_{ki}, \forall i$ are crisp.

In Interval AHP, it is assumed that the given comparisons are inconsistent since the weights of alternatives are uncertain (Sugihara & Tanaka, 2001; Sugihara et al., 2004). An alternative is compared to the other $n-1$ alternatives. For example, denoting the weights of alternative 1 in giving comparisons a_{k12} and a_{k13} as w_{k1}^2 and w_{k1}^3, respectively, they are not always equal, $w_{k1}^2 \neq w_{k1}^3$. These weights of alternative 1 depend on which alterative it is compared to. Then, the interval weight of alternative 1, $W_{k1} = [\underline{w}_{k1}, \overline{w}_{k1}]$, includes these weights as $\underline{w}_{k1} \leq w_{k1}^2 \leq \overline{w}_{k1}$ and $\underline{w}_{k1} \leq w_{k1}^3 \leq \overline{w}_{k1}$. In this way, the uncertain weight of an alternative is denoted as an interval. The weight of alternative i is denoted as interval $W_{ki} = [\underline{w}_{ki}, \overline{w}_{ki}]$, whose width $\overline{w}_{ki} - \underline{w}_{ki}$ represents its uncertainty. In other words, the decision maker k uses a real value in the interval weight W_{ki} in the given comparison a_{kij}, where $j \neq i$. The problem to obtain the interval weights is formulated as the following linear programming (LP) problem.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

$$
\begin{aligned}
&\min \quad \sum_i (\overline{w}_{ki} - \underline{w}_{ki}),\\
\text{s.t.} \quad &\sum_{i \neq j} \overline{w}_{ki} + \underline{w}_{kj} \geq 1, \forall j,\\
&\sum_{i \neq j} \underline{w}_{ki} + \overline{w}_{kj} \leq 1, \forall j,\\
&\frac{\underline{w}_{ki}}{\overline{w}_{kj}} \leq a_{kij} \leq \frac{\overline{w}_{ki}}{\underline{w}_{kj}}, \forall i, j,\\
&\underline{w}_{ki} \geq \epsilon, \forall i.
\end{aligned}
\tag{3}
$$

The first two constraints are for the normalization of intervals based on interval probability (de Campos, Huete, & Moral, 1994; Tanaka, Sugihara, & Maeda, 2004). The redundancy of the intervals to make their sum 1 is excluded. For instance, the 1st inequality for j requires $\underline{w}_{kj}$ to not be too small. When the weights are crisp, $\overline{w}_{ki} = \underline{w}_{ki} = w_{ki}, \forall i$, two inequalities are replaced into $\sum_i w_{ki} = 1$. The next inequalities are the inclusion constraints. These inequalities require the obtained interval weights to include the given comparisons as

$$
a_{kij} \in \frac{W_{ki}}{W_{kj}} = \frac{[\underline{w}_{ki}, \overline{w}_{ki}]}{[\underline{w}_{kj}, \overline{w}_{kj}]} = \left[\frac{\underline{w}_{ki}}{\overline{w}_{kj}}, \frac{\overline{w}_{ki}}{\underline{w}_{kj}}\right],
\tag{4}
$$

where the fraction of intervals is defined as its maximum range. In the case of interval comparison, $[\underline{a}_{kij}, \bar{a}_{kij}]$, it replaces crisp comparison a_{kij} in Equation 4. By minimizing the widths of the interval weights, the obtained weights are as close as possible to the given comparisons because of the inclusion constraints Equation 4. It is, in other words, minimizing uncertainty of the decision. If the comparisons are consistent as in Equation 2, the upper and lower bounds of the interval weights are equal, $\overline{w}_{ki} = \underline{w}_{ki}$, so that the weights are crisp as $w_{ki} = \overline{w}_{ki} = \underline{w}_{ki}, \forall i$. They are equal to those by the geometric mean and eigenvector methods. The more inconsistent the given comparisons are, the wider the obtained interval weights become. The inconsistency of the given comparisons is measured by the optimal objective function value of Equation 3.

3. Group decision from individual decisions

3.1 Relation between group and individual decisions

In the case of a group of m decision makers, there are m sets of individual interval weights. They are independently obtained from their comparison matrices by Equation 3. The group decision is also denoted as interval weights $W_i = [\underline{w}_i, \overline{w}_i]$ and they are normalized as

$$
\begin{aligned}
&\sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1, \forall j,\\
&\sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1, \forall j.
\end{aligned}
\tag{5}
$$

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

Then, based on the idea that a decision maker accepts the group decision if it has something in common with his/her decision, the group interval weight should satisfy the following conditions.

$$\underline{w_i} \leq \overline{w}_{ki}, \underline{w}_{ki} \leq \overline{w}_i \; \forall k \leftrightarrow \underline{w_i} \leq \min_k \overline{w}_{ki}, \max_k \underline{w}_{ki} \leq \overline{w}_i, \tag{6}$$

where the group decision intersects to all the individual decisions as $W_i \cap W_{ki} \neq \emptyset \; \forall k$. In Equation 6, the upper and lower bounds of the group interval weight are approximated by the lower and upper bounds of the individual interval weights, respectively. It is based on the conjunction approximation. It is noted that the group interval weight complements the individual interval weights from the possibility viewpoint.

The group decision includes and is included in the individual decisions as the extreme cases of Equation 6. Assuming that the group decision includes all the individual decisions as $W_i \supseteq W_{ki} \; \forall k$, Equation 6 is rewritten as

$$\begin{aligned} &\underline{w_i} \leq \underline{w}_{ki} \leq \overline{w}_{ki}, \underline{w}_{ki} \leq \overline{w}_{ki} \leq \overline{w}_i, \forall k \leftrightarrow \underline{w_i} \leq \underline{w}_{ki} \leq \overline{w}_{ki} \leq \overline{w}_i, \forall k \\ &\leftrightarrow \underline{w_i} = \min_k \underline{w}_{ki}, \max_k \overline{w}_{ki} = \overline{w}_i. \end{aligned} \tag{7}$$

Such a group decision is based on the upper approximation of the individual decisions. All the possible evaluations in the individual decisions are included in the group decision. The group interval weight whose bounds are $\underline{w_i} < \min_k \underline{w}_{ki}$ and $\max_k \overline{w}_{ki} < \overline{w}_i$ is redundant, since it includes the weight which is out of any decision maker's interval weight. By Equation 6, some evaluations in an individual decision may be ignored compared to the others' decisions, but the more precise group decision than that is obtained by Equation 7. While, based on their lower approximation, the group decision is included in all the individual decisions as $W_i \subseteq W_{ki}, \forall k$, Equation 6 is rewritten as

$$\begin{aligned} &\underline{w_i} \leq \overline{w}_i \leq \overline{w}_{ki}, \underline{w}_{ki} \leq \underline{w_i} \leq \overline{w}_i, \forall k \leftrightarrow \underline{w}_{ki} \leq \underline{w_i} \leq \overline{w}_i \leq \overline{w}_{ki}, \forall k \\ &\leftrightarrow \max_k \underline{w}_{ki} = \underline{w_i}, \overline{w}_i = \min_k \overline{w}_{ki}, \text{ if } \max_k \underline{w}_{ki} \leq \min_k \overline{w}_{ki}. \end{aligned} \tag{8}$$

If $\max_k \underline{w}_{ki} > \min_k \overline{w}_{ki}$, there is no group weight included in all the individual weights. Equations 6 or 7 result in a group decision for any individual decisions, but Equation 8 requires the individual decisions have something in common with each other. This study defines the general condition of the group decision by Equation 6, and uses Equations 7 and 8 as its extreme cases.

3.2 Satisfaction and dissatisfaction of decision maker

By Equation 6 the individual and group decisions intersect each other and the group interval weight has something in common with each individual interval weight. A decision maker is satisfied with the group decision if s/he can support most of it. Then, the similarity of the group decision to his/her decision measures his/her satisfaction. While, s/he is not satisfied with the group decision if s/he cannot support some of it. Then, the difference of the group decision from his/her decision measures his/her dissatisfaction.

In Figure 1, an individual interval weight is illustrated as the top line and the possible group weights which satisfy Equation 6 are illustrated as the following four lines. His/her

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

satisfaction and dissatisfaction of an alternative are dented as α and β, respectively. The lower two lines show the group weights based on the lower and upper approximations of the individual weights, respectively.

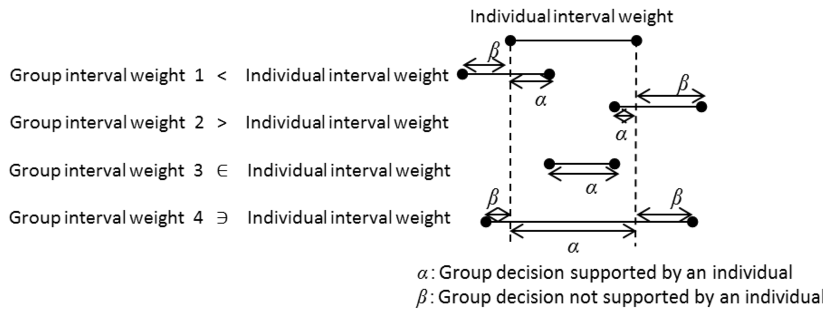

Figure 1. Individual and group decisions

Decision maker k is satisfied with the group weight of alternative i with degree α_{ki}, which is defined as follows.

$$\alpha_{ki} = \min\{(\overline{w}_i - \underline{w}_{ki}), (\overline{w}_{ki} - \underline{w}_i), (\overline{w}_i - \underline{w}_i), (\overline{w}_{ki} - \underline{w}_{ki})\}. \quad (9)$$

This range in the group interval weight is included in the individual one. Since s/he supports this range, his/her satisfaction is measured as the sum of these ranges of all alternatives, $\alpha_k = \sum_i \alpha_{ki}$. Then, the sum of the satisfaction of all decision makers should be maximized for the better group decision as

$$\max \sum_k \alpha_k = \max \sum_{ki} \alpha_{ki}. \quad (10)$$

On the other hand, the dissatisfaction degree β_{ki} is defined as follows.

$$\beta_{ki} = \max\{(\underline{w}_{ki} - \underline{w}_i), (\overline{w}_i - \overline{w}_{ki}), 0, (\underline{w}_{ki} - \underline{w}_i + \overline{w}_i - \overline{w}_{ki})\}. \quad (11)$$

This range in the group interval weight is not included in the individual one so that it is not supported by him/her. Then, the sum of the dissatisfaction with all alternatives should be minimized as

$$\min \sum_k \beta_k = \min \sum_{ki} \beta_{ki}. \quad (12)$$

As shown in Figure 1, the group interval weight of alternative i is divided into the ranges satisfying and dissatisfying decision maker k as

$$\overline{w}_i - \underline{w}_i = \alpha_{ki} + \beta_{ki}. \quad (13)$$

It is a trade-off between the increase of satisfaction and the decrease of dissatisfaction. When the group interval weight of an alternative is uncertain so that its width is wide, both the satisfaction and dissatisfaction tend to be large.

3.3 Model to obtain group decision

The group decision is denoted as the normalized interval weights as Equation 5 and should have something in common with each individual decision as Equation 6 for a consensus. Under these conditions, the satisfaction and dissatisfaction of the decision makers are maximized and minimized, respectively, as Equation 10 with Equation 9 and Equation 12 with Equation 11. Then, the problem to obtain a group decision is formulated as follows.

$$
\begin{aligned}
&\max \ \sum_{ki} \alpha_{ki}, \\
&\min \ \sum_{ki} \beta_{ki}, \\
\text{s.t.} \quad & \underline{w}_i \leq \overline{w}_{ki}, \forall i,k, \\
& \underline{w}_{ki} \leq \overline{w}_i, \forall i,k, \\
& \overline{w}_i - \underline{w}_{ki} \geq \alpha_{ki}, \forall i,k, \\
& \overline{w}_{ki} - \underline{w}_i \geq \alpha_{ki}, \forall i,k, \\
& \overline{w}_i - \underline{w}_i \geq \alpha_{ki}, \forall i,k, \\
& \overline{w}_{ki} - \underline{w}_{ki} \geq \alpha_{ki}, \forall i,k, \\
& \underline{w}_{ki} - \underline{w}_i \leq \beta_{ki}, \forall i,k, \\
& \overline{w}_i - \overline{w}_{ki} \leq \beta_{ki}, \forall i,k, \\
& 0 \leq \beta_{ki}, \forall i,k, \\
& \underline{w}_{ki} - \underline{w}_i + \overline{w}_i - \overline{w}_{ki} \leq \beta_{ki}, \forall i,k, \\
& \sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1, \forall j, \\
& \sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1, \forall j, \\
& \epsilon \leq \underline{w}_i \leq \overline{w}_i, \forall i,
\end{aligned}
\tag{14}
$$

where the variables are the upper and lower bounds of the group interval weight, $\underline{w}_i$ and $\overline{w}_i$, and the individual satisfaction and dissatisfaction, α_{ki} and β_{ki}. The upper and lower bounds of a group interval weight are approximated by the lower and upper bounds of all the individual interval weights by Equation 3.

As mentioned in Section 2, the conjunction approximation as the general condition of the group decision by Equation 6 includes the extreme conditions as the upper and lower approximations by Equations 7 and 8, respectively. Based on the upper approximation as $W_{ki} \subseteq W_i = [\min_k \underline{w}_{ki}, \max_k \overline{w}_{ki}]$ by Equation 7 illustrated as the lowest line in Figure 1, decision maker k is completely satisfied with the group decision. In this case, his/her satisfaction becomes the maximum, $\alpha_{ki} = \overline{w}_{ki} - \underline{w}_{ki}$ which is the width of his/her interval weight. In order for a group interval weight to include all the individual interval weights, the width of the group interval weight tends to be wide, especially when the individual weights are not very similar. A part of the group weight supported by an individual may not be supported by the other individuals. Therefore, as a result of maximizing the satisfaction of all individuals, the dissatisfaction is increased. On the other hand, decision maker k is not dissatisfied with the group decision when the group

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

decision is based on the lower approximation as $W_{ki} \supseteq W_i = [\max_k \underline{w}_{ki}, \min_k \overline{w}_{ki}]$ by Equation 8 illustrated as the 2nd line from the lowest in Figure 1. In this case, his/her dissatisfaction becomes the minimum, $\beta_{ki} = 0$. When all the individual interval weights are common with each other as $\max_k \underline{w}_{ki} \leq \min_k \overline{w}_{ik}$ as in Equation 8, the dissatisfaction of all decision makers is 0. Though, this is seldom satisfied, since the individual decisions are often different. If the individual weight is crisp as $w_{ki} = \underline{w}_{ki} = \overline{w}_{ki}$, it can support a point of the group interval weight. His/her satisfaction and dissatisfaction are $\alpha_{ki} = 0$ and $\beta_{ki} = \overline{w}_i - \underline{w}_i$, respectively. Therefore, the ranges of the satisfaction and dissatisfaction are $0 \leq \alpha_{ki} \leq \overline{w}_{ki} - \underline{w}_{ki}$ and $0 \leq \beta_{ki} \leq \overline{w}_i - \underline{w}_i$, respectively.

For calculation, two objective functions in Equation 14 are aggregated by the weighting approach as

$$\max \quad \lambda \sum_{ki} \alpha_{ki} - (1-\lambda) \sum_{ki} \beta_{ki}, \tag{15}$$

where λ and $(1-\lambda)$ are the weights for satisfaction and dissatisfaction, respectively. Focusing on the satisfaction with $\lambda = 1$, the group decision is obtained as the upper approximation of the individual decisions by Equation 7. While focusing on the dissatisfaction with $\lambda = 0$, the group decision approaches the lower approximation of the individual decisions by Equation 8.

The inconsistency among the given comparisons by a decision maker is reflected in his/her interval weights given by Equation 3. The individual interval weights include all the possibilities in his/her judgments. It may be uncomfortable for a decision maker to accept the weights out of his/her possible decision. S/he tends to disagree with the group decision if it includes the weight which s/he cannot support by his/her possible decision. From this viewpoint, it is reasonable to primarily minimize the dissatisfaction and secondarily maximize the satisfaction in order to be $1 - \lambda > \lambda$.

In Equation 14, the sums of the satisfaction and dissatisfaction with all alternatives of all decision makers are maximized and minimized, respectively. Since the satisfaction and dissatisfaction of each decision maker are not considered, it happens that the group decision which satisfies the specific decision maker may be obtained as the optimal solution of LP problem (Equation 14). For the fairness of the decision makers, it is useful to determine the appropriate thresholds of the satisfaction and dissatisfaction of a decision maker. However, it is not easy, since the satisfaction and dissatisfaction depend on the inconsistency in the given comparisons. Instead, it can be done by minimizing the deviation of the most and the least satisfaction and dissatisfaction of all the decision makers as follows.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

$$
\begin{aligned}
& \min \quad \lambda(\overline{\alpha}-\underline{\alpha})+(1-\lambda)(\overline{\beta}-\underline{\beta}) \\
s.t. \quad & \text{Constraints in (14)}, \\
& \underline{\beta} \le \sum_i \beta_{ki} \le \overline{\beta}, \forall k, \\
& \underline{\alpha} \le \sum_i \alpha_{ki} \le \overline{\alpha}, \forall k, \\
& \sum_i \beta_{ki} \le \sum_i \beta^*_{ki}, \forall k, \\
& \sum_i \alpha_{ki} \ge \sum_i \alpha^*_{ki}, \forall k,
\end{aligned} \tag{16}
$$

where $\alpha^*_{ki}, \forall k, i$ and $\beta^*_{ki}, \forall k, i$ are the optimal solutions of Equation 14. The sums of the satisfaction and dissatisfaction are maximized and minimized primarily in Equation 14 and under the condition the maximum deviations among the satisfaction and dissatisfaction of all decision makers are minimized in Equation 16. As a result, the group decision considers the fairness of the individual decision makers when some decision maker gives more inconsistent comparisons than the others.

4. Numerical example

Three decision makers give the following crisp comparison matrices of 4 alternatives independently.

$$
A_1 = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1/2 & 1 & 2 & 4 \\ 1/4 & 1/2 & 1 & 2 \\ 1/8 & 1/4 & 1/2 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 3 & 3 & 4 \\ 1/3 & 1 & 3 & 3 \\ 1/3 & 1/3 & 1 & 4 \\ 1/4 & 1/3 & 1/4 & 1 \end{bmatrix},
$$
$$
A_3 = \begin{bmatrix} 1 & 1 & 4 & 6 \\ 1 & 1 & 1 & 2 \\ 1/4 & 1 & 1 & 3 \\ 1/6 & 1/2 & 1/3 & 1 \end{bmatrix}. \tag{17}
$$

By Equation 3, each decision is obtained as the following interval weights of alternatives.

$$
W_1 = \begin{bmatrix} 0.533 \\ 0.267 \\ 0.133 \\ 0.067 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.571 \\ [0.190, 0.214] \\ [0.071, 0.190] \\ [0.048, 0.143] \end{bmatrix},
$$
$$
W_3 = \begin{bmatrix} 0.390 \\ [0.244, 0.390] \\ [0.098, 0.244] \\ [0.065, 0.122] \end{bmatrix}. \tag{18}
$$

All the decision makers evaluate alternative 1 the best and agree with the weights of alternatives 3 and 4 as 0.133 and 0.067, respectively. They are the common weights of all individual weights.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

In comparison, the crisp weights are obtained by geometric mean method, where the weight is obtained as a geometric mean of the comparisons, $w_{ki} = \left(\prod_j a_{kij}\right)^{1/n} / \sum_i \left(\prod_j a_{kij}\right)^{1/n}$.

$$w_1 = \begin{bmatrix} 0.533 \\ 0.267 \\ 0.133 \\ 0.067 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 0.494 \\ 0.265 \\ 0.165 \\ 0.048 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 0.467 \\ 0.251 \\ 0.196 \\ 0.086 \end{bmatrix}. \tag{19}$$

The comparisons by decision maker 1, A_1, are consistent as in Equation 2 so that his/her decision, W_1, is denoted as crisp weights. They are equal to those by geometric mean and eigenvector methods. As for the other decision makers, A_2 and A_3, the rough rankings by Equations 18 and 19 are not contradicted. When compared to the plausible evaluations by the crisp weights in Equation 19, the interval weights by Interval AHP in Equation 18 show us the possible evaluations. It is possible that alternative 3 in A2 is evaluated as less important by the interval weights than by the crisp weights. Similarly in A3, we find that alternative 2 may be evaluated as equal to alternative 1. The given comparisons are condensed into the crisp weights for the plausible decision. The interval weights give the possible decision including the given comparisons. The interval weights are useful enough to aid in decision making, since a decision is often at the decision maker's discretion.

Since the individual decision reflects all the possibilities in the given comparisons, it is reasonable that the weight for the dissatisfaction, $1-\lambda$, is more than that of the satisfaction, λ. Then, λ is supposed to be 0.1 and 0.4, where $1-\lambda > \lambda$, and in addition 0.9. First, Equation 14 is solved and the satisfaction and dissatisfaction of each decision maker are obtained. Then, Equation 16 is solved within them and the group interval weights are obtained.

$$\begin{aligned} W(\lambda = 0.1) &= \begin{bmatrix} [0.390, 0.571] \\ [0.214, 0.267] \\ [0.133, 0.200] \\ [0.067, 0.143] \end{bmatrix} \text{ (width: } 0.377), \\ W(\lambda = 0.4) &= \begin{bmatrix} [0.390, 0.571] \\ [0.214, 0.267] \\ [0.098, 0.200] \\ [0.066, 0.143] \end{bmatrix} \text{ (width: } 0.413), \\ W(\lambda = 0.9) &= \begin{bmatrix} [0.390, 0.571] \\ [0.190, 0.390] \\ [0.071, 0.244] \\ [0.048, 0.143] \end{bmatrix} \text{ (width: } 0.649). \end{aligned} \tag{20}$$

In comparison, the group crisp weights as the geometric means of the corresponding three individual crisp weights in Equation 19 are as follows.

ISSN 1936-6744
http://dx.doi.org/10.13033/ijahp.v7i3.277

$$w = \begin{bmatrix} 0.497 \\ 0.261 \\ 0.163 \\ 0.076 \end{bmatrix} \tag{21}$$

Since the given comparisons are the same, the group weights by two methods are similar and the crisp group weights in Equation 21 are included in the interval group weights in Equation 20. The interval group weights show us the possibilities of evaluations from the individual decisions. We find that the evaluation of alternative 1 is the most uncertain because of its wide width. As for the relationship between alternatives 3 and 4, there is a possibility that alternative 4 is evaluated better than alternative 3.

The individual weights of alternative 1 by all decision makers are crisp in Equation 18 so that his/her satisfaction cannot be more than 0 and his/her dissatisfaction is equal to the width of its group interval weight, 0.181, which does not depend on λ. When the satisfaction is primarily maximized with $\lambda = 0.9$, all the individual interval weights are included in the group interval weight with the minimum width. The group decision is from their minimum to their maximum, $W_i = [\min\{\underline{w}_{1i}, \underline{w}_{2i}, \underline{w}_{3i}\}, \max\{\overline{w}_{1i}, \overline{w}_{2i}, \overline{w}_{3i}\}]$. When the importance of the satisfaction, λ, increases from 0.1 to 0.9, the widths of the group interval weights become wide from 0.377 to 0.649. The group decision becomes uncertain so as to satisfy the decision makers by increasing the supported ranges. From the opposite viewpoint of the dissatisfaction, when its importance, $1 - \lambda$, increases from 0.1 to 0.9, the width of the group interval weight decreases from 0.649 to 0.377 by reducing the unsupported ranges. The group interval weights with various importance of satisfaction λs are not very different.

In cases where there are two kinds of the importance of satisfaction and dissatisfaction $(\lambda, 1 - \lambda)$ as $(0.1, 0.9)$ and $(0.4, 0.6)$, the satisfaction and dissatisfaction of decision maker k, α_k and β_k, respectively, are compared in Equation 22.

$$\begin{aligned} &(0.1,0.9): (\alpha_1, \alpha_2, \alpha_3) = (0, 0.133, 0.145), (\beta_1, \beta_2, \beta_3) = (0.377, 0.244, 0.232), \\ &(0.4,0.6): (\alpha_1, \alpha_2, \alpha_3) = (0, 0.169, 0.181), (\beta_1, \beta_2, \beta_3) = (0.413, 0.244, 0.232). \end{aligned} \tag{22}$$

With $(\lambda, 1 - \lambda) = (0.1, 0.9)$, the sum of the satisfaction and dissatisfaction of each decision maker is 0.377 and that with $(\lambda, 1 - \lambda) = (0.4, 0.6)$ is 0.413. They are equal to the sums of the widths of the group interval weights as in Equation 13 and Figure 1. The importance of the satisfaction, λ, increases from 0.1 to 0.4, the satisfaction of decision makers 2 and 3 also increase by 0.036. Correspondingly, the dissatisfaction of decision maker 1, whose satisfaction cannot go over 0, increases by 0.036 since the importance of the dissatisfaction, $1 - \lambda$, decreases from 0.9 to 0.6. There is a trade-off between the increase of the satisfaction and the decrease of the dissatisfaction.

5. Conclusion

The individual and group decisions in this study are both denoted as the normalized interval weights of alternatives. At first, the individual decisions are independently obtained from the crisp comparisons given by the corresponding decision makers. Then, the group decision is obtained so as to have something in common with each individual decision. The proposed model minimizes the deviations of the upper and lower bounds of the group interval weight from those of each individual interval weight. Therefore, the obtained group decision is the conjunction approximation of the individual decisions. A decision maker supports a part of the group interval weight but s/he does not support the other part of it. In other words, the group interval weight is divided into the ranges satisfying and dissatisfying a decision maker. For a better group decision, the satisfaction and dissatisfaction of all decision makers should be maximized and minimized, respectively. However, since these two objectives have a trade-off relationship, they are aggregated by the weighting approaches. As the extreme case of the conjunction approximation, the group decision can be obtained as the upper or lower approximation of the individual decisions. The former group decision includes all the individual decisions by primarily maximizing the satisfaction. The latter group decision is included in all the individual decisions by primarily minimizing the dissatisfaction. The group interval weights reflect the possibilities in the given crisp comparisons and the relationship among the individuals. Since they show us the possible evaluations of alternatives with the satisfaction or dissatisfaction degree, they are useful in decision making.

In this paper, the group interval weights are obtained as the approximation of the individual interval weights from the crisp comparison matrices. In future work, the group weight approximation process can be combined into determining the individual weights from the comparisons. Moreover, the comparisons can be extended into interval and fuzzy to be more suitable for our intuitive judgments.

REFERENCES

Aczel, J., & Saaty, T. L. (1983). Procedure for synthesizing ratio judgments. *Journal of Mathematical Psychology*, *27*, 93-102. doi:10.1016/0022-2496(83)90028-7

Arbel, A. (1989). Approximate articulation of preference and priority derivation. *European Journal of Operational Research, 43*, 317-326. doi: 10.1016/0377-2217(89)90231-2

Altuzarra, A., Moreno-Jimenez, J. M., & Salvador, M. (2007). A Bayesian priorization procedure for AHP-group decision making. *European Journal of Operational Research, 182(1),* 364-382. doi:10.1016/j.ejor.2006.07.025

Basak, I., & Saaty, T. L. (1993). Group decision making using Analytic Hierarchy Process. *Mathematical and Computer Modelling, 17 (4/5),* 101-109. doi:10.1016/0895-7177(93)90179-3

de Campos, L. M., Huete, J. F., & Moral, S. (1994). Probability intervals: a tool for uncertain reasoning. *International Journal of Uncertainty, 2 (2),* 167-196. doi: 10.1142/S0218488594000146

E. Dopazo, K. L., Chouinard, S., & Guisse, J. (2014). A parametric model for determining consensus priority vectors from fuzzy comparison matrices. *Fuzzy Sets and Systems, 246*, 49-61. doi:10.1016/j.fss.2013.07.022

Dyer, R. F., & Forman, E. H. (1992). Group decision support with the Analytic Hierarchy Process. *Decision Support Systems, 8,* 94-124.

Entani, T., & Inuiguchi, M. (2010). Lower approximations of group of intervals. *Proceedings of SCIS and ISIS*, 9-14.

Forman, E., & Peniwati, K. (1998). Aggregating individual judgments and priorities with the Analytic Hierarchy Process. *European Journal of Operational Research, 108,* 165-169. doi: http://dx.doi.org/10.1016/S0377-2217(97)00244-0

Saaty, T. L. (1980). The Analytic Hierarchy Process. New York: McGraw-Hill.

Saaty, T. L., & Vargas L. G. (1987). Uncertainty and rank order in the analytic hierarchy process. *European Journal of Operational Research, 32*, 107-117. doi:10.1016/0377-2217(87)90275-X

Sugihara, K., Ishii, H., & Tanaka, H. (2004). Interval priorities in AHP by interval regression analysis. *European Journal of Operational Research, 158 (3),* 745-754.

Sugihara, K., & Tanaka, H. (2001). Interval evaluations in the Analytic Hierarchy Process by possibilistic analysis. *Computational Intelligence, 17 (3),* 567-579.

Tanaka, H., Sugihara, K., & Maeda, Y. (2004). Non-additive measures by interval probability functions. *Information Sciences, 164,* 209-227. doi:10.1016/j.ins.2003.06.001

Xu, Z. (2013). Group decision making model and approach based on interval preference ordering. *Computers & Industrial Engineering, 64(3),* 797–803.

Yeh, C.-H., & Chang, Y.-H. (2009). Modeling subjective evaluation for fuzzy group multicriteria decision making. *European Journal of Operational Research, 194(2),* 464-473 doi:10.1016/j.ejor.2007.12.029