International Journal of Analysis and Applications
ISSN 2291-8639
Volume 12, Number 1 (2016), 71-79
http://www.etamaths.com

NEW WEIGHTED OSTROWSKI TYPE INEQUALITIES FOR MAPPINGS WHOSE nTH DERIVATIVES ARE OF BOUNDED VARIATION

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Abstract. We establish a new generalization of weighted Ostrowski type inequality for mappings of bounded variation. Spacial cases of this inequality reduce some well known inequalities. With the help of obtained inequality, we give applications for the kth moment of random variables.

1. Introduction

In 1938, Ostrowski established the integral inequality which is one of the fundemental inequalitıes of mathematic as follows (see, [22]):

Let $f:[a,b]\to\mathbb{R}$ be a differentiable mapping on (a,b) whose derivative $f^{'}:(a,b)\to\mathbb{R}$ is bounded on (a,b), i.e., $\|f'\|_\infty=\sup\limits_{t\in(a,b)}|f'(t)|<\infty$. Then, the inequality holds:

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)dt\right|\leq\left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\|f'\|_\infty \tag{1.1}$$

for all $x\in[a,b]$. The constant $\frac{1}{4}$ is the best possible.

This inequality is well known in the literature as the *Ostrowski inequality.*

The inequality (1.1) has attracted remarkable attention from mathematicians and researchers. Because of this, over the years researchers have devoted much time and effort to the improvement and generalization of (1.1) for several functions (bounded function, function of bounded variation, etc.).

Firstly, we start introducing concept of bounded variation:

Definition 1. *Let $P: a=x_0<x_1<...<x_n=b$ be any partition of $[a,b]$ and let $\Delta f(x_i)=f(x_{i+1})-f(x_i)$ Then $f(x)$ is said to be of bounded variation if the sum*

$$\sum_{i=1}^{n}|\Delta f(x_i)|$$

is bounded for all such partitions.

Let f be of bounded variation on $[a,b]$, and $\sum(P)$ denotes the sum $\sum\limits_{i=1}^{n}|\Delta f(x_i)|$ corresponding to the partition P of $[a,b]$. The number

$$\bigvee_a^b(f):=\sup\left\{\sum(P):P\in P([a,b])\right\},$$

is called the total variation of f on $[a,b]$. Here $P([a,b])$ denotes the family of partitions of $[a,b]$.

A similar result (1.1) is obtained by Dragomir in [14] for functions of bounded variation as follow:

2010 *Mathematics Subject Classification.* 26D07, 26D15, 26A45.

Key words and phrases. Ostrowski inequality; function of bounded variation; Riemann-Stieltjes integral; random variable.

Theorem 1. *Let $f:[a,b]\to\mathbb{R}$ be a mapping of bounded variation on $[a,b]$. Then*

$$\left|\int_a^b f(t)dt-(b-a)f(x)\right|\leq\left[\frac{1}{2}(b-a)+\left|x-\frac{a+b}{2}\right|\right]\bigvee_a^b(f) \tag{1.2}$$

holds for all $x\in[a,b]$. The constant $\frac{1}{2}$ is the best possible.

For recent new results regarding Ostrowski's type inequalities for functions of bounded variation see [3],[7],[9]-[11], [13]-[19], [21], [25].

In [20], Liu proved the following weighted Ostrowski type inequality for functions of bounded variation:

Theorem 2. *Let $f:[a,b]\to\mathbb{R}$ be a mapping of bounded variation, $g:[a,b]\to(0,\infty)$ continious and positive mapping on (a,b). Then for any $x\in[a,b]$ and $\alpha\in[0,1]$ we have*

$$\begin{aligned}&\left|(1-\alpha)\left(\int_a^b g(u)du\right)f(x)\right.\\&\left.+\alpha\left[\left(\int_a^x g(u)du\right)f(a)+\left(\int_x^b g(u)du\right)f(b)\right]-\int_a^b f(t)g(t)dt\right|\\ \leq\;&\left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\left[\frac{1}{2}\int_a^b g(u)du+\left|\int_a^x g(u)du-\frac{1}{2}\int_a^b g(u)du\right|\right]\bigvee_a^b(f)\end{aligned} \tag{1.3}$$

where $\bigvee_a^b(f)$ is the total variation of f on the interval $[a,b]$. The constant $\left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]$ is the best possible.

In [5], Budak and Sarıkaya gave the following weighted Ostrowski's type inequalities for mapping of bounded variation.

Theorem 3. *Let $I_n: a=x_0<x_1<...<x_n=b$ be a division of the interval $[a,b]$ and α_i $(i=0,1,...,n+1)$ be $n+2$ points so that $\alpha_0=a$, $\alpha_i\in[x_{i-1},x_i]$ $(i=1,...,n)$, $\alpha_{n+1}=b$. If $f:[a,b]\to\mathbb{R}$ is of bounded variation on $[a,b]$ and $w:[a,b]\to(0,\infty)$ be continious and positive mapping on (a,b), then we have the inequalities:*

$$\begin{aligned}&\left|\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_a^b f(t)w(t)dt\right|\\ \leq\;&\left[\frac{1}{2}v(L)+\max_{i\in\{0,1,\dots,n-1\}}\frac{1}{2}\left|\int_{x_i}^{\alpha_{i+1}}w(u)du-\int_{\alpha_{i+1}}^{x_{i+1}}w(u)du\right|\right]\bigvee_a^b(f)\\ \leq\;&v(L)\bigvee_a^b(f)\end{aligned} \tag{1.4}$$

where $v(L):=\max\{L_i|\ i=0,...,n-1\}$, $L_i=\int_{x_i}^{x_{i+1}}w(u)du$ $(i=0,1,...,n-1)$ and $\bigvee_a^b(f)$ is the total variation of f on the interval $[a,b]$.

A weighted generalization of trapezoid inequality for mappings of bounded variation was considered by Tseng et. al. [24]. Recently, researchers gave some weigted Ostrowski type inequalities for functions of bounded variation in [5], [8], [26]. In [1] and [2], the authors proved some generalizations of weighted companion of Ostrowski type inequality for mappings of bounded variation

In this paper, we establish a generalized weighted Ostrowski type integral inequality for mappings whose nth derivatives are of bounded variation. Then,we recapture some results given in earlier works by using this inequalities. Finally, some applications for the kth moment are given.

2. Main Results

In order to prove weighted integral inequalities, we need the following lemma:

Lemma 1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be $n+1$ times differentiable function on I°, $a, b \in I^{\circ}$ with $a < b$ and let $w : [a, b] \to \mathbb{R}$ be nonnegative and continuous on $[a, b]$. Then the following equality holds:*

$$\sum_{k=0}^{n} \frac{M_k(x)}{k!} f^{(k)}(x) - \int_a^b w(t) f(t)\, dt = \int_a^b P_w(x,t)\, df^{(n)}(t) \tag{2.1}$$

where $n \in \mathbb{N}$, $M_k(x)$ is defined by

$$M_k(x) = \int_a^b (u-x)^k w(u)\, du, \quad k = 0, 1, 2, ...$$

and

$$P_n(x,t) := \begin{cases} \frac{1}{n!} \int_a^t (u-t)^n w(u)\, du, & a \le t < x \\ \\ \frac{1}{n!} \int_b^t (u-t)^n w(u)\, du, & x \le t \le b. \end{cases} \tag{2.2}$$

Proof. Using the integration by parts in Riemann-Stieltjes integral, we have

$$\begin{aligned} & \int_a^b P_n(x,t)\, df^{(n)}(t) \\ = \; & \frac{1}{n!} \int_a^x \left(\int_a^t (u-t)^n w(u)\, du \right) df^{(n)}(t) + \frac{1}{n!} \int_x^b \left(\int_b^t (u-t)^n w(u)\, du \right) df^{(n)}(t) \\ = \; & \frac{1}{n!} \left(\int_a^b (u-x)^n w(u)\, du \right) f^{(n)}(x) + \int_a^b P_{n-1}(x,t) f^{(n)}(t)\, dt \end{aligned}$$

By integration by parts $n-$times, we get

$$\begin{aligned} \int_a^b P_{n-1}(x,t) f^{(n)}(t)\, dt \quad = \quad & \frac{M_{n-1}(x)}{(n-1)!} f^{(n-1)}(x) + ... + \frac{M_2(x)}{2!} f''(x) \\ & + M_1(x) f'(x) + M_0(x) f(x) - \int_a^b w(t) f(t)\, dt \end{aligned}$$

which completes the proof. □

Now, we deduce generalized weighted inequality of Ostrowski type for mappings whose nth derivatives are of bounded variation..

Theorem 4. *Suppose that all the assumptions of Lemma 1 hold. Additionally, we assume that $f^{(n)}$ is of bounded variation on $[a,b]$, then we have the inequality*

$$\begin{aligned}
&\left|\sum_{k=0}^{n}\frac{M_k(x)}{k!}f^{(k)}(x)-\int_a^b w(t)f(t)\,dt\right| \\
&\leq \frac{1}{n!}\left[\frac{1}{2}\left[\int_a^x (x-u)^n w(u)\,du+\int_x^b (u-x)^n w(u)\,du\right]\right. \\
&\quad \left.+\frac{1}{2}\left|\int_a^x (x-u)^n w(u)\,du-\int_x^b (u-x)^n w(u)\,du\right|\right]\bigvee_a^b(f^{(n)})
\end{aligned}$$

for all $x\in[a,b]$.

Proof. If we take absolute value of both sides of the equality (2.1), we get

$$\begin{aligned}
&\left|\sum_{k=0}^{n}\frac{M_k(x)}{k!}f^{(k)}(x)-\int_a^b w(t)f(t)\,dt\right| \\
&\leq \frac{1}{n!}\left[\left|\int_a^x\int_a^t (t-u)^n w(u)\,du df^{(n)}(t)\right|+\frac{1}{n!}\left|\int_x^b\int_t^b (u-t)^n w(u)\,du df^{(n)}(t)\right|\right].
\end{aligned}$$

It is well known that if $g,f:[a,b]\to\mathbb{R}$ are such that g is continuous on $[a,b]$ and f is of bounded variation on $[a,b]$, then $\int_a^b g(t)df(t)$ exists and

$$\left|\int_a^b g(t)df(t)\right|\leq\sup_{t\in[a,b]}|g(t)|\bigvee_a^b(f). \tag{2.3}$$

On the other hand, by using (2.3), we obtain

$$\begin{aligned}
&\left|\sum_{k=0}^{n}\frac{M_k(x)}{k!}f^{(k)}(x)-\int_a^b w(t)f(t)\,dt\right| \\
&\leq \frac{1}{n!}\left[\sup_{t\in[a,x]}\left|\int_a^t (t-u)^n w(u)\,du\right|\bigvee_a^x(f^n)+\sup_{t\in[x,b]}\left|\int_t^b (u-t)^n w(u)\,du\right|\bigvee_x^b(f^{(n)})\right] \\
&= \frac{1}{n!}\left(\int_a^x (x-u)^n w(u)\,du\right)\bigvee_a^x(f^n)+\frac{1}{n!}\left(\int_x^b (u-x)^n w(u)\,du\right)\bigvee_x^b(f^{(n)}) \\
&\leq \frac{1}{n!}\left[\frac{1}{2}\left[\int_a^x (x-u)^n w(u)\,du+\int_x^b (u-x)^n w(u)\,du\right]\right. \\
&\quad \left.+\frac{1}{2}\left|\int_a^x (x-u)^n w(u)\,du-\int_x^b (u-x)^n w(u)\,du\right|\right]\bigvee_a^b(f^{(n)}).
\end{aligned}$$

This completes the proof. □

Remark 1. *If we take $w(u)=1$ and $n=0$ in Theorem 4, then we get the clasical Ostrowski inequality (1.2) for function of bounded variation .*

Remark 2. *If we choose $n=1$ in Theorem 4, then we obtain*

$$
\begin{aligned}
&\left|\left(\int_a^b (x-u)\,w(u)\,du\right) f'(x)+\left(\int_a^b w(u)\,du\right) f(x)-\int_a^b w(t)\,f(t)\,dt\right| \\
\leq\ &\left[\frac{1}{2}\left[\int_a^x (x-u)\,w(u)\,du+\int_x^b (u-x)\,w(u)\,du\right]\right. \\
&\left.+\frac{1}{2}\left|\int_a^x (x-u)\,w(u)\,du-\int_x^b (u-x)\,w(u)\,du\right|\right]\bigvee_a^b (f')
\end{aligned}
$$

which was given by Budak and Sarikaya in [6].

Remark 3. *If we choose $n=0$ in Theorem 4, then we have the inequality*

$$
\begin{aligned}
&\left|\left(\int_a^b w(t)\,dt\right) f(x)-\int_a^b w(t)\,f(t)\,dt\right| \\
\leq\ &\left[\frac{1}{2}\int_a^b w(t)\,dt+\left|\int_a^x w(t)dt-\frac{1}{2}\int_a^b w(t)\,dt\right|\right]\bigvee_a^b (f)
\end{aligned}
$$

which was proved by Liu. in [20].

Corollary 1. *With the assumptions as in Theorem 4, we have the result*

$$
\begin{aligned}
&\left|\sum_{k=0}^{n}\frac{(b-x)^{k+1}-(a-x)^{k+1}}{(k+1)!} f^{(k)}(x)-\int_a^b f(t)\,dt\right| \\
\leq\ &\frac{1}{(n+1)!}\left[\frac{1}{2}(b-a)+\left|x-\frac{a+b}{2}\right|\right]^{n+1}\bigvee_a^b (f^{(n)})
\end{aligned}
\tag{2.4}
$$

Proof. The proof is obvious from the property of maximum $\max\{a^n,b^n\}=(\max\{a,b\})^n$ for $a,b>0$, $n\in\mathbb{N}$, if we take $w(u)=1$. □

Remark 4. *If we choose $n=1$ in Corollary 1, we have the inequality*

$$
\begin{aligned}
&\left|\left(\frac{a+b}{2}-x\right) f'(x)+f(x)-\frac{1}{b-a}\int_a^b f(t)dt\right| \\
\leq\ &\frac{b-a}{2}\left[\frac{1}{2}+\left|\frac{x-\frac{a+b}{2}}{b-a}\right|\right]^2\bigvee_a^b (f')
\end{aligned}
$$

which was given by Budak and Sarikaya in [6].

Corollary 2. *In (2.4), if we choose,*

i) $x=\frac{a+b}{2}$, then we have

$$
\left|\sum_{k=0}^{n}\frac{(b-a)^{k+1}\left[1+(-1)^k\right]}{2^{k+1}(k+1)!} f^{(k)}\left(\frac{a+b}{2}\right)-\int_a^b f(t)\,dt\right|\leq\frac{(b-a)^{n+1}}{2^{n+1}(n+1)!}\bigvee_a^b (f^{(n)}),
\tag{2.5}
$$

ii) $x=a$, then we have

$$
\left|\sum_{k=0}^{n}\frac{(b-a)^{k+1}}{(k+1)!} f^{(k)}(a)-\int_a^b f(t)\,dt\right|\leq\frac{(b-a)^{n+1}}{(n+1)!}\bigvee_a^b (f^{(n)}),
$$

iii) $x=b$, *then we have*

$$\left|\sum_{k=0}^{n}\frac{(-1)^k(b-a)^{k+1}}{(k+1)!}f^{(k)}(b)-\int_a^b f(t)\,dt\right|\leq\frac{(b-a)^{n+1}}{(n+1)!}\bigvee_a^b(f^{(n)}).$$

Remark 5. *If we choose* $n=1$ *in (2.5), then we have the inequalities*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\leq\frac{b-a}{8}\bigvee_a^b(f'),$$

which was given by Liu in [21].

Corollary 3. *Under the assumption of Theorem 4. Suppose that* $f\in C^{n+1}[a,b]$, *then we have*

$$\begin{aligned}&\left|\sum_{k=0}^{n}\frac{M_k(x)}{k!}f^{(k)}(x)-\int_a^b w(t)f(t)\,dt\right|\\ \leq\;&\frac{1}{n!}\left[\frac{1}{2}\left[\int_a^x(x-u)^n w(u)\,du+\int_x^b(u-x)^n w(u)\,du\right]\right.\\ &\left.\frac{1}{2}\left|\int_a^x(x-u)^n w(u)\,du-\int_x^b(u-x)^n w(u)\,du\right|\right]\left\|f^{(n+1)}\right\|_1.\end{aligned}$$

Here as subsequently $\|.\|_1$ *is the* L_1-*norm*

$$\left\|f^{(n+1)}\right\|_1:=\int_a^b f^{(n+1)}(t)dt.$$

Corollary 4. *Under the assumption of Theorem 4. Let* $f^{(n)}$ *be a Lipschitzian with the constant* $L>0$. *Then the inequality holds:*

$$\begin{aligned}&\left|\sum_{k=0}^{n}\frac{M_k(x)}{k!}f^{(k)}(x)-\int_a^b w(t)f(t)\,dt\right|\\ \leq\;&\frac{1}{n!}\left[\frac{1}{2}\left[\int_a^x(x-u)^n w(u)\,du+\int_x^b(u-x)^n w(u)\,du\right]\right.\\ &\left.\frac{1}{2}\left|\int_a^x(x-u)^n w(u)\,du-\int_x^b(u-x)^n w(u)\,du\right|\right](b-a)L\end{aligned}$$

Corollary 5. *Under the assumption of Theorem 4. Let* $f^{(n)}$ *be a monotone mapping on* $[a,b]$. *Then we have*

$$\begin{aligned}&\left|\sum_{k=0}^{n}\frac{M_k(x)}{k!}f^{(k)}(x)-\int_a^b w(t)f(t)\,dt\right|\\ \leq\;&\frac{1}{n!}\left[\frac{1}{2}\left[\int_a^x(x-u)^n w(u)\,du+\int_x^b(u-x)^n w(u)\,du\right]\right.\\ &\left.\frac{1}{2}\left|\int_a^x(x-u)^n w(u)\,du-\int_x^b(u-x)^n w(u)\,du\right|\right]\left[f^{(n)}(b)-f^{(n)}(a)\right].\end{aligned}$$

3. Some applications for the moments

We now deal with applications of the result developed in the previous section, to obtain some new inequalities involving moments. Applying the mathematical inequalities, some estimations for the moments of random variables were recently studied (see, [4],[12],[18] and [23]).

Set X to denote a random variable whose probability density function is $w : [a,b] \to [0,\infty)$ on the interval of real numbers I $(a, b \in I,\ a < b)$. Denoted by $M_r(x)$ the rth central moment of the random variable X, defined as

$$M_r(x) = \int_a^b (u - E(x))^r \, w(u) \, du, \quad r = 0, 1, 2, ...$$

where $E(x)$ is the mean of the random variables X. It may be noted that $M_0(x) = 1$, $M_1(x) = 0$, $M_2(x) = \sigma^2(X)$ where $\sigma^2(X)$ is the variance of the random variables X.

Now, we reconsider the identity (3.1) by changing conditions given in Lemma 1. Herewith, we deduce an identity involving rth moment.

Lemma 2. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be $n+1$ times differentiable function on I°, $a, b \in I^\circ$ with $a < b$ and and let X be a random variable whose p.d.f. is $w : [a,b] \to [0,\infty)$. Then the following equality holds:*

$$\sum_{k=0}^{n} \frac{M_k(x)}{k!} f^{(k)}(x) - \int_a^b w(t) f(t) \, dt = \int_a^b P_w(x,t) \, df^{(n)}(t) \tag{3.1}$$

where $n \in \mathbb{N}$, $M_k(x)$ is the kth moment, and $P_n(x,t)$ is defined as in (2.2).

Theorem 5. *Suppose that all the assumptions of Lemma 2 hold. If $f^{(n)}$ is of bounded variation on $[a,b]$, then we have the inequality*

$$\begin{aligned} &\left| \sum_{k=0}^{n} \frac{M_k(x)}{k!} f^{(k)}(x) - \int_a^b w(t) f(t) \, dt \right| \\ \leq\ & \frac{1}{n!} \left[\frac{b-a}{2} + \left| x - \frac{a+b}{2} \right| \right]^n \bigvee_a^b (f^{(n)}) \end{aligned}$$

for all $x \in [a,b]$.

Proof. By similar methods in the proof of Theorem 4, we obtain

$$\begin{aligned} &\left| \sum_{k=0}^{n} \frac{M_k(x)}{k!} f^{(k)}(x) - \int_a^b w(t) f(t) \, dt \right| \\ \leq\ & \frac{1}{n!} \left(\int_a^x (x-u)^n w(u) \, du \right) \bigvee_a^x (f^n) + \frac{1}{n!} \left(\int_x^b (u-x)^n w(u) \, du \right) \bigvee_x^b (f^{(n)}) \\ \leq\ & \left[\int_a^x \frac{(x-u)^n}{n!} w(u) \, du + \int_x^b \frac{(u-x)^n}{n!} w(u) \, du \right] \bigvee_a^b (f^n) \end{aligned}$$

We observe that

$$\begin{aligned}
&\int_a^x \frac{(x-u)^n}{n!} w(u)du + \int_x^b \frac{(u-x)^n}{n!} w(u)du \\
\leq\ & \frac{1}{n!}\left[\sup_{u\in[a,x]} (x-u)^n \int_a^x w(u)du + \sup_{u\in[x,b]} (u-x)^n \int_x^b w(u)du\right] \\
=\ & \frac{1}{n!}\left[(x-a)^n \int_a^x w(u)du + (b-x)^n \int_x^b w(u)du\right] \\
\leq\ & \frac{1}{n!} \max\left\{(x-a)^n, (b-x)^n\right\} \int_a^b w(u)du
\end{aligned}$$

Because g is a p.d.f., $\int_a^b w(u)du = 1$. Using the identity

$$\max\{X, Y\} = \frac{X+Y}{2} + \left|\frac{Y-X}{2}\right|,$$

we get

$$\max\left\{(x-a)^n, (b-x)^n\right\} \int_a^b g(u)du = \left[\frac{b-a}{2} + \left|x - \frac{a+b}{2}\right|\right]^n.$$

which completes the proof. □

Remark 6. *If we choose $n = 1$ in theorem 7, we have the inequality*

$$\left| f(x) - \int_a^b w(t) f(t)\, dt \right| \leq \left[\frac{b-a}{2} + \left|x - \frac{a+b}{2}\right|\right] \bigvee_a^b (f').$$

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