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Topological Evaluation of Four Para-Line Graphs Absolute Pentacene Graphs Using Topological Indices

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Abstract. A real-number to molecular structure mapping is a topological index. It is a graph invariant method for describing physico-chemical properties of molecular structures specific substances. In that article, We examined pentacene's chemical composition. The research on the subsequent indices is reflected in our paper, we conducted an analysis of several indices including general randic connectivity index, first general zagreb index, general sum-connectivity index, atomic bond connectivity index, geometric-arithmetic index, fifth class of geometric-arithmetic indices, hyper-zagreb index, first and second multiple zagreb indices for a four para-lines graphs of linear [n]-pentacene and multi-pentacene.

1. Introduction and preliminaries

All substances molecule possesses qualities, both chemical and physical, and certain may also exhibit physiologically active characteristics. Several pharmaceutical companies are really hunting for novel antibacterial chemicals. For this reason, hundreds of compounds are examined, however costly examinations for biology. In order to circumvent such issue, additional methods for investigating potential antibiotics employ the relationship between structural features and biological activity or features of chemical and physical nature. Topological indices, or molecular descriptors, provide insights into the physicochemical properties of molecules. They are valuable tools for understanding and explaining

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the characteristics of chemical compounds. Several graph invariants have been created in recent years and have been used in many academic fields such as structural chemistry, theoretical chemistry, environmental chemistry, toxicology, and pharmacology. Because of the substantial industrial need, researchers are urged to study topological indices. More than 400 topological indexes have been opened a consequence of research. Chemical compounds' topological structures and chemical characteristics are tightly related, since each compound's shape is critical to determining its functionality. Topological indices are often used in multilinear regression modelling, chemical documentation, drug design, QSAR/QSPR modelling, and database selection. Molecular descriptors are utilized to describe the physicochemical properties of molecular structures. These descriptors can be classified into three main types. degree-based indices [1–5], distance-based indices [6–11] and spectrum-based indices [12–15]. Studies that have been documented in the literature (see [16–18]) use indicators that are based on both distances and degrees.

Due to pentacene's important functions in both electrical devices and organic solar cells, a popular hydrocarbon semiconductor, it is necessary to optimise organic solar cells for less expensive energy sources [19]. The Georgia Institute of Technology researchers have developed method to produce portable artificial solar cells. Pentacene has been shown to be a very efficient means of converting sunlight into energy. In contrast to other materials, pentacene functions well as a semiconductor due to its crystalline properties. Pentacene's relevance motivated us to do topological study on it, and as a result, we have made several important discoveries that could be helpful for analysing pentacene's physical and chemical characteristics. See [20, 21] for further topological research on pentacene.

Consider an easy graph G consisting of a edge set $E(G)$ and vertex set $V(G)$, where loops and several edges present are excluded. The set $x \in V(G)$, N_x of neighbors in G is represented by N_x, and the valence (degree) of x is equal to $d_{x_1} = |N_x|$ and $S_{x_1} = \sum_{y \in N_x} d_{x_2}$. By inserting a vertex between every edge of the given graph, the edges are divided into two, resulting in the graph being subdivided. This operation, known as graph subdivision and denoted as $S(G)$, leads to the formation of a line graph where adjacent edges in G become connected vertices in the new graph. The resulting graph, denoted as $L(G)$, represents the line graph of the subdivision graph. In this article, the four para-line graph of G is represented by $L(S(G))$ (referred to as $G^\star$). Conversely, $G^\star$ can be constructed from G using the following procedure:

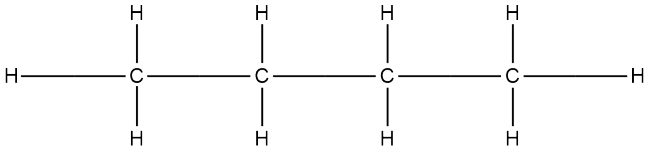
1. Replace each vertex $x_1 \in V(G)$ with K_{x_1}, complete graph on d_{x_1} vertices;

2. There is an edge connecting the vertex K_{x_1} and the vertex K_{x_2} in $G^\star$ if and only if there is an edge that coincides with x_1 and x_2 in G;

3. For each vertex x_2 in K_{x_1}, in $G^\star$, the valency (degree) of $_{x_2}$ is equal to the valency (degree) of x_1 in G.

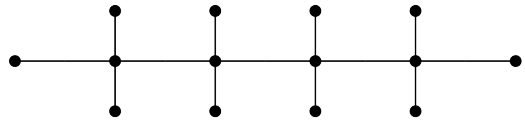
Structural chemistry commonly utilizes these diagrams. The research focus on four para-line graphs has diminished in recent times, but there is a shift happening. One appealing aspect of these graphs

is their straightforward construction process. The carbon skeleton, in which each atom acts as the vertex and each link between nearby atoms as the edge, may be used to generate any chemical compound. For example, Butane is an organic compound with the formula (C_4H_{10}). Butane is a saturated hydrocarbon containing 4 of carbon atoms, with an unbranched structure. Butane is mainly used as a gasoline blend, alone or mixed with propane. It is also used as a feedstock for the production of ethylene and butadiene. Butane, like propane, is obtained from natural gas or refineries, and the two gases usually occur together. Butane is stored under pressure as a liquid. When the curler is turned on, butane is released and turns into a gas. Figure 1(a) depicts the the molecular graph and its structure of butane. Furthermore, Figure 2(b) and (c) exhibit the four para-line graphs derived from the molecular plot of butane. now figure is;

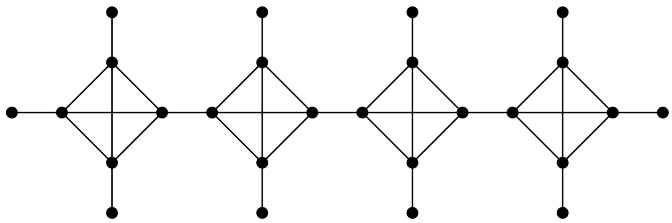


(a)

Figure 1. (a) The molecular architecture of butane



(b)



(c)

Figure 2. (b) The molecular architecture of butane (c) four para-line graph of butane to accurately represent it

The general randic connectivity index G is defined as [12].

$$R_\alpha(G) = \sum_{x_1x_2 \in E(G)} (d_{x_1} d_{x_2})^\alpha \tag{1.1}$$

The first universal Zagreb index was presented by Li and Zhao [22]:

$$M_\alpha(G) = \sum_{x_1 \in V(G)} (d_{x_1})^\alpha \tag{1.2}$$

The the general sum connectivity index of the G chart was introduced in 2010 [23]:

$$\chi_\alpha(G) = \sum_{x_1x_2 \in E(G)} (d_{x_1} + d_{x_2})^\alpha \tag{1.3}$$

The index (ABC) was proposed by Estrada [24]. It is expressed as follows for a graph G:

$$ABC(G) = \sum_{x_1x_2 \in E(G)} \sqrt{\frac{d_{x_1} + d_{x_2} - 2}{d_{x_1} d_{x_2}}} \tag{1.4}$$

The geometric-arithmetic index (GA) was introduced by Vukicevic and Furtula [25]. It is denoted as GA and is defined as follows for a graph G resently A. Asghar et.al [31]:

$$GA(G) = \sum_{x_1x_2 \in E(G)} \frac{2\sqrt{(d_{x_1} d_{x_2})}}{(d_{x_1} + d_{x_2})} \tag{1.5}$$

Ghorbani et al. [26] described another index belonging to the 4th class of indices, denoted as (ABC), which is defined as follows resently Zaib Hassan Niazi et.al [32]:

$$ABC_4(G) = \sum_{x_1x_2 \in E(G)} \sqrt{\frac{d_{S_1} + S_{x_2} - 2}{S_{x_1} S_{x_2}}} \tag{1.6}$$

Graovac et al. [27] introduced a fifth class of geometric-arithmetic indices denoted as GA_5, which is defined as follows:

$$GA_5(G) = \sum_{x_1x_2 \in E(G)} \frac{2\sqrt{(S_{x_1} S_{x_2})}}{(S_{x_1} + S_{x_2})} \tag{1.7}$$

Established the hyper-zagreb index in 2013 as follows resently Mukhtar Ahmad et.al [33]:

$$HM(G) = \sum_{x_1x_2 \in E(G)} (d_{x_1} + d_{x_2})^2 \tag{1.8}$$

In 2012, Ghorbani and Azimi introduced two new types of zagreb graph indices. The first is the first multiple zagreb index, denoted as $PM_1(G)$. The second multiple zagreb index is used, denoted as $PM_2(G)$. Additionally, the first and second zagreb polynomials, $M_1(G, p)$ and $M_2(G, p)$, respectively, are characterised as:

$$PM_1(G) = \Pi_{x_1x_2 \in E(G)} (d_{x_1} + d_{x_2}) \tag{1.9}$$

$$PM_2(G) = \Pi_{x_1x_2 \in E(G)} (d_{x_1} \times d_{x_2}) \tag{1.10}$$

$$M_1(G, p) = \sum_{x_1x_2 \in E(G)} P^{(d_{x_1} + d_{x_2})} \tag{1.11}$$

$$M_2(G, p) = \sum_{x_1x_2 \in E(G)} P^{(d_{x_1} \times d_{x_2})} \tag{1.12}$$

2. **Topological index of four para-line graphs**

For an index that Schultz offered, Ranjini created the independent relations. Under the watchful eye of the Schultz index, these researchers looked at the subdivision of a number of graphs, including helm, ladder, tadpole, and wheel [28]. They also looked at the ladder, tadpole, and wheel four para-line graph under the zagreb index [29]. In 2015, Xu and Su conducted an analysis of two indices specific to ladder, tadpole, and wheel graphs constructed using tare lines and named the total connectivity index of the sum and the co-index [30]. Nadim et al. calculated the atomic bond connectivity index and fifth class of geometric arithmetic indices for four para-line tadpole, wheel, and ladder graphs. They also investigated several other indices, including randic general connectivity index, first zagreb general index, summation general connectivity index, atomic bond connectivity index, geometric arithmetic index, fifth class of geometric arithmetic indices, hyperzagreb index, the first and second multiple zagreb index for a four paralinear graphs of linear [n]-pentacene and multiple pentacene., lattice plot in nanotorus $TUC_4C_8[p,q]$ and 2D nanotube.

In our study, we computed various indices, including randic general connectivity index, first zagreb general index, summation general connectivity index, atomic bond connectivity index, geometric arithmetic index, fifth class of geometric arithmetic indices, hyperzagreb index.

2.1. **Molecular characteristics of the linear [n]-pentacene four para-line graph.** Figure 3 depicts the linear [n]-pentacene molecular graph, which is indicated by the symbol T_n. T_n consists of $28n-2$ edges and $22n$ vertices.

Theorem 2.1. Consider a four para-line graph $G^\star$ derived from the graph T_n.

$M_\alpha(G^\star) = (5n+2)2^{\alpha+2} + 3^{\alpha+1}(12n-4)$.

Proof. In Figure 3, the graph $G^\star$ is displayed. There are $56n-4$ vertices in total in $G^\star$, this has $36n-12$ vertices of degree and $20n+8$ vertices of degree, where

$M_\alpha(G^\star) = (5n+2)2^{\alpha+2} + 3^{\alpha+1}(12n-4)$.

Theorem 2.2 Consider a four para-line graph $G^\star$ derived from the graph T_n.

1. $R_\alpha(G^\star) = (10n+10)16^\alpha + (20n-4)20^\alpha + (44n-16)25^\alpha$.

2. $\chi_\alpha(G^\star) = (10n+10)8^\alpha + (20n-4)9^\alpha + (44n-16)10^\alpha$.

3. $ABC(G^\star) = (15\sqrt{2} + \frac{88}{3})n + 3\sqrt{2} - \frac{32}{3}$.

4. $GA(G^\star) = (54+8\sqrt{6})n - 6 - \frac{8}{5}\sqrt{6}$.

Proof. The total number of edges in $G^\star$ is determined by the formula $74n-10$. The edges in $G^\star$ can be divided into three sets, $E_1(G^\star)$, $E_2(G^\star)$, and $E_3(G^\star)$, which do not intersect with each other. The edge partition $E_1(G^\star)$ contains $10n+10$ edges x_1, x_2, where $d_{x_1} = d_{x_2} = 4$, edge the partition $E_2(G^\star)$ contains $20n-4$ edges x_1, x_2, where $d_{x_1} = 4$ and $d_{x_2} = 5$, and The edge partition $E_3(G^\star)$ consists of $44n-16$ edges. This partition includes edges x_1 and x_2, where $d_{x_1} = d_{x_2} = 5$. By utilizing we get the required outcomes using formulae (1), (3), (4), and (5).

Theorem 2.3 Consider a four para-line graph $G^\star$ derived from the graph T_n.

Figure 3. Linear Pentacene

1. $ABC_4(G^\star) = (\sqrt{110} + 4\sqrt{2} + 2\sqrt{30} + \frac{16}{3})n + \frac{5}{2} + \frac{2}{5} - \frac{8}{5}\sqrt{2} - \frac{2}{3}\sqrt{30} - \frac{1}{5}\sqrt{110} - \frac{32}{9}$

2. $GA_5(G^\star) = (30 + \frac{80}{13}\sqrt{10} + \frac{288}{17}\sqrt{2})n - 2 + \frac{16}{9}\sqrt{5} - \frac{16}{13}\sqrt{10} - \frac{96}{17}\sqrt{2}$

Proof. Assuming that the set of edges depends on the sum of the degrees of the neighbors of the end vertices, we can partition edges that divide $(G^\star)$ into seven distinct sets: $E_6(G^\star)$, $E_7(G^\star)$, ..., $E_{12}(G^\star)$. Thus, we have $E(G^\star) = \bigcup_{i=6}^{12} E_i(G^\star)$. The edge assortment $E_6(G^\star)$ comprises 12 edges x_1x_2, where $S_{x_1} = S_{x_2} = 6$, the edge collection $E_7(G^\star)$ holds 6 edges x_1x_2, where $S_{x_1} = 6$ and $S_{x_2} = 7$, the edge collection $E_8(G^\star)$ holds $11n - 5$ edges x_1x_2, where $S_{x_1} = S_{x_2} = 7$, set of edges $E_9(G^\star)$ contains $22n - 5$ edges x_1x_2, where $S_{x_1} = 7$ and $S_{x_2} = 10$, edge the collection $E_{10}(G^\star)$ contains $10n$ edges x_1x_2, where $S_{x_1} = S_{x_2} = 10$, the edge set $E_{11}(G^\star)$ contains $26n - 9$ edges x_1x_2, where $S_{x_1} = 10$ and $S_{x_2} = 11$ and the set of edges $E_{12}(G^\star)$ is satisfied $13n - 9$ edges x_1x_2, where $S_{x_1} = S_{x_2} = 11$. By utilizing we can get the required outcomes using formulae 6 and 7.

Theorem 2.4 Consider a four para-line graph $G^\star$ derived from the graph T_n

1. $HM(G) = 6480n - 1464$.

2. $PM_1(G^\star) = 8^{10n+10} \times 9^{20n-4} \times 10^{44n-16}$.

3. $PM_2(G) = 16^{10n+10} \times 20^{20n-4} \times 25^{44n-16}$.

Proof. Consider a four para-line graph $G^\star$ of a linear pentacene. Based on the angles of the final vertex, the collection of edges $E(G^\star)$ might be categorised as three distinct groups. The first category, $E_1(G^\star)$, consists of $10n+10$ edges x_1x_2, where $d_{x_1} = d_{x_2}$ =4. The second category, $E_2(G^\star)$, includes $20n-4$ edges x_1x_2, where $d_{x_1} = 4$ and d_{x_2} =5. The third category, $E_3(G^\star)$, comprises $44n-16$ edges x_1x_2, where $d_{x_1} = d_{x_2} = 5$. Let $|E_1(G)| = e_{4,4}$, $|E_2(G)| = e_{4,5}$, and $|E_3(G)| = e_{5,5}$. Therefore,

1. $HM(G) = \sum_{x_1x_2 \in E(G)} (d_{x_1} + d_{x_2})^2$

$HM(G) = \sum_{x_1x_2 \in E_1(G)} [d_{x_1} + d_{x_2}]^2 + \sum_{x_1x_2 \in E_2(G)} [d_{x_1} + d_{x_2}]^2 + \sum_{x_1x_2 \in E_3(G)} [d_{x_1} + d_{x_2}]^2$

$HM(G) = 64|E_1(G)| + 81|E_2(G)| + 100|E_3(G)|$

$HM(G) = 64(10n + 10) + 81(20n - 4) + 100(44n - 16)$

$HM(G) = 460n + 460 + 1620n - 324 + 4400n - 1600$

This implies that

$HM(G) = 6480n - 1464$.

2. $PM_1(G) = \Pi_{x_1x_2 \in E_1(G)} (d_{x_1} + d_{x_2}) \times \Pi_{x_1x_2 \in E_2(G)} (d_{x_1} + d_{x_2}) \times \Pi_{x_1x_2 \in E_3(G)} (d_{x_1} + d_{x_2})$

$PM_1(G) = 8^{|E_1(G)|} \times 9^{|E_2(G)|} \times 10^{|E_1(G)|}$

$PM_1(G) = 8^{10n+10} \times 9^{20n-4} \times 10^{44n-16}$

3. $PM_2(G) = \Pi_{x_1x_2\in E_1(G)}(d_{x_1} \times d_{x_2}) \times \Pi_{x_1x_2\in E_2(G)}(d_{x_1} \times (d_{x_2}) \times \Pi_{x_1x_2\in E_3(G)}(d_{x_1} \times d_{x_2})$
$PM_2(G) = 16^{|E_1(G)|} \times 20^{|E_2(G)|} \times 25^{|E_1(G)|}$
$PM_2(G) = 16^{|E_1(G)|} \times 20^{|E_2(G)|} \times 25^{|E_1(G)|}$
$PM_2(G) = 16^{10n+10} \times 20^{20n-4} \times 25^{44n-16}$

2.2. **Molecular descriptors of four paraline graphs for multiple pentacenes.** The chemical diagram $T_{m,n}$ representing multiple pentacene is depicted in Figure 4. This graph consists of $22mn$ vertices and $33mn - 2m - 5n$ edges.

Theorem 2.5 Consider a four para-line graph $G^\star$ derived from the graph $T_{m,n}$.
$M\alpha(G^\star) = (5n + 2)2^{\alpha+2} + 3^{\alpha+1}(12n - 4)$.

Proof. Figure 5 shows the graph $G^\star$ in a visual format. It has $56n-4$ worth of vertices in total, of which $20n+8$ and $36n-12$ have degrees of 3 and 4, respectively. Using formula 2, we can calculate $M\alpha(G^\star)$.

Theorem 2.6 Consider a four para-line graph $G^\star$ derived from the graph $T_{m,n}$.
1. $R_\alpha(G^\star) = (10n + 6m + 4)16^\alpha + (4m + 20n - 8)20^\alpha + (99mn - 20m - 55n + 4)25^\alpha$.
2. $\chi_\alpha(G^\star) = (10n + 6m + 4)8^\alpha + (4m + 20n - 8)9^\alpha + (99mn - 20m - 55n + 4)10^\alpha$.
3. $ABC(G^\star) = (15\sqrt{2} - \frac{110}{3})n + (5\sqrt{2} - \frac{40}{3})m - 2\sqrt{2} + 66mn + \frac{8}{3}$.
4. $GA(G^\star) = (-45 + 8\sqrt{6})n + (\frac{8}{5}\sqrt{6} - 14)m + 99mn + 8 - \frac{16}{5}\sqrt{6}$.

Proof. The division graph $S(T_{m,n})$ comprises a total of $198mn - 20m - 50$ vertices and $99mn - 10m - 25n$ edges. There are $8m + 20nverticesofdegree2and66$mn-12m-30n vertices of degree 3, according to the vertex division. The edge set $E(G^\star)$ of the four para-line graph $G^\star$ consists of $99mn - 20m - 55n + 4$ edges. Based on the angles of the end vertices, these edges are divided into three groups, i.e, $E(G^\star) = E_1(G^\star) \cup E_2(G^\star) \cup E_3(G^\star)$. The edge separation $E_1(G^\star)$ consists of $10n+6m+4$ edges x_1x_2. where $d_{x_1} = d_{x_2} = 4$. Edge Separation $4m+20n-8$ with $E_2(G^\star)$ Edge x_1x_2, where $d_{x_1} = 4$ and $d_{x_2} = 5$. Lastly, Separating the edges $E_3(G^\star)$ comprises $99mn - 20m - 55n + 4$ edges x_1x_2, where $d_{x_1} = d_{x_2} = 5$. By applying the required outcome may be produced using formulae (1), (3), (4) and (5).

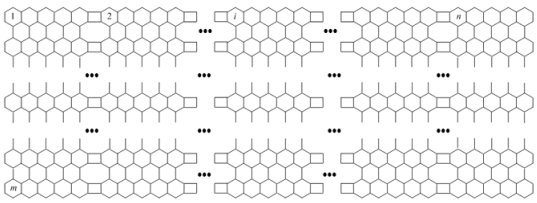


Figure 4. Multiple Pentacene

Theorem 2.7 Consider a four para-line graph $G^\star$ derived from the graph $T_{m,n}$.
1. $ABC_4(G^\star) = (44m + \sqrt{14} + 4\sqrt{2} + \sqrt{110} + 2\sqrt{30} - \frac{116}{3})n + (\frac{1}{2}\sqrt{6}$

$$+\tfrac{1}{5}\sqrt{110}+\tfrac{2}{5}\sqrt{35}-\tfrac{112}{9}+\tfrac{2}{3}\sqrt{30})m+2\sqrt{6}-\tfrac{8}{5}\sqrt{2}-\tfrac{2}{5}\sqrt{110}-\tfrac{4}{3}\sqrt{30}+\tfrac{80}{9}.$$

2. $GA_5(G^\star) = (\frac{80}{13}\sqrt{10}+99m+\frac{288}{17}\sqrt{2}-69)n +$
$$(-26+\tfrac{16}{13}\sqrt{10}+\tfrac{16}{9}\sqrt{5}+\tfrac{96}{17}\sqrt{2})m-\tfrac{192}{17}\sqrt{10}-\tfrac{32}{13}\sqrt{10}+24$$

Proof. Seven distinct edge sets may be formed from the set of edges by taking into account the degree sum of end vertices' neighbours. $E_i(G^\star)$, where $i = 6, 7, ..., 12$. Thus, we have $E(G^\star) = \bigcup_{i=6}^{12} E_i(G^\star)$. The edge partition $E_6(G^\star)$ contains $2m+8$ edges x_1x_2, where $S_{x_1} = S_{x_2} = 6$. The edge partition $E_7(G^\star)$ consists of $4m$ edges x_1x_2, where $S_{x_1} = 6$ and $S_{x_2} = 7$. Edge partition $E_8(G^\star)$ contains $10n-4$ edges x_1x_2. where $S_{x_1} = S_{x_2} = 7$. Edge partition $E_9(G^\star)$ contains $20n+4m-8$ edges x_1x_2. where $S_{x_1} = 8$ and $S_{x_2} = 9$. Edge partition $E_{10}(G^\star)$ consists of $10n$ edges x_1x_2. where $S_{x_1} = S_{x_2} = 9$. Edge partition $E_{11}(G^\star)$ contains $8m+24n-16$ edge x_1x_2. where $S_{x_1} = 10$ and $S_{x_2} = 11$. Finally, edge partition $E_{12}(G^\star)$ contains $99mn-28m-87n+20$ edge x_1x_2. where $S_{x_1} = S_{x_2} = 11$. By utilizing formulas (6) and (7), we obtain the desired result.

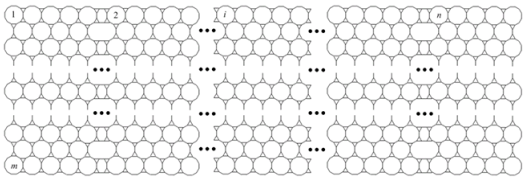


Figure 5. Four para-line graph multiple of pentacene

By performing computations on the chemical structures of multiple-pentacene, we obtain the following indices: $HM(G), PM_1(G), PM_2(G)$.

Theorem 2.8 Consider a four para-line graph $G^\star$ derived from the graph $T_{m,n}$.

1. $HM(G^\star) = 9900mn-1292m-3420n+8$

2. $PM_1(G^\star) = 8^{10n+6m+4} \times 9^{4m+20n-8} \times 10^{99mn-20m-55n+4}$.

3. $PM_2(G^\star) = 16^{10n+6m+4} \times 20^{4m+20n-8} \times 25^{99mn-20m-55n+4}$

4. $M_1(G,p) = (10n+6m+4)P^8 + (4m+20n-8)P^9 + (99mn-20m-55n+4)P^{10}$.

5. $M_2(G,p) = (10n+6m+4)P^{16} + (4m+20n-8)P^{20} + (99mn-20m-55n+4)P^{25}$.

Proof. Consider a graph $G^\star$ with its edges broken down into three parts categories due to the degrees of the final vertex. The initial category, denoted as $E_1(G)$, consists of $10n+6m+4$ edges x_1x_2, which both vertices x_1 and x_2 have a degree of 4. The second category, denoted as $E_2(G)$, contains $4m+20n-8$ edges x_1x_2, which x_1 has a degree of 4 and x_2 has a degree of 5. The third category, denoted as $E_3(G)$, includes $99mn-20m-55n+4$ edges x_1x_2, where both vertices x_1 and x_2 have a degree of 5. We can observe that the cardinality of $E_1(G)$ is equal to $e_{4,4}$, $E_2(G)$ is equal to $e_{4,5}$, and $E_3(G)$ is equal to $e_{5,5}$.

1. $HM(G^\star) = \sum_{x_1x_2\in E(G)}(d_{x_1}+d_{x_2})^2$

$HM(G^\star) = \sum_{x_1x_2\in E_1(G)}[d_{x_1}+d_{x_2}]^2 + \sum_{x_1x_2\in E_2(G)}[d_{x_1}+d_{x_2}]^2 + \sum_{x_1x_2\in E_3(G)}[d_{x_1}+d_{x_2}]^2$

$HM(G^\star) = 64|E_1(G)| + 81|E_2(G)| + 100|E_3(G)|$

$HM(G^\star) = 64(10n+6m+4) + 81(4m+20n-8) + 100(99mn-20m-55n+4)$

$HM(G^\star) = 460n+384m+256+324m+1620n-648 +9900mn-2000m-5500n+400$

This implies that

$HM(G^\star) = 9900mn-1292m-3420n+8$

Since,

2. $PM_1(G^\star) = \Pi_{x_1x_2\in E(G)}(d_{x_1}+d_{x_2})$

$PM_1(G^\star) = \Pi_{x_1x_2\in E_1(G)}(d_{x_1}+d_{x_2}) \times \Pi_{x_1x_2\in E_2(G)}(d_{x_1}+d_{x_2}) \times \Pi_{x_1x_2\in E_3(G)}(d_{x_1}+d_{x_2})$

$PM_1(G^\star) = 8^{10n+6m+4} \times 9^{4m+20n-8} \times 10^{99mn-20m-55n+4}$.

Now that

3. $PM_2(G^\star) = \Pi_{x_1x_2\in E(G)}(d_{x_1} \times d_{x_2})$

$PM_2(G^\star) = \Pi_{x_1x_2\in E_1(G)}(d_{x_1}\ times d_{x_2}) \times \Pi_{x_1x_2\in E_2(G)}(d_{x_1} \times d_{x_2}) \times \Pi_{x_1x_2\in E_3(G)}(d_{x_1} \times d_{x_2})$

$PM_2(G^\star) = 16^{|E_1(G)|} \times 20^{|E_1(G)|} \times 25^{|E_1(G)|}$

$PM_2(G^\star) = 16^{10n+6m+4} \times 20^{4m+20n-8} \times 25^{99mn-20m-55n+4}$.

4. $M_1(G,p) = \sum_{x_1x_2\in E(G)} P^{(d_{x_1}+d_{x_2}}$

$M_1(G,p) = \sum_{x_1x_2\in E_1(G)} P^{(d_{x_1}+d_{x_2})} + \sum_{x_1x_2\in E_2(G)} P^{(d_{x_1}+d_{x_2})} \sum_{x_1x_2\in E_1(G)} P^{(d_{x_1}+d_{x_2})}$

$M_1(G,p) = \sum_{x_1x_2\in E_1(G)} P^8 + \sum_{x_1x_2\in E_2(G)} P^9 + \sum_{x_1x_2\in E_1(G)} P^{10}$

$M_1(G,p) = |E_1(G)|P^8 + |E_2(G)|P^9 + |E_3(G)|P^{10}$

$M_1(G,p) = (10n+6m+4)P^8 + (4m+20n-8)P^9 + (99mn-20m-55n+4)P^{10}$.

5. $M_2(G,p) = \sum_{x_1x_2\in E(G)} P^{(d_{x_1}+d_{x_2}}$

$M_2(G,p) = \sum_{x_1x_2\in E_1(G)} P^{(d_{x_1}\times d_{x_2})} + \sum_{x_1x_2\in E_2(G)} P^{(d_{x_1}\times d_{x_2} \sum_{x_1x_2\in E_1(G)} P^{(d_{x_1}\times d_{x_2})}$

$M_2(G,p) = \sum_{x_1x_2\in E_1(G)} P^{16} + \sum_{x_1x_2\in E_2(G)} P^{20} + \sum_{x_1x_2\in E_1(G)} P^{20}$

$M_2(G,p) = |E_1(G)|P^{16} + |E_2(G)|P^{20} + |E_3(G)|P^{25}$

$M_2(G,p) = (10n+6m+4)P^{16} + (4m+20n-8)P^{20} + (99mn-20m-55n+4)P^{25}$.

This makes the proof whole.

3. **Conclusion and Future Studies**

In our research article, we investigated indices randic general connectivity index, first zagreb general index, summation general connectivity index, atomic bond connectivity index, geometric arithmetic index, fifth class of geometric arithmetic indices, hyperzagreb index, The initial and secondly multiple [n]-pentacene zagreb indices for a four paraline graphs of these two types of pentacenes. These indices play a crucial role in chemical informatics, specifically in the analysis of organic compounds. The randic index ($R\alpha$) is commonly used to explore the physicochemical properties of alkanes, such as boiling point, surface area, and enthalpy of formation. It provides valuable insights into the characteristics of organic molecules. The ABC index is a useful tool for predicting the stability of hydrocarbons, encompassing both linear and branched alkanes. The stability of cycloalkanes can be assessed by the indicated index, which is associated with their strain energy stability. This provides significant insights

into the overall stability of cycloalkanes. In terms of predicting physicochemical characteristics, chemical reactivity, and biological activities, the GA index demonstrates superior performance compared to the ABC index. Our investigation of pentacene was approached from a philosophical standpoint rather than relying solely on empirical observations. Our theoretical understanding of pentacenes can substantially benefit in understanding their physical properties, chemical activity and biological activity. A variety from physical feature-related data may be correlated with the chemical structure of pentacenes according to this study's major results, which may be useful for the power industry.

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