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Fuzzy Initial and Final Segments in ADL's

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Abstract. In this paper, we define the concepts of fuzzy initial and final segments in an Almost Distributive Lattice (ADL) and certain properties of these are discussed. It is proved that the set of fuzzy initial segments forms a complete lattice and that the set of fuzzy final segments of an ADL A forms a complete lattice if and only if A is a bounded distributive lattice.

1. Introduction

A fuzzy subset A of a non empty set X is a mapping of X into $[0, 1]$. The notion of fuzzy subsets originally introduced by Zadeh in his pioneering work [17]. Since Rosenfield [7] applied this concept to the theory of groups, many researchers are turned and engaged in fuzzyfying various concepts of abstract algebra. Kuroki [2], Malik and Mordeson [4] studied fuzzy ideals and bi-ideals in semi groups. Wang-jin Liu [3] studied fuzzy ideals of a ring followed by Mukherjee and Sen [5]. Swamy and Swamy [15] have introduced the concept of a fuzzy prime ideal of a ring and developed theory of fuzzy ideals. Further, Swamy and DV Raju [13] have introduced the concepts of fuzzy ideals and congruences of lattices. Further Swamy, Sundar Raj and Natnael [8, 9, 10] have applied some concepts from theory of ADLs and introduced the notion of fuzzy ideals (filters) and prime (maximal) ideals (filters). Natnael, Srikanya and Sundar Raj [6] constructed an L-fuzzy prime spectrum of ADLs.

George Boole's attempt to formalize propositional logic led to the concept of Boolean algebra which is a complemented distributive lattice. M.H. Stone [10] has proved that any Boolean algebra can be made into a Boolean ring (a ring with unity, in which every element is idempotent) and vice-versa and established a strong duality between Boolean algebras (rings) and Boolean spaces (compact

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Hausdorff totally disconnected spaces). Swamy and Rao [14] have introduced a common abstraction of these ring theoretic and lattice theoretic generalizations of Boolean rings and Boolean algebras in the form of an Almost Distributive lattice (abbreviated as ADL) as an algebra $(A, \wedge, \vee, 0)$ of type $(2, 2, 0)$ satisfying the following identities:

(1) $0 \wedge a = 0$

(2) $a \vee 0 = a$

(3) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(4) $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$

(5) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

(6) $(a \vee b) \wedge b = b$.

We first recall certain elementary definitions and results concerning Almost Distributive Lattices. These are collected from [14].

Example 1.1. *Let R be a commutative regular ring with identity (i.e. R is a commutative ring with identity in which, for each $a \in R$, there exists an (unique) idempotent a_0 in R such that $aR = a_0R$). For any $a, b \in R$, define*

$a \wedge b = a_0 b$ *and* $a \vee b = a + b - a_0 b$.

Then $(R, \wedge, \vee, 0)$ is an ADL.

Example 1.2. *Let X be a nonempty set and fix an arbitrarily chosen element $0 \in X$. For any a and $b \in X$, define*

$$a \wedge b = \begin{cases} 0 & \text{if } a = 0 \\ b & \text{if } a \neq 0 \end{cases} \quad \text{and} \quad a \vee b = \begin{cases} b & \text{if } a = 0 \\ a & \text{if } a \neq 0. \end{cases}$$

Then $(X, \wedge, \vee, 0)$ is an ADL and is called a discrete ADL.

Definition 1.1. *Let $A = (A, \wedge, \vee, 0)$ be an ADL. For any a and $b \in A$, define $a \leq b$ if and only if $a = a \wedge b$ ($\Leftrightarrow a \vee b = b$).*

It is known that $\leq$ is a partial order in A.

Theorem 1.1. *The following hold for any elements a,b and c in an ADL A.*

(1) $a \wedge 0 = 0 = 0 \wedge a$ *and* $a \vee 0 = a = 0 \vee a$

(2) $a \wedge a = a = a \vee a$

(3) $a \wedge b \leq b \leq b \vee a$

(4) $a \wedge b = a \Leftrightarrow a \vee b = b$ *and* $a \wedge b = b \Leftrightarrow a \vee b = a$

(5) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ *(i.e., $\wedge$ is associative)*

(6) $a \vee (b \vee a) = a \vee b$

(7) $a \leq b \Rightarrow a \wedge b = a = b \wedge a$ *and* $a \vee b = b = b \vee a$

(8) $a \wedge b \wedge c = b \wedge a \wedge c$

(9) $(a \vee b) \wedge c = (b \vee a) \wedge c$

(10) $a \vee b = b \vee a$ *and* $a \wedge b = b \wedge a$ *whenever* $a \wedge b = 0$

(11) $a \vee b = b \vee a$ *and* $a \wedge b = b \wedge a$ *(and hence A is a distributive lattice), whenever* $a \leq x$ *and* $b \leq x$ *for some* $x \in A$.

(12) $a \wedge b = b \wedge a \Leftrightarrow a \vee b = b \vee a$

(13) $a \wedge b = \inf\{a, b\} \Leftrightarrow a \vee b = \sup\{a, b\}$.

(14) *The set* $\{y \in A : y \leq a\}$ *is a bounded distributive lattice under the induced operations* $\wedge$ *and* $\vee$ *with* 0 *is the least element and* a *is the largest element.*

(15) m *is maximal in* $(A, \leq) \Leftrightarrow m \wedge a = a$ $(\Leftrightarrow m \vee a = m)$ *for all* $a \in A$.

In this paper, we introduce the concepts of fuzzy initial and final segments in an ADL A and discuss certain properties of these. Throughout this paper L stands for a non-trivial complete lattice $(L, \wedge, \vee, 0, 1)$ satisfying the infinite meet distributive law:

$$a \wedge \Big(\bigvee_{s \in S} s\Big) = \bigvee_{s \in S} (a \wedge s)$$

for all $a \in L$ and $S \subseteq L$ such a lattice is called a frame. Infact, the interval $[0, 1]$ is a frame under usual ordering. Also A stands for an ADL $(A, \wedge, \vee, 0)$ with a maximal element unless otherwise stated. As usual by L-fuzzy subset (simply, fuzzy subset) of an ADL A, we mean a mapping $\lambda : A \to L$.

2. Fuzzy Initial Segments

First we recall from [16] that a non empty subset I of an ADL A is called an (crisp) initial segment of A if $a \wedge x \in I$ for all $x \in I$ and $a \in A$ (equivalently, $a \in$A and $a \leq x \in I$ implies $a \in I$)

Definition 2.1. *A fuzzy subset* λ *of* A *is said to be a fuzzy initial segment of* A *if,*
$\lambda(x_0) = 1$ *for some* $x_0 \in A$
and $x \leq y \Rightarrow \lambda(y) \leq \lambda(x)$ *for all* $x, y \in A$.

Note that if λ is a fuzzy initial segment of an ADL $(A, \wedge, \vee, 0)$ then $\lambda(0) = 1$. It can be easily verified that a fuzzy subset λ of A is a fuzzy initial segment if and only if, for any $\alpha \in L$, λ_α is an initial segment of A, where $\lambda_\alpha = \{x \in A : \alpha \leq \lambda(x)\}$, called the $\alpha-$cut of λ.

An initial segment I of an ADL A need not satisfy the condition that,
$x \wedge a \in I$ for any $x \in I$ and $a \in A$ $(*)$
For example, let D be a 3-element discrete ADL, say $\{0, x, y\}$. Then $I = \{0, x\}$ is an initial segment of D; but $x \wedge y = y \notin I$. However for any non empty subset I of an ADL A satisfying $*$ and, for any $x \in I$ and $a \in I$, we have
$a \wedge x = a \wedge x \wedge x = (x \wedge a) \wedge x$ (*by* 1.4(8))
and, since $x \wedge a \in I$, we get $(x \wedge a) \wedge x \in I$. (*by* $*$)
So that I is an initial segment of A. This can be extended to fuzzy sets in the following.

Theorem 2.1. *Let λ be a fuzzy subset of A satisfying $\lambda(x_0) = 1$ for some $x_0 \in A$ and $\lambda(x \vee y) \leq \lambda(x) \wedge \lambda(y)$ for all $x, y \in A$. Then λ is a fuzzy initial segment of A.*

Proof. Let $x, y \in A$ with $x \leq y$. Then,
$\lambda(y) = \lambda(x \vee y) \leq \lambda(x) \wedge \lambda(y) \leq \lambda(x)$
Therefore λ is a fuzzy initial segment of A. □

The converse of above theorem is not true. For, consider the example given in the following.

Example 2.1. *Let $A = \{0, x, y\}$ be a dicrete ADL and $L = \{0, s, 1\}$ be a chain with $0 < s < 1$. Define $\lambda : D \longrightarrow L$ by $\lambda(0) = 1$, $\lambda(x) = 1$ and $\lambda(y) = s$. Then λ is a fuzzy initial segment of D; but, $\lambda(x \vee y) = \lambda(x) = 1 \nleq s = 1 \wedge s = \lambda(x) \wedge \lambda(y)$.*

For any fuzzy subset λ and μ of A, we write $\lambda \leq \mu$ to mean $\lambda(x) \leq \mu(x)$ in the ordering of L, for all $x \in A$. It can be easily verified that $\leq$ is a partial order on the set of all fuzzy subsets of A and is called the point wise ordering. In [16], *it is proved that the set intersection of any class of initial segments of an ADL A is again an initial segment, so that the set of all initial segments of A is a complete lattice under the usual set inclusion ordering.*

Theorem 2.2. *The set of all fuzzy initial segments of A is a complete lattice with respect to the point wise ordering in which, for any set $\{\lambda_i\}_{i \in \Delta}$ of fuzzy initial segments of A,*
g.l.b $\{\lambda_i\}_{i \in \Delta} = \bigwedge_{ß \in \Delta} \lambda_i$, the point wise infimum
l.u.b $\{\lambda_i\}_{i \in \Delta} = \bigvee_{i \in \Delta} \lambda_i = \bigwedge \left\{\lambda : \lambda \text{is a fuzzy initial segment of } A \text{ and } \lambda_i \leq \lambda \text{ for all } i \in \Delta\right\}$.
In fact, if $\lambda = \bigwedge_{i \in \Delta} \lambda_i$ and $\alpha \in L$, then $\lambda_\alpha = \bigcap_{i \in \Delta} \lambda_{i_\alpha}$.

Proof. Let $\{\lambda_i\}_{i \in \Delta}$ be a class of fuzzy initial segments of A and let $\lambda = \bigwedge_{ß \in \Delta} \lambda_i$, the point wise infimum of $\{\lambda_i\}_{i \in \Delta}$; that is,
$\lambda(x) = \bigwedge_{ß \in \Delta} \lambda_i(x) = g.l.b \{\ \lambda_i(x)\ :\ i \in \Delta\}$ in L, for any $x \in A$.
Since $\lambda_i(0) = 1$ for all $i \in \Delta$, it follows that $\lambda(0) = 1$. Let $x, y \in A$ such that $x \leq y$. Then $\lambda_i(y) \leq \lambda_i(x)$, since each λ_i is fuzzy initial segment. So that $\bigwedge_{i \in \Delta} \lambda_i(y) \leq \bigwedge_{i \in \Delta} \lambda_i(x)$ and hence $\lambda(y) \leq \lambda(x)$. Therefore λ is a fuzzy initial segment of A. Also, λ is the $g.l.b \{\lambda_i\}_{i \in \Delta}$ under the point wise ordering. Thus the set of all fuzzy initial segments of A is a complete lattice under point wise ordering.

□

In the following, we describe the smallest fuzzy initial segment containing a given fuzzy subset.

Definition 2.2. *For any fuzzy subset λ of A, define $\bar{\lambda} : A \longrightarrow L$ by*
$\bar{\lambda}(x) = \bigwedge \left\{\mu(x) : \mu \text{ is a fuzzy initial segment of } A \text{ and } \lambda \leq \mu\right\}$.

By theorem 2.4, $\bar{\lambda}$ is a fuzzy initial segment of A and, with respect to the point wise ordering $\bar{\lambda}$ is the smallest fuzzy initial segment of A such that $\lambda \leq \bar{\lambda}$; in the sense that for any fuzzy initial segment μ of A, $\lambda \leq \mu \Leftrightarrow \bar{\lambda} \leq \mu$. $\bar{\lambda}$ is called the fuzzy initial segment of A generated by λ.
In the following, we give a point wise description of $\bar{\lambda}$ for any given fuzzy subset λ of A.

Theorem 2.3. *Let λ be a fuzzy subset of A. Then the fuzzy initial segment generated by λ is given by* $\bar{\lambda}(0) = 1$ *and* $\bar{\lambda}(x) = \bigvee\{\lambda(a) : x \leq a$ *for some* $a \in A\}$ *for any* $0 \neq x \in A$.

Proof. Clearly $\lambda \leq \bar{\lambda}$. If $x \leq y$ in A.Then for any $a \in A$,
$y \leq a \Rightarrow x \leq a \Rightarrow \lambda(a) \leq \bar{\lambda}(x)$.
which implies that $\bar{\lambda}(y) \leq \bar{\lambda}(x)$. Therefore $\bar{\lambda}$ is a fuzzy initial segment of A. Finally, if μ is a fuzzy initial segment of A such that $\lambda \leq \mu$, then for any $x \in A$ with $x \leq a,\ a \in A$, we have
$\lambda(a) \leq \mu(a) \leq \mu(x)$. Therefore $\bar{\lambda}(x) \leq \mu(x)$ for all $x \in A$ so that $\bar{\lambda} \leq \mu$. □

3. Fuzzy Final Segments

Recall that a non empty subset F of an ADL A is called a (crisp) final segment of A if $a \in A, x \in F$ and $x \leq a \Rightarrow a \in F$ ($equivalently$, $x \in F$ and $a \in A \Rightarrow x \vee a \in F$).

Definition 3.1. *A fuzzy subset λ of A is said to be a fuzzy final segment of A if*
$\lambda(x_0) = 1$ *for some* $x_0 \in A$ *and*
$x \leq y \Rightarrow \lambda(x) \leq \lambda(y)$ *for all* $x, y \in A$.

It can be easily verified that a fuzzy subset λ of A is a fuzzy final segment of A if and only if, for any $\alpha \in L$, λ_α is a (crisp) final segment of A. A final segment F of an ADL A need not satisfies that properly that, $x \in F$ and $a \in A \implies a \vee x \in F$. For example, in any discrete ADL D, for any $x, y \in D, \{x\}$ is a final segment; but $y \vee x = y \notin \{x\}$. However, a non empty subset F of an ADL A satisfying the property that, $x \in F$ and $a \in A \implies a \vee x \in F$ is a final segment of A (refer theorem 3.2 in [16]). This result can be seen by fuzzyfying in the following.

Theorem 3.1. *Let λ be a fuzzy subset of A satisfying* $\lambda(x_0) = 1$ *for some* $x_0 \in A$ *and* $\lambda(x \wedge y) \leq \lambda(x) \wedge \lambda(y)$ *for all* $x, y \in A$. *Then λ is a fuzzy final segment of A.*

Proof. Let $x, y \in A$ such that $x \leq y$. Then $\lambda(x) = \lambda(x \wedge y) \leq \lambda(x) \wedge \lambda(y) \leq \lambda(y)$. Therefore λ is a fuzzy final segment of A.

□

The converse of above theorem is not true. For, consider the following example.

Example 3.1. *Let* $A = \{0, x, y\}$ *be a discrete ADL and* $L = \{0, s, 1\}$ *be a chain with* $0 < s < 1$. *If we define* $\lambda(x) = s,\ \lambda(y) = 1$ *and* $\lambda(0) = 0$, *then λ is a fuzzy final segment of A; but* $\lambda(x \wedge y) = \lambda(y) = 1 \nleq s = \lambda(x) \wedge \lambda(y)$.

It can be easily verified that, for any non empty subset F of A,
F is a final(initial)segment of A if and only if χ_F is a fuzzy final(initial) segment of A,

$$\text{where } \chi_F(x) = \begin{cases} 1 & \text{if } x \in F \\ 0 & \text{if } x \notin F. \end{cases}$$

Note that for any $0 \neq \alpha \in L$, the α-cut of χ_F;

$$(\chi_F)_\alpha = \{x \in A : \alpha \leq \chi_F(x)\} = F$$

In contrast to the case of fuzzy initial segments, the point wise infimum of any class of fuzzy final segments may not be a fuzzy final segment. However, in the following we establish a set of equivalent conditions for the point wise infimum of any family of fuzzy final segments to be fuzzy final segment and inturn it is equivalent to saying that the set of all fuzzy final segments form a complete lattice.

Theorem 3.2. *let A be an ADL. Then the following statements are equivalent to each other.*

(1) *For any non empty family $\{\lambda_i : i \in \Delta\}$ of fuzzy final segments of A, $\bigwedge\limits_{i \in \Delta} \lambda_i$ (the point wise infimum of λ_i's) is a fuzzy final segment of A.*
(2) *A has a largest element.*
(3) *A has a unique maximal element.*
(4) *There exists smallest fuzzy final segment.*
(5) *A is a bounded distributive lattice.*
(6) *The set of all final segments of A is a complete lattice under $\subseteq$.*
(7) *The set of all fuzzy final segments of A is a complete lattice under point wise ordering $\leq$.*

Proof. $(1) \Rightarrow (2)$: For any $a \in A$, let $[a) = \{x \in A : a \leq x\}$. Then $[a)$ is a final segment containing a and hence $\chi_{[a)}$ is a fuzzy final segment of A. Then, by (1),
$\lambda = \bigwedge\limits_{a \in A} \chi_{[a)}$; the point wise infimum of $\chi_{[a)}$'s
is a fuzzy final segment of A. Let $0 \neq \alpha \in L$.
Then, for any $a \in A$, the α-cut of $\chi_{[a)} = \big(\chi_{[a)}\big)_\alpha = [a)$. Now

$$\lambda_\alpha = \big(\bigwedge_{a \in A} \chi_{[a)}\big)_\alpha = \bigcap_{a \in A} (\chi_{[a)})_\alpha = \bigcap_{a \in A} [a).$$

Since λ_α is non empty, there exist $m \in A$ such that $a \leq m$ for all $a \in A$. Then m is the largest element of A.
$(2) \Rightarrow (3)$: It is trivial, since the largest element will be the unique maximal element.
$(3) \Rightarrow (4)$: If m is the unique maximal element in A, then $x \vee m$ is maximal, and hence $x \vee m = m$, so that $x \leq m$ for all $x \in A$. Now $[m)$ is a final segment of A and hence $\chi_{[m)}$ is a fuzzy final segment of A. Let λ be any fuzzy final segment of A such that $\lambda(x_0) = 1$ for some $x_0 \in A$. Since $x_0 \leq m$, we get $1 = \lambda(x_0) \leq \lambda(m)$, so that $\lambda(m) = 1$. This implies that $\chi_{[m)} \leq \lambda$, for all fuzzy final segments λ of A. Thus $\chi_{[m)}$ is the smallest fuzzy final segment of A.

$(4) \Rightarrow (5)$: Suppose μ is the smallest fuzzy final segment of A such that $\mu(m) = 1$ for some $m \in A$. Now for each $x \in A$, $[x)$ is a final segment of A and hence $\chi_{[x)}$ is a fuzzy final segment of A. Since $\mu \leq \chi_{[x)}$, we get $1 = \mu(m) \leq \chi_{[x)}(m)$ and hence $\chi_{[x)}(m) = 1$, so that $m \in [x)$ and hence $x \leq m$ for all $x \in A$. Therefore m is the largest element in A. Thus A is a bounded distributive lattice (by theorem 1.4 (11)) .

$(5) \Rightarrow (6)$: Let $\{F_\alpha : \alpha \in \Delta\}$ be a class of final segments of A. Put $F = \bigcap\limits_{\alpha\in\Delta} F_\alpha$. Since the largest element of A is necessarily an element in every final segment of A, it implies that F is non empty and hence F is a final segment of A. Also, F is the $g.l.b$ $\{F_\alpha : \alpha \in \Delta\}$. Thus the set of all final segments form a complete lattice under $\subseteq$.

$(6) \Rightarrow (7)$: It is obvious.

□

The point wise infimum of any two fuzzy final segments of an ADL need not be a fuzzy final segment. However, we prove the following.

Theorem 3.3. *Let A be an ADL. Then the following are equivalent to each other.*

(1) *The point wise meet of any two fuzzy final segments of A is again a fuzzy final segment.*
(2) $(A, \leq)$ *is directed above.*
(3) *A is a distributive lattice.*
(4) *The set of all fuzzy final segments of A is a lattice.*
(5) *The set of all final segments of A is a lattice.*
(6) *The set of all final segments of A is closed under finite intersections.*

Proof. $(1) \Rightarrow (2)$: Let a and $b \in A$. Then $\chi_{[a)}$ and $\chi_{[b)}$ are fuzzy final segment of A, and by (1), $\chi_{[a)} \wedge \chi_{[b)}$ is also a fuzzy final segment of A, in perticular, there exists $x_0 \in A$ such that $\big(\chi_{[a)} \wedge \chi_{[b)}\big)(x_0) = \chi_{[a)}(x_0) \wedge \chi_{[b)}(x_0) = 1$. Then $x_0 \in [a) \cap [b)$, so that $a \leq x_0$ and $b \leq x_0$. Therefore $(A, \leq)$ is directed above.

$(2) \Rightarrow (3)$: It is consequence of theorem 1.4 (11)

$(3) \Rightarrow (1)$: Let λ and μ be fuzzy final segments of A. Choose x and $y \in A$ such that $\lambda(x) = 1$ and $\lambda(y) = 1$. By (3), $x \leq x \vee y$ and $y \leq x \vee y$ and hence

$\lambda(x) \leq \lambda(x \vee y)$ and $\mu(y) \leq \mu(x \vee y)$.

$\therefore$ $\lambda(x \vee y) = 1$ and $\mu(x \vee y) = 1$

For any $a, b \in A$ with $a \leq b$, we have

$\lambda(a) \wedge \mu(a) \leq \lambda(b) \wedge \mu(b)$

and hence $(\lambda \wedge \mu)(a) = (\lambda \wedge \mu)(b)$. Therefore $\lambda \wedge \mu$ is a fuzzy final segment of A.

$(1) \Rightarrow (4)$: Let λ and μ be fuzzy final segments of A. Then, by (1), $\lambda \wedge \mu$ is a fuzzy final segment of A and is the $g.l.b$ $\{\lambda, \mu\}$ with respect to the point wise ordering $\leq$. Also, by (1), clearly $\lambda \vee \mu$; the point wise supremum of λ and μ is a fuzzy final segment of A and which is the $l.u.b$ $\{\lambda, \mu\}$ with

respect to $\leq$. Thus the set of all fuzzy final segments of A is lattice under $\leq$.
(4) $\Rightarrow$ (1) : It is clear. (3) $\Rightarrow$ (5) $\Rightarrow$ (6) $\Rightarrow$ (3) is well known result (by theorem 3.4 in [16]). $\square$

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