

## Fuzzy Soft Boolean Rings

**Gadde Sambasiva Rao<sup>1</sup>, D. Ramesh<sup>1</sup>, Aiyared Iampan<sup>2,\*</sup>, B. Satyanarayana<sup>3</sup>**

<sup>1</sup>*Department of Engineering Mathematics, College of Engineering, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Andhra Pradesh-522302, India*

<sup>2</sup>*Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand*

<sup>3</sup>*Department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar, Andhra Pradesh-522510, India*

\* *Corresponding author: aiyared.ia@up.ac.th*

**Abstract.** This article introduces the idea of (fuzzy soft Boolean rings) FSBRs and investigates their algebraic properties. The concepts of fuzzy soft ideals (FSIs) of FSBRs, and idealistic fuzzy soft Boolean rings (IFSBRs) are then defined and discussed.

### 1. Introduction

In classical mathematics, exact solutions to mathematical models are required. If the model is so complex that an exact solution cannot be determined, we can get a rough estimate. In 1999, Russian researcher Molodtsov [8] pioneered the idea behind soft set (SS) theory and began developing the foundations of the corresponding theory as a novel approach to modelling uncertainty. The SS is an approximate representation of an object. There are numerous potential applications for SS theory. SS theory and its applications are currently advancing at a rapid pace. Maji et al. [7] proposed new SS definitions. Pei and Miao [9] looked into how SSs and information systems interact. By fusing SS and fuzzy set (FS) designs, Maji et al. [6] developed the concept of fuzzy soft sets (FSSs) in 2001. To continue the investigation, Ahmad and Kharal [2] obtainable some more properties of FSSs. There has been a surge of interest in the algebraic structure of SSs in recent years. Soft groups were defined

Received: Apr. 10, 2023.

2020 *Mathematics Subject Classification.* 03E72, 03G05, 28A60, 06D72.

*Key words and phrases.* boolean ring; fuzzy soft set; fuzzy soft boolean ring; fuzzy soft sub boolean ring; fuzzy ideal; fuzzy soft ideal; idealistic fuzzy soft boolean ring.

by Aktaş and Çağman [3], and some properties were derived from them. Additionally, they contrasted SSs with rough and FSs, two concepts that are related. The fundamental ideas of soft rings were first introduced by Acar et al. [1] in 2010, which are a generalized family of subrings. Liu [5] proposed the fuzzy ring concept in 1982. Following that, Dixit et al. [4] investigated the fuzzy ring and discovered some theoretical analogues. The algebraic characteristics of FSSs in Boolean ring (BR) theory are examined in this paper.

## 2. Preliminaries

To begin, we will present Maji et al. [6] and Ahmad and Kharal [2]'s fundamental definitions and notations.

**Definition 2.1.** Let  $\hat{E}$  denote a set of parameters,  $Z$  denote an initial universe, and  $\hat{I}$  represents the closed unit interval.  $Q(Z)$  represents  $Z$ 's power set. Then the pair  $(M, \hat{E})$  over  $Z$  is a soft set, where  $M : \hat{E} \rightarrow Q(Z)$  is a set valued function.

**Definition 2.2.** Let  $\hat{E}$  denote a set of parameters,  $Z$  denote an initial universe, and  $\hat{I}$  represents the closed unit interval, i.e.,  $\hat{I} = [0, 1]$ .  $Q(Z)$  represents  $Z$ 's power set, where  $M : C \rightarrow I^Z$  is set-valued function and  $I^Z$  represent the collection of all fuzzy sets on  $Z$ .

**Definition 2.3.** Consider  $(M, C)$  and  $(N, D)$  to be FSSs. Then  $(M, C)$  is a FSS of  $(N, D)$  and we can write  $(M, C) \subseteq (N, D)$  if (i)  $C \subseteq D$  (ii) for each  $\alpha \in C$ ,  $M_\alpha \leq N_\alpha$  implying that,  $M_\alpha$  is fuzzy subset of  $N_\alpha$ .

**Definition 2.4.** Let us assume  $(M, C)$  and  $(N, D)$  be two FSSs, with  $C \cap D \neq \emptyset$ . Then the FSS  $(O, E)$  is formed by the intersection of  $(M, C)$  and  $(N, D)$ , where  $E = C \cap D$  and  $O_\alpha = M_\alpha \wedge N_\alpha, \forall \alpha \in E$ . We can write  $(M, C) \cap (N, D) = (O, E)$ .

**Definition 2.5.** Let us assume  $(M, C)$  and  $(N, D)$  are two FSSs. The FSS  $(O, E)$  is formed by the union of  $(M, C)$  and  $(N, D)$ , where  $E = C \cup D$  and

$$(\forall \alpha \in E) \left( O_\alpha = \begin{cases} M_\alpha & \text{if } \alpha \in C - D \\ N_\alpha & \text{if } \alpha \in D - C \\ M_\alpha, \vee N_\alpha & \text{if } \alpha \in C \cap D \end{cases} \right). \quad (2.1)$$

Then we write  $(M, C) \cup (N, D) = (O, E)$ .

**Definition 2.6.** Let  $(M_j, C_j)_{j \in J}$  be a family of FSSs. The union of these FSSs is a FSS  $(O, E)$ , where  $E = \cup_{j \in J} C_j$  and  $O(\alpha) = \vee_{j \in J} M_j(\alpha), \forall \alpha \in E$ . Then we can write  $\cup_{j \in J} (M_j, C_j) = (O, E)$ .

**Definition 2.7.** Let  $(M_j, C_j)_{j \in J}$  be a family of FSSs, with  $\cap_{j \in J} C_j \neq \emptyset$ . A FSS is formed by the intersection of these FSSs  $(O, E)$ , where  $E = \cap_{j \in J} C_j$  and  $O(\alpha) = \wedge_{j \in J} M_j(\alpha), \forall \alpha \in E$ . Then we can write  $\cap_{j \in J} (M_j, C_j) = (O, E)$ .

**Definition 2.8.** Let  $(M, C)$  and  $(N, D)$  be two FSSs, subsequently,  $(M, C)$  AND  $(N, D)$  are represented by  $(M, C) \widehat{\wedge} (N, D)$  and it's indicated by  $(O, C \times D)$ , where  $O(\alpha, \beta) = O_{\alpha, \beta} = M_{\alpha} \wedge N_{\beta}$  for every  $(\alpha, \beta) \in C \times D$ .

**Definition 2.9.** Let  $(M, C)$  and  $(N, D)$  be two FSSs. Then  $(M, C)$  OR  $(N, D)$  are represented by  $(M, C) \widehat{\vee} (N, D)$  and it's indicated by  $(O, C \times D)$ , where  $O(\alpha, \beta) = O_{\alpha, \beta} = M_{\alpha} \vee N_{\beta}$  for every  $(\alpha, \beta) \in C \times D$ .

**Definition 2.10.** Consider  $(M, C)$  to be a FSS. The set  $\text{Supp}(M, C) = \{\alpha \in C : M(\alpha) = M_{\alpha} \neq \emptyset\}$  is known support of the FSS  $(M, C)$ . If the support of a FSS is greater than the empty set, it is said to be non-null.

### 3. Fuzzy Soft Boolean Rings

The concept of soft rings was proposed by Acar et al. [1]. In this section contains, we define FSBRs and discuss some of their fundamental properties.  $R$  denotes a BR from now on, and all FSSs are preferred over  $R$ .

**Definition 3.1.** Let us assume  $(M, C)$  is a non-null SS. Then  $(M, C)$  is referred to as a soft Boolean ring (SBR) over  $R$  if for each  $\alpha \in C$ ,  $M(\alpha)$  is a sub-BR of  $R$ .

**Definition 3.2.** Let us assume  $(M, C)$  is a non-null FSS. Then  $(M, C)$  is referred to as a FSBR over  $R$  if for each  $\gamma \in C$ ,  $M(\gamma) = M_{\gamma}$  is a F-sub-BR of  $R$ , i.e.,  $M_{\gamma}(\alpha - \beta) \geq \min(M_{\gamma}(\alpha), M_{\gamma}(\beta))$  and  $M_{\gamma}(\alpha \cdot \beta) \geq \min(M_{\gamma}(\alpha), M_{\gamma}(\beta)), \forall \alpha, \beta \in R$ .

**Example 3.1.** Let  $R = \{0, j, t, r\}$  be a nonempty set with two binary operations  $+$  and  $\cdot$  defined as follows:

$+$	0	j	t	r
0	0	j	t	r
j	j	0	r	t
t	t	r	0	j
r	r	t	j	0

$\cdot$	0	j	t	r
0	0	0	0	0
j	0	j	r	t
t	0	r	t	j
r	0	t	j	r

Let  $A = \{e_1^1, e_2^1, e_3^1\}$  be the set of parameters and now define a FSS  $(M, C)$  on a BR  $R$  by

$$M(e_1^1) = \{(0, 0.9), (j, 0.8), (t, 0.6), (r, 0.4)\},$$

$$M(e_2^1) = \{(0, 0.8), (j, 0.5), (t, 0.3), (r, 0.1)\},$$

$$M(e_3^1) = \{(0, 0.9), (j, 0.6), (t, 0.5), (r, 0.4)\}.$$

Here  $(M, C)$  is a FSS over  $R$ , which is also a F-sub-BR of  $R$ , for all  $\alpha \in C$ . Hence  $(M, C)$  is a FSBR over  $R$ .

**Example 3.2.** Because each SS can be thought of as a FSS, and each characteristic function of a BR is a F-sub-BR of  $R$ , we can think of an SBR as a FSBR.

**Theorem 3.1.** Let  $(M, C)$  and  $(N, D)$  are two FSBRs over  $R$ . If  $(M, C) \hat{\wedge} (N, D)$  is non-null, then it's a FSBR over  $R$ .

*Proof.* Let  $(M, C) \hat{\wedge} (N, D) = (O, C \times D)$ , where  $O(\alpha, \beta) = M_\alpha \wedge N_\beta, \forall (\alpha, \beta) \in C \times D$ . Since  $(O, C \times D)$ , is non-null, then there exists the pair  $(\alpha, \beta) \in C \times D$  such that  $O_{\alpha, \beta} = M_\alpha \wedge N_\beta \neq 0_R$ . We already know that  $M_\alpha, \forall \alpha \in C$  and  $N_\beta, \forall \beta \in D$  are F-sub-BR of  $R$ . Since then the intersection of two F-sub-BRs of  $R$  is also a F-sub-BR of  $R$ , then  $O(\alpha, \beta) = O_{\alpha, \beta}$  is a F-sub-BR of  $R$ . Hence  $(O, C \times D) = (M, C) \hat{\wedge} (N, D)$  is a FSBR over  $R$ .  $\square$

**Theorem 3.2.** Let  $(M, C)$  and  $(N, D)$  are two FSBRs over  $R$ . If  $(M, C) \hat{\cap} (N, D)$  is non-null, then it's a FSBR over  $R$ .

*Proof.* Let  $(M, C) \hat{\cap} (N, D) = (O, E)$ , where  $E = C \cap D$  and  $O_\alpha = M_\alpha \wedge N_\alpha, \forall \alpha \in E$ . Since  $(O, E)$  is non-null, then there exists  $\alpha \in E$  such that  $O_\alpha(\beta) \neq 0$  for some  $\beta \in R$ . We know  $M_\alpha \wedge N_\alpha$  is a F-sub-BR of  $R$ , because  $O_\alpha \neq 0_R$  and  $M_\alpha, N_\alpha$  are F-sub-BR of  $R$ . Therefore,  $(O, E) = (M, C) \hat{\cap} (N, D)$  is a FSBR over  $R$ .  $\square$

**Theorem 3.3.** Let  $(M_j, C_j)_{j \in J}$  be a family of FSBRs over  $R$ . There are also the following:

(i) If  $\hat{\wedge}_{j \in J} (M_j, C_j)$  is non-null, then it's a FSBR over  $R$ .

(ii) If  $\hat{\cap}_{j \in J} (M_j, C_j)$  is non-null, it's a FSBR over  $R$ .

*Proof.* (i) Let  $\hat{\wedge}_{j \in J} (M_j, C_j) = (O, E)$ , where  $E = \hat{\cap}_{j \in J} C_j$  and  $O_\alpha = \wedge_{j \in J} M_j(e_j), \forall \alpha = (\alpha_j)_{j \in J} \in E$ . Suppose that the FSS  $(O, E)$  is non-null. If  $\alpha = (\alpha_j)_{j \in J} \in \text{Supp}(O, E)$ , then  $O_\alpha = \wedge_{j \in J} M_j(\alpha_j) \neq 0_R$ . Since  $(M_j, C_j)$  is a FSBR over  $R, \forall j \in J, M_j(\alpha_j)$  is a F-sub-BR of  $R$ . As a result  $O_\alpha$  is a F-sub-BR of  $R$  for all  $\alpha \in \text{Supp}(O, E)$ . Consequently,  $\hat{\wedge}_{j \in J} (M_j, C_j) = (O, E)$  is a FSBR over  $R$ .

(ii) Let  $\hat{\cap}_{j \in J} (M_j, C_j) = (O, E)$ , where  $E = \cap_{j \in J} C_j$  and  $O_\alpha = \wedge_{j \in J} M_j(\alpha_j), \forall \alpha \in E$ . Suppose that the FSS  $(O, E)$  is non-null. If  $\alpha \in \text{Supp}(O, E)$ , then  $O_\alpha = \wedge_{j \in J} M_j(\alpha_j) \neq 0_R$ . Since  $(M_j, C_j)$  is a FSBR over  $R$ , then  $M_j(\alpha_j)$  is a F-sub-BR of  $R$  for all  $j \in J$ . As a result  $O_\alpha$  is a F-sub-BR of  $R$  for all  $\alpha \in \text{Supp}(O, E)$ . Therefore  $\hat{\cap}_{j \in J} (M_j, C_j) = (O, E)$  is a FSBR over  $R$ .  $\square$

**Definition 3.3.** Let  $(M, C)$  and  $(N, D)$  be two FSBRs over  $R$ . Then  $(N, D)$  is referred to as a FSSBR of  $(M, C)$  if the circumstances listed below are true:

- (i)  $D \subseteq C$ ,
- (ii)  $N_\alpha$  is a F-sub-BR of  $M_\alpha, \forall \alpha \in \text{Supp}(N, E)$ .

**Theorem 3.4.** Let  $(M, C)$  and  $(N, D)$  be two FSBRs over  $R$ . If  $(M, C) \cap (N, D)$  is non-null, then it's a FSSBR of  $(M, C)$  and  $(N, D)$ .

*Proof.*  $(M, C) \cap (N, D) = (O, E)$ , where  $E = C \cap D$  and  $O_\alpha = M_\alpha \wedge N_\alpha, \forall \alpha \in E$ . Since  $E = C \cap D \subseteq C$  and  $O_\alpha = M_\alpha \wedge N_\alpha$ , is a F-sub-BR of  $M_\alpha$ , then  $(O, E)$  is a FSSBR of  $(M, C)$ . Similarly, we obtain that  $(O, E)$  is a FSSBR of  $(N, D)$ . □

#### 4. Fuzzy Soft Ideals of Fuzzy Soft Boolean Rings

**Definition 4.1.** Assume  $(M, C)$  is a FSBR over  $R$ . A FSS  $(N, D)$  is a FSI of  $(M, C)$ , as indicated by  $(N, D) \hat{\cap} (M, C)$ . If it meets the following criteria:

- (i)  $D \subseteq C$ ,
- (ii)  $N_\gamma$  is a fuzzy ideal (FI) of a fuzzy BR  $M_\gamma$  for all  $\gamma \in \text{Supp}(N, D)$ , i.e.,  $N_\gamma$  is a FI, for each  $\gamma \in \text{Supp}(N, D)$ ,
  - (a)  $N_\gamma(\alpha - \beta) \geq N_\gamma(\alpha) \wedge N_\gamma(\beta)$ ,
  - (b)  $N_\gamma(\alpha\beta) \geq N_\gamma(\alpha) \wedge N_\gamma(\beta)$ ,
  - (c)  $N_\gamma(\alpha) \leq M_\gamma(\alpha), \forall \alpha, \beta \in R$ .

**Example 4.1.** Take a look at the BR  $(R, +, \cdot)$  established in Example 3.1.

$$M(e_1^1) = \{(0, 0.9), (j, 0.8), (t, 0.6), (r, 0.4)\},$$

$$M(e_2^1) = \{(0, 0.8), (j, 0.5), (t, 0.3), (r, 0.1)\},$$

$$M(e_3^1) = \{(0, 0.9), (j, 0.6), (t, 0.5), (r, 0.4)\}.$$

Here  $(M, C)$  is a FSS over  $R$ , which is also a F-sub-BR of  $R, \forall \alpha \in C$ . Hence  $(M, C)$  is a FSBR over  $R$ . Let  $D = \{e_2^1\}$  and  $N : D \rightarrow Q(R)$  be a function with a set of values defined by

$$N(e_2^1) = \{(0, 0.4), (1, 0.3), (2, 0.2), (3, 0.2)\}.$$

Obviously  $(N, D)$  is a FSS of  $R$ . We also see that  $D \subseteq C$  and  $N(\gamma)$  is a FI of  $M(\gamma), \forall \gamma \in I$ . As a result,  $(N, D)$  is a FSI of  $(M, C)$ .

**Theorem 4.1.** Let  $(N_1, D_1)$  and  $(N_2, D_2)$  be FSIs of a FSBR  $(M, C)$ . Then  $(N_1, D_1) \cap (N_2, D_2)$  is a FSI of  $(M, C)$  if it is non-null.

*Proof.* Let  $(N_1, D_1) \hat{\cap} (M, C), (N_2, D_2) \hat{\cap} (M, C)$ . By the Definition 2.4, we write  $(N_1, D_1) \cap (N_2, D_2) = (N, D)$ , where  $D = D_1 \cap D_2$  and  $N(\gamma) = N_1(\gamma) \wedge N_2(\gamma), \forall \gamma \in D$ . Since  $D_1 \subseteq C$  and  $D_2 \subseteq C$ , we have

$D_1 \cap D_2 = D \subseteq C$ . Suppose that  $(N, D)$  is non-null. If  $\gamma \in \text{Supp}(N, D)$ , then  $N(\gamma) = N_1(\gamma) \wedge N_2(\gamma) \neq 0_R$ . Since  $(N_1, D_1) \hat{\triangleright}(M, C)$  and  $(N_2, D_2) \hat{\triangleright}(M, C)$ ,  $N_1(\gamma)$  and  $N_2(\gamma)$  are both FIs of  $M(\gamma)$ , we conclude  $M(\gamma)$ . As a result,  $N(\gamma)$  is a FI of  $M(\gamma)$ ,  $\forall \gamma \in \text{Supp}(N, D)$ . Therefore,  $(N_1, D_1) \hat{\cap}(N_2, D_2) = (N, D)$  is a FSI of  $(M, C)$ .  $\square$

**Theorem 4.2.** Let  $(N_1, D_1)$  and  $(N_2, D_2)$  be FSIs of a FSBR  $(M, C)$ . If  $D_1$  and  $D_2$  are disjoint, then  $(N_1, D_1) \cup (N_2, D_2)$  is a FSI of  $(M, C)$ .

*Proof.* Let  $(N_1, D_1) \hat{\triangleright}(M, C)$ ,  $(N_2, D_2) \hat{\triangleright}(M, C)$ . By the Definition 2.5, we write  $(N_1, D_1) \cup (N_2, D_2) = (N, D)$ , where  $D = D_1 \cup D_2$  and  $\forall \gamma \in D$ ,

$$(\forall \alpha \in E) \left( N_\gamma = \begin{cases} N_1(\gamma) & \text{if } \alpha \in D_1 - D_2 \\ N_2(\gamma) & \text{if } \alpha \in D_2 - D_1 \\ N_1(\gamma) \vee N_2(\gamma) & \text{if } \alpha \in D_1 \cap D_2 \end{cases} \right). \quad (4.1)$$

Obviously, we have  $D \subseteq C$ . Since  $D_1$  and  $D_2$  are disjoint,  $\gamma \in D_1 - D_2$  or  $\gamma \in D_2 - D_1$ ,  $\forall \gamma \in \text{Supp}(N, D)$ . Let  $\gamma \in D_1 - D_2$ . Since  $(N_1, D_1) \hat{\triangleright}$  is a FI of  $M(\gamma)$ . Thus,  $\forall \gamma \in \text{Supp}(N, D)$ ,  $(N_1, D_1) \subseteq (M, C)$ . Consequently,  $(N, D)$  is a FSI of  $(M, C)$ .  $\square$

## 5. Idealistic Fuzzy Soft Boolean Rings

**Definition 5.1.** Let  $(M, C)$  be a non-null FSS. Then  $(M, C)$  is referred to as an IFSBR over  $R$ , if  $M_\gamma$  is a FI of  $R$ ,  $\forall \gamma \in \text{Supp}(M, C)$ . In other words, for each  $\gamma \in \text{Supp}(M, C)$ ,  $M_\gamma$  is a FI of  $R$  defined in [4], i.e.,  $M_\gamma(\alpha - \beta) \geq M_\gamma(\alpha) \wedge M_\gamma(\beta)$  and  $M_\gamma(\alpha \cdot \beta) \geq M_\gamma(\alpha) \vee M_\gamma(\beta)$ ,  $\forall \alpha, \beta \in R$ .

**Example 5.1.** Let  $R = \{0, j, t, r\}$  be a set with two binary operations  $+$  and  $\cdot$  as shown:

$+$	0	j	t	r
0	0	j	t	r
j	j	0	r	t
t	t	r	0	j
r	r	t	j	0
$\cdot$	0	j	t	r
0	0	0	0	0
j	0	j	r	t
t	0	r	t	j
r	0	t	j	r

Then  $(R, +, \cdot)$  is a BR. Let  $A = \{e_1^1, e_2^1\}$  represent the set of parameters.

$$M(e_1^1) = \{(0, 0.9), (j, 0.7), (t, 0.6), (r, 0.4)\},$$

$$M(e_2^1) = \{(0, 0.8), (j, 0.5), (t, 0.3), (r, 0.1)\}.$$

Here  $(M, C)$  is a FSS over  $R$ . Also, we can also see that  $M(\gamma)$  is a FI of  $R$ ,  $\forall \gamma \in C$ . As a result,  $(M, C)$  is an IFSBR over  $R$ .

**Theorem 5.1.** Assume  $(M, C)$  and  $(N, D)$  are two IFSBRs over  $R$ . Then  $(M, C) \cap (N, D)$  is an IFSBR over  $R$  if it is non-null.

*Proof.* Let  $(M, C) \cap (N, D) = (O, E)$ , where  $O = C \cap D$  and  $O_\gamma = M_\gamma \wedge N_\gamma, \forall \gamma \in E$ . Suppose that  $(O, E)$  is non-null. If  $\gamma \in \text{Supp}(O, E)$ , then  $O_\gamma = M_\gamma \wedge N_\gamma \neq 0_R$ . As a result,  $M_\gamma$  and  $N_\gamma$  are both FIs of  $R$ . As a result,  $O_\gamma$  is a FI of  $R$ ,  $\forall \gamma \in \text{Supp}(O, E)$ . Hence,  $(O, E) = (M, C) \cap (N, D)$  is an IFSBR over  $R$ .  $\square$

**Theorem 5.2.** Assume  $(M, C)$  and  $(N, D)$  are two IFSBRs over  $R$ . If  $C$  and  $D$  are disjoint, then  $(M, C) \cup (N, D)$  is an IFSBR over  $R$ .

*Proof.* Let  $(M, C) \cup (N, D) = (O, E)$ , where  $E = C \cup D$  and

$$(\forall \gamma \in E) \left( O_\gamma = \begin{cases} M_\gamma & \text{if } \gamma \in C - D \\ N_\gamma & \text{if } \gamma \in D - C \\ M_\gamma \vee N_\gamma & \text{if } \gamma \in C \cap D \end{cases} \right). \tag{5.1}$$

Let us suppose that  $C \cap D = \emptyset$ . Then either  $\gamma \in C - D$  or  $N_\gamma \in D - C, \forall \gamma \in \text{Supp}(O, E)$ .

If  $\gamma \in C - D, O_\gamma = M_\gamma$  is a FI of  $R$ . Because  $(M, C)$  is an IFSBR over  $R$ .

If  $\gamma \in D - C, O_\gamma = N_\gamma$  is a FI of  $R$ . Because  $(N, D)$  is an IFSBR over  $R$ .

Thus, for all  $\gamma \in \text{Supp}(O, D), O_\gamma$  is a FI of  $R$ . Consequently,  $(O, E) = (M, C) \cup (N, D)$  is an IFSBR over  $R$ .  $\square$

Theorem 5.2 is false generally if and only if  $C$  and  $D$  are not disjoint. Consequently, the theorem is not generally true. Because a ring's FI may not be the union of two different FIs of a ring  $R$ .

**Theorem 5.3.** Assume  $(M, C)$  and  $(N, D)$  are two IFSBRs over  $R$ . Then  $(M, C) \wedge (N, D)$  is an IFSBR over  $R$  if it is non-null.

*Proof.* Let  $(M, C) \wedge (N, D) = (O, C \times D)$ , where  $O(\alpha, \beta) = O_{\alpha, \beta} = M_\alpha \wedge N_\beta, \forall (\alpha, \beta) \in (C \times D)$ . Assume  $(O, C \times D)$  is non-null. If  $(\alpha, \beta) \in \text{Supp}(O, C \times D)$ , then  $O_{\alpha, \beta} = M_\alpha \wedge N_\beta \neq \emptyset$ . Since  $(M, C)$  and  $(N, D)$  are IFSBRs over  $R$ , we can conclude that  $M_\alpha$  and  $N_\beta$  are both FIs of  $R$ . As a result,  $O_{\alpha, \beta}$  is a FI of  $R, \forall (\alpha, \beta) \in \text{Supp}(O, C \times D)$ . Thus,  $(O, C \times D) = (M, C) \wedge (N, D)$  is an IFSBR over  $R$ .  $\square$

## 6. Conclusion

The concept of FSBRS is introduced and its individual properties are studied in this paper. The concepts of FSIs of a FSBRS and an IFSBR are also introduced. This research could be expanded to investigate the properties of FSSs in other algebraic structures.

**Acknowledgment:** This research project was supported by the Thailand Science Research and Innovation Fund and the University of Phayao (Grant No. FF66-UoE017).

**Conflicts of Interest:** The authors declare that there are no conflicts of interest regarding the publication of this paper.

### References

- [1] U. Acar, F. Koyuncu, B. Tanay, Soft Sets and Soft Rings, *Computers Math. Appl.* 59 (2010), 3458-3463. <https://doi.org/10.1016/j.camwa.2010.03.034>.
- [2] B. Ahmad, A. Kharal, On Fuzzy Soft Sets, *Adv. Fuzzy Syst.* 2009 (2009), 586507. <https://doi.org/10.1155/2009/586507>.
- [3] H. Aktaş, N. Çağman, Soft Sets and Soft Groups, *Inform. Sci.* 177 (2007), 2726-2735. <https://doi.org/10.1016/j.ins.2006.12.008>.
- [4] V.N. Dixit, R. Kumar, N. Ajmal, On Fuzzy Rings, *Fuzzy Sets Syst.* 49 (1992), 205-213. [https://doi.org/10.1016/0165-0114\(92\)90325-x](https://doi.org/10.1016/0165-0114(92)90325-x).
- [5] W. Liu, Fuzzy Invariant Subgroups and Fuzzy Ideals, *Fuzzy Sets Syst.* 8 (1982), 133-139. [https://doi.org/10.1016/0165-0114\(82\)90003-3](https://doi.org/10.1016/0165-0114(82)90003-3).
- [6] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy Soft Sets, *J. Fuzzy Math.* 9 (2001), 589-602.
- [7] P.K. Maji, R. Biswas, A.R. Roy, Soft Set Theory, *Computers Math. Appl.* 45 (2003), 555-562. [https://doi.org/10.1016/s0898-1221\(03\)00016-6](https://doi.org/10.1016/s0898-1221(03)00016-6).
- [8] D. Molodtsov, Soft Set Theory—First Results, *Computers Math. Appl.* 37 (1999), 19-31. [https://doi.org/10.1016/s0898-1221\(99\)00056-5](https://doi.org/10.1016/s0898-1221(99)00056-5).
- [9] D. Pei, D. Miao, From Soft Sets to Information Systems, in: 2005 IEEE International Conference on Granular Computing, IEEE, Beijing, China, 2005: pp. 617-621. <https://doi.org/10.1109/GRC.2005.1547365>.