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Mathematical Model of Hepatitis B Virus With Effect of Vaccination and Treatments

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Abstract. In this paper, a mathematical model of hepatitis B virus with vaccination and treatments is studied, Stability analysis discussed and the disease-free equilibrium and endemic equilibrium points obtained, the basic reproductive number $\mathcal{R}_0$ determined and became the threshold for equilibrium points stability. The study showed when $\mathcal{R}_0 < 1$ the disease-free equilibrium point was stable, whereas $\mathcal{R}_0 > 1$ the virus is endemic and the endemic equilibrium point is stable. The sensitivity analysis for the parameters that could reduce the spread of hepatitis B virus is studied. Finally the numerical simulation are established by using SageMath software package to show the effect of vaccination and treatments. We found that vaccination and also treatments give an effect on value of $\mathcal{R}_0$. Increasing the value of the vaccine in the immunized compartment or in the suspected compartment may decrease the value of $\mathcal{R}_0$ which mean reduce the spread of the disease.

1. Introduction

Infectious diseases are disorders caused by small organisms such as bacteria, viruses, fungi or parasites. Some infectious diseases transmitted from infected person to another by direct physical contact, airborne droplets, water or food, disease vectors, or mother to newborn. Hepatitis B is an infectious disease of the liver caused by the hepatitis B virus (HBV). In some people hepatitis B can become chronic leading to liver failure, liver cancer or cirrhosis. Most healthy adults who are newly infected will recover without any problems. But babies and young children may not be able to successfully get rid of the virus. 90% of healthy adults will get rid of the virus and recover without any problems; 10% will develop chronic hepatitis B. Young Children – Up to 50% of young children between 1 and 5 years

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who are infected will develop a chronic hepatitis B infection. Infants – 90% will become chronically infected; only 10% will be able to get rid of the virus [1].

Mathematical Model of Hepatitis B virus based on MSEIR model, the population divided into five compartments Immunized(M), Susceptible (S), Exposed (E), Infected (I) and Recovered (R). There are many previous studies on the spread of hepatitis B, Zhang (2015) study the application and optimal control for an HBV model with vaccination and treatments [2], Aniji (2019) developed the model by adding the Carrier compartment [3], Beay (2017) developed the model by adding migration compartments [4], Wiraningsih (2015) discussed the model with vaccination effect [5], cao (2015) studied the Global stability of an epidemic model with carrier state in heterogeneous network [6]. In this paper we use SEIR model and apply it to hepatitis B virus. We extend the model by introducing the immunized class. The paper outline as follows, in section 2, we present the description and assumption of the model, in section 3 we discus the stability analysis for disease free equilibrium and endemic equilibrium, in section 4 we show the relation between the basic reproductive number and the model parameters, in section 5 the explore the numerical simulation to show the dynamical behaviour of our results, we finished the paper with a conclusion in section 6.

2. Description of the model

The Population of our model is divided into five compartments: Immunized (M), Susceptible (S), Exposed (E), Infected (I) and Recovered (R). The interaction between the five compartments is shown in the following diagram:

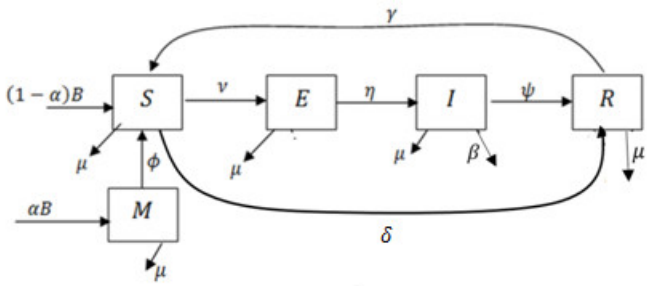


Figure 1. The transmission of hepatitis B virus model

The basic assumptions of the model:

- birth rate equal to the death rate.
- The population size is constant N.
- The rate of transmission from exposed to infective is η.
- The rate of transmission from infected to recovered compartment is ψ.
- The rate of transmission from susceptible to infected class is ν.
- The individual who take vaccine transfer from Susceptible class to Recovered class with rate δ.

- The individual who lost immune transfer from Recovered class to Susceptible class with rate γ

The differential equations that govern the model are

$$\begin{aligned}
\frac{dM}{dt} &= \alpha B - \phi M - \mu M \\
\frac{dS}{dt} &= (1-\alpha)B + \phi M + \gamma R - (\nu I + \mu + \delta)S \\
\frac{dE}{dt} &= \nu SI - \eta E - \mu E \\
\frac{dI}{dt} &= \eta E - \psi I - \beta I - \mu I \\
\frac{dR}{dt} &= \delta S + \psi I - \gamma R - \mu R
\end{aligned} \tag{1}$$

where, $M + S + E + I + R = N$ (total population).
The description of the parameters are shown in the following table

Parameter	Description
α	Immunized newborns
β	HB induced mortality
γ	Rate of transmission from R to S
μ	Rate of natural mortality
ν	Rate of transmission from S to E
η	Rate of transmission from E to I
δ	Rate of transmission from S to R
ψ	Rate of transmission from I to R
ϕ	Rate expiration of vaccine efficacy

By assumption that the total population is constant, (the birth rate = the death rate) then the model can be expressed in a simpler term in the form

$$\begin{aligned}
\frac{dM}{dt} &= \alpha\mu - \phi M - \mu M \\
\frac{dS}{dt} &= (1-\alpha)\mu + \phi M + \gamma R - (\nu I + \mu + \delta)S \\
\frac{dE}{dt} &= \nu SI - \eta E - \mu E \\
\frac{dI}{dt} &= \eta E - \psi I - \beta I - \mu I \\
\frac{dR}{dt} &= \delta S + \psi I - \gamma R - \mu R
\end{aligned} \tag{2}$$

3. Stability Analysis

3.1. **Equilibrium Solutions:** Let $E(M, S, E, I, R)$ be the equilibrium point of the system (2), at the equilibrium state, we have

$$\frac{dM}{dt} = \frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

that gives,

$$\alpha\mu - \phi M - \mu M = 0$$

$$(1-\alpha)\mu + \phi M + \gamma R - (\nu I + \mu + \delta)S = 0$$

$$\nu SI - \eta E - \mu E = 0 \tag{3}$$

$$\eta E - \psi I - \beta I - \mu I = 0$$

$$\delta S + \psi I - \gamma R - \mu R = 0$$

3.2. **Disease-Free Equilibrium DFE:.** Let $E_0(M_0, S_0, E_0, I_0, R_0)$ be a solution of the system (3), suppose both I and E must be zero, $(I_0 = 0, E_0 = 0)$. By substitute $(I = 0, E = 0)$ into the system (3) we get

$$\alpha\mu - (\phi + \mu)M = 0$$

$$(1-\alpha)\mu + \phi M + \gamma R - (\nu + \delta)S = 0 \tag{4}$$

$$\delta S + -(\gamma + \mu)R = 0$$

By solving these equations we get

$$M_0 = \frac{\alpha\mu}{\phi + \mu},$$
$$R_0 = \frac{\delta(\phi + (1-\alpha)\mu)}{\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2},$$

and

$$S_0 = \frac{\phi\gamma + \mu\gamma + \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2}{\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2}$$

therefore the disease - free equilibrium of the model is

$$E_0\left(\frac{\alpha\mu}{\phi + \mu}, \frac{\phi\gamma + \mu\gamma + \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2}{\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2}, 0, 0, \frac{\delta(\phi + (1-\alpha)\mu)}{\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2}\right)$$

3.3. **Basic reproductive number** ($\mathcal{R}_0$) **for DFE:.** According to calculation method of $\mathcal{R}_0$ [12], Let $X = [E, I]^T$ and set

$$\mathcal{F} = \begin{pmatrix} \nu SI \\ 0 \end{pmatrix}, \ \mathcal{V} = \begin{pmatrix} (\eta+\mu)E \\ -\eta E + (\psi+\mu+\beta)I \end{pmatrix}$$

then we can write

$$\frac{dX}{dt} = \mathcal{F} - \mathcal{V},$$

Compute F and V where,

$$F = \begin{pmatrix} \frac{\partial \mathcal{F}_1}{\partial E} & \frac{\partial \mathcal{F}_1}{\partial I} \\ \\ \frac{\partial \mathcal{F}_2}{\partial E} & \frac{\partial \mathcal{F}_2}{\partial I} \end{pmatrix}_{E_0} = \begin{pmatrix} 0 & \nu S_0 \\ 0 & 0 \end{pmatrix},$$

$$V = \begin{pmatrix} \frac{\partial \mathcal{V}_1}{\partial E} & \frac{\partial \mathcal{V}_1}{\partial I} \\ \\ \frac{\partial \mathcal{V}_2}{\partial E} & \frac{\partial \mathcal{V}_2}{\partial I} \end{pmatrix}_{E_0} = \begin{pmatrix} \eta+\mu & 0 \\ -\eta & \psi+\mu+\beta \end{pmatrix},$$

the next generation matrix $K = FV^{-1}$

$$K = \frac{1}{(\eta+\mu)(\psi+\mu+\beta)} \begin{pmatrix} 0 & \nu S_0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi+\mu+\beta & 0 \\ \eta & \eta+\mu \end{pmatrix},$$

$$K = \begin{pmatrix} \frac{\nu\eta S_0}{(\eta+\mu)(\psi+\mu+\beta)} & \frac{\nu S_0}{(\psi+\mu+\beta)} \\ 0 & 0 \end{pmatrix}$$

The eigen values of K are $\lambda_1 = 0$ and

$$\lambda_2 = \frac{\nu\eta S_0}{(\eta+\mu)(\psi+\mu+\beta)} = \frac{\nu\eta(\phi\gamma+\mu\gamma+\alpha\mu\gamma+\phi\mu+\mu^2-\alpha\mu^2)}{(\eta+\mu)(\psi+\mu+\beta)(\phi\gamma+\delta\phi+\mu\phi+\mu\gamma+\mu\delta+\mu^2)},$$

Hence,

$$\mathcal{R}_0 = \rho(K) = \frac{\nu\eta(\phi\gamma+\mu\gamma+\alpha\mu\gamma+\phi\mu+\mu^2-\alpha\mu^2)}{(\eta+\mu)(\psi+\mu+\beta)(\phi\gamma+\delta\phi+\mu\phi+\mu\gamma+\mu\delta+\mu^2)}, \tag{5}$$

Theorem 3.1. *The disease free-equilibrium point E_0 of the system (2) is asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.*

The value of $\mathcal{R}_0$ measures whether the disease will spread and become endemic or will disappear from the population. When $\mathcal{R}_0 < 1$, the disease will disappear and the exposed and infected compartments tend to zero as time goes on. When $\mathcal{R}_0 > 1$, the disease will spread and become endemic. This means that each compartments will be positive valued for a long time. In another words, in the case of $\mathcal{R}_0 > 1$ the endemic equilibrium point E^* exists and stable.

3.4. **Stability Analysis of DFE:.** we examine the behavior of the model near E_0, linearize the system near E_0, suppose the left hand side of system (2) be $F_1, \ldots, F_n$ respectively, then the Jacobin matrix is given as

$$J\left(\frac{F_1, F_2, F_3, F_4, F_5}{M, S, E, I, R}\right) =$$

$$\begin{bmatrix} -(\phi+\mu) & 0 & 0 & 0 & 0 \\ \phi & -(\nu I+\mu+\delta) & 0 & -\nu S & \gamma \\ 0 & \nu I & -(\eta+\mu) & \nu S & 0 \\ 0 & 0 & \eta & -(\psi+\beta+\mu) & 0 \\ 0 & \delta & 0 & \psi & -(\gamma+\mu) \end{bmatrix}$$

the Jacobian matrix of the disease-free equilibrium is given by:

$$J(E_0) = \begin{bmatrix} -(\phi+\mu) & 0 & 0 & 0 & 0 \\ \phi & -(\mu+\delta) & 0 & -\nu S_0 & \gamma \\ 0 & \nu I & -(\eta+\mu) & \nu S_0 & 0 \\ 0 & 0 & \eta & -(\psi+\beta+\mu) & 0 \\ 0 & \delta & 0 & \psi & -(\gamma+\mu) \end{bmatrix}$$

now let

$$J(E_0) = \begin{bmatrix} J_0 & J_2 \\ J_1 & J_3 \end{bmatrix}$$

, where $J_0 = \begin{bmatrix} -(\phi+\mu) & 0 \\ \phi & -(\mu+\delta) \end{bmatrix}$, $J_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\nu S_0 & \gamma \end{bmatrix}$

$J_1 = \begin{bmatrix} o & 0 \\ 0 & 0 \\ 0 & \delta \end{bmatrix}$ and $J_3 = \begin{bmatrix} -(\eta+\mu) & \nu S_0 & 0 \\ \eta & -(\psi+\beta+\mu) & 0 \\ 0 & \psi & -(\gamma+\mu) \end{bmatrix}$

The eigen values given by:

$$|J(E_0) - \lambda| = 0$$

implies that,

$$\begin{vmatrix} -(\phi+\mu)-\lambda & 0 \\ \phi & -(\mu+\delta)-\lambda \end{vmatrix} = 0 \tag{6}$$

and

$$\begin{vmatrix} -(\eta+\mu)-\lambda & \nu S_0 & 0 \\ \eta & -(\psi+\beta+\mu)-\lambda & 0 \\ 0 & \psi & -(\gamma+\mu)-\lambda \end{vmatrix} = 0 \tag{7}$$

from (6) we obtain

$$\lambda_1 = -\phi - \mu,$$

$$\lambda_2 = -\mu - \delta,$$

and from (7),

$$\lambda_3 = -\gamma - \mu,$$

$$(\eta + \mu + \lambda)(\psi + \beta + \mu + \lambda) - \nu\eta S_0 = 0,$$

that yields

$$\lambda^2 + (\eta + \psi + \beta + 2\mu)\lambda + (\eta + \mu)(\psi + \beta + \mu) - \nu\eta S_0 = 0,$$

or,

$$\lambda^2 + \omega_1\lambda + \omega_2 = 0 \tag{8}$$

where, $\omega_1 = \eta + \psi + \beta + 2\mu$, and $\omega_2 = (\eta + \mu)(\psi + \beta + \mu) - \nu\eta S_0$
from Routh-Hurwitz criterion [13], equation (8) have negative real part if and only if:

$$\omega_1 > 0 \text{ and } \omega_2 > 0,$$

since all the parameters η, ψ, β, μ are positive then $\omega_1 > 0$, and the condition ($\omega_2 > 0$) yields

$$(\eta + \mu)(\psi + \beta + \mu) > \nu\eta S_0$$

It can be seen that all the eigenvalues have negative real parts and therefore the disease-free equilibrium is Locally Asymptotically Stable.

3.5. **Stability Analysis of Endemic Equilibrium:** Denote the endemic equilibrium of the system (2) is $E^* = (M^*, S^*, E^*, I^*, R^*)$, which can obtain by putting

$$\frac{dM}{dt} = \frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

or,

$$\alpha\mu - \phi M - \mu M = 0$$

$$(1 - \alpha)\mu + \phi M + \gamma R - (\nu I + \mu + \delta)S = 0$$

$$\nu SI - \eta E - \mu E = 0 \tag{9}$$

$$\eta E - \psi I - \beta I - \mu I = 0$$

$$\delta S + \psi I - \gamma R - \mu R = 0$$

by solving these equations we obtain,

$$M^* = \frac{\alpha\mu}{(\phi + \mu)} = \sigma_0,$$

$$E^* = \frac{(\psi + \mu + \beta)}{\eta} I = \sigma_1 I,$$

$$S^* = \frac{(\eta + \mu)\sigma_1}{\nu} = \sigma_2,$$

$$R^* = \frac{\psi}{(\gamma+\mu)} I^* + \frac{\delta\sigma_2}{(\gamma+\mu)} = I^*\sigma_3 + \sigma_4,$$

and

$$I^* = \frac{(\mu+\delta)\sigma_2 - \gamma\sigma_4 - (1-\alpha)\mu - \phi\sigma_0}{(\sigma_3\gamma - \nu\sigma_2)}$$

where,

$$\sigma_0 = \frac{\alpha\mu}{(\phi+\mu)}, \sigma_1 = \frac{(\psi+\mu+\beta)}{\eta}, \sigma_2 = \frac{(\eta+\mu)\sigma_1}{\nu}, \sigma_3 = \frac{\psi}{(\gamma+\mu)} \text{ and } \sigma_4 = \frac{\delta\sigma_2}{(\gamma+\mu)}$$

the Jacobian matrix of the system (4.2) at E^*

$$J(E^*) = \begin{bmatrix} -(\phi+\mu) & 0 & 0 & 0 & 0 \\ \phi & -(\nu I^* + \mu + \delta) & 0 & -\nu S^* & \gamma \\ 0 & \nu I^* & -(\eta+\mu) & \nu S^* & 0 \\ 0 & 0 & \eta & -(\psi+\beta+\mu) & 0 \\ 0 & \delta & 0 & \psi & -(\gamma+\mu) \end{bmatrix}$$

now let

$$J(E_0) = \begin{bmatrix} J_0 & J_2 \\ J_1 & J_3 \end{bmatrix}$$

, where $J_0 = \begin{bmatrix} -(\phi+\mu) & 0 \\ \phi & -(\nu I^* + \mu + \delta) \end{bmatrix}$, $J_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\nu S^* & \gamma \end{bmatrix}$

$J_1 = \begin{bmatrix} 0 & \nu I^* \\ 0 & 0 \\ 0 & \delta \end{bmatrix}$ and $J_3 = \begin{bmatrix} -(\eta+\mu) & \nu S^* & 0 \\ \eta & -(\psi+\beta+\mu) & 0 \\ 0 & \psi & -(\gamma+\mu) \end{bmatrix}$

The eigen values given by:

$$|J(E^*) - \lambda| = 0$$

that implies that

$$\begin{vmatrix} -(\phi+\mu) - \lambda & 0 \\ \phi & -(\nu I^* + \mu + \delta) - \lambda \end{vmatrix} = 0 \tag{10}$$

and

$$\begin{vmatrix} -(\eta+\mu) - \lambda & \nu S^* & 0 \\ \eta & -(\psi+\beta+\mu) - \lambda & 0 \\ 0 & \psi & -(\gamma+\mu) - \lambda \end{vmatrix} = 0 \tag{11}$$

from (10) we obtain $\lambda_1 = -\phi - \mu$, and $\lambda_2 = -(\nu I^* + \mu + \delta)$,

since I^* is positive implies that λ_1, λ_2 are negative.

and from (11),

$$-(\eta+\mu+\lambda)[(\psi+\beta+\mu+\lambda)(\gamma+\mu+\lambda)] - \nu S^*(\gamma+\mu+\lambda)\eta = 0$$

or,

$$(\gamma+\mu+\lambda)[(\eta+\mu+\lambda)(\psi+\beta+\mu+\lambda)+\nu\eta S^*]=0,$$

that yields ,

$$\lambda_3=-\gamma-\mu,$$

and $\lambda_{4,5}$ are given from

$$\lambda^2+\omega_1\lambda+\omega_2=0,$$

where

$$\omega_1=\eta+2\mu+\psi+\beta$$

$$\omega_2=(\eta+\mu)(\psi+\beta+\mu)-\nu\eta S^*=0$$

thus,

$$\lambda^2+\omega_1\lambda=0,$$

$$\lambda_4=0, \qquad \lambda_5=-\omega_1$$

It can be seen that all the eigenvalues have negative real parts and therefore the endemic equilibrium is locally asymptotically stable.

4. Sensitivity Analysis of $\mathcal{R}_0$

In this section, we show the relation between the basic reproductive number $\mathcal{R}_0$ and the parameters that can reduce the spread of the disease α, δ and ψ. the value of the parameters are shown in the table 1

Parameter	**Description**	**Value**	References
α	Immunized newborns	0.05-1	[10]
β	HBV induced mortality	0.015	[10]
γ	Rate of transmission from R to S	0.06	[10]
μ	Rate of natural mortality	0.0121	[10]
ν	Rate of transmission from S to E	0.08	Assume
η	Rate of transmission from E to I	0.75	Assume
δ	Rate of transmission from S to R	0.1-1	[10]
ψ	Rate of transmission from I to R	0.1-1	[10]
ϕ	Rate expiration of vaccine efficacy	0.05	[10]

Table 1. Parameters Values of the model

4.1. **The relation between $\mathcal{R}_0$ and α:** We can write the basic reproductive number as

$$\mathcal{R}_0 = \frac{\nu\eta(\phi\gamma + \mu\gamma - \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2)}{(\eta+\mu)(\psi+\beta+\mu)(\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2)} = K_1 - \alpha K_2$$

where,

$K_1 = \frac{\nu\eta(\phi\gamma+\mu\gamma+\phi\mu+\mu^2)}{(\eta+\mu)(\psi+\beta+\mu)(\phi\gamma+\delta\phi+\mu\phi+\mu\gamma+\mu\delta+\mu^2)}$ and $K_2 = \frac{(\mu\gamma+\mu^2)\nu\eta}{(\eta+\mu)(\psi+\beta+\mu)(\phi\gamma+\delta\phi+\mu\phi+\mu\gamma+\mu\delta+\mu^2)}$ Suppose the values of the parameters are given as $\beta = 0.015, \gamma = 0.07, \mu = 0.0125, \nu = 0.08, \eta = 0.75, \delta = 0.02, \psi = 0.03$ and $\phi = 0.05$.

Table 2 and figure 2 show the relation between α and $\mathcal{R}_0$ for six values of α namely $\alpha = 0.15, \alpha = 0.25, \alpha = 0.35, \alpha = 0.45, \alpha = 0.55,$ and $\alpha = 0.65$.

Parameter α	$\mathcal{R}_0$	Free/Endemic
0.15	1.068428	Endemic
0.25	1.046399	Endemic
0.35	1.024369	Endemic
0.45	1.00234	Endemic
0.55	0.98031	Free
0.65	0.958281	Free

Table 2. The relation between α and $\mathcal{R}_0$

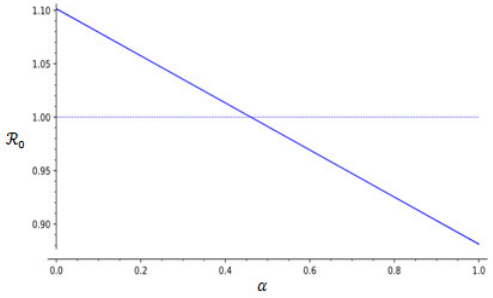


Figure 2. The relation between α and $\mathcal{R}_0$

4.2. **The relation between $\mathcal{R}_0$ and δ:** We can write the basic reproductive number as

$$\mathcal{R}_0 = \frac{\nu\eta(\phi\gamma + \mu\gamma - \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2)}{(\eta+\mu)(\psi+\beta+\mu)(\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2)} = \frac{C_1}{C_2 + \delta C_3}$$

where,

$C_1 = \nu\eta(\phi\gamma + \mu\gamma - \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2$

$C_2 = (\eta+\mu)(\psi+\beta+\mu)(\phi\gamma + \mu\phi + \mu\gamma + \mu^2)$

$C_3 = (\eta+\mu)(\psi+\beta+\mu)(\phi+\mu)$.

Suppose that the values of parameter are given as $\alpha = 0.05, \beta = 0.015, \gamma = 0.07, \mu = 0.0125, \nu = 0.08, \eta = 0.75, \psi = 0.03$ *and* $\phi = 0.05$. Table 3 and figure 3 show the effect of immunization for adults to the value of $\mathcal{R}_0$. For six values of δ, namely $\delta = 0.01, \delta = 0.05, \delta = 0.1, \delta = 0.15, \delta = 0.2,$ *and* $\delta = 0.25$.

Parameter δ	$\mathcal{R}_0$	Free/Endemic
0.01	2.265647	Endemic
0.05	1.581678	Endemic
0.1	1.148342	Endemic
0.15	0.901386	Free
0.2	0.741849	Free
0.25	0.630293	Free

Table 3. The relation between δ and $\mathcal{R}_0$

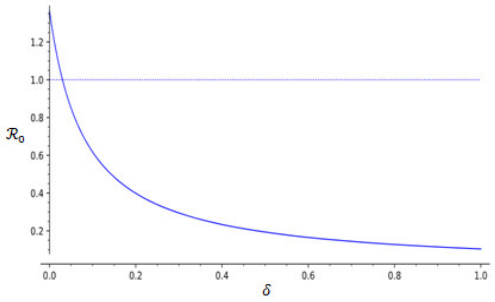


Figure 3. The relation between δ and $\mathcal{R}_0$

4.3. **The relation between $\mathcal{R}_0$ and ψ:** We can write the basic reproductive number as

$$\mathcal{R}_0 = \frac{\nu\eta(\phi\gamma + \mu\gamma - \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2)}{(\eta+\mu)(\psi+\beta+\mu)(\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2)} = \frac{A_1}{(A_2 + A_3\psi)}$$

where,

$$A_1 = \nu\eta(\phi\gamma + \mu\gamma - \alpha\mu\gamma + \phi\mu + \mu^2 - \alpha\mu^2$$

$$A_2 = (\eta+\mu)(\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2)(\beta+\mu)$$

$$A_3 = (\eta+\mu)(\phi\gamma + \delta\phi + \mu\phi + \mu\gamma + \mu\delta + \mu^2)$$

we suppose that the values of parameter are given as $\alpha = 0.05, \beta = 0.015, \gamma = 0.07, \mu = 0.0125, \nu = 0.08, \eta = 0.75, \delta = 0.02$ and $\phi = 0.05$.

Table 4 and figure 4 show the effect of treatments on infected individuals(ψ) to the value of $\mathcal{R}_0$. For six values of ψ, namely $\psi = 0.01, \psi = 0.05, \psi = 0.1, \psi = 0.15, \psi = 0.2,$ *and* $\psi = 0.25$.

Parameter ψ	$\mathcal{R}_0$	Free/Endemic
0.01	2.42524	Endemic
0.05	1.173503	Endemic
0.1	0.713306	Free
0.15	0.512375	Free
0.2	0.399765	Free
0.25	0.327735	Free

Table 4. The relation between ψ and $\mathcal{R}_0$

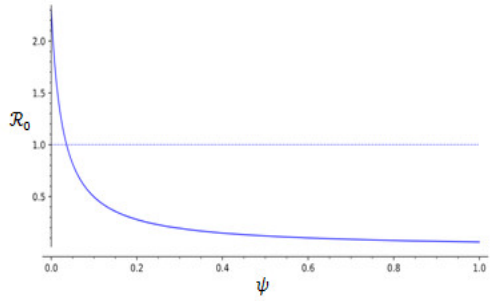
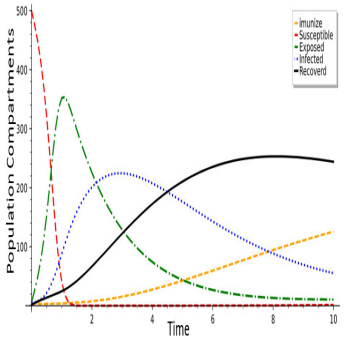


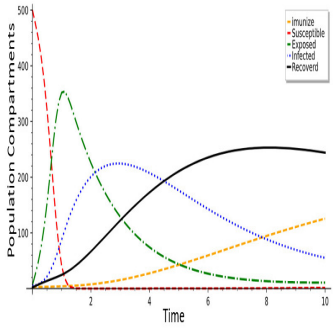
Figure 4. The relation between ψ and $\mathcal{R}_0$

5. Results and Discussion

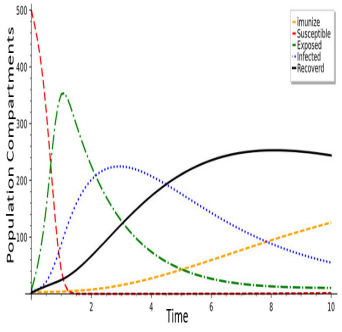
5.1. **Effect of immunization of newborns (α).** We suppose that the values of parameter of the model are given as β=0.015, δ=0.05, γ=0.09, μ=0.0125 ϕ=0.05, ν=0.08 ,η=0.55 ,ψ=0.3. There are five values of α, 0.05, 0.25, 0.45, 0.65, 0.85 and 1. The figures 5 show the effect of increasing value of α gives an increasing in immunized and recovered curves and a decreasing in infectious curve.



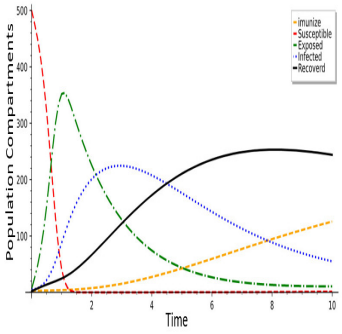
(a) The population compartments α = 0.05



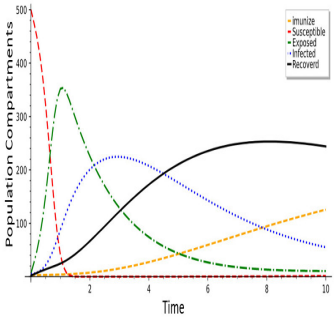
(b) The population compartments α = 0.25



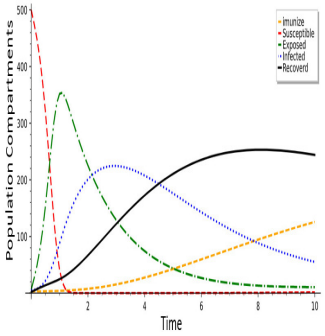
(c) The population compartments α = 0.45



(d) The population compartments α = 0.65



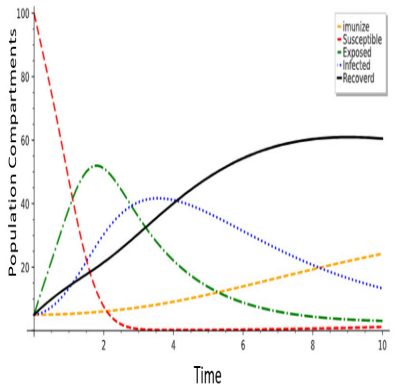
(e) The population compartments α = 0.85



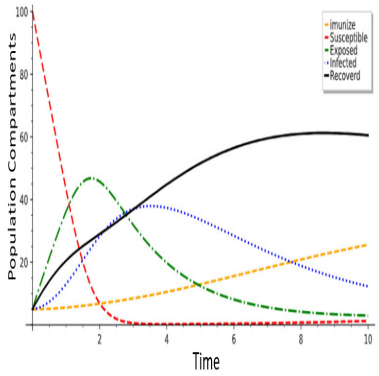
(f) The population compartments α = 1

Figure 5. The variation of the parameter α

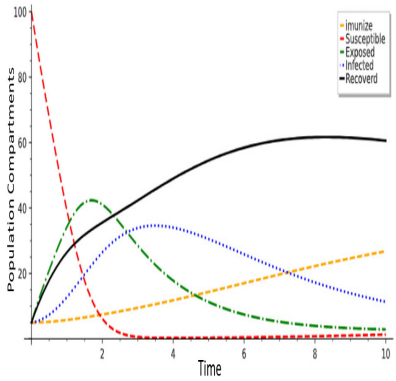
5.2. **Effect of immunization of adults (δ).** We suppose that the values of the parameters are given as α=0.1,β=0.015, γ=0.09, μ=0.0125, ϕ=0.07, ν=0.08 ,η=0.5 and ψ=0.3, There are five values of δ are 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. The figures 6 show the effect of increasing value of δ gives an increasing in immunized and recovered curves and a decreasing in infectious curve.



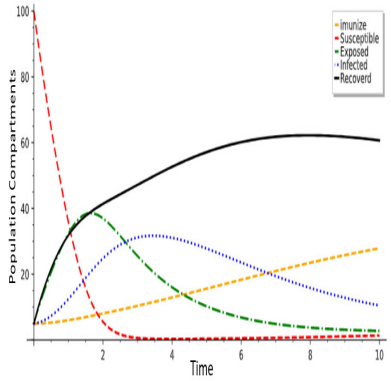
(a) The population compartments $\delta = 0.1$



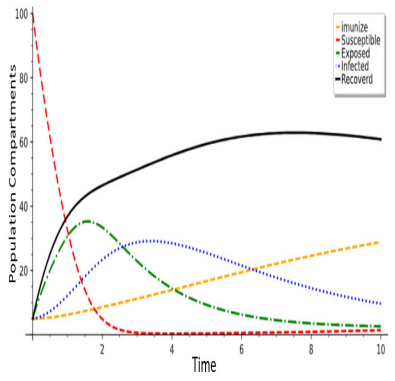
(b) The population compartments $\delta = 0.2$



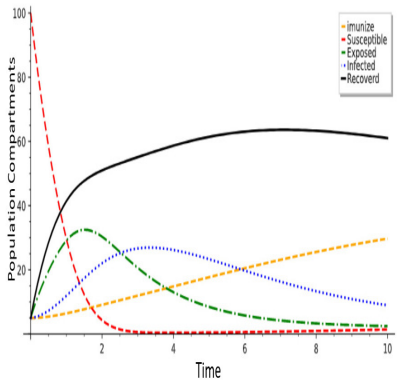
(c) The population compartments $\delta = 0.3$



(d) The population compartments $\delta = 0.4$



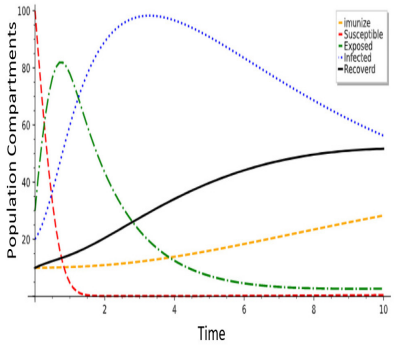
(e) The population compartments $\delta = 0.5$



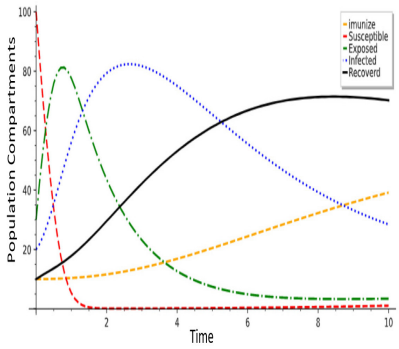
(f) The population compartments $\delta = 0.6$

Figure 6. The variation of the parameter δ

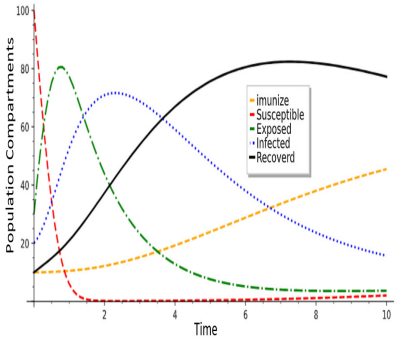
5.3. **Effect of treatment on infected to the spread of hepatitis B virus (ψ).** We suppose that the values of parameter of the model are given as $\alpha = 0.1, \beta = 0.015, , \gamma = 0.09, \mu = 0.0125, \phi = 0.07, \nu = 0.08, \eta = 0.65$ and $\delta = 0.05$, There are five values of ψ, are $\psi = 0.1, \psi = 0.2, \psi = 0.3, \psi = 0.4, \psi = 0.5$ and $\psi = 0.6$. The figure 7 show the effect of increasing value of ψ gives an increasing in immunized and recovered curves and a decreasing in infectious curve.



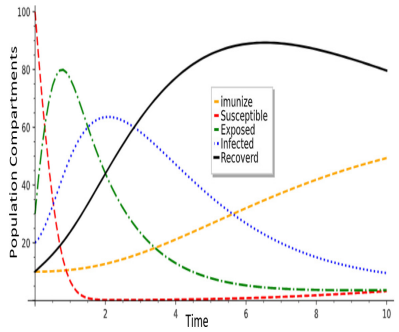
(a) The population compartments $\psi = 0.1$



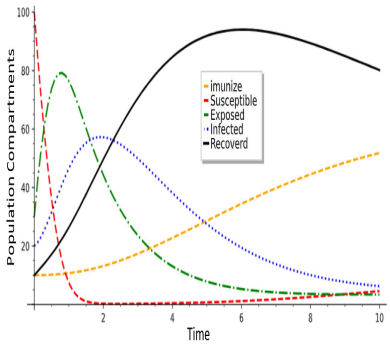
(b) The population compartments $\psi = 0.2$



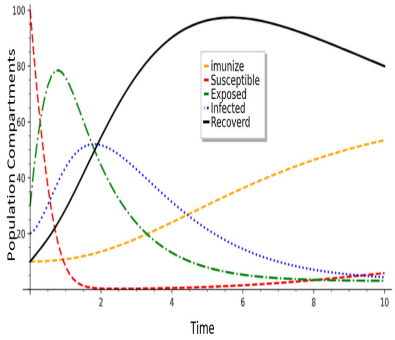
(c) The population compartments $\psi = 0.3$



(d) The population compartments $\psi = 0.4$



(e) The population compartments $\psi = 0.5$



(f) The population compartments $\psi = 0.6$

Figure 7. The variation of the parameter ψ

6. Conclusion

In the model of hepatitis B virus using the standard SEIR model has been developed by considering vaccination and treatments in compartments. This strategy aims to reduce the spread of hepatitis B virus. The model was divided into five compartments which include immunized, exposed, infected, and recovered compartments. The model has two cases free - equilibrium point and endemic - equilibrium point. The existence of endemic equilibrium point depends on the value of reproductive number $\mathcal{R}_0$. From the table 2 and table 3 we notice that increasing the value of α and δ leads to decreasing in the value of $\mathcal{R}_0$, which its becomes from value greater than one to value less than one, that means giving more vaccines for newborns and adults change the disease from endemic to free condition. From table 4 we notice that increasing in the value of ψ leads to decreasing in $\mathcal{R}_0$, which its becomes from value greater than one to value less than one, which means the treatments on infected has a big effect on changing from endemic case to free case.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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