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THE EFFECT OF UNITS LOST DUE TO DETERIORATION IN FUZZY ECONOMIC ORDER QUANTITY (FEOQ) MODEL

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Abstract. For several decades, the Economic Order Quantity (EOQ) model and its variations have received much attention from researchers. Recently, there has been an investigation into an EOQ model incorporating effect of units lost due to deterioration in infinite planning horizon with crisp decision environment. Accounting for holding and ordering cost, as has traditionally been the case of modeling inventory systems in fuzzy environment are investigated which are not precisely known and defined on a bounded interval of real numbers. The question is how reliable are the EOQ models when items stocked deteriorate one time. This paper introduces Fuzzy Economic Order Quantity (FEOQ) model in which it assumes that units lost due to deterioration is included in the objective function to properly model the problem in finite planning horizon. The numerical analysis shows that an appropriate fuzzy policy can benefit the retailer and that is significant, especially for deteriorating items is shown to be superior to that of crisp decision making. A computational algorithm using LINGO 13.0 and MATLAB (R2009a) software are developed to find the optimal solution. Sensitivity analysis of the optimal solution is also studied and managerial insights are drawn which shows the influence of key model parameters.

1. Introduction

As markets have become more and more competitive, high quality product has become a prevailing characteristic of modern production systems that are operating in complex, dynamic and uncertain

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environments. Good inventory management is often the mark of a well-known organization. Inventory levels must be carefully to balance the costs and reasonable levels of good customer service. Since the marketing environment is continuously changing, corporate strategy must be adjusted accordingly. In order to establish a good image and to enhance customers' royalty, many efforts such as upgrading the service facilities, maintaining the high quality products and increasing expenditure on advertisement could be made by a selling shop. A manager's intention should always be to explore the possibility of improving the current system so as to maximize (minimize) the total profit (cost). A subject in the area of inventory theory that has recently been receiving considerable attention is the class of inventory models with deterioration. With these models, the presence of retail inventory is assumed to have a motivating effect on the customer.

Many models have been proposed to deal with a variety of inventory problems. Comprehensive reviews of inventory models can be found in (Gupta and Gerchak, 1995), (Osteryoung et al., 1986) and (Waters, 1994). Product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs etc. suffer from depletion by decent spoilage. Decaying products are of two types. Product which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles is available in inventory literature considering deterioration. Interested readers may consult the survey paper of (Pattnaik, 2011) investigates an entropic order quantity model for perishable items with pre and post deterioration discounts under two component demand in finite horizon. (Pattnaik, 2011) discusses an economic order quantity model for perishable items with constant demand where instant deterioration discount is allowed to obtain maximum profit. (Goyal and Gunasekaran, 1995) and (Raafat, 1991) surveyed for perishable items to optimize the EOQ model. The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries today.

Comprehensive reviews of inventory models under deterioration can be found in (Bose et al., 1995). In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. (Jain and Silver, 1994) developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. (Gupta and Gerchak, 1995) examined the simultaneous selection product durability and order quantity for items that deteriorate over time. Their choice of product durability is modeled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. (Hariga, 1995) considered the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. (Hariga, 1994) argued that the analysis of (Bose et al. 1995) contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. (Padmanavan and Vrat, 1995) presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. (Pattnaik, 2011) developed an entropic order quantity model for deteriorating items where cash discounts are allowed but (Pattnaik, 2011) modified to obtain the decision parameters for perishable items where instant deterioration discount is allowed in EOQ model.

The most recent work found in the literature is that of (Hariga, 1996) who extended his earlier work by assuming a time-varying demand over a finite planning horizon. (Salameh, et al., 1999) studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked.

This paper considers a continuous review, using fuzzy arithmetic approach to the system cost for instantaneous production process. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in (Salameh et al., 1999). From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. (Vujosevic et al., 1996) introduced the EOQ model where inventory system cost is fuzzy. (Mahata and Goswami, 2006) then presented production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating items where permissible delay in payments are allowed. Later, (Tripathy and Pattnaik, 2009, 2011) also investigated fuzzy EOQ model with reliability consideration in instantaneous production plan. Again (Tripathy and Pattnaik, 2008, 2011) developed fuzzy entropic order quantity model for perishable items under two component demand and discounted selling price. For this reason, this paper considers the same since no researcher have discussed EOQ model by introducing the holding cost and ordering cost as with allowing promotion and wasting the percentage of the fuzzy number. The model provides an approach for quantifying these benefits which can be substantial, and should be reflected in fuzzy arithmetic system cost. The objective is to find optimal values of the policy variables when the criterion is to optimize the expected total payoff over a finite horizon.

The effect of deteriorating items on the instantaneous profit maximization replenishment model under promotion is considered in this paper. The market demand may increase with the promotion of the product over time when the units lost due to deterioration. In the existing literature about promotion it is assumed that the promotional effort cost is a function of promotion. (Pattnaik, 2010, 2011) studies profit maximization entropic order quantity model for deteriorated items with stock dependent demand where instant deterioration and post deterioration cash discounts respectively are allowed for acquiring more profit. In this paper, promotional effort and replenishment decision are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the planning horizon. The objective of this paper is to determine optimal promotional efforts and replenishment quantities in an instantaneous replenishment profit maximization model. This paper establishes and analyzes the fuzzy inventory model under profit maximization which extends the classical economic order quantity (EOQ) model. An efficient EOQ does more than just reduce cost. It also creates revenue for the retailer and the manufacturer. The evolution of the EOQ model concept tends toward revenue and demand focused strategic formation and decision making in business operations. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus. This paper focuses on the profit maximizing issues in a production model, based on cost reduction mechanisms and a revenue improvement stimulus and formulates the decision model by considering the effects of promotion and units lost due to deterioration in the replenishment policy.

In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality, these companies have mainly relied on qualitative judgment. The paper tackles to investigate the effect of the wasting the percentage of on-hand inventory due to deterioration for obtaining the optimum average payoff and the optimal values of the policy variables. The problem consists of the optimization of fuzzy EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. A policy iteration algorithm is designed and optimum solution is obtained using LINGO 13.0 version software. This paper postulates that measuring the behavior of production systems may be achievable by incorporating the idea of making optimum decision on replenishment with units lost due to deterioration in fuzzy decision space. A numerical experiment is conducted to analyze the magnitude of the approximation error. However, the adding of both the fuzzy system costs and the wastage to the percentage of on-hand inventory due to deterioration might lead to super gain for the retailer. The major assumptions used in the above research articles are summarized in Table1.

Table 1: Summary of the Related Researches

Author(s) and published Year	Structure of the model	Model	Demand	Demand patterns	Deterioration	Loss of Units Sales	Planning
Hariga (1994)	Crisp	Cost	Time	Non-stationary	Yes	No	Finite
Vujosevic etal. (1996)	Fuzzy	Cost	Constant (deterministic)	Constant	No	No	Infinite
Salameh etal (1999)	Crisp	Profit	Constant (deterministic)	Constant	Yes	Yes	Finite
Tripathy etal. (2009)	Fuzzy	Cost	Constant (deterministic)	Constant	No	No	Finite
Present Paper (2013)	Fuzzy	Profit	Constant (deterministic)	Constant	Yes	Yes	Finite

The remainder of the paper is organized as follows. In section 2 notations and assumptions are provided for the development of the model. The mathematical formulation is developed in section 3. Section 4 develops the fuzzy economic order quantity (FEOQ) model. In section 5, the solution procedure is given. In section 6, an illustrative numerical analysis is given to illustrate the procedure of solving the proposed model. The sensitivity analysis is carried out in section 7 to observe the changes in the optimal solution. Finally section 8 deals with the summary and the concluding remarks.

2. Notations and Assumptions

r = the consumption rate.
t_c = the cycle length.
h = the holding cost of one unit for one unit of time.
$HC(q)$ = the holding cost per cycle.
K = the setup cost per cycle.
c = the purchasing cost per unit.
p_s = the selling price per unit.
α = the percentage of on-hand inventory that is lost due to deterioration.
q = the order quantity.
q^{**} = the modified economic ordering / production quantity (FEOQ/FEPQ).
q^{*} = the traditional economic ordering quantity (EOQ).
$\varphi(t)$ = the on-hand inventory level at time t.
$\pi_1(q, \rho)$ = the net profit per cycle in crisp strategy.
$\pi(q, \rho)$ = the average profit per unit per cycle in crisp strategy.
$\tilde{\pi}_1(q,\rho)$ = the net profit per cycle in fuzzy decision space.
$\tilde{\pi}(q, \rho)$ = the average profit per unit per cycle in fuzzy decision space.

3. Mathematical Model

Denote $\varphi(t)$ as the on-hand inventory level at time t. During a change in time from point t to t+dt, where $t + dt > t$, the on-hand inventory drops from $\varphi(t)$ to $\varphi(t+dt)$. Then $\varphi(t+dt)$ is given as:

$$\varphi(t+dt) = \varphi(t) - r\,dt - \alpha\,\varphi(t)\,dt \qquad (1)$$

Equation (1) can be re-written as:

$$\frac{\varphi(t+dt) - \varphi(t)}{dt} = -r - \alpha\varphi(t) \qquad (2)$$

and $dt \to 0$, equation (2) reduces to:

$$\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r = 0 \qquad (3)$$

Equation (3) is a differential equation, solution is

$$\varphi(t) = \frac{-r}{\alpha} + \left(q + \frac{r}{\alpha}\right) \times e^{-\alpha t} \quad (4)$$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length t_c units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. The cycle length, t_c, is determined by first substituting t_c into equation (4) and then setting it equal to zero to get:

$$t_c = \frac{1}{\alpha} \ln\left(\frac{\propto q + r}{r}\right) \quad (5)$$

Equation (4) and (5) are used to develop the mathematical model. It is worthy to mention that as α approaches to zero, t_c approaches to $\frac{q}{r}$. Then the total number of units lost per cycle, L, is given as:

$$L = r\left[\frac{q}{r} - \frac{1}{\alpha} ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (6)$$

The total cost per cycle, TC (q), is the sum of the procurement cost per cycle, K+cq and the holding cost per cycle, HC (q). HC (q) is obtained from equation (4) as:

$$HC(q) = \int_0^{tc} h\varphi(t)dt = h\int_0^{\frac{1}{\propto}ln\left(\frac{\propto q + r}{r}\right)} \left[-\frac{r}{\alpha} + \left(q + \frac{r}{\alpha}\right) \times e^{-\alpha t}\right] dt$$

$$= h \times \left[\frac{q}{\propto} - \frac{r}{\alpha^2} ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (7)$$

$$TC(q) = K + cq + h \times \left[\frac{q}{\propto} - \frac{r}{\alpha^2} ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (8)$$

The total cost per unit of time, TCU (q), is given by dividing equation (8) by equation (5) to give:

$$TCU(q) = \left[K + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} ln\left(\frac{\alpha q + r}{r}\right)\right]\right] \times \left[\frac{1}{\alpha} \ln\left(\frac{\propto q + r}{r}\right)\right]^{-1}$$

$$= \frac{K\propto + (c\propto + h)q}{ln\left(1 + \frac{\propto q}{r}\right)} - \frac{hr}{\alpha} \quad (9)$$

As α approaches zero in equation (9) reduces to TCU (q) = $\frac{Kr}{q} + cr + \frac{hq}{2}$, whose solution is given by the traditional EOQ formula, $q^* = \sqrt{\frac{2Kr}{h}}$. The total profit per cycle is $\pi_1(q)$.

$$\pi_1(q) = (q - L) \times p_s - TC(q)$$

$$= (q - L) \times p_s - K - cq - h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \times ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (10)$$

Where L, the number of units lost per cycle due to deterioration, and TC (q) the total cost per cycle, are calculated from Eqs. (6) and (9) respectively. The average profit $\pi(q)$ per unit time is obtained by dividing t_c in $\pi_1(q)$.

$$\pi_1(q) = F_1(q) + F_2(q)h + F_3(q)K \quad (11)$$

Where, $F_1(q) = (q - L) \times p_s - cq \quad (12)$

$$F_2(q) = -\left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \times ln\left(\frac{\alpha q + r}{r}\right)\right] \quad (13)$$

and $F_3(q) = -1 \quad (14)$

4. Fuzzy Model

The holding cost and ordering cost are replaced by fuzzy numbers $\tilde{h}$ and $\tilde{K}$ respectively. By expressing $\tilde{h}$ and $\tilde{K}$ as the normal triangular fuzzy numbers (h_1, h_0, h_2) and (K_1, K_0, K_2), where, $h_1 = h - \Delta_1$, $ho = h$, $h_2 = h + \Delta_2$, $K_1 = K - \Delta_3$, $K_0 = K$, $K_2 = K + \Delta_4$ such that $0 < \Delta_1 < h$, $0 < \Delta_2$, $0 < \Delta_3 < K$, $0 < \Delta_4$, $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy ordering cost are considered as:

$$\mu_{\tilde{h}}(h) = \begin{cases} \dfrac{h - h_1}{h_0 - h_1}, & h_1 \le h \le h_0 \\ \dfrac{h_2 - h}{h_2 - h_0}, & h_0 \le h \le h_2 \\ 0 & , otherwise \end{cases} \quad (15)$$

$$\mu_{\tilde{k}}(K) = \begin{cases} \dfrac{K - K_1}{K_0 - K_1}, & K_1 \le K \le K_0 \\ \dfrac{K_2 - K}{K_2 - K_0}, & K_0 \le K \le K_2 \\ 0 & , \text{otherwise} \end{cases} \tag{16}$$

Then the centroid for $\tilde{h}$ and $\tilde{K}$ are given by

$M_{\tilde{h}} = \dfrac{h_1 + h_o + h_2}{3} = h + \dfrac{\Delta_2 - \Delta_1}{3}$ and $M_{\tilde{k}} = \dfrac{K_1 + K_o + K_2}{3} = K + \dfrac{\Delta_4 - \Delta_3}{3}$

respectively.

For fixed value of q, let $\pi_1(h, K) = F_1(q) + F_2(q)h + F_3(q)K = y$

Let $h = \dfrac{y - F_1 - F_3 K}{F_2}$, $\dfrac{\Delta_2 - \Delta_1}{3} = \psi_1$ and $\dfrac{\Delta_4 - \Delta_3}{3} = \psi_2$

By extension principle the membership function of the fuzzy profit function is given by

$$\begin{aligned} \mu_{\pi_1(\tilde{h}, \tilde{k})}(y) &= \sup_{(h,k) \in \pi_1^{-1}(y)} \left\{ \mu_{\tilde{h}}(h) \wedge \mu_{\tilde{k}}(K) \right\} \\ &= \sup_{k_1 \le k \le k_2} \left\{ \mu_{\tilde{h}}\left(\frac{y - F_1 - F_3 K}{F_2} \right) \wedge \mu_{\tilde{k}}(K) \right\} \end{aligned} \tag{17}$$

Now,

$$\mu_{\tilde{h}}\left(\frac{y - F_1 - F_3 K}{F_2} \right) = \begin{cases} \dfrac{y - F_1 - F_2 h_1 - F_3 K}{F_2 (h_0 - h_1)} & , \quad u_2 \le K \le u_1 \\ \dfrac{F_1 + F_2 h_2 + F_3 K - y}{F_2 (h_2 - h_0)} & , \quad u_3 \le K \le u_2 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{18}$$

where, $u_1 = \dfrac{y - F_1 - F_2 h_1}{F_3}$, $u_2 = \dfrac{y - F_1 - F_2 h_0}{F_3}$ and $u_3 = \dfrac{y - F_1 - F_2 h_2}{F_3}$

when $u_2 \le K$ and $K \le u_1$ then $y \le F_1 + F_2 h_0 + F_3 K_0$ and $y \ge F_1 + F_2 h_1 + F_3 K_1$.

It is clear that for every $y \in [F_1 + F_2 h_1 + F_3 K_1,\ F_1 + F_2 h_0 + F_3 K_0]$, $\mu_y(y) = PP'$.

From the Eqs. (15) and (18) the value of PP' may be found by solving the following equation:

$$\frac{K - K_1}{K_0 - K_1} = \frac{y - F_1 - F_2 h_1 - F_3 K}{F_2 (h_0 - h_1)}$$

or $$K = \frac{(y - F_1 - F_2 h_1)(K_0 - K_1) + F_2 K_1 (h_0 - h_1)}{F_2 (h_0 - h_1) + F_3 (K_0 - K_1)}$$

Therefore, $PP' = \dfrac{K - K_1}{K_0 - K_1} = \dfrac{y - F_1 - F_2 h_1 - F_3 K}{F_2 (h_0 - h_1) + F_3 (K_0 - K_1)} = \mu_1(y)$, (say). (19)

When $u_3 \le K$ and $K \le u_2$ then $y \le F_1 + F_2 h_2 + F_3 K_2$ and $y \ge F_1 + F_2 h_0 + F_3 K_0$. It is evident that for every $y \in [F_1 + F_2 h_0 + F_3 k_0 \ , F_1 + F_2 h_2 + F_3 K_2]$, $\mu_{\tilde{y}}(y) = PP''$. From the Eqs. (15) and (18), the value of PP'' may be found by solving the following equation:

$$\frac{K_2 - K}{K_2 - K_0} = \frac{F_1 + F_2 h_2 + F_3 K - y}{F_2 (h_2 - h_0)}$$

or, $$K = \frac{F_2 K_2 (h_2 - h_0) - (F_1 + F_2 h_2 - y)(K_2 - K_0)}{F_2 (h_2 - h_0) + F_3 (K_2 - K_0)}$$

Therefore, $PP'' = \dfrac{K_2 - K}{K_2 - K_0} = \dfrac{F_1 + F_2 h_2 + F_3 K_2 - y}{F_2 (h_2 - h_0) + F_3 (K_2 - K_0)} = \mu_2(y)$, (say). (20)

Thus the membership function for fuzzy total profit is given by

$$\mu_{\pi_1(\tilde{h},K)}(y) = \begin{cases} \mu_1(y); & F_1 + F_2 h_1 + F_3 K_1 \le y \le F_1 + F_2 h_0 + F_3 K_0 \\ \mu_2(y); & F_1 + F_2 h_0 + F_3 K_0 \le y \le F_1 + F_2 h_2 + F_3 K_2 \\ 0 \ ; & \text{otherwise} \end{cases} \tag{21}$$

Now, let $P_1 = \int_{-\infty}^{\infty} \mu_{\pi_1(\tilde{h},\tilde{k})}(y)\, dy$ and $R_1 = \int_{-\infty}^{\infty} y\, \mu_{\pi_1(\tilde{h},\tilde{k})}(y)\, dy$

Hence, the centroid for fuzzy total profit is given by $\tilde{\pi}_1(q) = M_{\tilde{TP}}(q) = \dfrac{R_1}{P_1}$

$$= F_1(q) + F_2(q)h + F_3(q)K + \psi_1 F_2(q) + \psi_2 F_2(q) \tag{22}$$

$$M_{\tilde{TP}}(q) = F_1 + (h + \psi_1)F_2 + (K + \psi_2)F_3 \tag{23}$$

where, $F_1(q)$, $F_2(q)$ and $F_3(q)$ are given by equations (12), (13) and (14).
Hence the profit maximization problem is

$$\text{Maximize } \tilde{\pi}_1(q) = M_{\tilde{TP}}(q) \quad \forall\, q \geq 0 \tag{24}$$

5. Solution Procedure (Optimization)

The fuzzy optimal ordering quantity q per cycle can be determined by differentiating equation (24) with respect to q, then setting these to zero.In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is concave in terms of ordering quantity q. The second order derivates of equation (24) with respect to q are strictly negative. Consider the following proposition.

Proposition- 1 The net profit $\tilde{\pi}_1$ (q) per cycle is concave in q.

Conditions for optimal q

$$\frac{d\tilde{\pi}_1(q)}{dq} = \frac{r}{\alpha q + r}\left(p_s + \frac{h+\psi_1}{\alpha}\right) - \left(c + \frac{h+\psi_1}{\alpha}\right) = 0 \tag{25}$$

The second order derivative of the net profit per cycle with respect to q can be expressed as

$$\frac{d^2\tilde{\pi}_1(q)}{dq^2} = -\frac{r}{(\alpha q+r)^2}(p_s\alpha + (h + \psi_1)) \tag{26}$$

Since r > 0 and $p_s\alpha + (h + \psi_1) > 0$ equation (26) is negative. Proposition 1 shows that the second order derivative of equation (24) with respect to q are strictly negative.

The objective is to determine the optimal values of q to maximize the unit profit function of equation (24). It is very difficult to derive the optimal values of q, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here

LINGO 13.0 version software is used to derive the optimal values of the decision variable.

6. Numerical Analysis

In order to illustrate the proposed model, I consider the following example:

Example- Given K = Rs. 200 per order, h = Rs. 5 per unit per unit of time, r = 1200 units per unit of time, c = Rs. 100 per unit, p_s = Rs. 125 per unit, α = 5%, Δ_1= 0.002, Δ_2= 0.02, Δ_3= 0.002 and Δ_4=0.2. The optimal solution that maximizes Eq. (24) and q^{**} is determined by using LINGO 13.0 version software and the results are tabulated in Table 2.

Table 2: Fuzzy Optimal Solutions of the Proposed Model

Model	Iteration	t^*	L^*	q^*	$\tilde{\pi}_1^*(q)$	$\tilde{\pi}^*(q)$
Fuzzy	72	0.1819965	0.9967023	219.3925	27804.76	5060.369003
Fuzzy	41	0.2580867	-	309.7040	28449.62	7342.468542

Comparative Analysis

From Table 3, it can be found that the retailer always includes the wastage to percentage of on-hand inventory due to deterioration in FEOQ model for obtaining the maximum net profit with less time consumption.

Table 3: Comparative Analysis with Percentage Change in Different Crisp Models with Fuzzy Model

Model	Deterioration	Iteration	q^{**}	t_c^*	L^*	$\tilde{\pi}_1^*(q)$	$\tilde{\pi}^*(q)$
Fuzzy	Yes	72	219.3925	0.1820	0.9967023	27804.76	5060.369003
Fuzzy	No	41	309.7040	0.2581	-	28449.62	7342.468542
% Change	-	-	-29.1606	-29.4847	-	-2.6667	-31.0808
Crisp	Yes	-	220	0.183	1.00221	27806.128	5074.5683

% Change	-	-	-0.2761	-0.5464	-0.54985	-0.0049198	-0.2798129
Crisp	No	41	309.839	0.258	-	28450.81	7345.9678
% Changes	-	-	-29.1915	-29.4574	-	-2.2708	-31.1137

7. Sensitivity Analysis

It is interesting to investigate the influence of α on retailer behaviour. The computational results shown in Table 4 indicates the following managerial phenomena: when the percentage of on-hand inventory that is lost due to deterioration α increases, the replenishment cycle length, the optimal replenishment quantity, optimal net profit per unit per cycle and optimal average profit per unit per cycle decrease respectively in fuzzy decision space. The optimal total number of units lost per cycle increases with increases in the percentage value of the major parameter α.

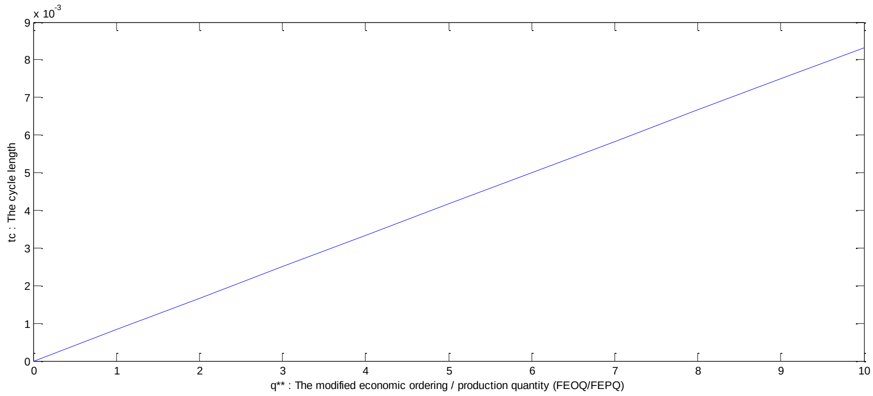

Fig. 1: The Cycle Length and the number of Order Quantity

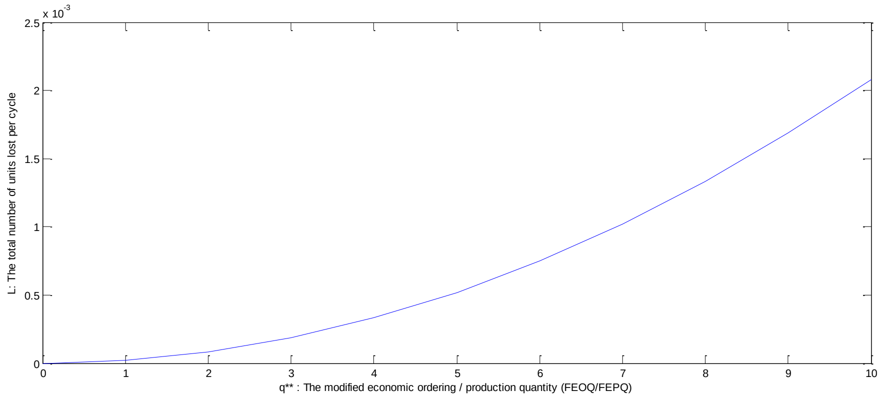

Fig. 2: The number of Units Lost due to Deterioration and the number of Order Quantity.

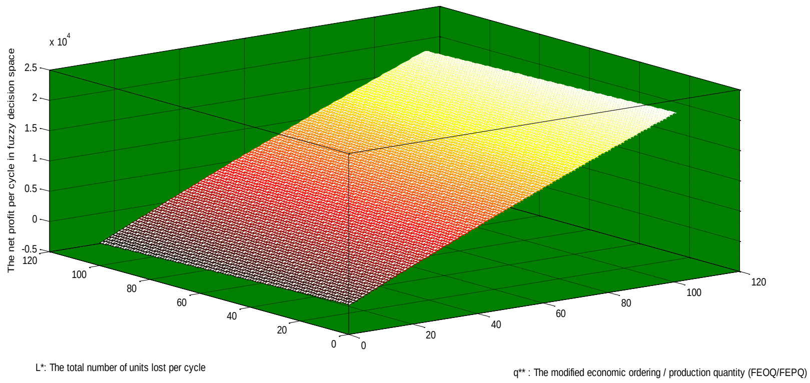

Fig. 3: Mesh plot of Total profit, the Total number of Units Lost and the number of Order Quantity.

The behavior of the fuzzy optimal average profit per unit per time and optimal order quantity is illustrated using MATLAB (R2009a) version software in Fig. 1 and the results indicate that the fuzzy average profit per unit is decreased as the percentage of on-hand inventory that is lost due to deterioration is increased as shown in Fig. 2. Fig. 3 shows that there is a linear positive correlation between fuzzy optimal cycle length and the fuzzy optimum average profit per unit per time.

Table-4 Sensitivity Analysis of the Parameter α

α%	Iteration	t^*	L^*	q^*	$\tilde{\pi}_1^*(q)$	$\tilde{\pi}^*(q)$	% Change in $\tilde{\pi}^*(q)$
.01	243	0.2354385	0.3328490	282.8591	28301.15	6663.180304	31.67380284
.02	84	0.2178438	0.5702988	261.9828	28164.55	6135.472597	21.24555726
.04	79	0.1919258	0.8863189	231.1973	27917.84	5358.153776	5.884645425
.10	86	0.1483303	1.3266640	179.3230	27309.08	4050.764029	-19.95121252
.15	92	0.1282753	1.4904550	155.4209	26890.65	3449.406196	-31.83488805
.30	74	0.0966578	1.6980650	117.6874	25880.23	2501.526095	-50.56633037

8. Conclusion and Extensions

In this paper, a modified FEOQ model is introduced which investigates the optimal order quantity assumes that a percentage of the on-hand inventory is wasted due to deterioration as a characteristic feature and the inventory conditions govern the item stocked. The profit maximization problem with a finite planning horizon into a fuzzy economic order quantity model is first formulated. This model allows us to establish optimal control policy. Numerical results indicate that a significant profit increase can be obtained by allowing the paper provides a useful property for finding the optimal profit and ordering quantity with deteriorated units of lost sales. A new mathematical fuzzy model is developed and compared to the traditional EOQ model numerically. The fuzzy economic order quantity $\tilde{q}^{**}$ and units lost due to deterioration $\tilde{L}^*$ for the modified model were found to be less than that of the traditional, q^*, i.e. $\tilde{q}^{**} < q^*$. But the modified net profit per unit per cycle is approaching to that of the traditional profit per unit per cycle in uncertain environment. Finally, wasting the percentage of on-hand inventory due to deterioration effect was demonstrated numerically to have an adverse effect on the average profit per unit per cycle. Hence the utilization of units lost due to deterioration makes the scope of the application broader. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analysis with respect to the major parameter α. The model in this study is a general framework that considers wasting/ no wasting the percentage of on-hand inventory due to deterioration simultaneously. Another interesting extension may be the incorporation of yield uncertainty. In the current model, it is assumed that there is no yield loss in process. In reality, yield uncertainty is not uncommon in various production situations, such as electronics fabrication and assembly.

In the future study, it is hoped to further incorporate the proposed model into several situations such as shortages are allowed and the consideration of multi-item problem. Further work in this area could reveal links between fuzzy and optimization allows extending discounted structured model and flexibility price model into the infinite realm. Furthermore, it may also take partial backlogging into account when determining the optimal replenishment policy.

REFERENCES

[1] Bose, S., Goswami, A. and Chaudhuri, K.S. (1995) "An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting", Journal of Operational Research Society, Vol. 46, pp. 775-782.

[2] Goyal, S.K. and Gunasekaran, A. (1995) "An integrated production-inventory-marketing model for deteriorating items', Computer Industrial Engineering, Vol. 28 pp. 755-762.

[3] Gupta, D. and Gerchak, Y. (1995) "Joint product durability and lot sizing models", European Journal of Operational Research, Vol. 84, pp. 371-384.

[4] Hariga, M. (1994) "Economic analysis of dynamic inventory models with non-stationary costs and demand", International Journal of Production Economics, Vol. 36, pp. 255-266.

[5] Hariga, M. (1995) "An EOQ model for deteriorating items with shortages and time-varying demand", Journal of Operational Research Society, Vol. 46, pp. 398-404.

[6] Hariga, M. (1996) "An EOQ model for deteriorating items with time-varying demand", Journal of Operational Research Society, Vol. 47, pp. 1228-1246.

[7] Jain, K. and Silver, E. (1994) "A lot sizing for a product subject to obsolescence or perishability", European Journal of Operational Research, Vol. 75, pp. 287-295.

[8] Mahata, G.C. and Goswami, A. (2006) "Production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating item under permissible delay in payments", Journal of Operational Research Society of India, Vol. 43, pp. 359-375.

[9] Osteryoung, J.S., Mccarty, D.E. and Reinhart, W.L. (1986), "Use of EOQ models for inventory analysis", Production Inventory Management, Vol. 3rd Qtr, pp. 39-45.

[10] Padmanabhan, G. and Vrat, P. (1995) "EOQ models for perishable items under stock dependent selling rate", European Journal of Operational Research, Vol. 86, pp. 281-292.

[11] Pattnaik, M. (2010) "An entropic order quantity (EnOQ) model under instant deterioration of perishable items with price discounts", International Mathematical Forum, Vol. 5 No. 52, pp. 2581-2590.

[12] Pattnaik, M. (2011) "A note on optimal inventory policy involving instant deterioration of perishable items with price discounts", The Journal of Mathematics and Computer Science, Vol. 3 No. 2, pp. 145-155.

[13] Pattnaik, M. (2011) "An entropic order quantity (EnOQ) model with post deterioration cash discounts", International Journal of Contemporary Mathematical Sciences, Vol. 6 No. 19, pp. 931-939.

[14] Pattnaik, M. (2011) "Entropic order quantity (EnOQ) model under cash discounts", Thailand Statistician Journal, Vol. 9, No. 2, pp. 129-141.

[15] Pattnaik, M. (2012) "A note on non linear profit-maximization entropic order quantity (EnOQ) model for deteriorating items with stock dependent demand rate", Operations and Supply Chain Management, Vol. 5 No. 2, pp. 97-102.

[16] Raafat, F. (1991) "Survey of literature on continuously deteriorating inventory models", Journal of Operational Research Society, Vol. 42, pp. 89-94.

[17] Salameh, M.K., Jaber, M.Y. and Noueihed, N. (1999) "Effect of deteriorating items on the instantaneous replenishment model", Production Planning Control, Vol. 10 No. 2, pp. 175-180.

[18] Tripathy, P.K. and Pattnaik, M. (2008) "An entropic order quantity model with fuzzy holding cost and fuzzy disposal cost for perishable items under two component demand and discounted selling price", Pakistan Journal of Statistics and Operational Research, Vol. 4 No. 2, pp. 93-110.

[19] Tripathy, P.K. and Pattnaik, M. (2009) "Optimal disposal mechanism with fuzzy system cost under flexibility & Reliability criteria in non-random optimization environment", Applied Mathematical Science, Vol. 3 No. 37, pp. 1823-1847.

[20] Tripathy, P.K. and Pattnaik, M. (2011) "A fuzzy arithmetic approach for perishable items in discounted entropic order quantity model", International Journal of Scientific and Statistical Computing, Vol. 1 No. 2, pp. 7-19.

[21] Tripathy, P.K. and Pattnaik, M. (2011) "A non-random optimization approach to a disposal mechanism under flexibility and reliability criteria", The Open Operational Research Journal, Vol. 5, pp. 1-18.

[22] Vujosevic, M., Petrovic, D. and Petrovic, R. (1996) "EOQ formula when inventory cost is fuzzy", International Journal of Production Economics, Vol. 45, pp. 499-504.

[23] Waters, C.D.J. (1994) *Inventory control and Management*, Wiley, Chichester.

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