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ROBUSTIFYING FORECAST PERFORMANCE THROUGH HYBRIDIZED ARIMA-GARCH-TYPE MODELING IN A DISCRETE-TIME STOCHASTIC SERIES

IMOH UDO MOFFAT [1,*], AND EMMANUEL ALPHONSUS AKPAN [2]

[1]*Department of Statistics, University of Uyo, Nigeria*

[2]*Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi, Nigeria*

[*]*Corresponding author: moffitto2011@gmail.com*

ABSTRACT. The study is aimed at investigating the robustness of forecast performance of a hybridized (ARIMA-GARCH-type) model over each single component using different periods of horizon to display consistency over time. Daily closing share prices were explored from the Nigerian Stock Exchange for First City Monument Bank and Wema Bank Plc, spanning from January 3, 2006 to December 30, 2016, with a total of 2,713 observations. ARIMA model, GARCH-type, and hybridized ARIMA-GARCH-type were considered. The hybridized ARIMA-GARCH-type was found to produce the best forecast performance in terms of robustness over each single component model and the robustness was found to be consistent over different time horizons for the datasets. The implication is that, it provides an essential remedy to the problem associated with model instability when forecasting a discrete-time stochastic series.

1. Introduction

Forecasting for future observations is one of the objectives of time series application. Time series forecasting takes different approaches depending on what it is targeted at. It could be in-sample forecast approach, where the same data used for model formulation are also employed for checking the predictive performance of such model, which is aimed at selecting the best fitted models. Also, it could be out-of-sample forecast approach, where the data are

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partitioned into two components (training and validation datasets), targeted at achieving forecast accuracy. Analytically, the later is advantageous over the former in that it is able to give information about the future values of observed time series, depict forecaster in real time, and strongly discourage model overfitting ([1], [2], [3], [4], [5]). Apart from predicting for future observations, forecasting is also a proven and valid tool for selecting a model with best predictive values ([6], [7], [8], [9], [10]). However, in a quest for best predictive performance, model instability has become a subject of discourse and tends to pose a serious threat to time series forecasting accuracy. Model instability refers to the tendency for estimated parameters to fluctuate over time. One obvious consequence of model instability is that it leads to wrong choice of model resulting in high variability of prediction error ([11], [12], [13], [14], [15]).

One way of overcoming the problem associated with forecast accuracy due to model instability is to account for possible instability in the model ([16], [17], [18]). On the other hand, instead of accounting for fluctuating parameters of a single model in order to achieve improved forecast accuracy, it is a good practice and in line with the recent innovation trend in time series to combat the menace of model instability on forecast accuracy through combined models. This alternative approach involves selecting diverse models and thereafter hybridizing the models to generate forecast, which can depict the provable forecast performance of individual models and at the same time provides robust accuracy. It is evident in the literature that diverse forecast hybridized approaches have supported the assertion that forecasts generated from combined models appeared to be improved and robust over the forecasts obtained by single models ([19, [20], [21], [22], [23], [24], [25]).

Meanwhile, in Nigeria, [7] showed that out-of-sample model selection approach outperformed the in-sample counterpart in describing the characterizations of future observations without necessarily considering the choice of true model by utilizing the advantage of combining both ARIMA and GARCH-type models to achieve forecast accuracy. [26] looked at possible combination of both ARMA and ARCH-type models to form a single model such as ARMA-ARCH that could completely capture the linear and non-linear features of financial data. Their findings revealed that such combination was sufficient for the time series under study. [27] investigated the carry-over effect of biased estimates of joint ARIMA-GARCH-type model parameters on forecast accuracy in the presence of outliers and their results showed that after adjusting for outliers, marginal improvement on the forecasts was observed. However, these previous studies in the discrete-time stochastic series failed to compare the advantages of the combined ARIMA-GARCH-type models over its individual components, dwelt on only one time horizon, and failed to predict on different time horizons leading to risky reliance. Therefore, this study seeks to bridge this gap by investigating the robustness of forecast performance of ARIMA-GARCH-type model over each single component using different time horizons to show consistency over time.

The motivation for this study is drawn from the fact that ARIMA models are not sufficient for modeling the return series due to the presence of heteroscedasticity, which leads to spurious forecasts. On the other hand, GARCH-type models are often misspecified with biased parameters and are susceptible to the presence of structural breaks and outliers. Thus, these challenges provide the pragmatic reason and form the symbolic platform for adopting the hybridized ARIMA-GARCH-type model in this study.

The remaining aspect of the study is organized as follows: Section 2 takes care of materials and methods; section 3 handles discussion of results, while section 4 concludes the study.

2 Materials and Methods

2.1 The Return Series

The return series, R_t can be obtained given that, P_t, is the price of a unit share at time, t and P_{t-1} is the share price at time t−1.

$$R_t = \nabla lnP_t = (1-B)lnP_t = lnP_t - lnP_{t-1.} \quad (1)$$

Here, R_t in equation (1) is regarded as a transformed series of the share price, P_t, meant to attain stationarity, where both the mean and the variance of the series are stable ([28], [29]), while B is the backshift operator.

2.2 ARIMA Models

ARIMA model is practically applied to capture the linear dependence in the return series ([30],[28], [26], [31], [32]). However, the fact that the series tends to appear in clusters, which actually results in the violation of assumption of constant variance. Also, the linear time series models do not seem to produce accurate out-of-sample forecasts, thus providing a more sensible argument for adopting heteroscedastic models ([31], [33]). A typical ARIMA model equation is presented in (2):

$$R_t = \mu_t + a_t, \tag{2}$$

where $\mu_t = \varphi_0 + \sum_{j=1}^{p} \varphi_j R_{t-j} + \sum_{i=1}^{q} \theta_i a_{t-i}$,

φ is an autoregressive parameter, and θ is a moving average parameter.

2.3 GARCH-type Models

The generalized autoregressive conditional heteroscedastic (GARCH-type) models were introduced to account for heteroscedasticity (changing variance) and to overcome the problems associated with violation of assumption of constant variance. The GARCH-type specification could be symmetric (for example ARCH) which rely on modeling the conditional variance as a linear function of squared past residuals or asymmetric (for example EGARCH) which allows for the signs of the innovations (returns) to have impact on the volatility apart from magnitude ([34], [35], [31], [36]). Moreover, the generalized autoregressive conditional heteroscedastic (GARCH-type) models were specified based on the normal distribution for the innovations but could not capture the heavy-tailed property ([34]). Therefore, in this study, the student-t distribution is adopted which was traditionally introduced to overcome the weaknesses of the normal distribution in accommodating the heavy-tailed property.

2.3.1 Autoregressive Conditional Heteroscedastic (ARCH) Model

ARCH(q) model as provided in [37] and specified as

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2, \tag{3}$$

where σ_t^2 is the conditional variance (heteroscedasticity), ω is the constant term andα_q is the coefficient of volatility clustering up to order q.

2.3.2 Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) Model

EGARCH(q,p) model applies the natural logarithm to ensure that the conditional variance is positive and thus overcome the requirement of parameter restrictions ([38]). The EGARCH (q, p) is defined as,

$$ln\sigma_t^2 = \omega + \sum_{k=1}^{r} \gamma_k a_{t-k} + \sum_{i=1}^{q} \alpha_i \left(|a_{t-i}| - \frac{2\sqrt{v-2}\,\Gamma(v+1)/2}{(v-1)\Gamma(v/2)\sqrt{\pi}} \right) + \sum_{j=1}^{p} \beta_j \, ln\sigma_{t-j}^2, \tag{4}$$

$\beta_j$is the garch coefficient measuring persistence, and$\gamma_k$is the asymmetric coefficient.

2.4 Hybridized Models

Heteroscedastic models are hybridized of both mean and variance equations. The mean equation represents the ARIMA model as shown in equation (5):

$$R_t = \varphi_0 + \sum_{j=1}^{p} \varphi_j R_{t-j} + \sum_{i=1}^{q} \theta_i a_{t-i} + a_t, \quad (5)$$

$$a_t = \sigma_t e_t, \quad (6)$$

where e_t is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero, that is, $E(e_t) = 0$ and variance 1,while a_t in (6) is the standardized residual term that follows ARCH(q) and EGARCH(q,p) models in (3) and (4), respectively. Putting it differently, equation (6) provides the link between the ARIMA and the GARCH-type models.

2.5 Model Evaluation Criteria

The methods of forecast evaluation based on forecast error include Mean Squared Error (MSE), and Mean Absolute Error (MAE). These criteria measure forecast accuracy. These measures are employed in this study because of their popularity.

The measures are computed as follows:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n} e_i^2 \quad (7)$$

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n} |e_i| \quad (8)$$

where e_i is the forecast error and n is the number of forecast error.

3. Results and Discussion

The data are divided into two groups; training and testing which is purposed at checking the forecasting performance. The training data set ranging from January 3, 2006 to November 24, 2016, consisting of 2690 was used for model formulation while the testing data set was used for evaluating the forecasting performance at 8, 16 and 23 horizons.

3.1 Plot Analysis

The plots of share prices in Figures 1-2 showed a fluctuating movement away from the common means indicating the presence of nonstationarity. However, Figures 3-4 represent the return series, which are the stationary, because they are clustered around the common means, indicating the presence of changing variance.

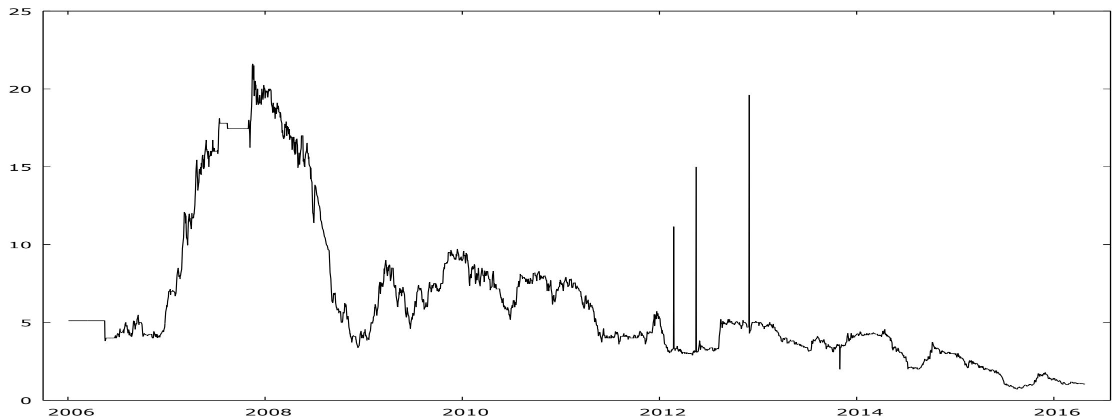

Figure 1: Share Price Series of First City Monument Bank

Source: Data Analysis

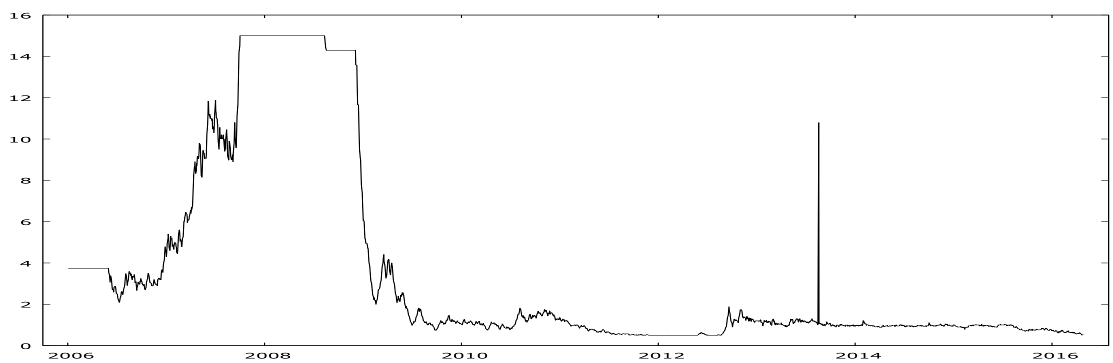

Figure 2: Share Price Series of Wema Bank

Source: Data Analysis

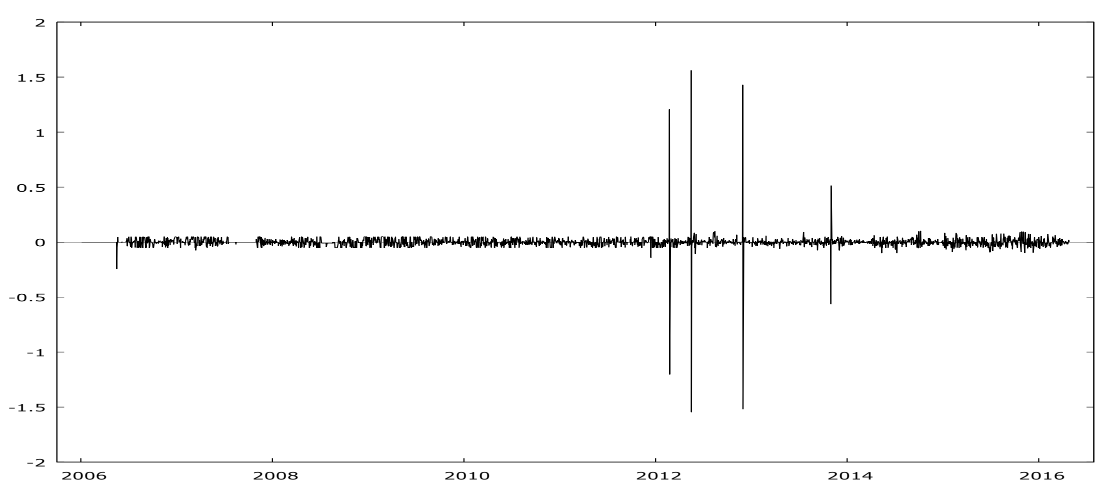

Figure 3: Return Series of First City Monument Bank

Source: Data Analysis

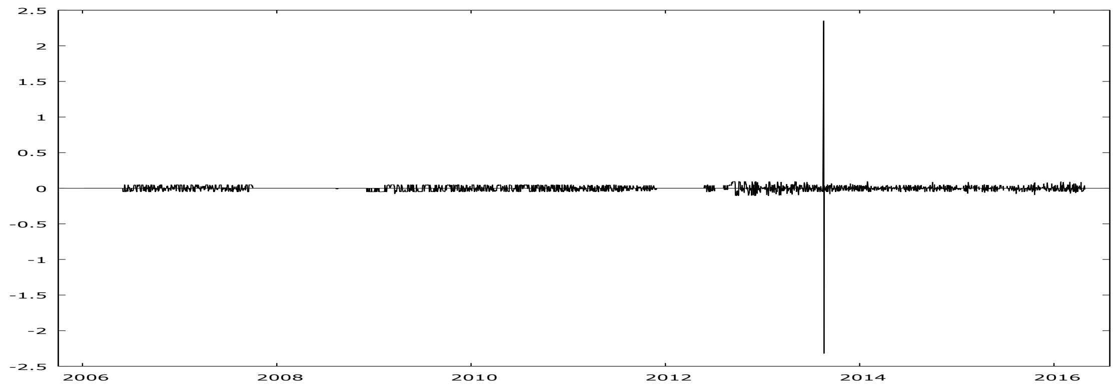

Figure 4: Return Series of Wema Bank

Source: Data Analysis

3.3 Evaluation of Forecast Performance

Considering the return series of First City Monument bank, ARIMA(0,1,1) model was found to be adequate in capturing the linear dependence in the data. On the other hand, ARCH(2)-t model was adequate in handling the heteroscedasticity in the data. However, combining the two models resulted in an ARIMA(0,1,1)-ARCH(2)-t model could jointly express both the linear and nonlinear properties of the series. Since our aim is to assess the forecast performance of the combined model in comparison to the individual component models at different horizons, the MSE and MAD were explored as the forecast performance evaluation measures and their values at different horizons are shown in Table I.

Table I: Evaluation of Forecast Performance for First City Monument Bank

Horizon		8		16		23	
Model		MSE	MAE	MSE	MAE	MSE	MAE
ARIMA	ARIMA(0,1,1)	$3.267e^{-4}$	0.01379	$4.792e^{-4}$	0.01681	$4.626e^{-4}$	0.01718
GARCH-type	ARCH(2)-t	$3.333e^{-4}$	0.01438	$4.778e^{-4}$	0.01711	$4.500e^{-4}$	0.01717
ARIMA-GARCH-type	ARIMA(0,1,1)-ARCH(2)-t	$2.327e^{-4}$	0.01330	$4.667e^{-4}$	0.01602	$4.300e^{-4}$	0.01710

Source: Data Analysis

To show the improvement of forecast performance of the joint ARIMA(0,1,1)-ARCH(2)-t model over the ARIMA(0,1,1) model, from Table II, it was found that ARIMA(0,1,1)-ARCH(2)-t model outperformed ARIMA(0,1,1) model by; 0.0094% and 0.049% as indicated by MSE and MAE, respectively at horizon 8; 0.00125% and 0.079% as shown by MSE and MAE, respectively at horizon 16; 0.00326% and 0.008% as indicated by MSE and MAE, respectively at horizon 23.

Table II: Percentage Improvement of Forecasting Performance between ARIMA and ARIMA-GARCH-type Models for First City Monument Bank

Horizon		8		16		23	
Model		MSE	MAE	MSE	MAE	MSE	MAE
ARIMA	ARIMA(0,0,1)	$3.267e^{-4}$	0.01379	$4.792e^{-4}$	0.01681	$4.626e^{-4}$	0.01718
ARIMA-GARCH-type	ARIMA(0,0,1)-ARCH(2)-t	$2.327e^{-4}$	0.01330	$4.667e^{-4}$	0.01602	$4.300e^{-4}$	0.01710
Percentage Difference		$0.94e^{-2}$	0.049	$0.125e^{-2}$	0.079	$0.326e^{-2}$	0.008

Source: Data Analysis

From Table III, it was found that ARIMA(0,1,1)-ARCH(2)-t model also outperformed ARCH(2)-t model by 0.01006% and 11.1% (as indicated by the respective values of MSE and MAE) at horizon 8; 0.00111% and 0.109% (as indicated by the respective values of MSE and MAE) at horizon 16; while 0.002% and 0.007% (as indicated by the respective values of MSE and MAE) at horizon 23.

Table III: Percentage Improvement of Forecasting Performance between GARCH and ARIMA-GARCH-type Models for First City Monument Bank

Horizon		8		16		23	
Model		MSE	MAE	MSE	MAE	MSE	MAE
GARCH-type	ARCH(2)-t	$3.333e^{-4}$	0.01438	$4.778e^{-4}$	0.01711	$4.5e^{-4}$	0.01717
ARIMA-GARCH-type	ARIMA(0,0,1)-ARCH(2)-t	$2.327e^{-4}$	0.01330	$4.667e^{-4}$	0.01602	$4.3e^{-4}$	0.01710
Percentage Difference		$1.006e^{-2}$	11.1	$0.111e^{-2}$	0.109	$0.2e^{-2}$	0.007

Source: Data Analysis

For the return series of Wema bank, ARIMA(2,1,1) model was successfully adequate in handling the linear dependence in the data. EGARCH(1,1)-t model, on the other hand, was adequate in expressing the heteroscedasticity in the data. Combining the two models resulted in an ARIMA(2,1,1)-EGARCH(1,1)-t model, which was able to jointly capture both the linear and nonlinear properties of the series. Assessing the forecast performance of the combined model in comparison to the individual component models at different time horizons, the MSE and MAD were explored as the forecast performance evaluation measures and their values at different horizons are shown in Table IV.

Table IV: Evaluation of Forecast Performance for Wema Bank

Horizon		8		16		23	
Model		MSE	MAE	MSE	MAE	MSE	MAE
ARIMA	ARIMA(2,1,1)	$7.690e^{-4}$	0.02354	$7.821e^{-4}$	0.02352	$8.848e^{-4}$	0.02226
GARCH-type	EGARCH(1,1)-t	$7.559e^{-4}$	0.02296	$7.753e^{-4}$	0.02323	$8.803e^{-4}$	0.02206
ARIMA-GARCH-type	ARIMA(2,1,1)-EGARCH(1,1)-t	$7.446e^{-4}$	0.02148	$7.503e^{-4}$	0.02131	$8.750e^{-4}$	0.02202

Source: Data Analysis

From Table V, it was found that ARIMA(2,1,1)-EGARCH(1,1)-t model outperformed ARIMA(2,1,1) model by 0.00244% and 0.206% (as indicated by the respective values of MSE and MAE) at horizon 8; 0.00318% and 0.221% (as indicated by the respective values of MSE and MAE) at horizon 16; while 0.00098% and 0.024% (as indicated by the respective values of MSE and MAE) at horizon 23.

Table V: Percentage Improvement of Forecasting Performance between ARIMA and ARIMA-GARCH-type Models for Wema Bank

Horizon		8		16		23	
Model		MSE	MAE	MSE	MAE	MSE	MAE
ARIMA	ARIMA(2,0,1)	$7.690e^{-4}$	0.02354	$7.821e^{-4}$	0.02352	$8.848e^{-4}$	0.02226
ARIMA-GARCH-type	ARIMA(2,0,1)-EGARCH(1,1)-t	$7.446e^{-4}$	0.02148	$7.503e^{-4}$	0.02131	$8.750e^{-4}$	0.02202
Percentage Difference		$0.244e^{-2}$	0.206	$0.318e^{-2}$	0.221	$0.098e^{-2}$	0.024

Source: Data Analysis

From Table VI, it was found that ARIMA(2,1,1)-EGARCH(1,1)-t model outperformed EGARCH(1,1)-t model by 0.00113% and 0.148% (as indicated by the respective values of MSE and MAE) at horizon 8; 0.0025% and 0.192% (as shown by the respective values of MSE and MAE) at horizon 16; while 0.00053% and 0.004% (as indicated by the respective values of MSE and MAE) at horizon 23.

Table VI: Percentage Improvement of Forecasting Performance between ARIMA and ARIMA-GARCH-type Models for Wema Bank

Horizon		8		16		23	
Model		MSE	MAE	MSE	MAE	MSE	MAE
GARCH-type	EGARCH(1,1)-t	$7.559e^{-4}$	0.02296	$7.753e^{-4}$	0.02323	$8.803e^{-4}$	0.02206
ARIMA-GARCH-type	ARIMA(2,0,1)-EGARCH(1,1)-t	$7.446e^{-4}$	0.02148	$7.503e^{-4}$	0.02131	$8.750e^{-4}$	0.02202
Percentage Difference		$0.113e^{-2}$	0.148	$0.25e^{-2}$	0.192	$0.053e^{-2}$	0.004

Source: Data Analysis

So far, ARIMA(0,1,1)-ARCH(2)-t model appeared to be more robust than each of ARIMA(0,1,1) and ARCH(2)-t models for the return series of First City Monument bank, while ARIMA(2,1,1)-EGARCH(1,1)-t model seemed to be more robust than each of ARIMA(2,1,1) and EGARCH(1,1)-t models for the return series of Wema bank by producing the least forecast errors (as measured by MSE and MAE at 8, 16 and 23 time horizons), while the percentage difference between each of the component models and the combined model provides the quantity of improvement measured. These findings are in tandem with the studies of [19], [20], [21], [22], [23], [24], [25]. Evidently, the study has provided the needed improvement on the work of [27] by showing that the robustness of forecast performance of ARIMA-GARCH-type model over each component using different horizon periods is consistent over time.

4. Conclusion

In summary, our findings revealed that the forecast performances of the combined models are better and more robust than those of individual components. Actually, the robustness of their performances is consistent over different time horizons, which is a clear indication of an insignificant variability of the prediction error. This is particularly, a remedy to model instability, which is a process that results in high variability of prediction error. Conversely, it is recommended that model instability should be accounted for in each of the component models and their forecast performances compared to that of the combined model in order to assess whether they all result in near robustness.

Conflicts of Interest: The author(s) declare that there are no conflicts of interest regarding the publication of this paper.

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