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CUBIC GRAPHS WITH APPLICATION

SHEIKH RASHID[1], NAVEED YAQOOB[2,*], MUHAMMAD AKRAM[3], MUHAMMAD GULISTAN[1]

[1] *Department of Mathematics, Hazara University, Mansehra, Pakistan*

[2] *Department of Mathematics, College of Science Al-Zulfi, Majmaah University, Al-Zulfi, Saudi Arabia*

[3] *Department of Mathematics, University of the Punjab, New Campus, Lahore, Pakistan*

[*] *Corresponding author:* nayaqoob@ymail.com

Abstract. We introduce certain concepts, including cubic graphs, internal cubic graphs, external cubic graphs, and illustrate these concepts by examples. We deal with fundamental operations, Cartesian product, composition, union and join of cubic graphs. We discuss some results of internal cubic graphs and external cubic graphs. We also describe an application of cubic graphs.

1. Introduction

Cubic sets are one of the real generalizations of fuzzy sets [27] provided by Jun et al. [9–11, 15, 26] during the last five years. They developed cubic set theory in many directions and for more detail about cubic sets one can see [12]. Kang and Kim [13] studied mappings of cubic sets. Muhiuddin et al. [18] presented the idea of stable cubic sets.

Fuzzy graphs were studied by Rosenfeld [23] and give a few theoretical ideas in spite of the fact that the fundamental thought was presented by Kauffmann [14] in 1973. Bhattacharya [6] gave some remarks on fuzzy graphs. A book written by Mordeson and Nair [17] is devoted especially to the study of fuzzy graphs

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and fuzzy hypergraphs. Akram et al. gave the idea of interval-valued fuzzy graphs [1,2], intuitionistic fuzzy graphs [3] and bipolar fuzzy graphs [4,5]. Borzooei and Rashmanlou [7] studied Cayley interval-valued fuzzy threshold graphs. Buckley [8] introduced self-centered graphs. Sunitha et al. [25] characterized g-self centered fuzzy graphs. Mishra et al. [16] studied coherent category of interval-valued intuitionistic fuzzy graphs. Pal et al. [19] and Pramanik et al. [21,22] added some useful results to the theory of interval-valued fuzzy graphs. Parvathi et al. [20] provided some different operations on intuitionistic fuzzy graphs and Sahoo and Pal [24] studied product of intuitionistic fuzzy graphs.

In this paper we study some operations on cubic graphs. Internal and external cubic graphs are studied with some example. We provided some conditions for union and join of external and internal cubic graphs.

2. Preliminaries

Here we recall some basic helping material from the existing literature.

Definition 2.1. A graph is denoted by $\Omega^* = (P, Q)$, where P denotes the set of vertices of Ω^* and Q stands for the set of edges of Ω^*.

Definition 2.2. [12] Let T be a non-empty set. By a cubic set in T we mean a structure

$$\Lambda = \{\langle t, \widetilde{\varpi}_\Lambda(t), \mu_\Lambda(t)\rangle \,|t \in T\}$$

in which $\widetilde{\varpi}_\Lambda$ is an interval-valued fuzzy set in T and μ_Λ is a fuzzy set in T.

A cubic set $\Lambda = \{\langle t, \widetilde{\varpi}_\Lambda(t), \mu_\Lambda(t)\rangle \,|t \in T\}$ is simply denoted by $\Lambda = \langle \widetilde{\varpi}_\Lambda, \mu_\Lambda\rangle$.

Definition 2.3. [12] Let T be a non-empty set. A cubic set $\Lambda = \langle \widetilde{\varpi}_\Lambda, \mu_\Lambda\rangle$ in T is said to be an internal cubic (resp., external cubic) set if

$$\varpi^-_\Lambda(t) \le \mu_\Lambda(t) \le \varpi^+_\Lambda(t) \;\; \text{(resp., } \mu_\Lambda(t) \notin (\varpi^-_\Lambda(t), \varpi^+_\Lambda(t)))$$

for all $t \in T$.

Definition 2.4. [12] For any $\Lambda_i = \{\langle t, \widetilde{\varpi}_{\Lambda_i}(t), \mu_{\Lambda_i}(t)\rangle \,|t \in T\}$ where $i \in I$, we define

(a) $\underset{i\in I}{\cup_P}\Lambda_i = \left\{\left\langle t, \left(\underset{i\in I}{\cup}\widetilde{\varpi}_{\Lambda_i}\right)(t), \left(\underset{i\in I}{\vee}\mu_{\Lambda_i}\right)(t)\right\rangle |t \in T\right\}$ (P-union)

(b) $\underset{i\in I}{\cup_R}\Lambda_i = \left\{\left\langle t, \left(\underset{i\in I}{\cup}\widetilde{\varpi}_{\Lambda_i}\right)(t), \left(\underset{i\in I}{\wedge}\mu_{\Lambda_i}\right)(t)\right\rangle |t \in T\right\}$ (R-union)

3. Cubic graphs

We develop the theory of a cubic graph and some operations on cubic graph.

Definition 3.1. Let $M^* = \langle P, Q \rangle$ be a graph. A cubic graph of a graph $M^* = \langle P, Q \rangle$, is the structure $M = \langle \alpha, \beta \rangle$, where $\alpha = \langle \widetilde{\varpi}_\alpha, \mu_\alpha \rangle$ is the cubic set representation for the vertex P and $\beta = \langle \widetilde{\varpi}_\beta, \mu_\beta \rangle$ denotes the cubic set representation for the edge Q, with

$$\begin{aligned} \widetilde{\varpi}_\alpha &: P \to D[0,1],\ \mu_\alpha : P \to [0,1], \\ \text{and } \widetilde{\varpi}_\beta &: Q \to D[0,1],\ \mu_\beta : Q \to [0,1], \end{aligned}$$

such that

$$\begin{aligned} \widetilde{\varpi}_\beta(p_i p_j) &\preceq r\min\{\widetilde{\varpi}_\alpha(p_i), \widetilde{\varpi}_\alpha(p_j)\}, \\ \mu_\beta(p_i p_j) &\leq \max\{\mu_\alpha(p_i), \mu_\alpha(p_j)\}, \end{aligned}$$

for all $(p_i, p_j) \in Q \subseteq P \times P$.

Example 3.1. Let us consider a graph $\Omega^* = (P, Q)$ such that $P = \{p_1, p_2, p_3, p_4\}$, $Q = \{p_1p_2, p_2p_3, p_3p_4, p_4p_1\}$. Let α be a cubic set of P and let β be a cubic set of Q defined by

P	$\widetilde{\varpi}_\alpha$	μ_α
p_1	$[0.1, 0.5]$	0.7
p_2	$[0.3, 0.7]$	0.2
p_3	$[0.2, 0.4]$	0.2
p_4	$[0.1, 0.8]$	0.7

Q	$\widetilde{\varpi}_\beta$	μ_β
p_1p_2	$[0.1, 0.4]$	0.4
p_2p_3	$[0.1, 0.3]$	0.1
p_3p_4	$[0.1, 0.4]$	0.5
p_4p_1	$[0.1, 0.4]$	0.3

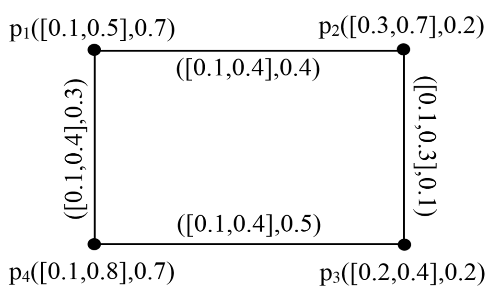

Figure 1. Cubic graph

By routine calculations, it can be observed that the graph shown in Fig. 1 is a cubic graph.

Example 3.2. Consider a graph $\Omega^* = (P, Q)$. Let α be a cubic set of P and let β be a cubic set of Q defined by

$$\mu_\alpha(p_i) = \frac{\varpi_\alpha^-(p_i) + \varpi_\alpha^+(p_i)}{2} \quad \textit{and} \quad \mu_\beta(e_i) = \frac{\varpi_\beta^-(e_i) + \varpi_\beta^+(e_i)}{2}.$$

Then $M = \langle \alpha, \beta \rangle$ is a cubic graph of Ω^*.

Remark 3.1. If $\widetilde{\varpi}_\beta(p_ip_j) = [0, 0]$ and $\mu_\beta(p_ip_j) = 0$, then the cubic graph $M = \langle\alpha, \beta\rangle$ has no edge.

Definition 3.2. Let $M_1 = \langle\alpha_1, \beta_1\rangle$ and $M_2 = \langle\alpha_2, \beta_2\rangle$ be two cubic graphs of the graphs Ω_1^* and Ω_2^*, respectively. The Cartesian product of M_1 and M_2 is denoted by $M_1 \times M_2 = \langle\alpha_1 \times \alpha_2, \beta_1 \times \beta_2\rangle$ and is defined as follows:

(i) $\begin{cases} (\widetilde{\varpi}_{\alpha_1} \times \widetilde{\varpi}_{\alpha_2})(p_1, p_2) = r\min\{\widetilde{\varpi}_{\alpha_1}(p_1), \widetilde{\varpi}_{\alpha_2}(p_2)\} \\ (\mu_{\alpha_1} \times \mu_{\alpha_2})(p_1, p_2) = \max\{\mu_{\alpha_1}(p_1), \mu_{\alpha_2}(p_2)\} \end{cases}$

for all $(p_1, p_2) \in P = P_1 \times P_2$,

(ii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} \times \widetilde{\varpi}_{\beta_2})((q, q_2)(q, p_2)) = r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\beta_2}(q_2p_2)\} \\ (\mu_{\beta_1} \times \mu_{\beta_2})((q, q_2)(q, p_2)) = \max\{\mu_{\alpha_1}(q), \mu_{\beta_2}(q_2p_2)\} \end{cases}$

for all $q \in P_1$, and $q_2p_2 \in Q_2$,

(iii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} \times \widetilde{\varpi}_{\beta_2})((q_1, r)(p_1, r)) = r\min\{\widetilde{\varpi}_{\beta_1}(q_1p_1), \widetilde{\varpi}_{\alpha_2}(r)\} \\ (\mu_{\beta_1} \times \mu_{\beta_2})((q_1, r)(p_1, r)) = \max\{\mu_{\beta_1}(q_1p_1), \mu_{\alpha_2}(r)\} \end{cases}$

for all $r \in P_2$, and $q_1p_1 \in Q_1$.

Example 3.3. Consider two cubic graphs $M_1 = \langle\alpha_1, \beta_1\rangle$ and $M_2 = \langle\alpha_2, \beta_2\rangle$ as shown in figure 2.

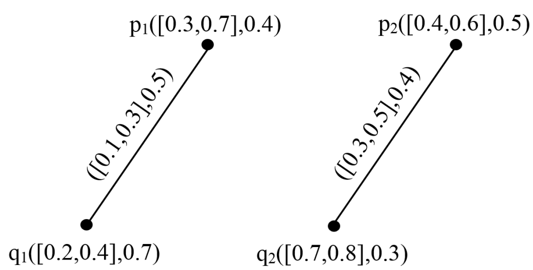

Figure 2. Cubic graphs M_1 and M_2

Then, their corresponding Cartesian product $M_1 \times M_2$ is shown in figure 3.

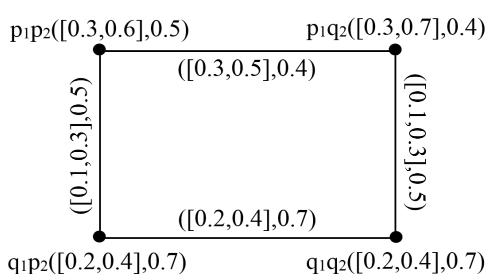

Figure 3. Cubic graph $M_1 \times M_2$

Clearly, $M_1 \times M_2$ is a cubic graph.

Proposition 3.1. The Cartesian product of two cubic graphs is a cubic graph.

Proof. The conditions for $\alpha_1 \times \alpha_2$ are obvious, therefore, we verify only conditions for $\beta_1 \times \beta_2$. Let $q \in P_1$, and $q_2 p_2 \in Q_2$. Then

$$\begin{aligned}
(\widetilde{\varpi}_{\beta_1} \times \widetilde{\varpi}_{\beta_2})((q, q_2)(q, p_2)) &= r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\beta_2}(q_2 p_2)\} \\
&\preceq r\min\{\widetilde{\varpi}_{\alpha_1}(q), r\min\{\widetilde{\varpi}_{\alpha_2}(q_2), \widetilde{\varpi}_{\alpha_2}(p_2)\}\} \\
&= r\min\{r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\alpha_2}(q_2)\}, r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\alpha_2}(p_2)\}\} \\
&= r\min\{(\widetilde{\varpi}_{\alpha_1} \times \widetilde{\varpi}_{\alpha_2})(q, q_2), (\widetilde{\varpi}_{\alpha_1} \times \widetilde{\varpi}_{\alpha_2})((q, p_2)\}
\end{aligned}$$

$$\begin{aligned}
(\mu_{\beta_1} \times \mu_{\beta_2})((q, q_2)(q, p_2)) &= \max\{\mu_{\alpha_1}(q), \mu_{\beta_2}(q_2 p_2)\} \\
&\leq \max\{\mu_{\alpha_1}(q), \max\{\mu_{\alpha_2}(q_2), \mu_{\alpha_2}(p_2)\}\} \\
&= \max\{\max\{\mu_{\alpha_1}(q), \mu_{\alpha_2}(q_2)\}, \max\{\mu_{\alpha_1}(q), \mu_{\alpha_2}(p_2)\}\} \\
&= \max\{(\mu_{\alpha_1} \times \mu_{\alpha_2})(q, q_2), (\mu_{\alpha_1} \times \mu_{\alpha_2})((q, p_2)\}
\end{aligned}$$

Similarly, we can prove it for $r \in P_2$, and $q_1 p_1 \in Q_1$. □

Definition 3.3. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two cubic graphs of the graphs Ω_1^* and Ω_2^*, respectively. The composition of M_1 and M_2 is denoted by $M_1[M_2] = \langle \alpha_1 \circ \alpha_2, \beta_1 \circ \beta_2 \rangle$ and is defined as follows:

(i) $\begin{cases} (\widetilde{\varpi}_{\alpha_1} \circ \widetilde{\varpi}_{\alpha_2})(p_1, p_2) = r\min\{\widetilde{\varpi}_{\alpha_1}(p_1), \widetilde{\varpi}_{\alpha_2}(p_2)\} \\ (\mu_{\alpha_1} \circ \mu_{\alpha_2})(p_1, p_2) = \max\{\mu_{\alpha_1}(p_1), \mu_{\alpha_2}(p_2)\} \end{cases}$

for all $(p_1, p_2) \in P = P_1 \times P_2$,

(ii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} \circ \widetilde{\varpi}_{\beta_2})((q, q_2)(q, p_2)) = r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\beta_2}(q_2 p_2)\} \\ (\mu_{\beta_1} \circ \mu_{\beta_2})((q, q_2)(q, p_2)) = \max\{\mu_{\alpha_1}(q), \mu_{\beta_2}(q_2 p_2)\} \end{cases}$

for all $q \in P_1$, and $q_2 p_2 \in Q_2$,

(iii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} \circ \widetilde{\varpi}_{\beta_2})((q_1, r)(p_1, r)) = r\min\{\widetilde{\varpi}_{\beta_1}(q_1 p_1), \widetilde{\varpi}_{\alpha_2}(r)\} \\ (\mu_{\beta_1} \circ \mu_{\beta_2})((q_1, r)(p_1, r)) = \max\{\mu_{\beta_1}(q_1 p_1), \mu_{\alpha_2}(r)\} \end{cases}$

for all $r \in P_2$, and $q_1 p_1 \in Q_1$.

(iv) $\begin{cases} (\widetilde{\varpi}_{\beta_1} \circ \widetilde{\varpi}_{\beta_2})((q_1, q_2)(p_1, p_2)) = r\min\{\widetilde{\varpi}_{\alpha_2}(q_2), \widetilde{\varpi}_{\alpha_2}(p_2), \widetilde{\varpi}_{\beta_1}(q_1 p_1)\} \\ (\mu_{\beta_1} \circ \mu_{\beta_2})((q_1, q_2)(p_1, p_2)) = \max\{\mu_{\alpha_2}(q_2), \mu_{\alpha_2}(p_2), \mu_{\beta_1}(q_1 p_1)\} \end{cases}$

for all $q_2, p_2 \in P_2$, $q_2 \neq p_2$ and $q_1 p_1 \in Q_1$.

Example 3.4. From Example 3.3, consider two cubic graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ as shown in figure 2. Then, their corresponding composition $M_1[M_2]$ is shown in figure 4.

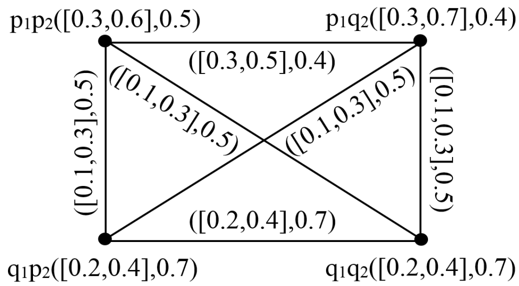

FIGURE 4. Cubic graph $M_1[M_2]$

Clearly, $M_1[M_2]$ is a cubic graph.

Proposition 3.2. The composition of two cubic graphs is a cubic graph.

Definition 3.4. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two cubic graphs of the graphs Ω_1^* and Ω_2^*, respectively. The P-union of two cubic graphs M_1 and M_2 is denoted by $M_1 \cup_P M_2 = \langle \alpha_1 \cup_p \alpha_2, \beta_1 \cup_p \beta_2 \rangle$ and is defined as follows:

(i) $(\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(p) = \begin{cases} \widetilde{\varpi}_{\alpha_1}(p) & \text{if } p \in P_1 - P_2 \\ \widetilde{\varpi}_{\alpha_2}(p) & \text{if } p \in P_2 - P_1 \\ r\max\{\widetilde{\varpi}_{\alpha_1}(p), \widetilde{\varpi}_{\alpha_2}(p)\} & \text{if } p \in P_1 \cap P_2 \end{cases}$

(ii) $(\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(p) = \begin{cases} \mu_{\alpha_1}(p) & \text{if } p \in P_1 - P_2 \\ \mu_{\alpha_2}(p) & \text{if } p \in P_2 - P_1 \\ \max\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\} & \text{if } p \in P_1 \cap P_2 \end{cases}$

(iii) $(\widetilde{\varpi}_{\beta_1} \cup_p \widetilde{\varpi}_{\beta_2})(p_ip_j) = \begin{cases} \widetilde{\varpi}_{\beta_1}(p_ip_j) & \text{if } p_ip_j \in Q_1 - Q_2 \\ \widetilde{\varpi}_{\beta_2}(p_ip_j) & \text{if } p_ip_j \in Q_2 - Q_1 \\ r\max\{\widetilde{\varpi}_{\beta_1}(p_ip_j), \widetilde{\varpi}_{\beta_2}(p_ip_j)\} & \text{if } p_ip_j \in Q_1 \cap Q_2 \end{cases}$

(iv) $(\mu_{\beta_1} \cup_p \mu_{\beta_2})(p_ip_j) = \begin{cases} \mu_{\beta_1}(p_ip_j) & \text{if } p_ip_j \in Q_1 - Q_2 \\ \mu_{\beta_2}(p_ip_j) & \text{if } p_ip_j \in Q_2 - Q_1 \\ \max\{\mu_{\beta_1}(p_ip_j), \mu_{\beta_2}(p_ip_j)\} & \text{if } p_ip_j \in Q_1 \cap Q_2 \end{cases}$

Definition 3.5. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two cubic graphs of the graphs Ω_1^* and Ω_2^*, respectively. The R-union of two cubic graphs M_1 and M_2 is denoted by $M_1 \cup_R M_2 = \langle \alpha_1 \cup_R \alpha_2, \beta_1 \cup_R \beta_2 \rangle$ and is defined as follows:

(i) $(\widetilde{\varpi}_{\alpha_1} \cup_R \widetilde{\varpi}_{\alpha_2})(p) = \begin{cases} \widetilde{\varpi}_{\alpha_1}(p) & \text{if } p \in P_1 - P_2 \\ \widetilde{\varpi}_{\alpha_2}(p) & \text{if } p \in P_2 - P_1 \\ r\max\{\widetilde{\varpi}_{\alpha_1}(p), \widetilde{\varpi}_{\alpha_2}(p)\} & \text{if } p \in P_1 \cap P_2 \end{cases}$

(ii) $(\mu_{\alpha_1} \cup_R \mu_{\alpha_2})(p) = \begin{cases} \mu_{\alpha_1}(p) & \text{if } p \in P_1 - P_2 \\ \mu_{\alpha_2}(p) & \text{if } p \in P_2 - P_1 \\ \min\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\} & \text{if } p \in P_1 \cap P_2 \end{cases}$

(iii) $(\widetilde{\varpi}_{\beta_1} \cup_R \widetilde{\varpi}_{\beta_2})(p_ip_j) = \begin{cases} \widetilde{\varpi}_{\beta_1}(p_ip_j) & \text{if } p_ip_j \in Q_1 - Q_2 \\ \widetilde{\varpi}_{\beta_2}(p_ip_j) & \text{if } p_ip_j \in Q_2 - Q_1 \\ r\max\{\widetilde{\varpi}_{\beta_1}(p_ip_j), \widetilde{\varpi}_{\beta_2}(p_ip_j)\} & \text{if } p_ip_j \in Q_1 \cap Q_2 \end{cases}$

(iv) $(\mu_{\beta_1} \cup_R \mu_{\beta_2})(p_ip_j) = \begin{cases} \mu_{\beta_1}(p_ip_j) & \text{if } p_ip_j \in Q_1 - Q_2 \\ \mu_{\beta_2}(p_ip_j) & \text{if } p_ip_j \in Q_2 - Q_1 \\ \min\{\mu_{\beta_1}(p_ip_j), \mu_{\beta_2}(p_ip_j)\} & \text{if } p_ip_j \in Q_1 \cap Q_2 \end{cases}$

Example 3.5. Consider two cubic graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ as shown in figure 5.

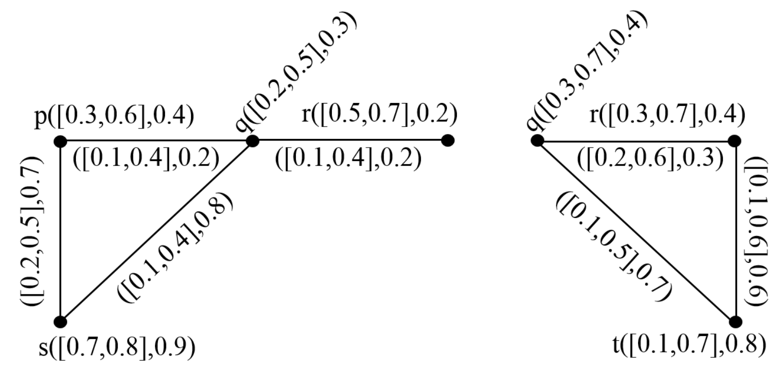

FIGURE 5. Cubic graphs M_1 and M_2

Then, their corresponding P-union $M_1 \cup_P M_2$ is shown in figure 6.

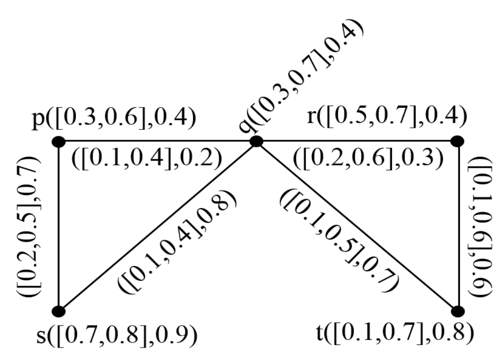

FIGURE 6. Cubic graph $M_1 \cup_P M_2$

Also, their corresponding R-union $M_1 \cup_R M_2$ is shown in figure 7.

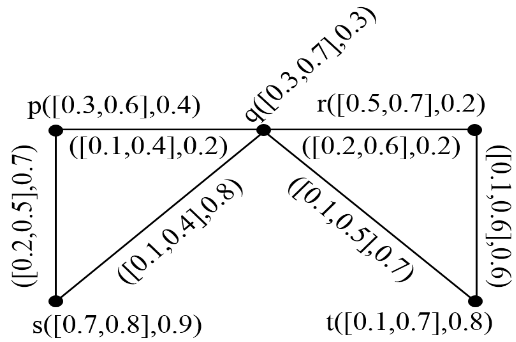

Figure 7. Cubic graph $M_1 \cup_R M_2$

Clearly, $M_1 \cup_P M_2$ and $M_1 \cup_R M_2$ are cubic graphs.

Proposition 3.3. The P-union and R-union of two cubic graphs is a cubic graph.

Proof. Since all the conditions for $\alpha_1 \cup_p \alpha_2$ are automatically satisfied therefore, we verify only conditions for $\beta_1 \cup_p \beta_2$. In the case, when $qp \in Q_1 \cap Q_2$. Then

$$\begin{aligned}(\widetilde{\varpi}_{\beta_1} \cup_p \widetilde{\varpi}_{\beta_2})(qp) &= r\max\{\widetilde{\varpi}_{\beta_1}(qp), \widetilde{\varpi}_{\beta_2}(qp)\} \\ &\preceq r\max\{r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\alpha_1}(p)\}, r\min\{\widetilde{\varpi}_{\alpha_2}(q), \widetilde{\varpi}_{\alpha_2}(p)\}\} \\ &= r\min\{r\max\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\alpha_2}(q)\}, r\max\{\widetilde{\varpi}_{\alpha_1}(p), \widetilde{\varpi}_{\alpha_2}(p)\}\} \\ &= r\min\{(\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(q), (\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(p)\}.\end{aligned}$$

$$\begin{aligned}(\mu_{\beta_1} \cup_p \mu_{\beta_2})(qp) &= \max\{\mu_{\beta_1}(qp), \mu_{\beta_2}(qp)\} \\ &\leq \max\{\max\{\mu_{\alpha_1}(q), \mu_{\alpha_1}(p)\}, \max\{\mu_{\alpha_2}(q), \mu_{\alpha_2}(p)\}\} \\ &= \max\{\max\{\mu_{\alpha_1}(q), \mu_{\alpha_2}(q)\}, \max\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\}\} \\ &= \max\{(\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(q), (\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(p)\}.\end{aligned}$$

If $qp \in Q_1$ and $qp \notin Q_2$, then

$$\begin{aligned}(\widetilde{\varpi}_{\beta_1} \cup_p \widetilde{\varpi}_{\beta_2})(qp) &\preceq r\min\{(\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(q), (\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(p)\} \\ (\mu_{\beta_1} \cup_p \mu_{\beta_2})(qp) &\leq \max\{(\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(q), (\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(p)\}.\end{aligned}$$

If $qp \in Q_2$ and $qp \notin Q_1$, then

$$\begin{aligned}(\widetilde{\varpi}_{\beta_1} \cup_p \widetilde{\varpi}_{\beta_2})(qp) &\preceq r\min\{(\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(q), (\widetilde{\varpi}_{\alpha_1} \cup_p \widetilde{\varpi}_{\alpha_2})(p)\} \\ (\mu_{\beta_1} \cup_p \mu_{\beta_2})(qp) &\leq \max\{(\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(q), (\mu_{\alpha_1} \cup_p \mu_{\alpha_2})(p)\}.\end{aligned}$$

Hence the P-union of two cubic graphs is a cubic graph. The case for R-union of two cubic graphs can be seen in a similar way. □

Definition 3.6. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two cubic graphs of the graphs Ω_1^* and Ω_2^*, respectively. The P-join of two cubic graphs M_1 and M_2 is denoted by $M_1 +_P M_2 = \langle \alpha_1 +_P \alpha_2, \beta_1 +_P \beta_2 \rangle$ and is defined as follows:

(i) $\begin{cases} (\widetilde{\varpi}_{\alpha_1} +_P \widetilde{\varpi}_{\alpha_2})(p) = (\widetilde{\varpi}_{\alpha_1} \cup_P \widetilde{\varpi}_{\alpha_2})(p) \\ (\mu_{\alpha_1} +_P \mu_{\alpha_2})(p) = (\mu_{\alpha_1} \cup_P \mu_{\alpha_2})(p) \end{cases}$
for $p \in P_1 \cup P_2$,

(ii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} +_P \widetilde{\varpi}_{\beta_2})(qp) = (\widetilde{\varpi}_{\beta_1} \cup_P \widetilde{\varpi}_{\beta_2})(qp) \\ (\mu_{\beta_1} +_P \mu_{\beta_2})(qp) = (\mu_{\beta_1} \cup_P \mu_{\beta_2})(qp) \end{cases}$
for $qp \in Q_1 \cap Q_2$,

(iii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} +_P \widetilde{\varpi}_{\beta_2})(qp) = r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\alpha_2}(p)\} \\ (\mu_{\beta_1} +_P \mu_{\beta_2})(qp) = \min\{\mu_{\alpha_1}(q), \mu_{\alpha_2}(p)\} \end{cases}$
for $qp \in Q^*$, where Q^* is the set of all edges joining the vertices of P_1 and P_2.

Definition 3.7. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two cubic graphs of the graphs Ω_1^* and Ω_2^*, respectively. The R-join of two cubic graphs M_1 and M_2 is denoted by $M_1 +_R M_2 = \langle \alpha_1 +_R \alpha_2, \beta_1 +_R \beta_2 \rangle$ and is defined as follows:

(i) $\begin{cases} (\widetilde{\varpi}_{\alpha_1} +_R \widetilde{\varpi}_{\alpha_2})(p) = (\widetilde{\varpi}_{\alpha_1} \cup_R \widetilde{\varpi}_{\alpha_2})(p) \\ (\mu_{\alpha_1} +_R \mu_{\alpha_2})(p) = (\mu_{\alpha_1} \cup_R \mu_{\alpha_2})(p) \end{cases}$
for $p \in P_1 \cup P_2$,

(ii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} +_R \widetilde{\varpi}_{\beta_2})(qp) = (\widetilde{\varpi}_{\beta_1} \cup_R \widetilde{\varpi}_{\beta_2})(qp) \\ (\mu_{\beta_1} +_R \mu_{\beta_2})(qp) = (\mu_{\beta_1} \cup_R \mu_{\beta_2})(qp) \end{cases}$
for $qp \in Q_1 \cap Q_2$,

(iii) $\begin{cases} (\widetilde{\varpi}_{\beta_1} +_R \widetilde{\varpi}_{\beta_2})(qp) = r\min\{\widetilde{\varpi}_{\alpha_1}(q), \widetilde{\varpi}_{\alpha_2}(p)\} \\ (\mu_{\beta_1} +_R \mu_{\beta_2})(qp) = \max\{\mu_{\alpha_1}(q), \mu_{\alpha_2}(p)\} \end{cases}$
for $qp \in Q^*$, where Q^* is the set of all edges joining the vertices of P_1 and P_2.

Example 3.6. Consider two cubic graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ as shown in figure 8.

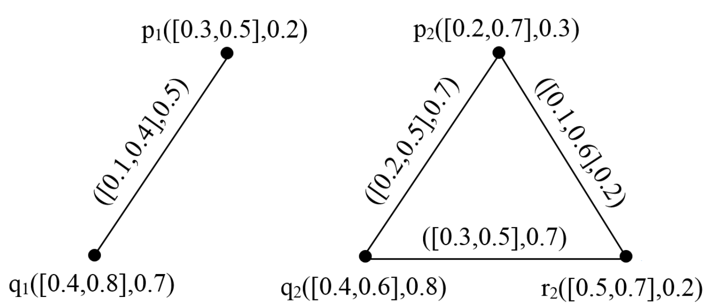

FIGURE 8. Cubic graphs M_1 and M_2

Then, their corresponding P-join $M_1 +_P M_2$ is shown in figure 9.

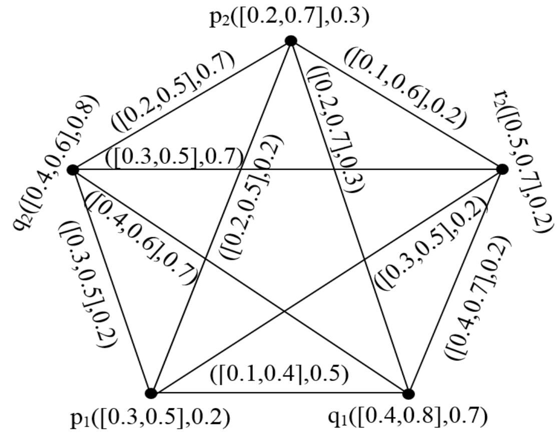

FIGURE 9. Cubic graph $M_1 +_P M_2$

Also, their corresponding R-join $M_1 +_R M_2$ is shown in figure 10.

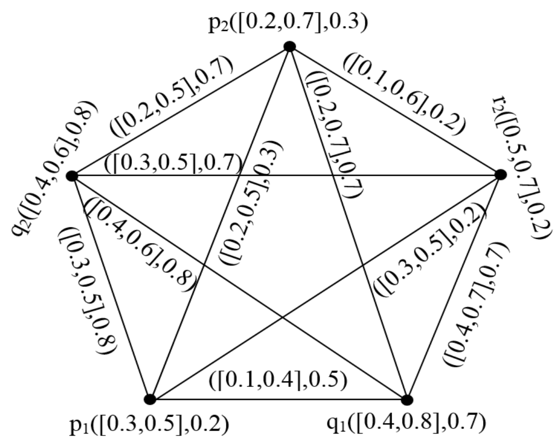

FIGURE 10. Cubic graph $M_1 +_R M_2$

Clearly, $M_1 +_P M_2$ and $M_1 +_R M_2$ are cubic graphs.

Proposition 3.4. The P-join and R-join of two cubic graphs is a cubic graph.

4. Internal and external cubic graphs

Here in this section we discuss some results related with internal and external cubic graphs.

Definition 4.1. A cubic graph $M = \langle \alpha, \beta \rangle$ is said to be an

(i) internal cubic graph (IC-graph) if

$$\mu_\alpha(p_i) \in [\varpi_\alpha^-(p_i), \varpi_\alpha^+(p_i)] \text{ and } \mu_\beta(e_i) \in [\varpi_\beta^-(e_i), \varpi_\beta^+(e_i)]$$

for each $p_i \in P$ and $e_i \in Q$.

(ii) external cubic graph (EC-graph) if

$$\mu_\alpha(p_i) \notin (\varpi_\alpha^-(p_i), \varpi_\alpha^+(p_i)) \text{ and } \mu_\beta(e_i) \notin (\varpi_\beta^-(e_i), \varpi_\beta^+(e_i))$$

for each $p_i \in P$ and $e_i \in Q$.

Example 4.1. The cubic graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ are internal and external cubic graphs, respectively, as shown in figure 11.

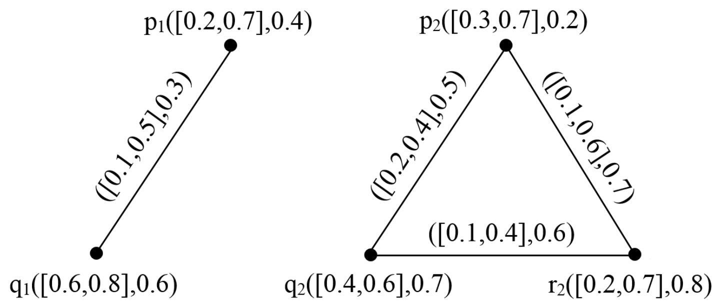

FIGURE 11. IC-graph M_1 and EC-graph M_2

Theorem 4.1. Let $\{M_i = \langle \alpha_i, \beta_i \rangle \,|i \in I\}$ be a family of IC-graphs. Then $\underset{i\in I}{\cup_P} M_i$ is an IC-graph.

Proof. Since M_i is an IC-graph, we have $\varpi_\alpha^-(p) \leq \mu_\alpha(p) \leq \varpi_\alpha^+(p)$ and $\varpi_\beta^-(e) \leq \mu_\beta(e) \leq \varpi_\beta^+(e)$ for $i \in I$. This implies that

$$\left(\underset{i\in I}{\cup} \varpi_\alpha^-\right)(p) \leq \left(\underset{i\in I}{\vee} \mu_\alpha\right)(p) \leq \left(\underset{i\in I}{\cup} \varpi_\alpha^+\right)(p),$$

and

$$\left(\underset{i\in I}{\cup} \varpi_\beta^-\right)(e) \leq \left(\underset{i\in I}{\vee} \mu_\beta\right)(e) \leq \left(\underset{i\in I}{\cup} \varpi_\beta^+\right)(e).$$

Hence $\underset{i\in I}{\cup_P} M_i$ is an IC-graph. □

The following example shows that the R-union of IC-graphs need not be an IC-graph (EC-graph).

Example 4.2. Consider two IC-graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ as shown in figure 12.

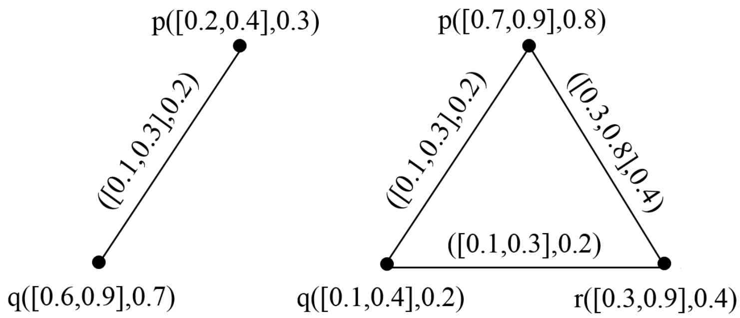

FIGURE 12. IC-graphs M_1 and M_2

Then, their corresponding R-union $M_1 \cup_R M_2$ is shown in figure 13.

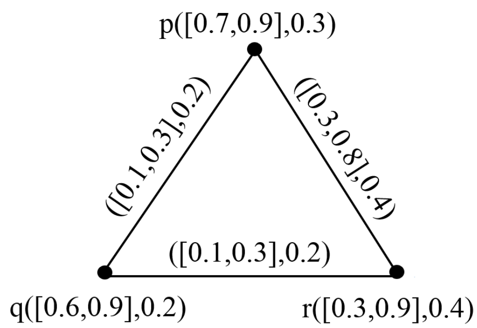

FIGURE 13. R-union of IC-graphs M_1 and M_2

It is easy to see that the cubic graph $M_1 \cup_R M_2$ is neither IC-graph nor EC-graph.

We provide a condition for the R-union of two IC-graphs to be an IC-graph.

Theorem 4.2. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be IC-graphs such that

$$\max\{\varpi^-_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\} \leq \min\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\}$$

and

$$\max\{\varpi^-_{\beta_1}(e), \varpi^-_{\beta_2}(e)\} \leq \min\{\mu_{\beta_1}(e), \mu_{\beta_2}(e)\}$$

for all $p \in P$ and $e \in Q$. Then the R-union of two IC-graphs M_1 and M_2 is an IC-graph.

Proof. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two IC-graphs which satisfy the conditions

$$\max\{\varpi^-_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\} \leq \min\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\}$$

and

$$\max\{\varpi^-_{\beta_1}(e), \varpi^-_{\beta_2}(e)\} \leq \min\{\mu_{\beta_1}(e), \mu_{\beta_2}(e)\}$$

for all $p \in P$ and $e \in Q$. Since $\mu_{\alpha_1}(p) \in [\varpi^-_{\alpha_1}(p), \varpi^+_{\alpha_1}(p)]$, $\mu_{\beta_1}(e) \in [\varpi^-_{\beta_1}(e), \varpi^+_{\beta_1}(e)]$ and $\mu_{\alpha_2}(p) \in [\varpi^-_{\alpha_2}(p), \varpi^+_{\alpha_2}(p)]$, $\mu_{\beta_2}(e) \in [\varpi^-_{\beta_2}(e), \varpi^+_{\beta_2}(e)]$. This implies that

$$\min\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\} \leq (\varpi^+_{\alpha_1} \cup \varpi^+_{\alpha_2})(p) \text{ and } \min\{\mu_{\beta_1}(e), \mu_{\beta_2}(e)\} \leq (\varpi^+_{\beta_1} \cup \varpi^+_{\beta_2})(e)$$

Thus from the given condition we get

$$(\varpi^-_{\alpha_1} \cup \varpi^-_{\alpha_2})(p) = \max\{\varpi^-_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\} \leq \min\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\} \leq (\varpi^+_{\alpha_1} \cup \varpi^+_{\alpha_2})(p),$$

and

$$(\varpi^-_{\beta_1} \cup \varpi^-_{\beta_2})(e) = \max\{\varpi^-_{\beta_1}(e), \varpi^-_{\beta_2}(e)\} \leq \min\{\mu_{\beta_1}(e), \mu_{\beta_2}(e)\} \leq (\varpi^+_{\beta_1} \cup \varpi^+_{\beta_2})(e).$$

This shows that $M_1 \cup_R M_2$ is an IC-graph. □

The following example shows that the P-union and R-union of EC-graphs need not be an EC-graph (IC-graph).

Example 4.3. Consider two EC-graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ as shown in figure 14.

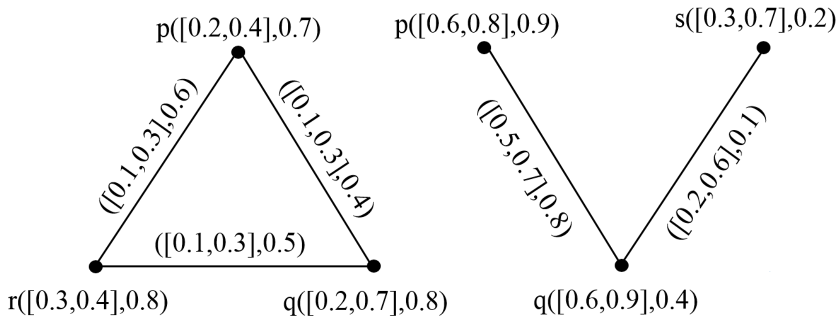

Figure 14. EC-graphs M_1 and M_2

Then, their corresponding P-union $M_1 \cup_P M_2$ is shown in figure 15.

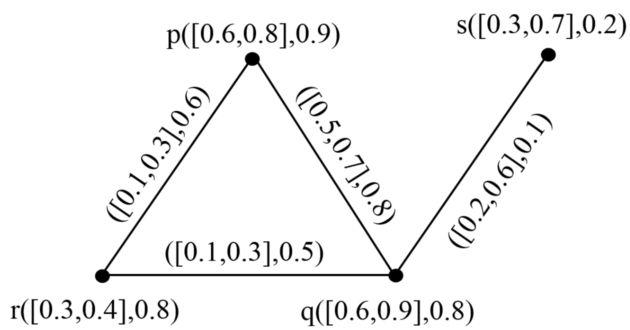

Figure 15. P-union of EC-graphs M_1 and M_2

Also, the corresponding R-union $M_1 \cup_R M_2$ is shown in figure 16.

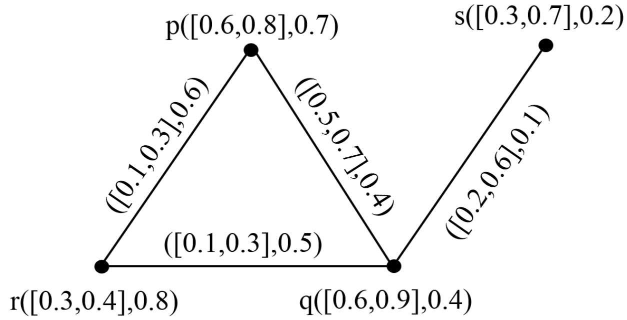

FIGURE 16. R-union of EC-graphs M_1 and M_2

It is easy to see that the cubic graph $M_1 \cup_P M_2$ and $M_1 \cup_R M_2$ are neither EC-graph nor IC-graph.

We provide a condition for the P-union of two EC-graphs to be an EC-graph.

Theorem 4.3. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two EC-graphs such that

$$\min \left\{ \begin{array}{c} \max\{\varpi^+_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\}, \\ \max\{\varpi^-_{\alpha_1}(p), \varpi^+_{\alpha_2}(p)\} \end{array} \right\} > \max\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\} \geq \max \left\{ \begin{array}{c} \min\{\varpi^+_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\}, \\ \min\{\varpi^-_{\alpha_1}(p), \varpi^+_{\alpha_2}(p)\} \end{array} \right\}$$

and

$$\min \left\{ \begin{array}{c} \max\{\varpi^+_{\beta_1}(e), \varpi^-_{\beta_2}(e)\}, \\ \max\{\varpi^+_{\beta_1}(e), \varpi^-_{\beta_2}(e)\} \end{array} \right\} > \max\{\mu_{\beta_1}(e), \mu_{\beta_2}(e)\} \geq \max \left\{ \begin{array}{c} \min\{\varpi^+_{\beta_1}(e), \varpi^-_{\beta_2}(e)\}, \\ \min\{\varpi^+_{\beta_1}(e), \varpi^-_{\beta_2}(e)\} \end{array} \right\}$$

for all $p \in P$ and $e \in Q$. Then the P-union of two EC-graphs is an EC-graph.

We provide a condition for the R-union of two EC-graphs to be an EC-graph.

Theorem 4.4. Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be two EC-graphs such that

$$\min \left\{ \begin{array}{c} \max\{\varpi^+_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\}, \\ \max\{\varpi^-_{\alpha_1}(p), \varpi^+_{\alpha_2}(p)\} \end{array} \right\} > \min\{\mu_{\alpha_1}(p), \mu_{\alpha_2}(p)\} \geq \max \left\{ \begin{array}{c} \min\{\varpi^+_{\alpha_1}(p), \varpi^-_{\alpha_2}(p)\}, \\ \min\{\varpi^-_{\alpha_1}(p), \varpi^+_{\alpha_2}(p)\} \end{array} \right\}$$

and

$$\min\left\{\begin{array}{c}\max\{\varpi^{+}_{\beta_1}(e),\varpi^{-}_{\beta_2}(e)\},\\ \max\{\varpi^{+}_{\beta_1}(e),\varpi^{-}_{\beta_2}(e)\}\end{array}\right\} \;>\; \min\{\mu_{\beta_1}(e),\mu_{\beta_2}(e)\}$$
$$\geq \max\left\{\begin{array}{c}\min\{\varpi^{+}_{\beta_1}(e),\varpi^{-}_{\beta_2}(e)\},\\ \min\{\varpi^{+}_{\beta_1}(e),\varpi^{-}_{\beta_2}(e)\}\end{array}\right\}$$

for all $p \in P$ and $e \in Q$. Then the R-union of two EC-graphs is an EC-graph.

Theorem 4.5. Let $M = \langle\alpha,\beta\rangle$ be a cubic graph which is not an EC-graph. Then there exist $p_i \in P$ and $e_i \in Q$ such that

$$\mu_\alpha(p_i) \in (\varpi^{-}_{\alpha}(p_i),\varpi^{+}_{\alpha}(p_i)) \text{ and } \mu_\beta(e_i) \in (\varpi^{-}_{\beta}(e_i),\varpi^{+}_{\beta}(e_i)).$$

Proof. Straightforward. □

Theorem 4.6. Let $M = \langle\alpha,\beta\rangle$ be a cubic graph of Ω^*. If $M = \langle\alpha,\beta\rangle$ is both an IC-graph and an EC-graph, then

$$\mu_\alpha(p_i) \in U(\widetilde{\varpi}_\alpha) \cup L(\widetilde{\varpi}_\alpha)$$

and

$$\mu_\beta(e_i) \in U(\widetilde{\varpi}_\beta) \cup L(\widetilde{\varpi}_\beta)$$

for all $p_i \in P$ and $e_i \in Q \subseteq P \times P$. Where

$$U(\widetilde{\varpi}_\alpha) = \{\varpi^{+}_{\alpha}(p_i)|p_i \in P\},\quad L(\widetilde{\varpi}_\alpha) = \{\varpi^{-}_{\alpha}(p_i)|p_i \in P\}$$

and

$$U(\widetilde{\varpi}_\beta) = \{\varpi^{+}_{\beta}(e_i)|e_i \in Q\}, L(\widetilde{\varpi}_\beta) = \{\varpi^{-}_{\beta}(e_i)|e_i \in Q\}.$$

Proof. Assume that $M = \langle\alpha,\beta\rangle$ is both an IC-graph and an EC-graph. Then by definition we have

$$\mu_\alpha(p_i) \in [\varpi^{-}_{\alpha}(p_i),\varpi^{+}_{\alpha}(p_i)],\ \mu_\beta(e_i) \in [\varpi^{-}_{\beta}(e_i),\varpi^{+}_{\beta}(e_i)]$$

and

$$\mu_\alpha(p_i) \notin (\varpi^{-}_{\alpha}(p_i),\varpi^{+}_{\alpha}(p_i)),\ \mu_\beta(e_i) \notin (\varpi^{-}_{\beta}(e_i),\varpi^{+}_{\beta}(e_i)).$$

Thus $\mu_\alpha(p_i) = \varpi^{-}_{\alpha}(p_i)$ or $\mu_\alpha(p_i) = \varpi^{+}_{\alpha}(p_i)$ and $\mu_\beta(e_i) = \varpi^{-}_{\beta}(e_i)$ or $\mu_\beta(e_i) = \varpi^{+}_{\beta}(e_i)$. Hence $\mu_\alpha(p_i) \in U(\widetilde{\varpi}_\alpha) \cup L(\widetilde{\varpi}_\alpha)$ and $\mu_\beta(e_i) \in U(\widetilde{\varpi}_\beta) \cup L(\widetilde{\varpi}_\beta)$ for all $p_i \in P$ and $e_i \in Q \subseteq P \times P$. □

Consider two cubic graphs $M_1 = \langle\alpha_1,\beta_1\rangle$ and $M_2 = \langle\alpha_2,\beta_2\rangle$ in Ω^*. If we exchange μ_{α_1} by μ_{α_2} and μ_{β_1} by μ_{β_2} we get the cubic graph as $\widehat{M_1} = \left\langle\widehat{\alpha_1},\widehat{\beta_1}\right\rangle$ and $\widehat{M_2} = \left\langle\widehat{\alpha_2},\widehat{\beta_2}\right\rangle$, respectively.

For any two IC-graphs (or EC-graphs) M_1 and M_2, two cubic graphs $\widehat{M_1}$ and $\widehat{M_2}$ may not be IC-graph and EC-graph.

Example 4.4. Consider two IC-graphs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ as shown in figure 17.

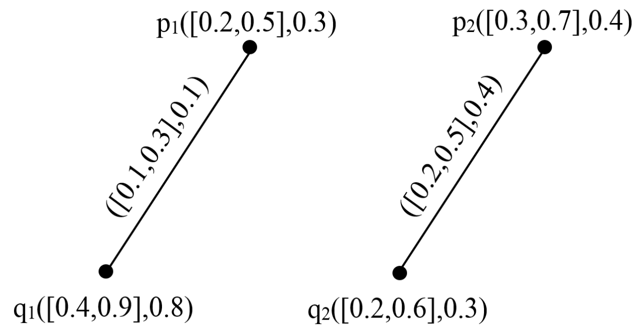

Figure 17. IC-graphs M_1 and M_2

Then, their corresponding $\widehat{M_1}$ and $\widehat{M_2}$ are shown in figure 18.

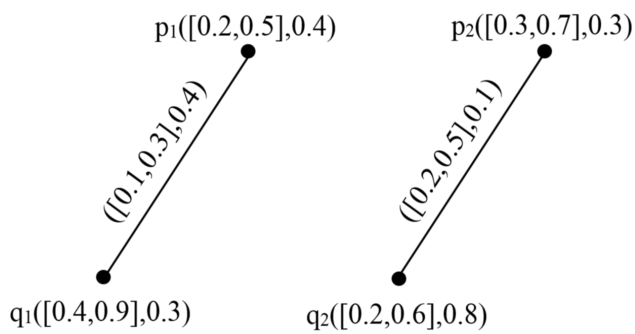

Figure 18. Cubic graphs $\widehat{M_1}$ and $\widehat{M_2}$

It is easy to see that the cubic graphs $\widehat{M_1}$ and $\widehat{M_2}$ are neither IC-graph nor EC-graph. Similarly, we can provide and example for two EC-graphs that are neither IC-graph nor EC-graph.

5. Application

Fuzzy graph theory is a platform which has wide range of applications in mathematics, computer science etc. Cubic graph is a more general approach, which can be used in decision making very effectively.

Suppose we have a set of three countries like, $P = \{X, Y, Z\}$ as a vertex set and the membership of each member of the set denotes the strength of that country over the neighbouring country with respect to future and present time by considering its economic strength. Now we want to observe the effect of strength of one country at the another country with respect to economy. Let we have a cubic set for each country based on certain information and data with respect to economy

$$\alpha = \begin{cases} \langle X : [0.6, 0.8], 0.9\rangle \\ \langle Y : [0.5, 0.9], 0.7\rangle \\ \langle Z : [0.3, 0.7], 0.8\rangle \end{cases}$$

where interval membership predicts the economy of a certain country for the future and the other membership shows economy of a certain country for the present time based on certain information and data with respect to economy. Now on the basis of α, we have the set β of edges as follows

$$\beta = \begin{cases} \langle XY : [0.5, 0.8], 0.9\rangle \\ \langle YZ : [0.3, 0.7], 0.8\rangle \\ \langle ZX : [0.3, 0.7], 0.9\rangle \end{cases}$$

where interval membership predicts the effect of economy of a certain country for the future and the other membership shows the effect of economy of a certain country for the present time at the other country. The corresponding cubic graph is shown in figure 19.

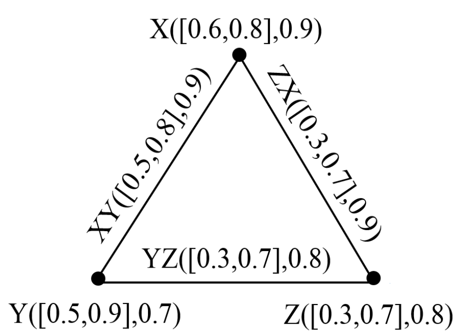

Figure 19. Cubic graph

So finally we concluded that economy of a certain country effect very much on the economy of the neighboring countries.

6. Conclusions

Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological and social systems. We come up here with the idea of cubic graphs and we define different operations of cubic graphs. We also provide a short application of cubic graph. In future we are planning to generalize our notions to (1) Cubic line graphs, (2) Cubic hypergraphs, and (3) Cubic soft graphs.

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