

# HARMONIC DISTORTION ASSESSMENT BY AREA BASED APPROACH AT SINGLE PHASING OF AN INDUCTION MOTOR

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**ABSTRACT:** This paper presents harmonic distortion assessment by area based approach at single phasing of an induction motor. This has been achieved by assessing voltage and current signals using area based approach. Some specific reference signals have been defined, after which, the real power system data are plotted with this reference signal and areas. Formed by the real power system data with the reference signal, the reference signal and areas have been calculated, where the contributions of fundamental waveform and the harmonic components in real and reactive powers are being assessed separately. Single phasing is done on induction machine as well as the total harmonic distortion factors have been calculated afterwards. Moreover, the significant change is observed in harmonic distortion due to single phasing.

**ABSTRAK:** Kertas kerja ini membentangkan penilaian herotan harmonik menggunakan kaedah keluasan kawasan pada pemfasaan tunggal motor aruhan. Menggunakan kaedah keluasan kawasan, penilaian terhasil dengan memantau isyarat arus dan voltan. Sesetengah isyarat rujukan tertentu dikenal pasti, di mana, data sistem kuasa sebenar diplotkan berdasarkan isyarat rujukan ini. Kawasan kemudiannya dibentuk dengan adanya data sistem kuasa sebenar dengan menggunakan pengiraan isyarat rujukan. Pengiraan ini memberikan bentuk gelombang asas dan komponen harmonik sebenar di mana kuasa reaktif ditentukan secara berasingan. Pemfasaan tunggal ditentukan menggunakan mesin aruhan dan faktor jumlah herotan harmonik diambil kira. Perubahan yang ketara dikenal pasti dalam herotan harmonik yang disebabkan oleh pemfasaan tunggal.

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**KEYWORDS:** Area Based Approach; Harmonics; Single-Phasing.

## 1. INTRODUCTION

Maintaining the near sinusoidal waveform of power distribution bus voltages and currents at rated magnitude and frequency is important for good quality of electric power in respect of harmonics. Harmonics are generated from various types of sources like actual loads, equipment and components, subsystems of transmission and distribution systems, etc. With the increasing use of solid state control devices, power converter, nonlinear loads like switch-mode-power-supplier and phase control rectifiers in distribution and transmission networks, the amount of such disturbances is also increasing. Different type of faults occurred in electrical machine also contribute in harmonic distortion. It leads

catastrophic consequences such as long production downtimes, mal-function of devices and shortened equipment life. Use of induction motor has increased and it demands an early and accurate determination of motor fault for its reliable operation. Single phasing is one common fault often observed in industrial application.

Research work is going on to study the sources and effects of faults. A detail review on measurement of harmonic contribution in impedances and different issues of their practical implementation has been done [1]. Modeling is done for large load areas considering harmonics in distributions networks [2]. Modeling is also done for assessing harmonics and inter-harmonic distortion in multi-converter system [3]. Measurement technique is introduced for determination of harmonic/ inter-harmonics of time varying frequencies [4]. Different passive models are introduced for motor control [5-11]. A critical impedance based method has been developed for identifying harmonic sources [5]. Passivity-based control system is developed for Euler-Lagrange systems [7]. A lot of experimental investigations have been done on the passivity based induction motor control [8-10]. Modeling of adjustable speed drives for power system harmonic analysis [11] has been done. For this purpose different electric quantities under sinusoidal, non sinusoidal, balanced or unbalanced conditions [12] have been defined. For motor control and fault diagnosis, analysis of air-gap flux, current and vibration signals as a function of the combination of static and dynamic air-gap eccentricity in 3-phase induction motors [13] has been done. For fault diagnosis, use of motor current signature analysis (MCSA) [14] [15] has become very popular. Condition monitoring of induction motor is also being done by MCSA [15]. A detail review of induction motor signature analysis as a medium for Fault detection has been provided in [16]. Some online diagnosis of induction motors using MCSA [14, 17, 18] have been introduced. Wavelet and PSD are being used as fault detection technique [19]. Induction motor stator faults diagnosis has been achieved by a current Concordia pattern-based fuzzy decision system [20]. Fault detection and diagnosis in an induction machine drive is done using pattern recognition approach based on Concordia stator mean current vector [21]. Monitoring and diagnosis of induction motor electrical faults using a current Park's vector pattern approach [22-25]. Current Concordia based assessment of single phasing of an induction motor by feature pattern extraction method and radar analysis [26] has been introduced. Assessment of crawling of an induction motor has also been done by stator current Concordia analysis [27].

In this paper, stator current of induction motor at single phasing are assessed by area based approach. Some specific reference signals having sinusoidal nature and different frequencies have been defined. Areas formed by the real power system data with the reference signal have been calculated. Mathematical relations have been established between those areas with amplitude and phase angles of voltage and current wave forms wherefrom contributions of fundamental waveform and harmonic components have been assessed separately. Active power, reactive power and total harmonic distortion factors have been measured. Distortion factors of obtained from normal and fault condition have been compared.

## **2. AREA BASED APPROACH FOR HARMONIC ASSESSMENT**

### **2.1 Power in Terms of Area**

Consider  $v(t)$  be the voltage and  $i(t)$  be the current in a power system. In presence of harmonics, these waveforms can be written as follows

$$v(t) = \sum_{n=1,2,3,\dots,n_V} V_n \sin(n\omega t - \phi_n) \quad (1)$$

$$i(t) = \sum_{n=1,2,3,\dots,n_I} I_n \sin(n\omega t - \theta_n) \quad (2)$$

where,  $n$  denotes order of harmonics,  $n_V$  and  $n_I$  be the highest order of harmonics present in voltage waveform and current waveform respectively. Fundamental frequency ( $\omega$ ) is assumed to be constant in single complete cycle. If these waveforms are plotted in voltage – current plane (v-i plane), covered area can be written as follows

$$A_{TOTAL}^{v-i} = \int v di = k_1 \sum_n n V_n I_n \sin \phi_n \quad (3)$$

where,  $\phi_n = \phi_n - \theta_n$  and  $k_1 = 0.1\omega$

From (1),

$$\sum_{n=1,2,3,\dots} \frac{1}{2} n V_n I_n \sin(\phi_n) = \sum_n n Q_n = \frac{1}{2} K_1 A_{TOTAL}^{v-i} \quad (4)$$

where,  $K_1 = \frac{1}{k_1}$

If voltage and current are plotted in vi-t plane, covered area can be written as follows

$$A_{TOTAL}^{vi-t} = \int v i dt = \int \sum_{n=1,2,3,\dots} V_n \sin(n\omega t - \phi_n) \sum_{n=1,2,3,\dots} I_n \sin(n\omega t - \theta_n) dt = k_2 \sum_n V_n I_n \cos \phi_n \quad (5)$$

Where,  $\phi_n = \phi_n - \theta_n$  and  $k_2$  is a constant

From (5),

$$\sum_{n=1,2,3,\dots} \frac{1}{2} V_n I_n \cos(\phi_n) = \sum_n P_n = \frac{1}{2} K_2 A_{TOTAL}^{vi-t} \quad (6)$$

where,  $K_2 = \frac{1}{k_2}$

Fundamental and harmonic components of same order, which are present in both voltage and current waveform, contribute to the active and reactive power (4, 6). Equation (3) shows that area in v-i plane contains the information of sine of angle between the voltage and current wave form whereas, (5) shows that the area under the curve formed by vi in vi-t plane gives the information of cosine of the phase angle difference between voltage and current waveform. In the proposed scheme, a reference signal (M) of sinusoidal nature has been defined and voltage (v) and current (i) signals have been plotted with the reference signal (M) in v-M, vM-t, i-M and iM-t planes respectively, areas have been calculated wherefrom amplitude and phase angles of voltage and current signals of fundamental as well as harmonic components have been isolated.

## 2.2 Reference Signals

For assessment of distortion factor, contribution of fundamental and harmonic components should be isolated. To extract powers contributed only by fundamental waveform a reference signal of unity amplitude is defined with the following properties

$$M_1(t) = \sin \omega_1 t \quad (7)$$

Here  $\omega_1$  is frequency of the reference signal. The frequency of the reference signal  $\omega_1$  should be equal to the fundamental frequency  $\omega$  of the real signal collected from the power system, which is initially an unknown quantity. To select the value of  $\omega_1$ , first take,  $\omega_1 = 50 \pm \Delta\omega$ , where,  $\Delta\omega$  is a small incremental value. If this signal is plotted against any one of the signals which have been derived from the real power system data, then the starting and ending point will not meet each other. There will be a finite gap between these two points as shown in Fig. 1.

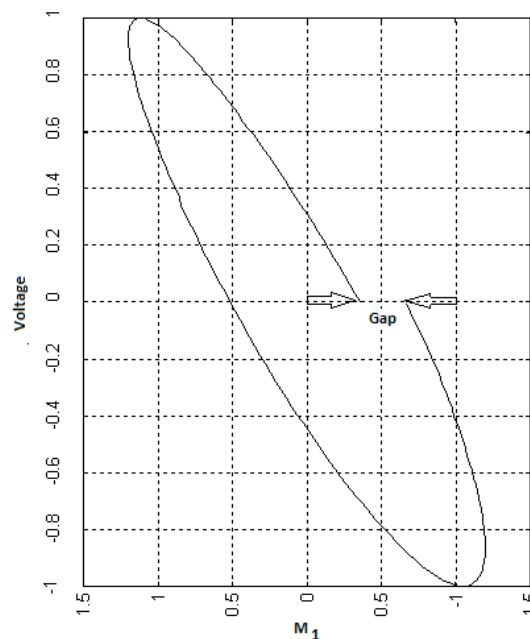


Fig. 1 Curves formed by  $M_1$  and voltage having unequal fundamental frequency.

If  $\omega_1$  is changed gradually by changing  $\Delta\omega$  in such a way that the starting and ending points come closer to each other then, it can be said that  $\omega_1$  is approaching towards  $\omega$ . When starting and ending points come at minimum distance from each other, ideally zero, then it can be said that  $\omega_1 = \omega$ . Then the reference signal will become

$$M_1(t) = \sin \omega t \quad (8)$$

It should be noted that the reference signal does not consist of harmonic components which may be in the real data. As a result of this, area formed by reference and real signals will not depend on any other frequency except fundamental.

Real voltage and current signals are plotted with the reference signal (8) and the areas covered by the curve in one complete cycle have been calculated wherefrom amplitude and phase angle of fundamental component, have been derived for assessment of contribution of

fundamental component in distortion. Similarly, reference signal for assessment of harmonic component is defined as

$$M_n(t) = \sin n\omega t \quad (9)$$

### 2.3 Fundamental Component in Phase R

Covered by vR and M1 waveforms in vR –M1 plane can be written as follows

$$A_{R1}^{v-M} = k_1 V_{R1} \sin \phi_{R1} \text{ or, } V_{R1} \sin \phi_{R1} = K_1 A_{R1}^{v-M} \quad (10)$$

Covered area in V<sub>R</sub> M<sub>1</sub> -t can be written as follows:

$$A_{R1}^{vM-t} = k_2 V_{R1} \cos \phi_{R1} \text{ or, } V_{R1} \cos \phi_{R1} = K_2 A_{R1}^{vM-t} \quad (11)$$

From (10) and (11),

$$V_{R1} = \sqrt{(K_1 A_{R1}^{v-M})^2 + (K_2 A_{R1}^{vM-t})^2} \quad (12)$$

$$\phi_{R1} = \tan^{-1} \left( \frac{K_1 A_{R1}^{v-M}}{K_2 A_{R1}^{vM-t}} \right) \quad (13)$$

Covered area in iR – M1 plane can be written as follows:

$$A_{R1}^{i-M} = k_1 I_{R1} \sin \theta_{R1} \\ \text{or, } I_{R1} \sin \theta_{R1} = K_1 A_{R1}^{i-M} \quad (14)$$

Covered area in iR M1 -t plane can be written as follows:

$$A_{R1}^{iM-t} = k_2 I_{R1} \cos \theta_{R1} \\ \text{or, } I_{R1} \cos \theta_{R1} = K_2 A_{R1}^{iM-t} \quad (15)$$

From (14) and (15),

$$I_{R1} = \sqrt{(K_1 A_{R1}^{i-M})^2 + (K_2 A_{R1}^{iM-t})^2} \quad (16)$$

$$\theta_{R1} = \tan^{-1} \left( \frac{K_1 A_{R1}^{i-M}}{K_2 A_{R1}^{iM-t}} \right) \quad (17)$$

### 2.4 Harmonic Component in Phase R

Covered area by curve v<sub>R</sub> and M<sub>n</sub> waveforms in v<sub>R</sub> –M<sub>n</sub> can be written as follows:

$$A_{Rn}^{v-M} = k_1 V_{Rn} \sin \phi_{Rn} \\ \text{or, } V_{Rn} \sin \phi_{Rn} = K_1 A_{Rn}^{v-M} \quad (18)$$

Covered area in V<sub>R</sub> M<sub>n</sub> -t can be written as follows:

$$A_{Rn}^{vM-t} = k_2 V_{Rn} \cos \phi_{Rn} \\ \text{or, } V_{Rn} \cos \phi_{Rn} = K_2 A_{Rn}^{vM-t} \quad (19)$$

From (18) and (19),

$$V_{Rn} = \sqrt{(K_1 A_{Rn}^{v-M})^2 + (K_2 A_{Rn}^{vM-t})^2} \quad (20)$$

$$\varphi_{Rn} = \tan^{-1} \left( \frac{K_1 A_{Rn}^{v-M}}{K_2 A_{Rn}^{iM-t}} \right) \tag{21}$$

Covered area in  $i_R - M_n$  plane can be written as follows

$$A_{Rn}^{i-M} = k_1 I_{Rn} \sin \theta_{Rn}$$

or,  $I_{Rn} \sin \theta_{Rn} = K_1 A_{Rn}^{i-M}$  (22)

Covered area formed in  $i_R M_n -t$  plane can be written as follows

$$A_{Rn}^{iM-t} = k_2 I_{Rn} \cos \theta_{Rn}$$

or,  $I_{Rn} \cos \theta_{Rn} = K_2 A_{Rn}^{iM-t}$  (23)

From (22) and (23),

$$I_{Rn} = \sqrt{(K_1 A_{Rn}^{i-M})^2 + (K_2 A_{Rn}^{iM-t})^2} \tag{24}$$

$$\theta_{Rn} = \tan^{-1} \left( \frac{K_1 A_{Rn}^{i-M}}{K_2 A_{Rn}^{iM-t}} \right) \tag{25}$$

### 2.5 Harmonic Distortion in Phase R

Harmonics distortion is expressed in terms of total harmonic distortion factor. For phase R, using (12), (16), (20) and (24) distortion factors can be written as follows:

$$THD_{RV} = \frac{\sqrt{\sum_{n=2,3,\dots} (V_{Rn})^2}}{V_{R1}} = \frac{\sqrt{\sum_{n=2,3,4,\dots} \sqrt{(K_1 A_{Rn}^{v-M})^2 + (K_2 A_{Rn}^{vM-t})^2}}}{\sqrt{(K_1 A_{R1}^{v-M})^2 + (K_2 A_{R1}^{vM-t})^2}} \tag{26}$$

$$THD_{RI} = \frac{\sqrt{\sum_{n=2,3,\dots} (I_{Rn})^2}}{I_{R1}} = \frac{\sqrt{\sum_{n=2,3,4,\dots} \sqrt{(K_1 A_{Rn}^{i-M})^2 + (K_2 A_{Rn}^{iM-t})^2}}}{\sqrt{(K_1 A_{R1}^{i-M})^2 + (K_2 A_{R1}^{iM-t})^2}} \tag{27}$$

### 3. EXPERIMENTATION

Block diagram for experimentation is shown in Fig. 2. An induction motor (Ventwell Corporation make, 415 volts, 3 phase, 50 Hz, Squirrel cage, 1 HP, 1425 rpm) is fed by balanced supply and is run at no-load. The line currents are stepped down with current transformers (CT) and voltages are stepped down by potential transformer. They are then sampled and digitized and captured through data acquisition system (DAS). Each phase is connected through separate switch. Suddenly one phase (in Fig. 2 it is phase B) is switched off and single phasing is done. Voltage and current signals of healthy phases are observed for analysis.

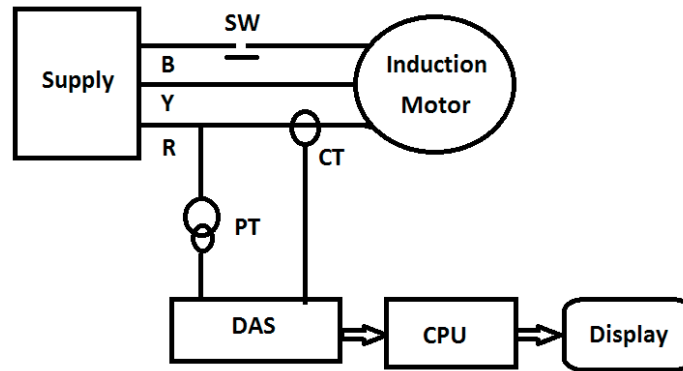


Fig. 2: Block diagram for experimentation.

#### 4. RESULTS

The motor is first run at normal condition and harmonics distortions are measured. Then, three cases single phasing are done by switching off phase R, phase Y and phase B respectively. For each case harmonics distortions of healthy phase are measured using area based approach. The result is compared with the normal value as presented in Table 1. Significant rise of harmonics distortions are observed.

Table 1: Distortions factors at normal and fault condition.

Condition	Current distortion in healthy phase	Voltage distortion in healthy phase
Normal	1.1586	1.1592
Single phasing at phase R	16.4655	8.3450
Single phasing at phase Y	16.4620	8.2550
Single phasing at phase B	16.5685	8.6575

Accuracy increases with the increase of sampling rate of the data acquisition system. The same technique may be extended in Clarke plane which will reduce the computational effort.

#### 5. CONCLUSION

In this approach fundamental frequency ( $\omega$ ) has been assumed constant only for one complete cycle and may change in next cycle, which is more advantageous in real system analysis than other methods where ( $\omega$ ) is to be constant throughout the process. Small error of the method strengthens the acceptability of the above methods. Single phasing has shown significant rise of harmonics distortion that has been detected by area based approach. This is done by capturing voltage and current signals. Some specific reference signals are defined, then, real power system data are plotted with this reference signal and areas are then calculated wherefrom contributions of fundamental waveform and harmonic components in real and reactive powers have been assessed separately. Single phasing is done on induction machine in each phase and harmonic distortion factors are calculated. Significant change of distortion is observed due to single phasing. The application area may be extended in Clarke plane for less computational effort.

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