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ENERGY AND STRUCTURE STATES OF LOW-LYING BANDS IN ^{156}Gd

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ABSTRACT: The experimental results of the literary and electronic nuclear database for ^{156}Gd were summarized and analyzed. Inertial parameters of rotating core were determined using the Harris method. The theoretical values of energy and wave functions were calculated within the framework of a phenomenological model that takes into account Coriolis mixing of state rotational bands. The calculated energy values were compared with existing experimental data, which were in good agreement.

ABSTRAK: Hasil dapatan kajian melalui eksperimen pangkalan data nuklear dan elektronik bagi ^{156}Gd diringkaskan dan dianalisis. Parameter inersia putaran berputar ditentukan menggunakan kaedah Harris. Nilai teori fungsi tenaga dan gelombang dikira dalam kerangka model fenomenologi yang mengambil kira campuran Coriolis pada band putaran keadaan. Nilai pengiraan tenaga dibandingkan dengan data eksperimen memberikan persetujuan yang baik.

KEYWORDS: *nuclei; energy; states; rotational bands; phenomenological model*

1. INTRODUCTION

The gadolinium isotope with a mass A=156 is one of the most studied nuclei. The main reason is the large cross section (n,γ) - reaction in ^{156}Gd , which provides big opportunities for studying the emission spectra in this reaction. Full results on this nucleus are given in references [1,2]. Several other nuclear processes supplement information on the levels and rotation bands in ^{156}Gd . In the reaction of $(\alpha,2n)$ data were obtained on the states of rotational bands with $K^\pi = 0_1^+$ – to $I = 26^+$, $K^\pi = 0_2^+$ – to $I = 14^+$, $K^\pi = 0_3^+$ – to $I = 10^+$, $K^\pi = 0_4^+$ – to $I = 6^+$, $K^\pi = 0_5^+$ – to $I = 4^+$, $K^\pi = 2_1^+$ – to $I = 15^+$ and $K^\pi = 2_2^+$ – to $I = 4^+$.

A wealth of experimental information was obtained as a result of studying the reaction $(n,n'\gamma)$ [3]. Based on the totality of the experimental data, one can think that for ^{156}Gd all or almost all levels were found up to excitation energy of 2 MeV. In ^{156}Gd five rotational bands with $K^\pi = 0^+$ are known. Therefore, systematic study of the properties of these levels is very important to search the corresponding levels in the neighboring nuclei.

^{156}Gd and ^{158}Gd are the first nuclei in which a new type of excitation was discovered – collective states with $I^\pi = 1^+$ [4]. At present, there have been several collective states experimentally observed in ^{156}Gd with $I^\pi = 1^+$. Their excitation energies and reduced probabilities $B(M1)\uparrow$ are determined [5].

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Nonadiabaticity is observed in the energies and especially in the electromagnetic characteristics of the excited states of rotational bands with $K^{\pi}=0^{+}$ and $K^{\pi}=2^{+}$ of these nuclei [6-9]. In the present work, to explain these nonadiabatics, we use a phenomenological model that takes into account Coriolis mixing of low–lying rotational bands [10, 11]. The energy levels and wave functions of the rotational band states are calculated. The nonadiabatics observed in the energies and wave functions of rotational states are discussed.

More details of the phenomenological model used are given in Ref. [10] and this model was also successfully applied to study the mixing of state bands of both positive and negative parity in [11-14].

2. DESCRIPTION OF THE MODEL

According to the phenomenological model of the nucleus [10], the Hamilton operator has the following appearance:

$$H = H_{\text{rot}}(I^2) + H_{K,K'} \tag{1}$$

$$H_{K,K'} = \omega_K \delta_{K,K'} - \omega_{\text{rot}}(I) < K \left| \hat{j}_x \right| K' > \chi(I,K)\delta_{K,K'\pm 1} \tag{2}$$

$\omega_{\text{rot}}(I) = dE_{\text{rot}}(I)/dI$ – angular frequency of rotation of the core; ω_K – the head energies of the rotational bands; $< K \left| \hat{j}_x \right| K' >$ – matrix element of Coriolis interaction between the states of rotational bands; $\chi(I,K)$ – the coefficients are as follows:

$$\chi(I,0) = 1, \qquad \chi(I,1) = \left| 1 - \frac{2}{I(I+1)} \right|^{1/2}.$$

The Eigen wave function of the Hamilton operator (1) has the following appearance:

$$\begin{aligned} |IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2}} \Bigg\{ \sqrt{2}\Psi^I_{\text{gr},K} D^I_{M,0}(\theta) + \\ + \sum_{K'} \frac{\Psi^I_{K',K}}{\sqrt{1+\delta_{K',0}}} \left[D^I_{M,K'}(\theta) b^+_{K'} + (-1)^{I+K} D^I_{M,-K}(\theta) b^+_{-K'} \right] \Bigg\} |0\rangle, \end{aligned} \tag{3}$$

here $D^I_{M,K'}(\theta)$ are generalized spherical functions; $|0\rangle$ is the vacuum for the b^+_K operators (in other words, the ground state of the nucleus in the intrinsic reference system); $\Psi^I_{K,K'}$ are the amplitudes of mixing states of different bands with the same angular momentum I due to the Coriolis interaction.

Eigen energies and wave functions of the rotational states are determined by diagonalization of the Hamiltonian operator (2):

$$H^{\sigma}_{K,K'} \Psi^I_{K,n} = \varepsilon^I_n \Psi^I_{K,n} \tag{4}$$

The total energies of the states can be found by the following formula:

$$E_K(I) = E_{\text{rot}}(I) + \varepsilon_K(I) \tag{5}$$

There are different methods for determining the energy of the core rotational motion of the nucleus, for example, Harris proposed to determine the following two-parameter formula [15]:

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$$\sqrt{I(I+1)} = \Im_0 \omega_{\text{rot}}(I) + \Im_1 \omega_{\text{rot}}^3(I) \tag{6}$$
$$E_{\text{rot}}(I) = \frac{1}{2}\Im_0 \omega_{\text{rot}}^2(I) + \frac{3}{4}\Im_1 \omega_{\text{rot}}^4(I),$$

here $\Im_0$ and $\Im_1$ – inertia parameters of the rotational core.

3. CALCULATION AND RESULTS

According to Bohr–Mottelson and Bengtsson–Frauendorf, at small values of the spin of the nucleus, the rotational energy of the nucleus corresponds to the ground rotational states energy [16,17]. Therefore, the inertial parameters of the core were determined by the Harris method using the experimental energy states of ground band up to spin $I \le 8$.

Table 1: Values of the model parameters used in calculating level energies

$\Im_0$	$\Im_1$	ω_{2_1}	ω_{2_2}	$\langle 0_1 \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$	$\langle 0_2 \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$	$\langle 0_3 \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$	$\langle 0_4 \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$	$\langle 0_5 \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$	$\langle 2_1^+ \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$	$\langle 2_2^+ \lvert \hat{j}_x \rvert 1_\nu^+ \rangle$
33.3	169.47	1.076	1.739	0.416	0.45	0.25	0.91	0.42	0.40	0.1

NOTE: $\Im_0\,(\hbar^2/\text{MeV})$, $\Im_1\,(\hbar^4/\text{MeV}^3)$ are the inertial parameters of the rotating core (Harris parameters), $\omega_{2_{1,2}}$ (MeV) are energies of the $K^\pi = 2^+_{1,2}$ band heads, and $< K \left| \hat{j}_x \right| K' >$ (MeV) are the matrix elements of the Coriolis interaction.

Figure 1 shows the values of the moment of inertia obtained by the Harris method, which are compared with the effective values obtained using the following formula

$$\Im_{\text{эфф}}(I) = \frac{\sqrt{I(I+1)}}{\omega_{\text{эфф}}(I)}, \text{ here } \omega_{\text{эфф}}(I) = \frac{E^{\text{экс}}(I+1) - E^{\text{экс}}(I-1)}{2} \tag{7}$$

It can be seen from the comparison that there is a difference between them at high values of angular rotation frequency. I.e. at the large values of the total angular momentum I, deviations from the adiabatic theory are observed in the energies of the ground band. Such nonadiabaticity is more pronounced in the energies of the vibration bands with $K^\pi = 0^+$ and $K^\pi = 2^+$. To determine the causes of nonadiabaticity in the energies of the states of low-lying rotational bands, we use the phenomenological model presented above.

The energy of the rotational motion of the nucleus, for the bands included in the basis of the Hamiltonian operator (1), is considered the same and its values are determined by formula (6). The rotational motion at low frequencies is not related to its internal structure, i.e., the rotation of the nucleus does not affect the Hamiltonian basic model (1). Ground, five beta band ($K^\pi = 0^+$), two gamma $K^\pi = 2^+$ and fifteen $K^\pi = 1_\nu^+$ rotational bands are introduced in the Hamiltonian model. The Schrödinger equation of size (23X23) was solved.

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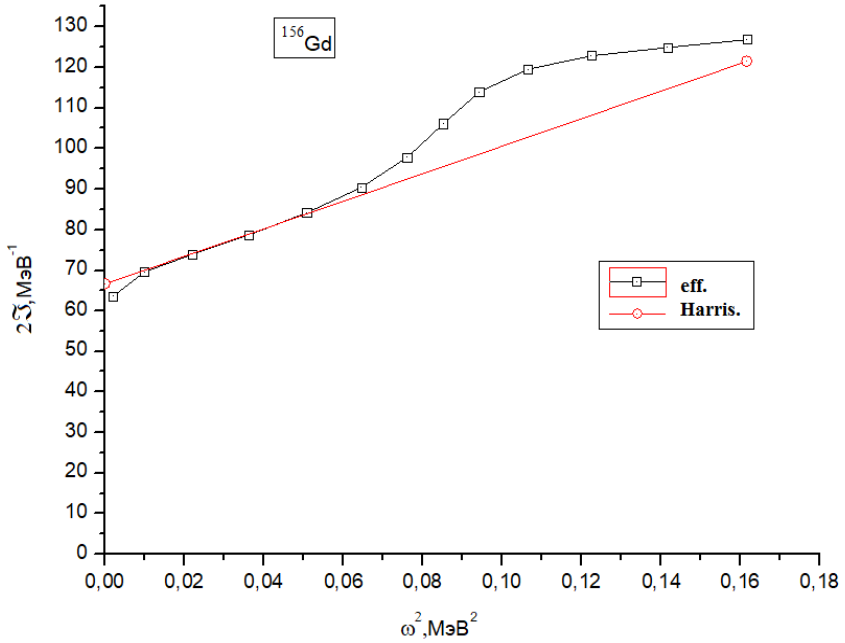

Fig. 1: Comparison of the values of the moments of inertia determined by the effective and Harris method.

The parameters of the model are determined in the following sequence:

1. For the head energies ω_K of the ground $(K^\pi = 0_1^+)$ and $K^\pi = 0_n^+$ bands are using the experimental values $E_{K_n}^{\text{exp.}}(I=0)$ because the head energies of these bands $I=0$ (in this case $\omega_{\text{rot}}(0)=0$) and these states are not mixed by the Coriolis forces;
2. The head energies ω_{1_ν} of the $K^\pi = 1_\nu^+$ bands found using their experimental values $\omega_{1_\nu} = E_{1_\nu}^{\text{эксп}}(I=1) - E_{\text{rot}}(I=1)$;
3. The head energies ω_{2_1}, ω_{2_2} of the $K^\pi = 2^+$ bands and $< 2_{1,2} \left| \hat{j}_x \right| 1_\nu^+ >$ are free parameters, their values were obtained by the least squares method, from the condition of better agreement of the theoretical energy of states with odd spins $K^\pi = 2_1^+$ and $K^\pi = 2_2^+$ rotational bands with the experimental data;
4. The parameters $< 0_n \left| \hat{j}_x \right| 1_\nu^+ >$ describing the Coriolis interaction of the states of the $K^\pi = 0_n$, and $K^\pi = 1_\nu^+$ bands are determined from the good agreement of the calculated theoretical energies of the states with even spins of the ground (0_1), $K^\pi = 0_2^+, 0_3^+, 0_4^+, 0_5^+$ and $K^\pi = 2_{1,2}^+$ of bands with experimental values.

The selected numerical values of the model parameters are given in Table 1. Figure 2 shows the calculated energy values in the framework of the model which are compared with experimental results.

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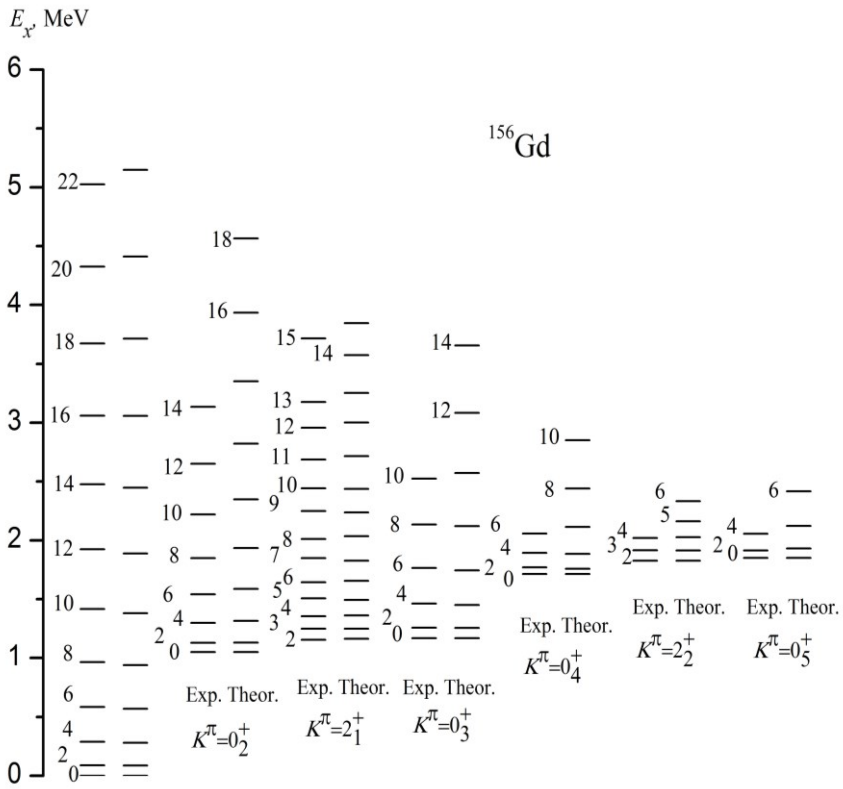

Fig. 2: Comparison of theoretical and experimental energy values.

The results of the comparison show that at small values of spin, the compatibility between them is good, and the difference increases with increasing spin. This difference suggests that additional effects must be taken into account when studying the properties of high-spin states. For example, it may be necessary to take into account the effect of the rotational motion of the nucleus on the intrinsic energy when the rotation of the nucleus is high.

The wave functions of the states $K^\pi = 0_2^+$, $K^\pi = 0_3^+$, $K^\pi = 0_4^+$ and $K^\pi = 2_1^+$ bands obtained by solving the Schrödinger equation (2) are present in tables 2, 3, 4 and 5 respectively. The Hamiltonian basis includes 15 bands with $K^\pi = 1_\nu^+$, but the table shows one component of $K^\pi = 1_1^+$. The components $\psi^I_{1_\nu K}$ of the other $K^\pi = 1_\nu^+$ bands are determined using the following formula

$$\psi^I_{1_\nu K} = \psi^I_{1_1 K} \frac{\omega_{1_1} - \omega_K}{\omega_{1_\nu} - \omega_K} . \tag{8}$$

In Table 2, it can be seen that in the band with $K^\pi = 0_2^+$, the other components $\psi^I_{K0_2}$ of the mixed bands increase with increasing spin I. This picture can be observed in other rotation bands. This result, which is due to Coriolis interaction, is the main cause of the nonadiabatics manifested in the energy and electromagnetic characteristics of the exited states.

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Table 2: The amplitude of the mixing of the base states $\psi^{I}_{K0^{+}_{2}}$ for $K^{\pi}=0^{+}_{2}$ band

					K			
I	**gr**	0_2^+	0_3^+	0_4^+	0_5^+	1_1^+	2_1^+	2_2^+
2	-0.0052	0.9927	0.0261	0.0176	0.0068	0.0279	0.0927	0.0011
4	-0.0183	0.9516	0.0808	0.0599	0.0231	0.0555	0.2534	0.0043
6	-0.0385	0.8926	0.1404	0.1186	0.0460	0.0829	0.3496	0.0087
8	-0.0638	0.8353	0.1872	0.1815	0.0708	0.1068	0.3890	0.0135
10	-0.0922	0.7835	0.2177	0.2395	0.0942	0.1260	0.3988	0.0179
12	-0.1222	0.7378	0.2352	0.2884	0.1143	0.1404	0.3951	0.0216
14	-0.1529	0.6983	0.2440	0.3274	0.1306	0.1506	0.3859	0.0246
16	-0.1836	0.6645	0.2476	0.3577	0.1436	0.1576	0.3750	0.0270
18	-0.2141	0.6355	0.2482	0.3808	0.1537	0.1621	0.3639	0.0288
20	-0.2444	0.6106	0.2469	0.3984	0.1615	0.1649	0.3533	0.0302

Table 3: The amplitude of the mixing of the base states $\psi^{I}_{K0^{+}_{3}}$ for $K^{\pi}=0^{+}_{3}$ band

					K			
I	**gr**	0_2^+	0_3^+	0_4^+	0_5^+	1_1^+	2_1^+	2_2^+
2	-0.0025	-0.0275	0.9986	0.0118	0.0044	0.0161	-0.0183	0.0007
4	-0.0072	-0.0794	0.9923	0.0330	0.0122	0.0260	-0.0618	0.0023
6	-0.0120	-0.1374	0.9795	0.0549	0.0204	0.0319	-0.1110	0.0040
8	-0.0162	-0.1895	0.9625	0.0732	0.0272	0.0347	-0.1562	0.0054
10	-0.0194	-0.2321	0.9446	0.0872	0.0324	0.0356	-0.1938	0.0064
12	-0.0218	-0.2655	0.9278	0.0974	0.0362	0.0356	-0.2238	0.0072
14	-0.0236	-0.2916	0.9129	0.1050	0.0391	0.0350	-0.2476	0.0078
16	-0.0250	-0.3121	0.9001	0.1107	0.0412	0.0342	-0.2666	0.0082
18	-0.0260	-0.3285	0.8890	0.1151	0.0429	0.0333	-0.2819	0.0086
20	-0.0269	-0.3419	0.8795	0.1186	0.0442	0.0324	-0.2944	0.0088

Table 4: The amplitude of the mixing of the base states $\psi^{I}_{K0^{+}_{4}}$ for $K^{\pi}=0^{+}_{4}$ band

					K			
I	**gr**	0_2^+	0_3^+	0_4^+	0_5^+	1_1^+	2_1^+	2_2^+
2	-0.0119	-0.0347	-0.0239	0.9486	0.1105	0.1673	-0.0186	0.0396
4	-0.0302	-0.0952	-0.0675	0.8892	0.1910	0.2087	-0.0596	0.0546
6	-0.0471	-0.1625	-0.1199	0.8468	0.2258	0.2115	-0.1055	0.0569
8	-0.0604	-0.2265	-0.1744	0.8123	0.2393	0.2019	-0.1497	0.0566
10	-0.0698	-0.2812	-0.2257	0.7809	0.2427	0.1886	-0.1881	0.0554
12	-0.0760	-0.3249	-0.2708	0.7521	0.2414	0.1748	-0.2192	0.0539
14	-0.0798	-0.3584	-0.3090	0.7262	0.2379	0.1620	-0.2436	0.0524
16	-0.0822	-0.3838	-0.3408	0.7035	0.2337	0.1507	-0.2623	0.0510
18	-0.0836	-0.4030	-0.3671	0.6839	0.2294	0.1409	-0.2767	0.0497
20	-0.0844	-0.4177	-0.3890	0.6670	0.2254	0.1324	-0.2879	0.0486

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Table 5: The amplitude of the mixing of the base states $\psi^{I}_{K2_1^+}$ for $K^{\pi}=2_1^+$ band

		K						
I	**gr**	$\mathbf{0_2^+}$	$\mathbf{0_3^+}$	$\mathbf{0_4^+}$	$\mathbf{0_5^+}$	$\mathbf{1_1^+}$	$\mathbf{2_1^+}$	$\mathbf{2_2^+}$
2	0.0021	0.0954	-0.0144	-0.0076	-0.0029	-0.0116	-0.9949	-0.0005
3	-	-	-	-	-	-0.0209	-0.9987	-0.0013
4	0.0052	0.2667	-0.0352	-0.0191	-0.0073	-0.0168	-0.9620	-0.0014
5	-	-	-	-	-	-0.0328	-0.9967	-0.0032
6	0.0070	0.3819	-0.0460	-0.0253	-0.0096	-0.0164	-0.9217	-0.0019
7	-	-	-	-	-	-0.0424	-0.9945	-0.0053
8	0.0078	0.4465	-0.0510	-0.0283	-0.0108	-0.0149	-0.8920	-0.0021
9	-	-	-	-	-	-0.0504	-0.9921	-0.0076
10	0.0082	0.4841	-0.0536	-0.0298	-0.0114	-0.0136	-0.8721	-0.0022
11	-	-	-	-	-	-0.0572	-0.9898	-0.0097
12	0.0085	0.5078	-0.0551	-0.0307	-0.0117	-0.0124	-0.8585	-0.0023
13	-	-	-	-	-	-0.0630	-0.9875	-0.0119
14	0.0087	0.5237	-0.0560	-0.0313	-0.0119	-0.0115	-0.8489	-0.0023
15	-	-	-	-	-	-0.0682	-0.9854	-0.0139
16	0.0088	0.5350	-0.0566	-0.0317	-0.0121	-0.0108	-0.8418	-0.0024
17	-	-	-	-	-	-0.0727	-0.9833	-0.0158
18	0.0089	0.5435	-0.0571	-0.0320	-0.0122	-0.0102	-0.8363	-0.0024
19	-	-	-	-	-	-0.0768	-0.9812	-0.0177
20	0.0089	0.5500	-0.0574	-0.0322	-0.0123	-0.0097	-0.8321	-0.0024
2	0.0021	0.0954	-0.0144	-0.0076	-0.0029	-0.0116	-0.9949	-0.0005

4. CONCLUSION

Theoretical calculations were carried out for ^{156}Gd nucleus in the framework of the phenomenological model taking into account Coriolis mixing of low–lying rotation bands with positive parity.

Nonadiabaticities observed in the energies is explained by the Coriolis mixing of low excited rotational states. To describe all the adiabatic rotational bands, the same moments of inertia have been used. The energy spectra of the positive parity states have been calculated. The results of calculation of energy spectra for ground $(K^{\pi}=0_1^+)$, $K^{\pi}=0_2^+$, $K^{\pi}=0_3^+$, $K^{\pi}=0_4^+$, $K^{\pi}=0_5^+$, $K^{\pi}=2_1^+$, and $K^{\pi}=2_2^+$ band states is compared with the existing experimental data correspondingly and their compatibility is given. The mixing effects of the lower bands have been shown to be significant even at small spins.

In the high spin states of ground band, the difference between the theory and experiment was observed. This may be due to the fact that for large values of the angular frequency of rotation of the nucleus, it is necessary to take into account the effects of rotation on intrinsic energy.

The wave function of states of the rotational bands is calculated. The regularities of the change in the state components of the mixing bands are studied depending on the total angular momentum.

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