Using Semi-Analytic Method to Decreasing Dangers of Lead

L. N. M. Tawfiq and M. N. M. Tawfiq
Department of Mathematics, College of Education Ibn Al - Haitham , University of Baghdad
Received in : 7 November 2010
Accepted in : 8 February 2011

Abstract

The aim of this paper is to present a method for solving of system of first order initial value problems of ordinary differential equation by a semi-analytic technique with constructing polynomial solutions for decreasing dangers of lead. The original problem is concerned using two-point osculatory interpolation with the fit equals numbers of derivatives at the end points of an interval [0 , 1].
Key word : Osculator interpolation , Initial value problem

Introduction

Lead is a metal found naturally in the earth's crust. In nature , it is found more often in chemical compounds than as a pure metal. When released into the air, it may travel long distances before settling to the ground , where it can contaminate water and soil .

Lead has been used extensively throughout human history because it is easy both to extract and to work with. Where used in some Cosmetics and hair dyes, paint houses , Plumbing and solder , Gutters , window glazing , Leaded gasoline , crayons , Printers (ink) , batteries , plastics and used in the manufacturing of products others . [1]

Where the deficiencies in some mineral nutrients, specifically calcium, iron and zinc, where involvement in building bones, may increase the amount of lead absorbed . As a result, Children, Pregnant Women, old Women and old man are more efficient at absorbing lead, than adult's because they need more calcium. Additionally, the kidneys and the intestine, less able to eliminate lead .

Lead absorption rates vary ; the gastrointestinal tracts of adults typically absorb 10 - 15 percent of ingested lead , while those of pregnant women and children can absorb up to 50 percent .

Lead poisoning occurs when there are adverse health effects due from lead in the body .Some of the more prominent symptoms of lead poisoning include headaches, irritability, abdominal pain , vomiting , anemia (general weakness) , weight loss, poor attention span, noticeable learning difficulty, Nervous system and kidney damage. However, at very low exposure levels, lead may not produce specific symptoms, but can produce subtle adverse effects on children's development .

There is a medical test to show whether I've been exposed to lead through a simple blood test, there are treatments to remove lead from the body although medication that used CAedta intravenously must have the attention into treatment because of probably the metal exciding ZINC from body.

Therefore, suggesting treatment application CaNa2 EDTA from mouth way over a period of four weeks , if high levels of lead in blood between 90-100 mg / de , treatment application dimercaprol, also known as BAL the dose 500 and 1500 CAedta where inject in the time self ,but all cured types used at case ,lead poisoning where thrown metal into out of ,but there metals desirable in body may thrown therefore the safeguard best from the remedy

Lead in the human body : A case study

Rabinowitz , Wetherill and Kopple in [2] made a carefully controlled study of the lead intake and exertion of a healthy volunteer in an industrial urban setting . The data from this study were used to estimate the rate constants for the compartment model (1):

(Blood) $y'_1 = -(k_{01} + k_{21} + k_{31}) y_1 + k_{12} y_2 + k_{13} y_3 + I_1$

(Tissue) $y'_2 = k_{21} y_1 - (k_{02} + k_{12}) y_2$ ...... (1)

(Bone) $y'_3 = k_{31}y_1 - k_{13} y_3$

Lead is measured in micrograms and time in days. For example, the rate term 49.3 in system (2) below is the ingestion rate I_1 of lead in micrograms per day, while the coefficient 0.0361 $(\text{day})^{-1}$ in the first rate equation of (2) is the sum of the three compartment transfer coefficients k_{02} , k_{21} and k_{31} of lead from the blood into, respectively, the excretory system, tissue and bones. The full IVP is given by :

$$y'_1 = -0.0361\, y_1 + 0.0124\, y_2 + 0.000035\, y_3 + 49.3 \;, \qquad y_1(0) = 0$$

$$y'_2 = 0.0111\, y_1 + 0.0286\, y_2 \;, \qquad y_2(0) = 0 \quad \ldots(2)$$

$$y'_3 = 0.0039\, y_1 - 0.00035\, y_3 \;, \qquad y_3(0) = 0$$

Assumed that initially there is no lead in the compartments .

Now, we solve system (2) using semi-analytic method ,i.e., use two-point osculatory interpolation [3]. Essentially this is a generalization of interpolation using Taylor polynomials and for that reason osculatory interpolation is sometimes referred to as two-point Taylor interpolation. The idea is to approximate a function y(x) by a polynomial P(x) in which values of y(x) and any number of its derivatives at given points are fitted by the corresponding function values and derivatives of P(x) .

In this paper, we are particularly concerned with fitting function values and derivatives at the two end points of a finite interval, say [0 , 1] ,wherein a useful and succinct way of writing osculatory interpolant $P_{2n+1}(x)$ of degree 2n + 1 was given for example by Phillips [4] as :

$$P_{2n+1}(x) = \sum_{j=0}^{n} \{ y^{(j)}(0)\, q_j(x) + (-1)^j\, y^{(j)}(1)\, q_j(1-x) \} \qquad \ldots\ldots\ldots\ldots(3)$$

$$q_j(x) = (x^j / j!)(1-x)^{n+1} \sum_{s=0}^{n-j} \binom{n+s}{s} x^s = Q_j(x) / j! \qquad \ldots\ldots\ldots\ldots(4)$$

so that (3) with (4) satisfies :

$$y^{(j)}(0) = P_{2n+1}^{(j)}(0) \;,\quad y^{(j)}(1) = P_{2n+1}^{(j)}(1) \;,\quad j = 0, 1, 2,\ldots, n .$$

implying that $P_{2n+1}(x)$ agrees with the appropriately truncated Taylor series for y(x) about x = 0 and x = 1. The error on [0, 1] is given by :

$$R_{2n+1} = y(x)-P_{2n+1}(x) = \frac{(-1)^{n+1} x^{(n+1)} (1-x)^{n+1} y^{(2n+2)}(\varepsilon)}{(2n+2)!}$$ where $\varepsilon \in (0, 1)$ and $\mathbf{y}^{(2n+2)}$ is assumed to be continuous.

Finally we observe that (3) can be written directly in terms of the Taylor coefficients a_i and b_i about x = 0 and x = 1 respectively, as :

$$P_{2n+1}(x) = \sum_{j=0}^{n} \{ a_j\, Q_j(x) + (-1)^j\, b_j\, Q_j(1-x) \} \qquad \ldots..\ (5)$$

Now, the simple idea behind the use of two-point polynomials is to replace y(x) in problem by a P_{2n+1} (equation(3) or (5)) which enables any unknown derivatives of y(x) to be computed . The first step therefore is to construct the P_{2n+1} . To do this we need the Taylor coefficients of $y_1(x)$ and $y_2(x)$ respectively about x = 0 :

$$y_1 = a_0 + a_1 x + \sum_{i=2}^{\infty} a_i x^i \qquad \ldots\ldots\ldots \qquad (6a)$$

$$y_2 = b_0 + b_1 x + \sum_{i=2}^{\infty} b_i x^i \qquad \ldots\ldots\ldots \qquad (6b)$$

where $y_1(0) = a_0$, $y_1'(0) = a_1$, … , $y_1^{(i)}(0) / i! = a_i$, $i = 2, 3, \ldots\ldots$
and $y_2(0) = b_0$, $y_2'(0) = b_1$, … , $y_2^{(i)}(0) / i! = b_i$, $i = 2, 3, \ldots\ldots$

then insert the series forms (6a) and (6b) respectively into (2) and equate coefficients of powers of x .
Also ,we need Taylor coefficients of $y_1(x)$ and $y_2(x)$ about $x = 1$, respectively

$$y_1 = c_0 + c_1 (x-1) + \sum_{i=2}^{\infty} c_i (x-1)^i \qquad \ldots\ldots\ldots \qquad (7a)$$

$$y_2 = d_0 + d_1 (x-1) + \sum_{i=2}^{\infty} d_i (x-1)^i \qquad \ldots\ldots\ldots \qquad (7b)$$

where $y_1(1) = c_0$, $y_1'(1) = c_1$, … , $y_1^{(i)}(1) / i! = c_i$, $i = 2, 3, \ldots\ldots$
and $y_2(1) = d_0$, $y_2'(1) = d_1$, … , $y_2^{(i)}(1) / i! = d_i$, $i = 2, 3, \ldots\ldots$
then insert the series forms (7a) and (7b) respectively into (2) and equate coefficients of powers of $(x - 1)$.

The resulting system of equations can be solved using MATLAB version 7.9 to obtain a_i , b_i , c_i and d_i for all $i \geq 2$, we see that c_is and d_is coefficients depend on indicated unknowns c_0 and d_0 .
The algebraic manipulations needed for this process .We are now in a position to construct a $P_{2n+1}(x)$ and $\widetilde{P}_{2n+1}(x)$ from (6) and (7) of the form (3) by the following:

$$P_{2n+1}(x) = \sum_{i=0}^{n} \{ a_i Q_i(x) + (-1)^i c_i Q_i(1-x) \} \qquad \ldots\ldots\ldots\ldots \qquad (8a)$$

and

$$\widetilde{P}_{2n+1}(x) = \sum_{i=0}^{n} \{ b_i Q_i(x) + (-1)^i d_i Q_i(1-x) \} \qquad \ldots\ldots\ldots\ldots \qquad (8b)$$

Where $Q_i(x)$ defined in (4) ,
We see that (8) have only two unknowns c_0 and d_0 .
Now, integrate equation (2) to obtain :

$$c_0 - a_0 = \int_0^1 f_1(x, y_1, y_2) \, dx \qquad \ldots\ldots\ldots\ldots \qquad (9a)$$

$$d_0 - b_0 = \int_0^1 f_2(x, y_1, y_2) \, dx \qquad \ldots\ldots\ldots\ldots \qquad (9b)$$

use P_{2n+1} and $\widetilde{P}_{2n+1}$ as a replacement of y_1 and y_2 respectively in (9) .

Since we have only the two unknowns c_0 and d_0 to compute for any n we only need to generate two equations from this procedure as two equations are already supplied by (9) and initial condition. Then solve this system of algebraic equations using MATLAB version 7.9 to obtain c_0 and d_0 ,so insert it into (8) thus (8) represent the solution of (2) .

Extensive computations have shown that this generally provides a more accurate polynomial representation for a given n .
Now from equations (3) and (4) we have :

$$P_9 = 0.00000000000000137t^9 - 0.00000000000028t^8 + 0.00000000005t^7 - 0.00000000802t^6 + 0.00000109t^5 - 0.00012516007t^4 + 0.0118401057t^3 - 0.889865000005t^2 + 49.3t$$

$$\widetilde{P}_9 = -0.0000000000000009t^9 + 0.00000000000019t^8 - 0.000000000035t^7 + 0.000000005387t^6 - 0.0000007071t^5 + 0.0000750482t^4 - 0.0059009635t^3 + 0.273615t^2$$

$$T_9 = -0.00000000000000012t^9 + 0.000000000000025t^8 - 0.0000000000045t^7 + 0.000000000709t^6 - 0.00000009771t^5 + 0.0000115542t^4 - 0.0011579461t^3 + 0.096135t^2$$

Table (1) give the results for n = 4 to different nodes in the domain .

Figure (2) illustrate buildup of lead in the bones , tissue , blood using semi-analytic solution . Also , A comparison between different methods are given in table (2) to illustrate the accuracy of suggested method .

With system (2) we can study the effect of changing the input rate I_1 = 49.3 micrograms / day of lead into the bloodstream, or the effect of a medication that increases the diffusion coefficient $k_{13} = 0.000035 \ (\text{day})^{-1}$ of lead out of the bones. Figure (2) displays the buildup of lead in the body compartments over a period of 800 days. Lead levels in the bloodstream and the issue appear to have nearly reached steady state after the first 200 days, but the lead level in the bones is far from a steady state. The transfer coefficients $k_{13} = 0.000035 \ (\text{day})^{-1}$ of lead for bones back into the blood stream is so small that the skeleton acts like a storage reservoir for lead .

Now, suppose that after 400 days the subject is placed in a completely lead-free environment (i.e., the term 49.3 in system (2) becomes 0 for t > 400). Figure (3) shows that the lead levels in the blood and tissue plunge dramatically, but the amount of lead in the bones, does not seem to drop very much, at least not in the next 400 days .

Another way to remove lead from the bones is to administer and anti lead medication that increases the rate at which lead leaves the skeletal system In particular, suppose that the rate coefficient k_{13} = 0.000035 increases by an order of magnitude to 0.00035. Figure (4) shows the very small effect this change has if the medication is administered from the 400th day onward. However, if the subject moves to a lead free environment and takes the medication from the 400th day onward, then there is a noticeable drop in the skeletal lead levels (Figure 5). Let us see what happens when a massive dose is given, so large that k_{13} increases from 0.000035 to 0.035. Suppose from the 400th day on that the heavy dose is given and the subject is in a lead free environment. The lead exists the bones very quickly. Following a slight rise after the 400th day, lead exists the blood and tissue as well (Figure 6) of course that much medication is probably more harmful than the lead .

Why might blood lead levels be higher in women after menopause ?

After menopause, women often develop osteoporosis , which is a progressive and serious loss of bone mass . As a result of loss of bone mass, lead stored in bone may be released into the blood .

What is lead "poisoning" compared to lead "exposure" ?

Lead exposure refers to the entry of lead into the body, through ingestion, inhalation, the skin or the placenta . Lead poisoning occurs when there are adverse health effects due to lead in the body .

Recommendations

After above study ,we introduce some recommendations and advices which help to minimize harms of lead in society and we get some of this information from [5] .

What role has the government played in reducing lead poisoning?

1. Using Benzene free of Lead which is called (Green Benzene)
2. Not renewing licenses for vehicles mechanically expired , with using transportation means that uses electricity , and the expansion in using collective transportation means such as (Mitro) with improving them
3. Surveillance on batteries factories the private as well as the governmental , which conduct the melting of Lead in addition to newspapers and magazines pressing , with transferring them to deserted regions
4. Paying the attention for not to construct new schools near to highways that is crowded with vehicles
5. It was proved through scientific study in the college of agriculture / Ain Shams university , that the root part absorbs the Lead more than the green system , where there are many kinds of vegetables which the root part is eaten , that is planted beside the highways the fact that leaded to the Lead to go into the human as well as the animal's body , therefore the work must aim for vegetables and fruits not to be planted near highways
6. Surveillance on all kinds of paints factories also the imported products with their inspection to make sure that they are free of Lead substance

What role has the family played in protecting children from lead poisoning ?[6]

1. Test your home for paint .if it content of lead or no
2. Hire a person with special training for correcting lead paint problems to remove lead paint from your home, someone who knows how to do this work safely.
3. Home Clean, window sills, and other surfaces weekly.
4. Don't bring lead dust into your home from work .
5. Make sure your children wash their hands before meals .
6. A child who gets enough iron and calcium will absorb less lead .
7. Don't store food or liquid in lead crystal glassware or imported or old pottery.
8. Household water will contain more lead if it has sat for a long time in the pipes
9. .therefore; let it run for 15–30 seconds before drinking it or cooking with it .

References

1. Lewis , J. (1985), Lead poisoning : a historical perspective , EPA Journal.
2. Robert, L. B. and Courtney , S. C. (1996), Differential Equations A Modeling perspective" , United States of America.
3. Girault ,V. and Scott ,L. R. (2002),Hermite Interpolation of Non smooth Functions Preserving Boundary Conditions, Mathematics of Computation, 71(239): 1043-1074 .
4. Phillips , G .M. (1973), Explicit forms for certain Hermite approximations, BIT 13 , 177-180 .
5. New England Journal of Medicine, (2003), April, 17(348): 1517-1526 .
6. Agency for Toxic Substances and Disease Registry. Lead ToxFAQs sheet . August(2007), www.atsdr.cdc.gov/tfacts13.html , viewed November 14 , (2007).

Table (1) : The result of the method for n = 4

	P_9	$\widetilde{P}_9$	T_9
b_{10}	48.42185102755592	48.42185102755592	48.42185102755592
b_{20}	0.2677883829038516	0.2677883829038516	0.2677883829038516
b_{30}	0.09498851115891924	0.09498851115891924	0.09498851115891924
t	P_9	$\widetilde{P}_9$	T_9
0	0	0	0
0.1	4.92111317760058	0.00273025653425228	0.000960193208372153
0.2	9.82449992093787	0.0108975121431540	0.00383615488695559
0.3	14.7102308217005	0.0244666301613681	0.00862097880837015
0.4	19.5783761737962	0.0434026523504553	0.0153077862420084
0.5	24.4290059746451	0.0676707980599557	0.0238897258380576
0.6	29.2621899264675	0.0972364633922961	0.0343599735120251
0.7	34.0779974375664	0.132065220371507	0.0467117323297653
0.8	38.8764976236032	0.172122816115731	0.0609382323930065
0.9	43.6577593088695	0.217375172013507	0.0770327307253737
1	48.4218510275513	0.267788382903812	0.0949885111589079

Table(2) : A comparison between semi-analytic and other methods

t	y_1:RK solution	y_1:ABM solution	P_9 by using Osculatory
0	0	0	0
0.1	4.92111317760100	4.9211131775747	4.92111317760058
0.2	9.82449992093843	9.8244999208060	9.82449992093787
0.3	14.7102308217013	14.710230821422	14.7102308217005
0.4	19.5783761737973	19.578376173370	19.5783761737962
0.5	24.4290059746466	24.429005974074	24.4290059746451
0.6	29.2621899264696	29.262189925928	29.2621899264675
0.7	34.0779974375690	34.077997437030	34.0779974375664
0.8	38.8764976236066	38.876497623070	38.8764976236032
0.9	43.6577593088736	43.657759308340	43.6577593088695
1	48.4218510275559	48.421851027025	48.4218510275513
t	y_3:RK solution	y_3:ABM solution	T_9 by using Osculatory
0	0	0	0
0.1	0.00096019320833	0.0009601932106	0.0009601932083
0.2	0.00383615488692	0.0038361548987	0.0038361548869
0.3	0.00862097880833	0.0086209788333	0.0086209788083
0.4	0.01530778624197	0.0153077862802	0.0153077862420
0.5	0.02388972583802	0.0238897258893	0.0238897258380
0.6	0.03435997351199	0.0343599735605	0.0343599735120
0.7	0.04671173232973	0.0467117323780	0.0467117323297
0.8	0.06093823239298	0.0609382324410	0.0609382323930
0.9	0.07703273072535	0.0770327307732	0.0770327307253
1	0.09498851115892	0.0949885112065	0.0949885111589
t	y_2:RK solution	y_2:ABM solution	$\widetilde{P}_9$ by using Osculatory
0	0	0	0
0.1	0.00273025653401	0.0027302565510	0.0027302565342
0.2	0.01089751214291	0.0108975122287	0.0108975121431
0.3	0.02446663016113	0.0244666303419	0.0244666301613
0.4	0.04340265235022	0.0434026526272	0.0434026523504
0.5	0.06767079805972	0.0676707984308	0.0676707980599
0.6	0.09723646339207	0.0972364637429	0.0972364633922
0.7	0.13206522037129	0.1320652207201	0.1320652203715
0.8	0.17212281611552	0.1721228164628	0.1721228161157
0.9	0.21737517201331	0.2173751723590	0.2173751720135
1	0.26778838290386	0.2677883832477	0.2677883829038

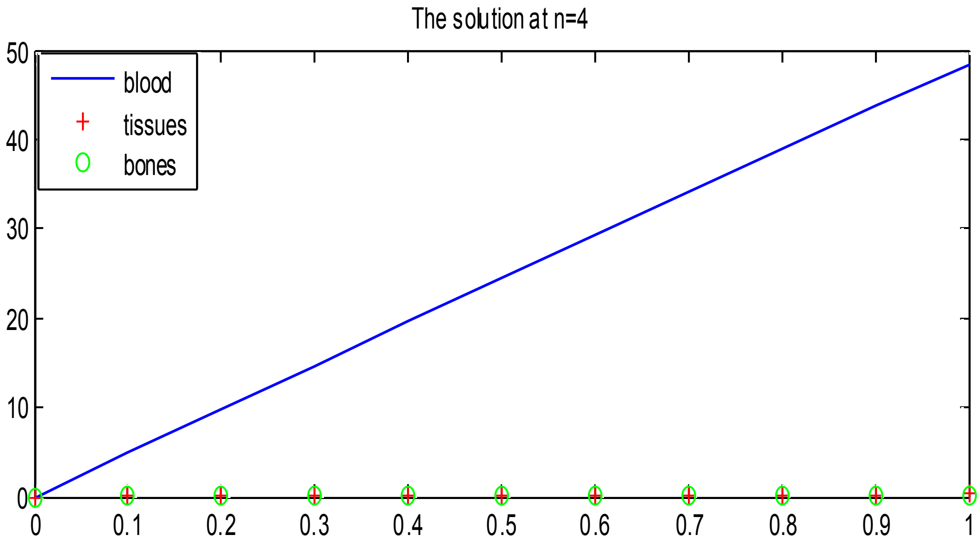

Fig.(1) : Buildup of lead in the bones , tissue , blood

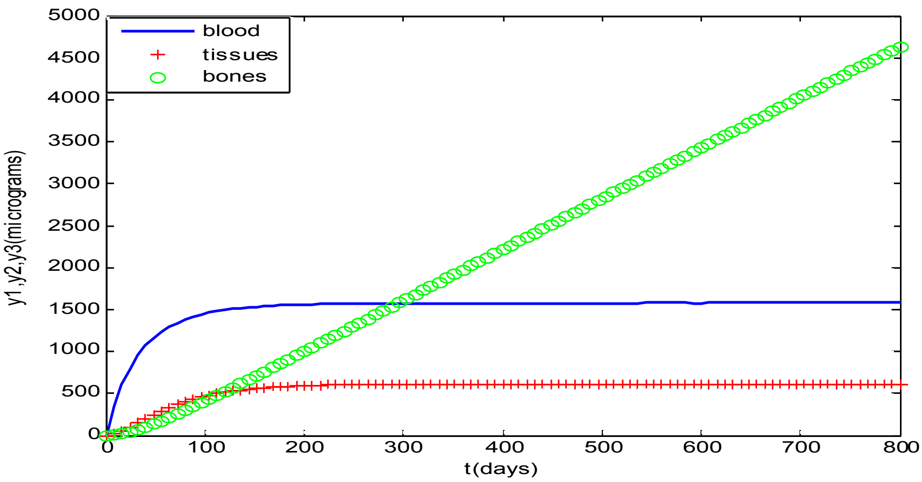

Fig.(2) : Buildup of lead in the bones(circle), blood(line), tissue(plus)

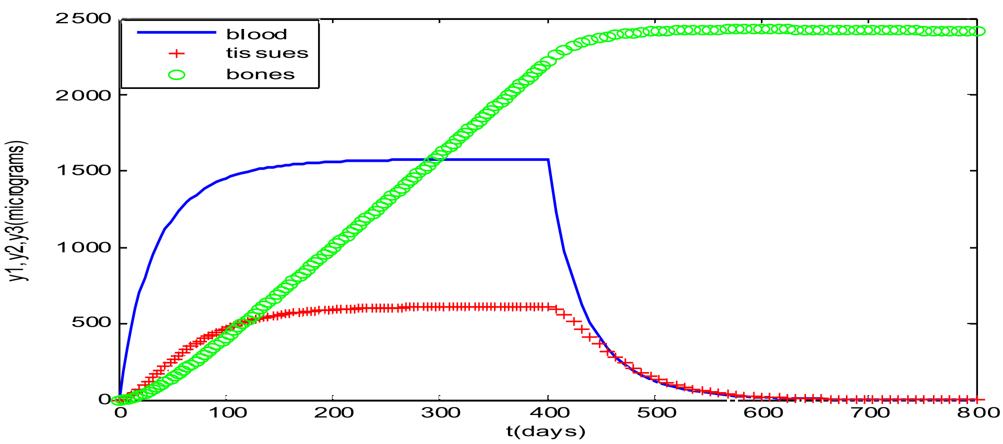

Fig.(3): lead intake stops on the 400 th day

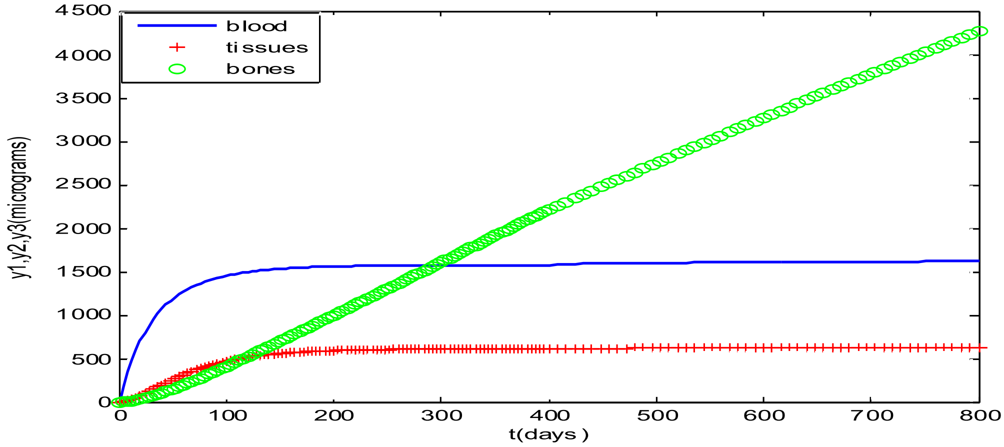

Fig.(4): Anti lead medication taken from the 400 th day on does not help much

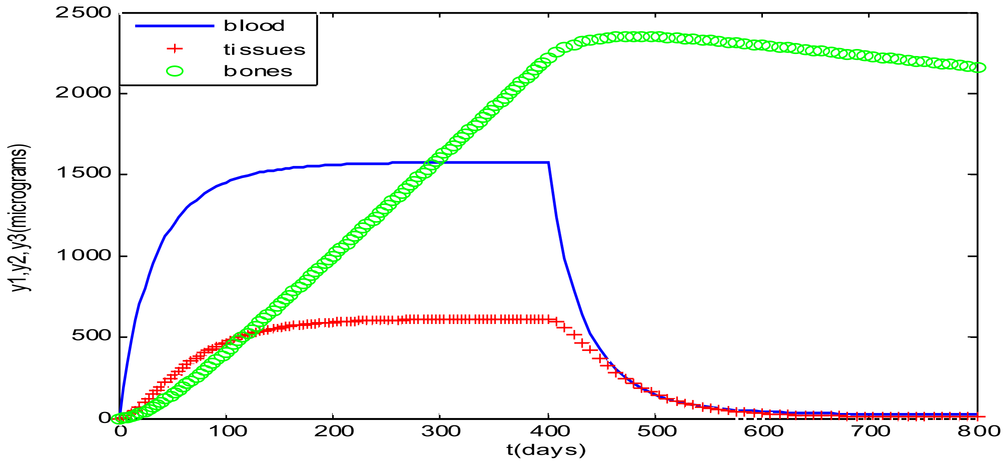

Fig.(5): lead-free environment and medication from the 400 th day on help a lot

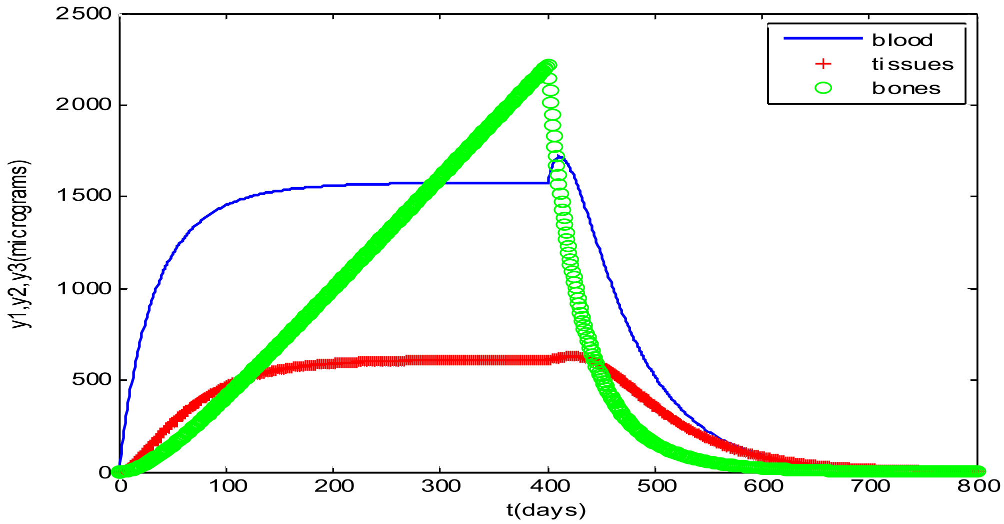

Fig.(6): lead-free environment and a heavy dose of antilead drug from the 400 th day on .Is the subject still alive?

استخدام الطريقة شبة التحليلية لتقليل أخطار الرصاص

لمى ناجي محمد توفيق و محمد ناجي محمد توفيق
قسم الرياضيات ، كلية التربية ابن الهيثم ،جامعة بغداد
استلم البحث في: 7 تشرين الثاني 2010
قبل البحث في: 8 شباط 2011

الخلاصة

الهدف من البحث هو أيجاد طريقة لحل منظومة معادلات تفاضلية اعتيادية من الرتبة الأولى لمسائل القيم الابتدائية باستعمال التقنية شبه التحليلية وذلك بإيجاد الحل بشكل متعددة حدود و ذلك للتقليل من أخطار الرصاص .

المسالة الأصلية تتعلق باستعمال الاندراج التماسي ذي النقطتين التي تتفق فيها الصورة وعدد متساو من المشتقات المعرفة عند نقطتي نهاية المدة [1 , 0] مع البيانات المعطاة .

الكلمات المفتاحية : الاندراج التماسي، مسائل القيم الابتدائية