On Max-Modules

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Abstract

In this paper ,we introduce a concept of Max– module as follows: M is called a Max-module if $\sqrt{ann_R N}$ is a maximal ideal of R, for each non– zero submodule N of M;

In other words, M is a Max– module iff (0) is a $*$- submodule, where a proper submodule N of M is called a $*$- submodule if $\sqrt{[N_R K]}$ is a maximal ideal of R, for each submodule K contains N properly.

In this paper, some properties and characterizations of max– modules and $*$- submodules are given. Also, various basic results a bout Max– modules are considered. Moreover, some relations between max- modules and other types of modules are considered.

Key word: Ring, Module, Max-Module

Introduction

Every ring considered in this paper will be assumed to be commutative with identity and every module is unitary. We introduce the following: An R– module M is called a max-module if $\sqrt{ann_R N}$ is a maximal ideal of R, for every non-zero submodule No f M, where $ann_R N = \{r: r \in R \text{ and } r N = 0\}$.

Our concern in this paper is to study max-modules and to look for any relation between max– modules and certain types of well– Known modules specially with primary modules.

This paper consists of three sections. Our main concern in §1, is to define and study $*$-submodules. Also we study the properties of a multiplication module that contains $*$-submodules. In §2, we study max– modules, and we give some characterizations for this concept. Also other basic results about this concept are given.

In §3, we study the relation between max– modules and primary modules. It is clear that every max-module is primary module, but the converse is not true in general. We give in (3.2), (3.3) conditions under which the two concepts are equivalent. Next we investigate the relationships between max, prime, semi– primary, quasi-primary finitely generated and uniform modules, see (3.4), (3.12).

1. SUBMODULES

In this section, we introduce the concept of $*$- submodule and we give some characterizations for this concept. And we end this section by studying the properties of a multiplication module that contains $*$- submodules.

Definition 1.1:

A proper submodule N of an R-module M is said to be a $*$- submodule if $\sqrt{[N_R K]}$ is a maximal ideal of R for each submodule Ko f M such that $K \supsetneqq N$.

Where $[N_R K] = \{r \in R: rK \subseteq N\}$.

Specially, an ideal I is a $*$- ideal of R if and only if I is a $*$- R– submodule of R– module R.

Examples and Remarks (1.2)

1- Recall that an R– submodule N of M is a quasi– primary submodule of M if $[N_R: K]$ is a primary ideal of R for each submodule K of M such that $K \supsetneq N$,[2]. It is well– Known that if $\sqrt{[N_R: K]}$ is a maximal ideal of R, then $[N_R:K]$ is a primary ideal of R, [1, prop. 4.9, P. 64]. Thus every $*$- R-submodule of M is a quasi– primary submodule.

2- The submodule Z of the Z-module Q is not a $*$- submodule since $\sqrt{Z_Z: Z+(1/6)} = \sqrt{6Z} = 6Z$ is not a maximal ideal of Z .

3- The intersection of any two $*$- submodules of an R– module need not be $*$-submodule for example. The Z– module Z_6 has two $*$- submodules, $N_1 = (\bar{2})$ and $N_2 = (\bar{3})$, but $N_1 \cap N_2 = (0)$ is not a $*$- submodule of Z_6, since $\sqrt{[(0)_Z: Z_6]} = \sqrt{6Z} = 6Z$ is not a maximal ideal of Z .

4- Every $*$- submodule is a semi-primary submodule.

Proof : Suppose N is a $*$- submodule of an R-module M. Hence $\sqrt{[N_R: K]}$ is a maximal ideal of R. Therefore $\sqrt{[N_R: K]}$ is a prime ideal of R, which implies that N is a semi– primary submodule of M by [2, definition 1.1].

However the converse is not true in general as the following example shows :

Let $M = Z \oplus Z_{12}$ as a Z– module and N=(0)=(0) ⊕ (0). It is clear that N is a semi–primary submodule of M, since $\sqrt{[(0)_Z: M]} = \sqrt{0} = 0$ is a prime ideal of Z. But (0) ⊕ (0) is not a $*$- submodule of M, since $\sqrt{[(0)\oplus(0)_Z: (0)\oplus Z_{12}]} = \sqrt{12Z} = 6Z$ which is not a maximal ideal of Z.

By using (1.2, (1)) and [2, Th. (3.1.3), chapter 3] we can give the following characterization for $*$- submodule.

Theorem 1.3

Let N be a proper submodule of an R-module M. If N is a $*$- submodule of M, then $\sqrt{[N_R: K]} = \sqrt{[N_R: rK]}$ for each submodule K of M such that $K \supsetneq N$, $rK \not\subset N$ and $r \in R$.

By using (1.2, (1)) and [2, prop.(3.1.4), chapter 3] we can give the following result :

Corollary 1.4

Let N be a proper submodule of an R- module M . If N is a $*$- submodule of M, then $\sqrt{[N_R: (rm)]} = \sqrt{[N_R: (m)]}$ for each m ∈ M\N, r ∈ R and $r \notin [N_R:(m)]$.

The converse of corollary (1.4) is not true in general for example: Let M = Z as a Z– module, let N = 6Z, r =5, $5 \notin [6Z_Z:(1)] = 6Z$ and $\sqrt{[6Z_Z: (5.1)]} = \sqrt{6Z} = 6Z = \sqrt{[6Z_Z: (1)]}$. But N is not a $*$- submodule of z.

Recall that an R-module M is called a multiplication module, if for every submodule N of M, there exists an ideal I of R such that IM = N, equivalenty; for every submodule N of M, N= $[N_R:M]$ M, see [3] .

An R- submodule N of M is called a prime R– submodule if and only if N≠M and whenever r ∈ N , for r ∈ R and x ∈ M, either $r \in [N_R:M]$ or x ∈ N, [10]. The prime radical P(N) of N in M is defined to be the intersection of all prime submodules P of M such that N ⊆ P i.e. P(N) = ∩ {P ⊆ M: Pis prime and N ⊆ P}.

It is Known that if M is multiplication module and N is a submodule of M, then $P(N) = \sqrt{[N_R: M]}$ M, [3, Th. 2. 12].

The following remark shows that a multiplication R-module which has a finitely generated $*$- submodule is finitely generated R– module.

Remark 1.5

Let M be a multiplication R-module. If M contains a finitely generated $*$- submodule N, then M is a finitely generated R– module.

Proof : Since N is a $*$- submodule, so N is a semi- primary submodule of M by (1.2, (3)). Therefore, M is finitely generated by [2, proposition 3.4, P. 135] .

Corollary 1.6

If N is a $*$- submodule of a multiplication R– module M, then rad (N) is a prime submodule of M.

Proof :Suppose that N is a $*$- submodule. Hence, N is a quasi– primary submodule by (1.2, (1)). But M is a multiplication R– module, so N is a primary submodule of M by [2, propostion (3.1.5), chapter 3]. Therefore, rad (N) is a prime submodule by [4, corollary 2.13, chapter 2] .

2. Basic Properties of Max-Modules

In this section, we introduce the concept of a max– module and give some characterizations and properties of this concept, we end the section by studying the relationships between max-rings and max-modules.

Definition 2.1

An R– module M is said to be a max– module if $\sqrt{ann_R N}$ is a maximal ideal of R, for each non– zero submodule N of M. Specially, a ring R is called a max– ring if and only if R is max– R– module. We give some examples and remarks:

Remarks and Examples 2.2

1- Z_{P^∞} as Z– module is a max– module.

N = IM for some ideal I of R. But M is faithful, $ann_R N = ann_R IM = ann_R I$ and so $\sqrt{ann_R N} = \sqrt{ann_R IM} = \sqrt{ann_R I}$ which is a maximal ideal of R. Therefore M is a max– module.

Proof : We know that every submodule of Z_{P^∞} is of the form $\langle \frac{1}{P^n} + z \rangle$, where n be a non-negative integer, so $\sqrt{ann_Z \langle \frac{1}{P^n} + Z \rangle} = \sqrt{P^n Z} = PZ$ is a maximal ideal of Z.

2- Z as a Z– module is not a max– module, since $\sqrt{ann_Z Z} = \sqrt{0} = 0$ is not a maximal ideal of Z.

3- Consider, the Z– module M = $Z_2 \oplus Z_{12}$ and the Z– submodule $N = (\bar{0}) \oplus (\bar{2})$. Then, $\sqrt{ann_Z N} = \sqrt{Z \cap 6Z} = \sqrt{6Z} = 6Z$, which is not a maximal ideal of Z. Therefore, M is not a max– module .

4- Q as a Z– module is not a max– module .

5- Every non– zero submodule of a max– module is a max– R-module.

6- Let M be a max– module, then $\sqrt{ann_R M}$ is a maximal ideal of R.

The following theorem gives a characterization for max– modules.

Theorem 2.3

Let M be an R-module, then M is a max– module if and only if (0) is a *- submodule.

Proof : Suppose that M is a max– module, to prove (0) is a $*$- submodule. Since M is a max, then $\sqrt{ann_R N}$ is a maximal ideal of R, for each non– zero submodule N of M. But $\sqrt{ann_R N} = \sqrt{[(0)_R : N]}$, for each non– zero submodule N of M so by definition (1.1) ,(0) is a $*$- submodule of M.

Conversely, if (0) is a $*$- submodule of M, to prove M is a max– module. Since (0) is a $*$-submodule,then definition (1.1) implies that $\sqrt{[(0)_R : N]}$ is a maximal ideal , for each non-zero submodule N of M. But $\sqrt{[(0)_R : N]} = \sqrt{ann_R N}$, so M is a max – module.

By using (1.2, (1)) and [2, theorem (3.3.6), chapter 3], we can give the following characterization for max-module.

Theorem 2.4

Let M be an R-module, if M is a max– module then $\sqrt{ann_R N} = \sqrt{ann_R rN}$ for each non–zero submodule N of M such that rN ≠ (0), r ∈ R .

By using (1.2, (1)) and [2, corollary (3.3.7), chapter 3], we can give the following result:

Corollary 2.5

Let M be an R-module, if M is a max– module then $\sqrt{ann_R(m)} = \sqrt{ann_R(rm)}$ for each 0 ≠ m ∈ M such that rm ≠ 0, r ∈ R.

Now, we state and prove the following result.

Proposition 2.6

Z_m as a Z– module is a max– module if and only if $m = p^n$ for some prime number p and n ∈ Z+.

Proof : If Z_m is a max– Z-module, to show that $m = p^n$ for some prime number p and n ∈ Z+. By (2.2, [5]), $\sqrt{ann_Z Zm} = \sqrt{mz} = PZ$ is a maximal ideal of z, therefore $m = p^n$ for some prime number p and n ∈ Z+.

Conversely, if $m = p^n$ for some p (prime number) and n ∈ Z+, to show that Zm a Z– module is a max– module. Let N be anon– zero submodule of Zm. Since N ⊆ Zm, $\sqrt{ann_Z N} \supseteq \sqrt{ann_Z Zm} = \sqrt{mz} = \sqrt{P^n Z} = PZ$ which is a maximal ideal, then $\sqrt{ann_Z N} = PZ$, and by definition (2.1), Zm as a Z- module is a max– module.

In the following result, we show that the converse of (2.2. [5]) is true.

Proposition 2.7

Let M be an R-module that satisfies ✪,then M is max– module if and only if $\sqrt{ann_R M}$ is a maximal ideal of R.

Where ✪: $ann_R M \underset{\neq}{\subset} [N :_R M]$, for each non- zero submodule N of M .

Proof : If M is a max– module, then by (2.2,[5]) $\sqrt{ann_R M}$ is a maximal ideal of R.

Conversely, if $\sqrt{ann_R M}$ is a maximal ideal of R, to prove that M is a max– module, ($\sqrt{ann_R N}$ is a maximal ideal of R, ∀ 0 ≠ N ⊆ M).

It is clear that $\sqrt{ann_R N} \supseteq \sqrt{ann_R M}$ .....(1).

Let $r \in \sqrt{ann_R N}$, so $r^n N = 0$ for some n ∈ Z+ . By ✪, there exists a ∈ R, a ≠ 0 such that aM ≠ 0 and aM ⊆ N. Hence $r^n aM \subseteq r^n N = 0$.

It follows that $r^n a \in \sqrt{ann_R M}$. But $\sqrt{ann_R M}$ is a maximal ideal, so $\sqrt{ann_R M}$ is a primary ideal by (1, proposition 4.6, P. 64), and a ∉ $ann_R M$ (since aM ≠ 0), so $(r^n)^k \in ann_R M$ for some K ∈ Z+ and hence $r \in \sqrt{ann_R M}$.

Thus, $\sqrt{ann_R N} \subseteq \sqrt{ann_R M}$ .....(2).

Therefore, by (1) and (2) we get $\sqrt{ann_R M} = \sqrt{ann_R N}$.

Thus $\sqrt{ann_R N}$ is a maximal ideal and so by definition (2.1), M is a max– module.

We note that if M is a max– module, then it is not necessary that R is a max– ring, for example: the Z– module Z_2 is max– module, but Z is not max– ring. Moreover , if R is a max– ring and M is an R– module, then M is not necessarily max– module, for example: Consider the Z_2– module Z_6, Z_2 is a max– ring, but Z_6, is not max– module.

Recall that an R– module M is called faithful R– module if $ann_R M = 0$.

However, in the class of faithful multiplication module, they are equivalent as the following result shows .

Proposition 2.8

If M is a faithful multiplication R– module, then M is a max– module if and only if R is a max– ring.

Proof : If M is a max– module. To prove R is a max– ring. Let I be a non– zero ideal of R. Then N = IM is a non– zero submodule of M. Hence $\sqrt{ann_R N}$ is a maximal ideal of R because M is a max– module. On the other hand, since M is a faithful multiplication R– module, then $ann_R N = ann_R I$, so $\sqrt{ann_R N} = \sqrt{ann_R I}$. Thus $\sqrt{ann_R I}$ is a maximal ideal and R is a max– ring.

Conversely, if R is a max– ring, to prove M is a max– module.

Let N be a non–zero submodule of M. Since M is a multiplication R– module,

3. Some Relations Between Max– Modules And Other Modules

In this section, we study the relationships between max-modules and primary modules and prime modules, semi-primary, quasi-primary, finitely generated and uniform modules.

We start with the following definitions which are needed.

Recall that an R-module M is said to be a primary module if (0) is a primary R– submodule of M, [2] .

Where a submodule N of an R– module M is called a primary submodule if $N \neq M$ and whenever $rx \in N$ for $r \in R$ and $x \in M$ we have either $x \in N$ or $r^n \in [N_R^{:}M]$ for some $n \in Z+$, where $[N_R^{:}M] = \{r : r \in R \wedge rM \in N\}$, [8] .

By using this concept, we have the following:

Remark 3.1

Every max– module is a primary module.

Proof : Let N be a non– zero submodule of an R– module M. Suppose that M is a max– module, to prove M is a primary module. Since M is a max– module, then $\sqrt{ann_R N}$ is a maximal ideal of R, for each non– zero submodule N of M by definition (2.1) and so $\sqrt{ann_R M}$ is a maximal ideal of R by (2.2,6).

But $\sqrt{ann_R N} \supseteq \sqrt{ann_R M}$ so $\sqrt{ann_R N} = \sqrt{ann_R M}$.

Therefore M is a primary R– module by (2,Theorem (2.1.3), chapter 2).

Note that, the converse of (3.1) is not true in general. For example, the Z–module M $= Z \oplus Z$ is a primary by [2, (2.1.2, (2)), Chapter 2], but it is not a max– module.

In the following proposition, we give a sufficient condition under which the converse of (3.1) is true.

Proposition 3.2

Let M is a module over a PID, and $0 \neq ann_R M$ is a primary ideal of R. If M is a primary R– module, then M is a max– module.

Proof : Let N be a non-zero R– submodule of M, to prove $\sqrt{ann_R N}$ is a maximal ideal. Since M is a module over a PID, then the only primary ideals in R are (0) and $<P^n>$ for some a prime element P and $n \in z+$.

But $0 \neq ann_R M$ is a primary ideal, so $ann_R M = <P^n>$, and this implies $\sqrt{ann_R M} = \sqrt{<P^n>} = <P>$ which is a maximal ideal.

But M is a primary, then $\sqrt{ann_R N} = \sqrt{ann_R M}$ by [2, Theorem (2.1.3), chapter 2]. Hence $\sqrt{ann_R N}$ is a maximal ideal and so by definition (2.1) M is a max– module.

In the following result, we give another condition for which a primary module be a max– module. But first we need the following definition.

The dimension of R, denoted by dim R, is defined to be: sup {n ∈ N: there exists a chain of prime ideals of R of length n, if the supremum exists, and ∞, otherwise}, [1].

Proposition 3.3

Let R be a 0– dimensional ring. Then a primary R– module M is a max– module.

Proof : Since M is a primary module, so $ann_R M$ is a primary ideal of R by (2, corollary 2.1.7, chapter 2) and hence $P = \sqrt{ann_R M}$ is a prime ideal. But dim R = 0 implies that p is a maximal ideal. On the other hand, $\sqrt{ann_R M} = \sqrt{ann_R N}$ for every non– zero submodule N of M (since M is a primary module), so that $\sqrt{ann_R N}$ is a maximal ideal. Therefore M is a max– module.

Now, we study the relation between max-modules and prime modules. But first we need the following definitions:

Recall an R– module M is said to be a prime module if (0) is a prime R–submodule of M, see [9] .

We notice that not every max– module is a prime– module, for example: The Z– module Z_4 is max by proposition (2.6), but it is not a prime Z-module by [5, (1.1.3 (3)), chapter 1].

The following proposition shows that ($ann_R M$ is a semi– prime ideal) is a sufficient condition for max– module to be prime.

Proposition 3.4

If M is a max-module and $ann_R M$ is a semi– prime ideal of R, then M is a prime R– module.

Proof : Since M is a max– module, then M is a primary R– module by (3.1). But $ann_R M$ is a semi– prime ideal of R, hence by [2, proposition (2.3.2), chapter 2], M is a prime R– module.

Next, a proper submodule N of M is called semi– prime submodule if for every r ∈ R, x ∈ M, K ∈ Z+, such that r^k x ∈ N, than rx ∈ N, see [7] .

By using this concept, we have the following:

Corollary 3.5

If M is a max– module and (0) is a semi– prime submodule , then M is a prime R-module.

Proof : Since (0) is a semi-prime submodule, so $ann_R M$ is a semi– prime ideal by [8, proposition (1-5), chapter 2], hence the result follows by (3.4).

Recall an R– module M is said to be a semi– primary if (0) is a semi– primary R– submodule, (2).

It is well known that every primary R– module is a semi– primary module [2, (3.5.3, (2)), chapter 3]. So that following result follows immediately from (3.1).

Corollary 3.6

Every max-module is a semi– primary R– module.

Note that the converse of (3.6) is not true in general. For example, the Z– module M =Z ⊕ Z_{12} is a semi– primary, but not a max– module.

Recall that an R-module M is said to be a quasi– primary module if $ann_R N$ is a primary ideal of R, for each non– zero submodule N of M, [2] .

However, we have the following:

Remark 3.7

Every max-module is a quasi– primary module.

proof : Since M is a max– module, then $\sqrt{ann_R N}$ is a maximal ideal of R for each non–zero submodule N of M. Hence $ann_R N$ is a primary ideal by [1, proposition 4.9, P. 64], and so M is a quasi– primary.

Note that, the converse of (3.7) is not true in general, for example, the Z– module Z is a quasi– primary since $ann_Z(N)$ = 0 is a prime ideal, for each non– zero N of Z, so it is a primary ideal. But it is not a max– module by [2.2, (2)].

We notice that not every max-module is finitely generated, for example: Z as a Z-module is a max– module but not finitely generated.

However, we have the following proposition :

Proposition 3.8

If M is a multiplication max-module, then M is a finitely generated module.

Proof : Since M is a max– module, then $\sqrt{ann_R M}$ is a maximal ideal by [2.2, (6)] and so $ann_R M$ is a primary ideal by [1, prop. 4.9, P. 64]. On the other hand M is a multiplication imply , M is a finitely generated by [5, prop.(2.7), chapter 2] .

Now, we study the relation between max-modules and uniform modules . But first we need the following definition:

Recall that an R– module M is said to be uniform module if every non– zero submodule of M is essential, [11] .

Where a submodule N of an R– module M is called essential proved that $N \cap K \neq 0$ for every non– zero submodule K of M, [11] .

Note that, it is not necessary that every uniform R– module is a max– module for example = Q as a Z– module is uniform. But it is not a max– module by [2.2, (4)] .

However, we have the following result.

Proposition 3.9

If M is a max– module such that $annR(N \cap U) = ann_R N + ann_R U$, for every non–zero submodules N and U of M, then uniform.

Proof : Since M is a max– module, so M is a primary module by (3.1), hence the result follows by [2, proposition (2.3.7), chapter 2] .

Now we can give the following result :

Proposition 3.10

Let M be an R-module and let $0 \neq x \in M$ such that:

1. Rx is an essential submodule of M.
2. $\sqrt{ann_R(x)}$ is a maximal ideal of R, and
3. $\sqrt{ann_R M} = \sqrt{ann_R(x)}$.

Then M is a max – module.

Proof : Let N be a non– zero submodule of M. Since Rx is an essential submodule of M, there exists $0 \neq t \in R$ such that $0 \neq tx \in N$ and hence $(tx) \subseteq N$. This implies that $ann_R N \subseteq ann_R(tx)$ and so , $\sqrt{ann_R N} \subseteq \sqrt{ann_R(tx)}$.

But $N \subseteq M$, then $\sqrt{ann_R M} \subseteq \sqrt{ann_R N}$ and hence $\sqrt{ann_R(x)} \subseteq \sqrt{ann_R N}$ (by condition 3).

Thus, $\sqrt{ann_R(x)} \subseteq ann_R N \subseteq \sqrt{ann_R(x)}$ ……(1).

Let $r \in \sqrt{ann_R(tx)}$, then $r^n tx = 0$ for some $n \in Z+$ and $r^n t \in ann_R(x)$.

But $tx \neq 0$; that is $t \notin ann_R(x)$ and by condition (2) $\sqrt{ann_R(x)}$ is a maximal ideal of R, so $ann_R(x)$ is a primary ideal of R, by [1, proposition 4.9, P. 64] .

Then $r \in \sqrt{ann_R(x)}$ and hence $\sqrt{ann_R N(tx)} \subseteq \sqrt{ann_R(x)}$.….(2).

Thus by (1) and (2), $\sqrt{ann_R(x)} = \sqrt{ann_R(tx)}$ and so $\sqrt{ann_R N} = \sqrt{ann_R(x)}$. Therefore (by condition 2) $\sqrt{ann_R N}$ is a maximal ideal of R and M is a max– module by definition (2.1).

The following result is a consequence of proposition (3.10).

Corollary 3.11

Let M be uniform R-module such that $\sqrt{ann_R(x)}$ is a maximal ideal of R and $\sqrt{ann_R M} = \sqrt{ann_R(x)}$ for some $x \neq 0$.

Then M is a max– module.

In the following corollary, we give a condition under which the converse of proposition (3.9) is true.

Corollary 3.12

If M is a uniform R– module such that $\sqrt{ann_R(x)}$ is a maximal ideal of R for some x $\in$ M. Then the following statements are equivalent.

1. $\sqrt{ann_R M} = \sqrt{ann_R(x)}$ for some x $\in$ M.
2. M is a max– module.

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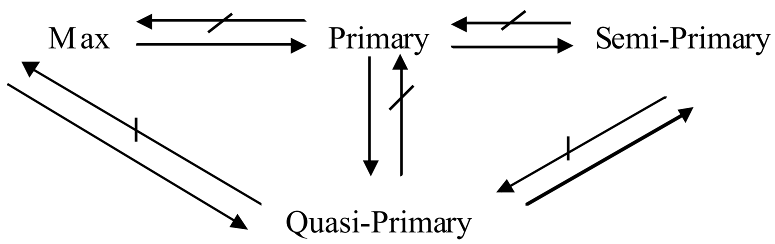
الخلاصة

لتكن R حلقة أبدالية ذات محايد، وليكن M مقاسا أحادياً على R. في هذا البحث قدمنا مفهوم مقاس من النوع Max كما يأتي: يطلق على M مقاساً (Max) إذا كان $Rad\ (ann_R N) = \sqrt{ann_R N}$ مثالياً أعظميا في R، لكل مقاس جزئي غير صفري N في M، بعبارة مكافئة، يكون M مقاساً (Max) إذا كان (0) مقاساً من النوع * وقد أطلقنا على أي مقاس جزئي فعلي N في M مقاساً من النوع * إذا كان $\sqrt{[N_R:K]}$ مثاليا أعظميا في R، لكل مقاس جزئي K في M يحتوي N فعلياً. في هذا البحث، أعطيت بعض الخواص و التميزات وكذلك دُرست العديد من النتائج الأساسية حول المقاسات من النوع (Max). فضلا عن هذا دُرست بعض العلاقات بينه وبين أنواع أخرى من المقاسات. والمخطط الآتي يوضح ملخص لما حصلت عليه:



الكلمات المفتاحية:الحلقة، الموديل، اكبر ، موديول