

## A New Design of Fractal Optical Modulation

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### Abstract

In this paper it was designed a new fractal optical modulation by using a new iteration of fractal function, the result was analyzed by MTF evaluation, and it compared with results of normal optical modulation.

The normal and fractal optical modulator is a circular disc which has a radius  $R=9\text{cm}$ , both of them consist of twenty sectors, ten sectors are opaque and the other ten sectors are transmitted for the light.

The fractal optical modulator contains two patterns, the pattern two can be used to detect the target, and pattern one can be used to lock the target

The best similarity of MTF behavior for normal and fractal Reticle was evaluating the power transparent depends on the size of the laser spot and the size of the sector, where the proportionality between them is directly.

**Keywords:** Fractal Optical Modulator, Chopping frequency, The Modulation Transfer Function MTF

### Introduction

There are many electro- optical tracking systems using the optical package of electromagnetic radiation spectrum which can be various types used to cover many of the civilian and military applications, these systems are classified into two types depending on the nature of work, which are (passive-mode & Active mode).

The main part in the electro- optical tracking systems which are used to determine the target locating is optical modulation disk (Reticle) [1].

Reticle produces forms of modulation that allows various instruments to differentiate objects or targets from their backgrounds and to produce appropriate signals that make possible a variety of applications, from measurement to guidance [2].

### The basic principles of Reticle

The optical modulation disk is the often used in the electro- optical tracking systems as optical filter for background discrimination. The design and movement of the Reticle is to enhance the object and suppress the background. The detect a point source in its environment refers to the efficiency of the Reticle, as shown in Fig (1) [3].

The Reticle pattern is determined by the requirement of the Reticle system. Fig(2) shows the simple Reticle consists of 12 sectors, 6 transparent and 6 opaque. The Reticle is rotating and the optical system slowly scans the scene from the left to the right. When the point object is in the field of view of the optical system, the Reticle pattern will generate a modulated output detector system signal consisting of square wave with frequency corresponding to the spinning rate times the number of transparent sectors. The output signal will be close to square wave as long as the point source fits within one sector. As the object becomes increasingly extended, the output signal becomes distorted, less modulated, and the modulation will become increasingly reduced as shown in Fig(2), the point source results in a square wave, while the extended cloud results in a signal with no or very little modulation. Each Reticle has aperture scanning which is adapted to it's application [4, 5].

**Reticle Design**

In this paper, two models has been designed for Reticle; the first design is normal way, so as to compare the results obtained from this model with the results of the second model, which was designed by using Fractal Function, a new technique of it.

**Normal Modulator Design**

The normal optical modulator is a circular disc which has a radius R, which assumes the number of sector is (twenty sectors), ten sectors are opaque and the other ten sectors are transmitted for the light as shown in Fig (3). One may consider these ten sectors also as opaque for the other regions of electro -magnetic wave spectrum.

By assuming the incident light is a perpendicular to the modulator which is moveable in a circular form. Hence the light beam will make discrete circles according to the number of sectors. Therefor the resultant will be a circumference of the circle.

**Fractal Modulator Design**

It is non-linear deterministic equations can self-generate irregular outputs. This system can be simulated when behavior is linear or nearly non-linear. When it increases, though smooth on short time scales, random and unpredictable behavior can be seen over longer periods.

Let  $(H(x),h(d))$  be a metric space, and let  $f : X \rightarrow X$  be a function.

Let  $S \subset X$ , then:-

$$f(S) = \{f(x) : x \in S\}.$$

The function  $f$  is one-to-one.

If  $x, y \in X$ , and  $f(x) = f(y)$ , so  $x = y$ , then the matrix space can be given by the equation:-

$$A' = TA \dots\dots\dots (1)$$

Where A is a point in initial area.

A' is a new point in under matrix operation (T)

The matrix (T) is given by:-

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \dots\dots\dots (2)$$

The transformation (W) in Euclidean plane can be given by:-[6]

$$W(x, y) = (ax + by + e, cx + dy + f) \dots\dots\dots (3)$$

The points **a**, **b**, **c**, and **d** define rotation and scaling operations to be applied to the point and are called affine transformation. The e and f points define a translation to be applied to the point. The transformation (W) can be defined in this formula [7, 8, 9]:-

$$W(x) = W \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \dots\dots\dots (4)$$

Or

$$W(x) = Ax + T \dots\dots\dots (5)$$

Where:-

$$Ax = \text{the matrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$T = \text{the horizontal vector} \begin{bmatrix} e \\ f \end{bmatrix}$$

By using this concept and IFS kit program, we have designed optical modulator as shown in Fig(10). this optical modulator consists of two pattern circles. Each circle is divided into ten transparents and ten opaque sectors (q).

The first pattern, is (inner pattern) designed in a circle with a radius of 0.1 cm, the maximum distance of this pattern is equal to 3 cm (from whole disc) , as shown in Fig (4) .

After conducting the operations of scaling rotation and iteration (for many times) it has been got the pattern as shown in Fig (5) and Fig (6), while Table 1 represents the data of the first pattern.

The second pattern (outer pattern) is designed in an equilateral triangle of side length 0.1 cm within the last third of the disk, the maximum points of this pattern is equal to 9 cm, where we left a blank space in the middle 3 cm in length, i.e., starting from a distance of 6 cm from the first pattern, As shown in Fig (7).After (many times) of conducting the operations of scaling rotation and iteration the result as shown in Fig.8 and Fig (9), while Table 2 represents the data of the second pattern.

**Modulation Transfer Function MTF**

The modulation transfer function is, as the name suggests, a measure of the transfer of modulation (or contrast) from the subject to the image. In other words, it measures how faithfully the lens reproduces (or transfers) detail from the object to the image produced by the lens.

Fig (11) Illustrates the black and white bars in row A of the test pattern below. This pattern consists of totally black bars on a totally white background. If we assign the number 255 to the totally white areas, and 0 to the totally black areas, and we plot a line profile of the test pattern, we get the graph shown in C. The regions at 0 correspond to the black lines; the regions at 255 correspond to the white lines

If it been taken a line profile of Image Pattern B, the Graph D above it will be the result. For the widest spaced set of black and white bars, the plot goes between 0 and 255. This corresponds to the performance of the lens when it recors low frequency detail. For the next set of patterns we can see that the plot no longer reaches either 255 or 0. The modulation in Target A is no longer faithfully reproduced in Image B. The formal definition of MTF [10]:  
 MTF = (maximum intensity - minimum intensity)/(maximum intensity + minimum intensity)

For the first pattern group, the MTF is 1. For the second pattern set, the MTF can be calculated to be 0.8. For the third set, the MTF is 0.5, and for the fourth set, the MTF is 0.1. If there were any finer patterns, with narrower black and white bars, the MTF would be 0 and the image of the pattern would be a uniform gray level, represented on the plot by a straight line at a value of 127. The point at which you can no longer see any variation in the image is the point at which the MTF is zero, and that's the definition of the "resolution" of the lens. In this case, the final pattern set with an MTF of 0.1 would be classified as "just resolved" by this lens.

**The Implementation Result and Discussion**

Obtained the results of this work through the establishment of a special program named "Disk optical modulator" using the language visual basic 6 contains many parameters and as shown in Fig (12)

When calculating the frequency has been converted to units (Rev / s), as well as for angular velocity  $w$ , The Law of frequency is given by

$$f_r = w / 2\pi \dots\dots\dots (6)$$

$$f_c = qfr \dots\dots\dots (7)$$

Where  $f_c$  chopping Frequency,  $f_r$  rotation Frequency and  $q$  number of sectors.

To calculate the MTF we used the following law

$$MTF = \frac{(A + B) - (A - B)}{(A + B) + (A - B)} \dots\dots\dots (9)$$

Where A the amplitude of increasing frequency and B the amplitude of incident frequency (assumed 0.5 mm) see Fig (13)

The results that were obtained based on a number of information assumed as shown in Table 3.

First, we may draw the relationship between the rotation frequency and Chopping frequency with number of sector depending on data in Table(4), we got the curve shown in Fig(14)

Fig(14) shows that both frequencies have become a sine function oscillating between zero and maximum value, and since the Reticle contains ten sections, therefore, the maximum value of chopping frequency greater than the maximum value of the rotation frequency of ten times and that is identical with the eq(7).

Table 5 shows the method of calculating the radius of the Fractal that contains ten sections and every section contains ten circles as described in Fig (15) .So this circle design was applied to other 9 remained circles.

Note: it has been taken one of the ten sections and it was the first section of the origin point  $(x_0, y_0 = 2.7, 0)$ , and the ten points are distributing as follow, with the note that  $e = 2.7, f = 0$  and taking into account the negative sign.

$X = ax_0 + by_0 + e$	$Y = cx_0 + dy_0 + f$	$R = \sqrt{(x - x_0)^2 + (y - y_0)^2}$
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$$R = \left( \frac{f_{max} - f_{min}}{2} \right) \dots\dots\dots (10)$$

Where R radius of sub circle

### Power transparent from the Reticle disk

If we assume the use of a source of laser-energy  $650000 \text{ watt/m}^2$ . with spot size  $0.3 \text{ mm}$ , a large part of the energy of this package will be lost as a result of the processes of reflection and absorption as they pass in the transparent disk with a permeability of  $\tau_r = 0.9$ , since reticle consists of ten sections of the window area of each section  $S_n$  (are shown in Table 6), the power transparent  $P$  of each sector is given by the

$$p = Q_r S_n \tau_r \dots\dots\dots (11)$$

So the movement of any section in a circular motion takes approximately  $0.0001$  seconds (for the disc consists of 10 sections of dark does not allow passage the power and 10 section window allows passage power) that would lead to cut the signal on an ongoing basis every  $0.0001$  seconds as shown in Table (7) and Figs (16, 17), which represent the relationship between power transparent and the time, also it shows, that the power transparent is directly proportional with size of sector. Then we evaluate the modulation transfer function MTF by using equation (9) for normal and fractal reticle as shown in Table 8

To explain the Table 8 we draw the relationship between the frequency and radius we get the curve graph shown in Figs (18, 19, 20) that give the frequency decreases with increasing radius in Normal and Fractal Reticle.

By drawing the relationship between MTF and chopping frequency  $f_c$ , It gives the behavior of MTF repeat itself, and remains similar in both two models in the case of fractal Reticle, (see Fig 21) while Fig 22 shows that the MTF curve is less dramatically with increasing frequency.

### Conclusions

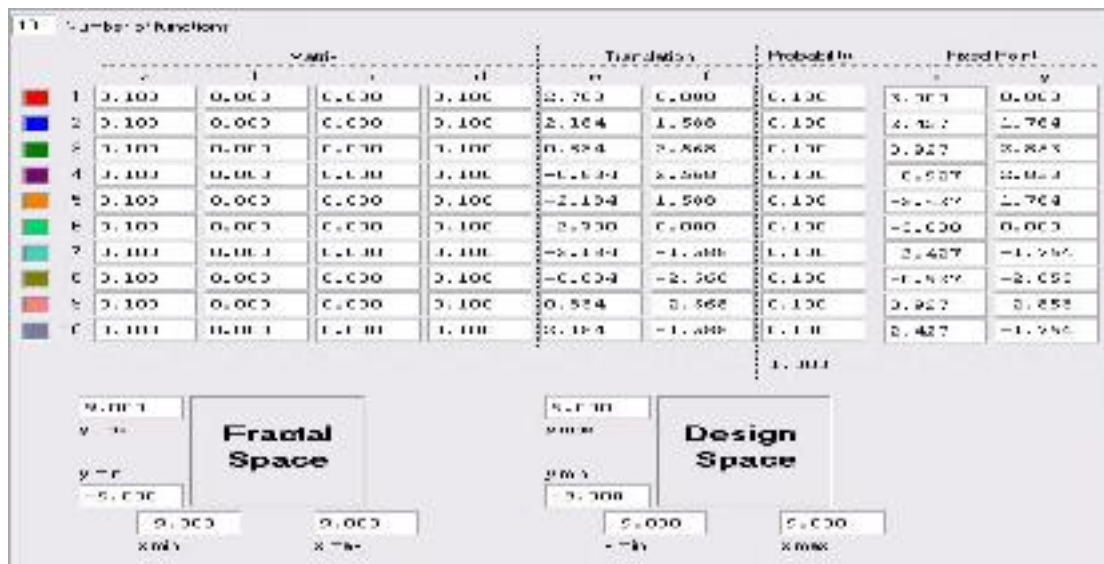
- 1- The rotation frequency is inversely proportional to the radius of rotation, in the case of the Normal Reticle note frequency less steadily with increasing radius in the beginning, but at radii large (at the end of the disk) we note a decrease of gradual frequency is similar to the decrease that was obtained in the case of Fractal Reticle
- 2- The MTF is inversely proportional with the rotational frequency
- 3- The best similarity of MTF behavior for normal and fractal Reticle was at the end of the Reticle
- 4- Power transparent depends on the size of the laser spot and the size of the sector, while the proportionality between them is directly
- 5- The type of supposed optical modulator can be defined by using the suitable spot size.
- 6- Pattern two of fractal reticle can be used to detect the target by using large size of spot size, and pattern one can be used to lock the target by using smaller size than spot size.
- 7- It is possible to design multi propos of modulator depending on multi patterns.

### References

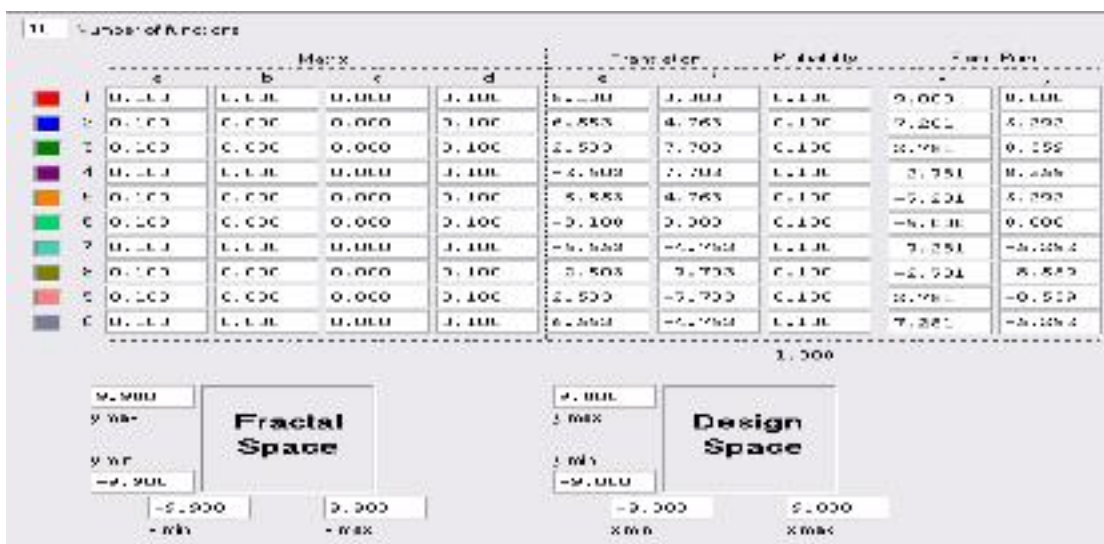
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**Table (1): The data of the first pattern**



**Table (2): The data of the second pattern**



**Table(3): The results of normal and Fractal Reticle disk**

State	Normal Reticle	Fractal Reticle	
		Pattern 1	Pattern2
radius	0.09 m	0.03 m	0.09 m
Time	0.002 sec	0.002 sec	0.002 sec
Number of sector	20	20	20
spot size of laser	0.5 mm <sup>2</sup>	0.5mm <sup>2</sup>	0.5mm <sup>2</sup>
Angle of sector	18 degree	18 degree	18 degree
Circumference	0.5652 m	0.1884 m	0.5652 m
Area of disk	0.025434 m <sup>2</sup>	0.002826 m <sup>2</sup>	0.025434 m <sup>2</sup>
Angular velocity	1744.44 rad/sec	5233.33 rad/sec	1744.44 rad/sec
Rotational frequency	277.77 rad/sec	833.33 rad/sec	277.77 rad/sec
Chopping frequency	2777.7 rad/sec	8333.3 rad/sec	2777.7 rad/sec

**Table(4): The relation between number of sector and frequency**

No. of sector	Normal Reticle		Fractal Reticle			
	Rotation frequency	Chopping frequency	Pattern 1		Pattern 2	
			Rotation frequency	Chopping frequency	Rotation frequency	Chopping frequency
1	0	0	0	0	0	0
2	277.77	2777.7	833.33	8333.3	277.77	2777.7
3	0	0	0	0	0	0
4	277.77	2777.7	833.33	8333.3	277.77	2777.7
5	0	0	0	0	0	0
6	277.77	2777.7	833.33	8333.3	277.77	2777.7
7	0	0	0	0	0	0
8	277.77	2777.7	833.33	8333.3	277.77	2777.7
9	0	0	0	0	0	0
10	277.77	2777.7	833.33	8333.3	277.77	2777.7
11	0	0	0	0	0	0
12	277.77	2777.7	833.33	8333.3	277.77	2777.7
13	0	0	0	0	0	0
14	277.77	2777.7	833.33	8333.3	277.77	2777.7
15	0	0	0	0	0	0
16	277.77	2777.7	833.33	8333.3	277.77	2777.7
17	0	0	0	0	0	0
18	277.77	2777.7	833.33	8333.3	277.77	2777.7
19	0	0	0	0	0	0
20	277.77	2777.7	833.33	8333.3	277.77	2777.7

**Table(5): Shows the method of calculating the radius of the Fractal**

Pattern 1			Pattern 2		
X	Y	R	X	Y	R
3	0	3	9	0	9
2.9427	0.1764	2.94798	8.8281	0.5292	8.8580
2.7927	0.2853	2.80723	8.3781	0.8559	8.4217
2.6073	0.2853	2.62286	7.8219	0.8559	7.8840
2.4573	0.1764	2.46362	7.3719	0.5292	7.3908
2.4	0	2.4	7.2	0	7.2
2.4573	-	2.46362	7.3719	-	7.3908
2.6073	-	2.62286	7.8219	-	7.8840
2.7927	-	2.80723	8.3781	-	8.4217
2.9427	-	2.94798	8.8281	-	8.8580

**Table (6): Data of sub sector for normal and fractal reticle**

State	Normal Reticle	Fractal Reticle	
		Pattern 1	Pattern2
radius of sub circle	0.09 m	0.0003 m	0.0009 m
Time	0.002 sec	0.002 sec	0.002 sec
Number of sector	20	20	20
spot size of laser	0.5 mm <sup>2</sup>	0.5mm <sup>2</sup>	0.5 mm <sup>2</sup>
Angle of sector	18 degree	18 degree	18 degree
Circumference sub sector	0.02826 m	0.001884m	0.005652 m
Area of sub sector	0.0012717 m <sup>2</sup>	282 x 10 <sup>-9</sup> m <sup>2</sup>	2543 x 10 <sup>-9</sup> m <sup>2</sup>
Power transparent	3.7197225 watt	0.00004133025	0.00037197225

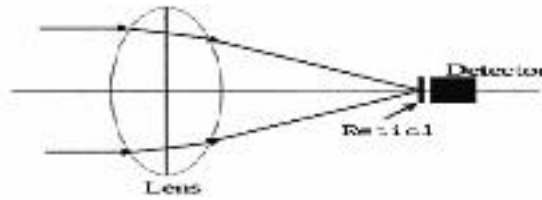


**Table (7): The power transparent of Reticle disk**

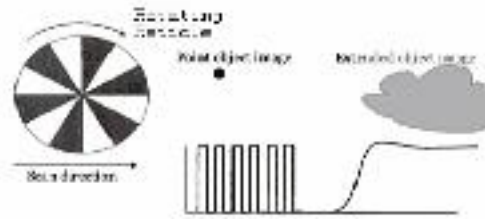
No. of sector	Time in sec	Normal reticle	Pattern 1 fractal reticle	Pattern 2 fractal reticle
		Power transparent in	Power transparent in	Power transparent in
1	0.0001	0	0	0
2	0.0002	3.7197225	0.00004133025	0.00037197225
3	0.0003	0	0	0
4	0.0004	3.7197225	0.00004133025	0.00037197225
5	0.0005	0	0	0
6	0.0006	3.7197225	0.00004133025	0.00037197225
7	0.0007	0	0	0
8	0.0008	3.7197225	0.00004133025	0.00037197225
9	0.0009	0	0	0
10	0.001	3.7197225	0.00004133025	0.00037197225
11	0.0011	0	0	0
12	0.0012	3.7197225	0.00004133025	0.00037197225
13	0.0013	0	0	0
14	0.0014	3.7197225	0.00004133025	0.00037197225
15	0.0015	0	0	0
16	0.0016	3.7197225	0.00004133025	0.00037197225
17	0.0017	0	0	0
18	0.0018	3.7197225	0.00004133025	0.00037197225
19	0.0019	0	0	0
20	0.002	3.7197225	0.00004133025	0.00037197225

**Table(8 ):The MTF of Normal and fractal Reticle**

Normal Reticle			Fractal Reticle					
R	Fc	MTF	Inner Pattern			Outer Pattern		
			R	Fc	MTF	R	Fc	MTF
0.009	27777.77	0.06	0.03	8333.33	0.2	0.09	2777.77	0.6
0.018	13888.88	0.12	0.02906	8602.89	0.193	0.08720	2866.97	0.581
0.027	9259.25	0.18	0.026477	9442.15	0.176	0.0794	3148.61	0.529
0.036	6944.44	0.24	0.02287	10931.35	0.152	0.0686	3644.31	0.457
0.045	5555.55	0.3	0.01946	12846.86	0.129	0.05840	4280.82	0.389
0.054	4629.62	0.36	0.018	13888.88	0.12	0.054	4629.62	0.36
0.063	3968.25	0.42	0.01946	12846.86	0.129	0.05840	4280.82	0.389
0.072	3472.22	0.48	0.02287	10931.35	0.152	0.0686	3644.31	0.457
0.081	3086.41	0.54	0.026477	9442.15	0.176	0.0794	3148.61	0.529
0.09	2777.77	0.6	0.02906	8602.89	0.193	0.08720	2866.97	0.581



Fig(1): simple Reticle optical system



Fig(2): The Reticle system scans the scene the incoming radiation is modulated.

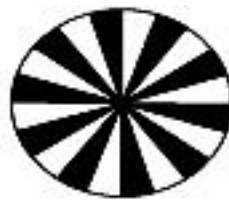


Fig (3) The Normal optical modulator



Fig(4): The initial shape of the first pattern



Fig(5): The first pattern after 1 iteration



Fig(6): The first pattern after 10 iteration



**Fig(7):** The initial shape of the second pattern



**Fig(8):** The second pattern after 1 iteration



**Fig(9):** The second pattern after 10 iteration



**Fig (10):** The fractal optical modulator

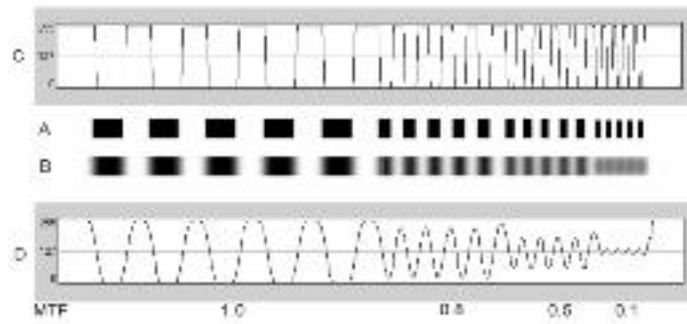


Fig. (11) [A] original test pattern [B] image of the test pattern [C] line profile of the original test pattern where 255=white and 0=black [D] line profile of the image of the test pattern where 255=white and 0= black



Fig.(12) The Disk optical modulator Program

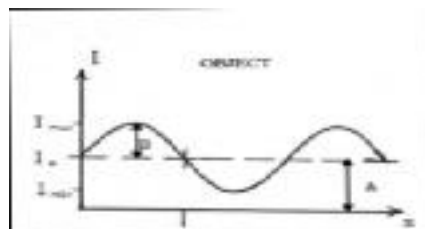


Fig. (13): The shape of wave

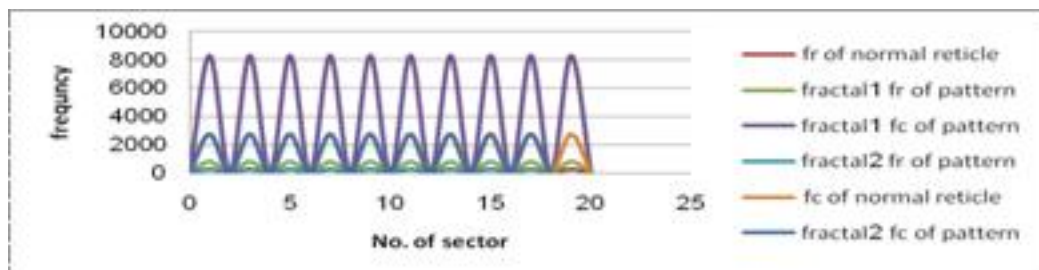


Fig.( 14) :The relation between No. of sector versus frequency

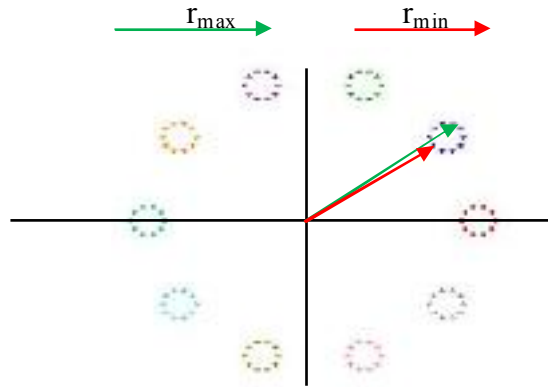


Fig. (15): The minimum and maximum radius of sub sector

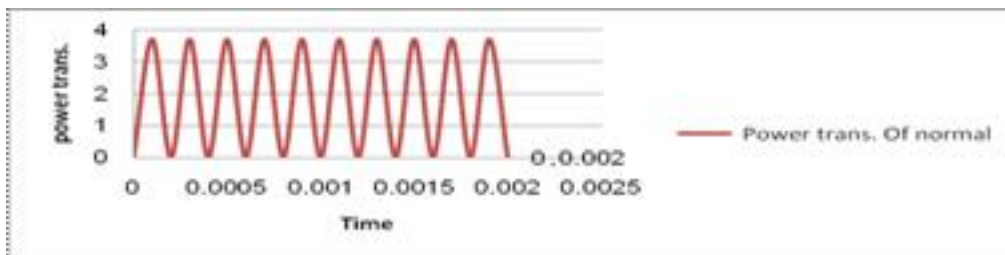


Fig. (16): The relationship between power transparent and the time for Normal Reticle

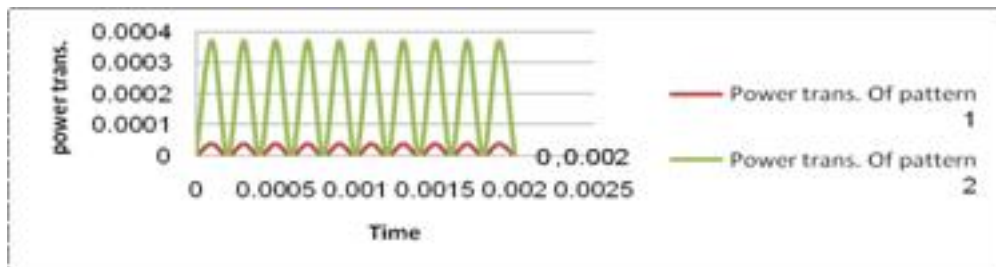


Fig. (17): The relationship between power transparent and the time for fractal Reticle

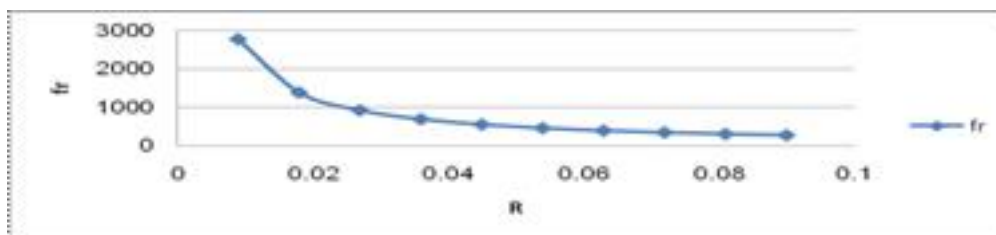


Fig (18) : Normal Reticle :The frequency decreases with increasing radius

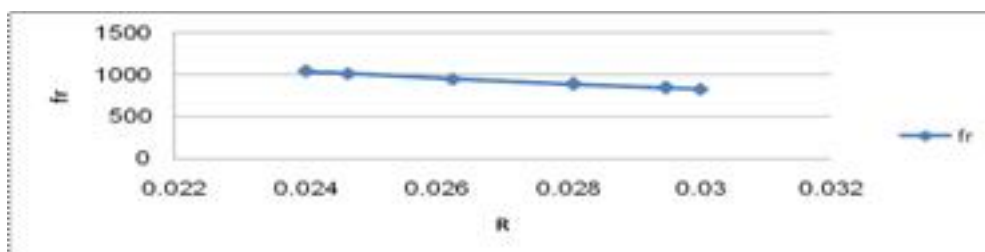


Fig. (19) : Fractal Reticle(pattern 1) :The frequency decreases with increasing radius

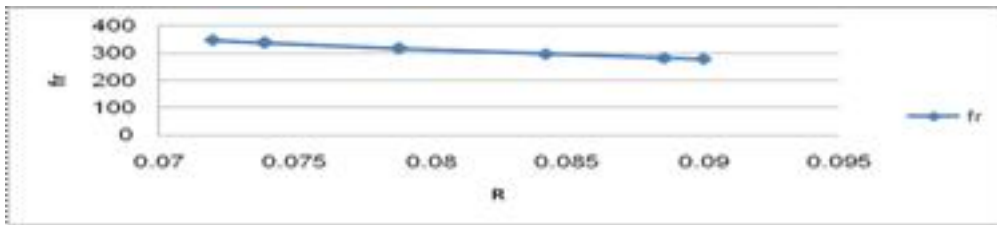


Fig. (20): Fractal Reticle(pattern 2) :The frequency decreases with increasing radius

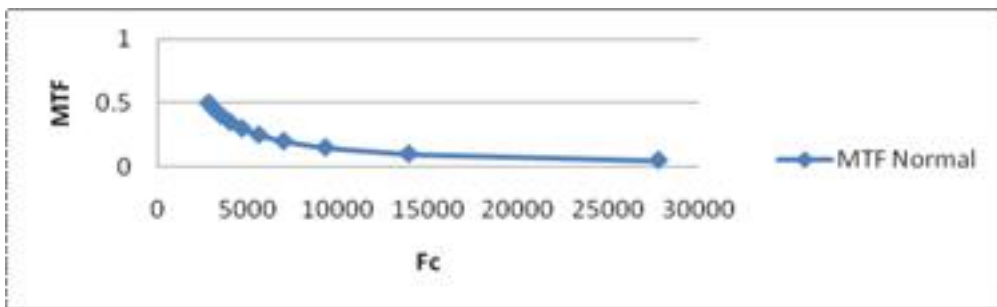


Fig. (21): The MTF versus fc with spot size 0.0005(Normal Reticle)

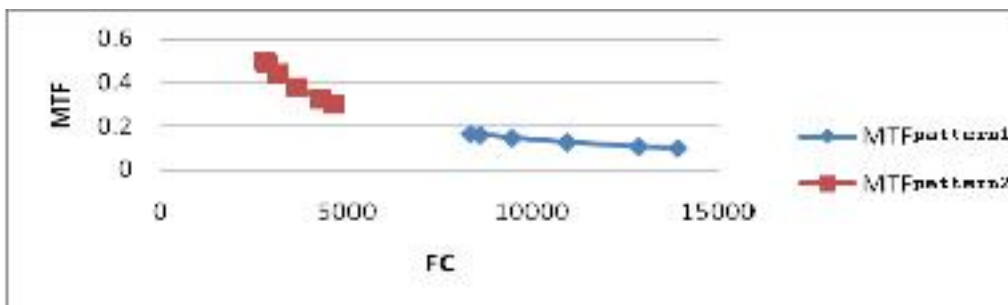


Fig. (22): The MTF versus fc with spot size 0.0005(fractal Reticle)

## تصميم جديد للتضمين البصري الكسوري

عبد الرزاق عبد السلام محمد ،خالد هلال حربي،ثائر عبد الكريم خليل العايش

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قبل البحث في: 23 ، اذار، 2011

## الخلاصة

في هذه البحث وضع تصميم جديد لقرص التضمين البصري الكسوري باستعمال تكرار جديد للدالة الكسورية ، وتم تحليل النتيجة عن طريق حساب دالة الانتقال الضمني ، وذلك بالمقارنة مع نتائج قرص التضمين البصري العادي.

وقرصا التضمين البصري العادي والكسوري عبارة عن قرص دائري ذي نصف قطر يساوي 9cm ، وكل منهما يتكون من 20 مقطعاً ، عشرة منها مضيئة والاخرى مظلمة.

وقرص التضمين الكسوري يحتوي على أنموذجين الانموذج الآخريستعمل للكشف عن الهدف ، والانموذج الاول يستعمل للقفل على الهدف.

وافضل تشابه لسلوك دالة الانتقال الضمني بين قرصا التضمين الاعتيادي والكسوري كان عند نهاية القرص العادي. كذلك فإن القدرة النافذة من القرصين تعتمد على حجم بقعة الليزر الساقطة وحجم المقطع الكسوري .وان التناسب بينهما طردي.

الكلمات المفتاحية : التضمين البصري الكسوري ، تردد القطع، دالة الانتقال الضمني

