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Semiprime Fuzzy Modules

M. A.Hamil
Department of Mathematics , College of Education -Ibn-Al-Haitham
University of Baghdad

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Abstract

In this paper we introduce the notion of semiprime fuzzy module as a generalization of semiprime module. We investigate several characterizations and properties of this concept.

Key Words: Prime fuzzy module, semiprime module, semiprime fuzzy module.

Introduction

The notion of fuzzy subsets of a set $S \neq \phi$ as a function from S into [0,1] was first developed by Zadeh [1]. The concept of fuzzy modules was introduced by Negoita and Ralescu in [2]. The concept of fuzzy submodule was introduced by Mashinch and Zahedi [3]. The concept of fuzzy ideal of a ring by Liu in [4]. Dauns in [5] introduced the notion of semiprime submodules as a generalization of semiprime ideals of a ring. Eman in [6] studied semiprime submodules. I.M.Hadi in [7] introduced the notion of semiprime fuzzy ideals of a ring also introduced semiprime fuzzy submodules of fuzzy module in [8]. Frias in [9] studies semiprime module. In this paper we introduce the notion of semiprime fuzzy modules as a generalization of R-semiprime modules and give many properties of this concept.

Throughout this paper R is commutative ring with unity, M is an R-module and X is a fuzzy module of an R-module M.

1- Preliminaries

In this section, we shall formulate the preliminary definitions and results that are required later in this paper.

1.1 Definition: [1]

Let S be a non-empty set. A fuzzy set A in S (a fuzzy subset of S) is a function from S into [0,1].

1.2 Definition: [2]

Let $x_t:S \longrightarrow [0,1]$ be a fuzzy set in S, where $x \in S$, $t \in [0,1]$ defined by:

$$x_t(y) = \begin{cases} t & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$$

for all $y \in S$. x_t is called a fuzzy singleton.

1.3 Proposition: [3]

Let a_t, b_k be two fuzzy singletons of a set S. If $a_t = b_k$, then $a = b$ and $t = k$, where $t, k \in [0,1]$.

1.4 Definition: [4]

Let A and B be two fuzzy sets in S, then:

1- $A = B$ iff $A(x) = B(x)$, for all $x \in S$.
2- $A \subseteq B$ iff $A(x) \le B(x)$, for all $x \in S$.
3- $(A \cap B)(x) = \min\{A(x), B(x)\}$, for all $x \in S$, [2].

1.5 Definition: [5]

Let A be any fuzzy set in S for all $t \in [0,1]$, the set $A_t = \{x \in S, A(x) \ge t\}$ is called a level subset of A.

1.6 Remark: [1]

The following properties of level subsets hold for each $t \in [0,1]$.

1- $(A \cap B)_t = A_t \cap B_t$.
2- $A = B$ iff $A_t = B_t$.

1.7 Definition: [1]

Let f be a mapping from a set M into a set N, let A be a fuzzy set in M and B be a fuzzy set in N. The image of A denoted by f(A) is the fuzzy set in N defined by:

$$f(A)(y) = \begin{cases} \sup\{A(z) : z \in f^{-1}(y)\} & \text{if } f^{-1}(y) \ne \phi \text{ for all } y \in N, \\ 0 & \text{otherwise} \end{cases}$$

And the inverse image of B, denoted by $f^{-1}(B)$ is the fuzzy set in M defined by:

$f^{-1}(B)(x) = B(f(x))$, for all $x \in M$.

1.8 Definition: [2]

Let M be an R-module. A fuzzy set X of M is called fuzzy module of an R-module M if:

1- $X(x - y) \ge \min\{X(x), X(y)\}$ for all $x, y \in M$.

2- $X(rx) \ge X(x)$, for all $x \in M$ and $r \in R$.

3- $X(0) = 1$.

1.9 Definition: [5]

Let X and A be two fuzzy modules of R-module M. A is called a fuzzy submodule of X if $A \subseteq X$.

1.10 Proposition: [6]

Let A be a fuzzy set of an R-module M. Then the level subset A_t, $t \in [0,1]$ is a submodule of M iff A is a fuzzy submodule of X, where X is a fuzzy module of an R-module M.

1.11 Remark: [5]

If X is a fuzzy module of an R-module M and $x_t \subseteq X$ then for all fuzzy singleton r_k of R, $r_k\, x_t = (rx)_\lambda$, where $\lambda = \min\{k,t\}$.

1.12 Definition: [7]

A fuzzy subset K of a ring R is called a fuzzy ideal of R if for each $x, y \in R$:

1- $K(x-y) \geq \min\{K(x),K(y)\}$.
2- $K(x \cdot y) \geq \max\{K(x),K(y)\}$.

1.13 Proposition: [7]

A fuzzy subset K of a ring R is a fuzzy ideal iff K_t, $t \in [0,1]$ is an ideal of R.

1.14 Definition: [2]

Let A and B be two fuzzy submodules of a fuzzy module X of an R-module M. The residual quotient A and B denoted by (A:B) is the fuzzy subset of R defined by:
$(A:B)(r) = \sup\{t \in [0,1], r_tB \subseteq A\}$ for all $r \in R$.
That is $(A:B) = \{r_t : r_tB \subseteq A, r_t$ is fuzzy singleton of $R\}$.

1.15 Theorem: [2]

Let A and B be two fuzzy submodules of a fuzzy module X of an R-module M. Then the residual quotient (A:B) of A and B is a fuzzy ideal of R.

1.16 Definition: [8]

Let A be fuzzy submodule of a fuzzy module X. The fuzzy annihilator of A denoted by F-annA is defined by:
$(F\text{-}annA)(r) = \sup\{t : t \in [0,1], r_tA \subseteq O_1\}$ for all $r \in R$. That is $F\text{-}annA = (O_1:A)$.

1.17 Definition: [9]

Let X and Y be two fuzzy modules of M_1 and M_2 respectively defined $X \oplus Y : M_1 \oplus M_2 \longrightarrow [0,1]$ by $(X \oplus Y)(a,b) = \min\{X(a),Y(b)\}$ for all $(a,b) \in X \oplus Y$.
$X \oplus Y$ is called a fuzzy external direct sum of X and Y.

1.18 Proposition: [9]

Let X and Y be fuzzy modules of M_1 and M_2 respectively then $X \oplus Y$ is a fuzzy module of $M_1 \oplus M_2$.

1.19 Remark: [9]

Let A and B be two fuzzy submodules of a fuzzy module X such that $X = A \oplus B$, then $X_s = A_s \oplus B_s$, for all $s \in [0,1]$.

1.20 Definition: [9]

A fuzzy module X of an R-module M is called a prime fuzzy module if F-annA = F-annX, for any nontrivial fuzzy submodule A of X.

2- Semiprime Fuzzy Modules

Firas in [9] introduced the concept of semiprime R-module (where M is called a semiprime module if for each $r \in R$, $x \in M$, $r^2x \subseteq M$ implies $rx \subseteq M$.. We shall fuzzify this concept in definition 2.3. But first we give the two definitions:

2.1 Definition: [7]

Let A be a non constant fuzzy ideal of a ring R. A is called semiprime fuzzy ideal if for any fuzzy singleton $x_t \in R$, $x_t^2 \in A$, implies $x_t \in A$.

2.2 Definition: [8]

Let A be a fuzzy submodule of a fuzzy module X of an R-module M such that $A \neq X$, A is called semiprime fuzzy submodule if for each fuzzy singletone $r_t \in R$, $x_s \subseteq X$, $r_t^2 x_s \subseteq A$ implies $r_t x_s \subseteq A$.

2.3 Definition:

Let X be a fuzzy module of an R-module M, X is called semiprime fuzzy module if for each non-zero fuzzy submodule A of X, F-annA is a semiprime fuzzy ideal of R.

2.4 Remarks:

1- Every prime fuzzy module X is a semiprime fuzzy module.

Proof: Let A be a fuzzy submodule of X. Since X is prime, hence F-annA is a prime fuzzy ideal by [17] which implies F-annA is a semiprime fuzzy ideal by [7]. Thus X is a semiprime fuzzy module.

2- If X is a semiprime fuzzy module, then F-annX is a semiprime fuzzy ideal.

Proof: It is clear by definition 2.3, so is omitted.

The following is a characterization of semiprime fuzzy module.

2.5 Proposition:

Let X be a fuzzy module of an R-module M. Then X is a semiprime fuzzy module if and only if X_t is a semiprime module, $\forall\ t \in [0,1]$.

Proof: ($\Rightarrow$) Let $N \subseteq X_t$, $t \in [0,1]$. To prove $ann_R N$ is a semiprime ideal of R. Let $a^2 \in ann_R N \subseteq X_t$. Since $a^2 \in ann_R N$, then $a^2 N = 0$, let $x \in N$, hence $a^2 x = 0$. Assume $X(x) = k$. Hence $x_k \in X$, so $\langle x_k \rangle \subseteq X$. But F-ann$\langle x_k \rangle$ is a semiprime fuzzy ideal. and $a_k^2 x_k = (a^2 x)_k \subseteq O_k \subseteq O_1$. Thus $a_k^2 \in$ F-ann$\langle x_k \rangle$. Since F-ann$\langle x_k \rangle$ is a semiprime fuzzy ideal. Thus $a_k \in$F-ann$\langle x_k \rangle$, hence $a_k x_k \subseteq O_1$, so $(ax)_k = O_k \subseteq O_1$. Thus $ax = 0$, for any $x \in N$.

($\Leftarrow$) Conversely, to prove F-annA is a semiprime fuzzy ideal of R for each non zero fuzzy submodule A of X. Let $r_k^2 \in$F-annA, so $r_k^2 x_t \subseteq O_1$, for all $x_t \in A$. This implies $(r^2 x)_\lambda = 0$, where $\lambda = \min\{k,t\}$ hence $r^2 x = 0$, $x \in A_t$. But $A_t \subseteq X_t$ and X_t is semiprime by hypothesis. Hence $rx = 0$. This implies $(rx)_\lambda \subseteq O_1$. That is $r_k x_t \subseteq O_1$. Therefore $r_k \in$ F-annA. By def. (2.3) we get the result.

The following proposition gives another characterization of semiprime fuzzy module.

2.6 Proposition:

Let X be a fuzzy module of an R-module M. Then X is a semiprime fuzzy module if and only if O_1 is a semiprime fuzzy submodule of X.

Proof: (⇒) Let $r_t^2 x_k \subseteq O_1$, for any fuzzy singleton r_t of R, $x_k \subseteq X$, hence $r_t^2 \in (O_1{:}x_k)$ = F-ann<x_k>. But F-ann<x_k> is a semiprime fuzzy ideal. Thus $r_t \in (O_1{:}x_k)$, so $r_t x_k \subseteq O_1$. Thus O_1 is a semiprime fuzzy submodule.

(⇐) To prove X is a semiprime fuzzy module. By proposition 2.5. It is enough to show that X_t is semiprime, ∀ t ∈ [0,1]. (i.e.) to prove {0} is semiprime submodule of X_t by [9,Th.4.1.8]. Let $r^2 = 0$, to prove r = 0, hence $r_t^2 = O_t \subseteq O_1$, $(r_t)^2 \subseteq O_1$, hence $r_t^2 \subseteq O_1$, which implies $r_t \subseteq O_1$, since O_1 is semiprime. Thus r = 0, hence X_t is a semiprime module, ∀ t ∈ [0,1]. Thus X is a semiprime fuzzy module.

Next we can give some examples of semiprime and not semiprime fuzzy module.

2.7 Examples:

1- Let $X{:}Z_6 \longrightarrow [0,1]$ defined by X(a) = 1, ∀ a ∈ Z_6. $X_t = Z_6$, which is a semiprime module, ∀ t ∈ [0,1]. Thus by prop. (2.5),X is a semiprime fuzzy module.

2- Let $X{:}Z_6 \longrightarrow [0,1]$ defined by

$$X(x) = \begin{cases} 1 & \text{if } x \in \{\bar{0}, \bar{3}\} \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

$X_0 = Z_6$, $X_{\frac{1}{2}} = Z_6$ which is semiprime and ∀ $t > \frac{1}{2}$, $X_t = \{\bar{0}, \bar{3}\}$ is a prime submodule, hence it is semiprime. Thus X_t is a semiprime module, ∀ t ∈ [0,1]. Thus X is a semiprime fuzzy module.

3- Let $X : Z_{12} \longrightarrow [0,1]$ defined by:

$$X(a) = \begin{cases} 1 & \text{if } a \in \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}\} \\ 0 & \text{otherwise} \end{cases}$$

It is clear that X is a fuzzy module and $X_0 = Z_{12}$, is not semiprime module. By prop.(2.5) X is not semiprime fuzzy module.

2.8 Lemma:

Let A and B be two fuzzy submodules of fuzzy module of an R-module M. If for each $x_t \in B$, [A:<x_t>] is a semiprime fuzzy ideal of R, then [A:B] is a semiprime fuzzy ideal of R.

Proof: Let $a_k^2 \subseteq$ [A:B], hence $a_k^2 B \subseteq A$. This implies $a_k^2 x_t \in A$, for all $x_t \in B$. Hence $a_k^2 \in$[A:<x_t>] which is a semiprime fuzzy ideal. Thus $a_k \in$ [A:<x_t>]. So $a_k x_t \in A$, hence $a_k B \subseteq A$. Thus [A:B] is a semiprime fuzzy ideal.

2.9 Proposition:

Let X be a fuzzy module of an R-module M. Then the following are equivalent:

1- X is semiprime.
2- [F-annA:B] is a semiprime fuzzy ideal of R for every nonzero fuzzy submodule A of X and for every non-zero fuzzy ideal B of R such that F-annA $\subseteq$ F-annB.
3- [F-annA:x_t] is a semiprime fuzzy ideal of R for every non-zero fuzzy submodule A of X and for every fuzzy singleton $x_t \in R$ such that $x_t \not\subseteq$ F-annA.
4- F-ann(x_k) is a semiprime fuzzy ideal of R for non-zero fuzzy singleton $x_k \in X$.

Proof: (1) $\Rightarrow$ (2) Let $r_k^2 \in$ [F-annA:B], hence $r_k^2 B \subseteq$ F-annA. So $r_k^2 b_s \subseteq$ F-annA, for all $b_s \in B$. Hence $r_k^2 b_s^2 \subseteq$ F-annA, so $(rb)_\lambda^2 \subseteq$ F-annA, where $\lambda = \min\{k,s\}$. Thus $(rb)_\lambda \subseteq$ F-annA, since F-annA is a semiprime fuzzy ideal. So $r_k b_s \in$ F-annA. Hence $r_k \in$ [F-annA:B]. Thus [F-annA:B] is a semiprime fuzzy ideal.

(2) $\Rightarrow$ (3) It is followed by putting $\langle x_t \rangle = B$.

(3) $\Rightarrow$ (4) It is easy to check that [F-annx_t:$\langle 1_1 \rangle$] = F-ann$\langle x_t \rangle$. But [F-ann$\langle x_t \rangle$:$\langle 1_1 \rangle$] is a semiprime fuzzy ideal by (3). Thus F-annx_t is a semiprime fuzzy ideal.

The following proposition shows that the direct sum of semiprime fuzzy modules is semiprime fuzzy module.

2.10 Proposition:

Let X and Y be two fuzzy modules of M_1 and M_2 R-modules respectively. Then X and Y are semiprime if and only if $X \oplus Y$ is a semiprime fuzzy module.

Proof: ($\Rightarrow$) If X and Y are semiprime, then X_t and Y_t are semiprime modules by proposition 2.5. Hence $X_t \oplus Y_t$ is a semiprime module by [9,prop.4.1.11]. But $X_t \oplus Y_t = (X \oplus Y)_t$ by remark 1.19. Thus $X \oplus Y$ is a semiprime fuzzy module by proposition 2.5.
($\Leftarrow$) The proof is similarly.

Now we turn our attention to image and inverse image of semiprime fuzzy module.
We have the following:

2.11 Proposition:

Let X and Y be two fuzzy modules of R-modules M_1 and M_2 respectively. Let $f: M_1 \longrightarrow M_2$ be R-homomorphism, then

3- Semiprime Fuzzy Modules and Other Related Fuzzy Modules

In this section we study the relationship between semiprime fuzzy module and divisible, uniform and F-regular fuzzy modules.

3.1 Definition: [17]

A fuzzy module X is divisible if $r_t X = X$, for all $r_t \neq O_t$ (r_t is a fuzzy singleton of R).

3.2 Definition: [17]

A fuzzy module is called uniform if $A \cap B \neq O_1$, for any non trivial fuzzy submodules A and B.

3.3 Definition: [17]

Let A be a fuzzy submodule of fuzzy module X. Then A is called an essential fuzzy submodule if $A \cap B \neq O_1$, for any nontrivial fuzzy submodule B of X.

3.4 Proposition:

Let X be a uniform fuzzy module. Then X is a prime fuzzy module if and only if X is semiprime fuzzy module.

Proof: ($\Rightarrow$) It is easy by prop.2.2.

($\Leftarrow$) To prove F-annX = F-annA, for any non trivial fuzzy submodule A of X [17,Def.3.1.1]. It is clear that F-annX $\subseteq$ F-annA. To prove F-annA $\subseteq$ F-annX.

Let $r_t \in$ F-annA and $r_t \notin$ F-annX. Thus there exists $x_k \in X$, $x_k \neq 0_k$ such that $r_t x_k \nsubseteq O_1$. Since X is uniform, $A \cap \langle r_t x_k \rangle \neq O_1$, then there exists $y_s \in A$ and $y_s \in \langle r_t x_k \rangle$ such that $y_s \neq O_1$. Thus $y_s = a_\ell r_t x_k$, a_ℓ is a fuzzy singleton of R. $O_1 = r_t y_s = a_\ell r_t^2 x_k$, it follows $r_t^2 \in$ F-ann$\langle a_\ell x_k \rangle$ since $a_\ell x_k \neq O_1$, so F-ann$\langle a_\ell x_k \rangle$ is a semiprime fuzzy ideal of R. Therefore $r_t \in$ F-ann$\langle a_\ell x_k \rangle$. This implies that $O_1 = r_t a_\ell x_k = y_s$. Thus $y_s = O_1$ which is a contradiction.

3.5 Proposition:

If X is a uniform fuzzy module, then X_t is a uniform module, $\forall$ t $\in$ (0,1].

Proof: Let N and W be submodules of X_t such that N $\neq$ O, W $\neq$ O. Define A: M $\longrightarrow$ [0,1], B:M $\longrightarrow$ [0,1] by

$$A(x) = \begin{cases} t & \text{if } x \in N, t \neq 0 \\ 0 & \text{otherwise} \end{cases}, \quad B(x) = \begin{cases} t & \text{if } x \in W, t \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

This implies A and B are fuzzy submodules of X and $A_t = N$, $B_t = W$, for all t $\in$ (0,1]. Since X is uniform, then $A \cap B \neq O_1$. But $(A \cap B)(x) = \begin{cases} t & \text{if } x \in N \cap W \\ 0 & \text{if } x \notin N \cap W \end{cases}$.

On the other hand, $(A \cap B)_t = A_t \cap B_t = N \cap W$. Hence $N \cap W \neq \{0\}$. Thus X_t is a uniform module, $\forall$ t $\in$ (0,1].

Recall that an R-submodule N of module M is called quasi-invertible if Hom($\frac{M}{N}$,M) = 0. And an R-module M is called quasi-Dedekind if every non-zero R-submodule of M is quasi-invertible(18).

3.6 Proposition:

Let X be a uniform and semiprime fuzzy module. Then X_t is a quasi-Dedekind module, $\forall$ t $\in$ (0,1].

Proof: X is uniform, implies X_t is uniform $\forall$ t $\in$ (0,1] and X is semiprime, then X_t is semiprime by prop. 2.5. Thus X_t is a quasi-Dedekind module $\forall$ t $\in$ (0,1] by [9,prop.2.4].

3.7 Proposition:

Let X be a fuzzy module of an R-module M such that every fuzzy submodule of X is divisible, then X is semiprime.

Proof: Let $O_1 \neq x_t \in X$. It is enough to show that F-ann$\langle x_t \rangle$ is a semiprime fuzzy ideal by prop.(2.9)(4). Let $r_k^2 \in$ F-ann$\langle x_t \rangle$, hence $r_k^2 x_t = O_1$. But $\langle x_t \rangle$ is a divisible fuzzy submodule. Then $\langle x_t \rangle = r_k \langle x_t \rangle$; r_k is a fuzzy singleton of R. Hence $x_t = r_k c_\ell x_t$, $c_\ell \in X$. So $r_k x_t = r_k r_k c_\ell x_t = r_k^2 c_\ell x_t$, but $r_k^2 c_\ell x_t = c_\ell r_k^2 x_t = c_\ell O_1 = O_1$. Thus $r_k x_t = O_1$. So $r_k \in$ F-ann$\langle x_t \rangle$. Thus F-ann$\langle x_t \rangle$ is a semiprime fuzzy ideal. Thus X is semiprime fuzzy module.

3.8 Corollary:

Let X be a fuzzy module and every fuzzy submodule of X is divisible. Then X is a prime fuzzy module.

Proof: By prop.3.8, X is semiprime. But X is divisible, then X is prime by [17,prop.3.1.13].

3.9 Proposition:

Let X be an F-regular fuzzy module of an R-module M ,where R is a principle ideal domain. Then X is a semiprime fuzzy module.

Proof: Since X is F-regular, then X_t is F-regular $\forall\ t \in (0,1]$ by [11]. Hence X_t is a semiprime module $\forall\ t \in (0,1]$ by [9,prop.4.2.6]. Thus X is a semiprime fuzzy module.

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المقاسات شبه الأولية الضبابية

ميسون عبد هامل

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الخلاصة

في هذا البحث قدم مفهوم المقاسات شبه الاولية الضبابية اعماما" للمقاسات شبه الاولية.ثم اعطيت العديد من التشخيصات والخواص لهذا المفهوم.

الكلمات المفتاحية: المقاس الأولي الضبابي ، المقاس شبه الأولي ، المقاس شبه الأولي الضبابي.