On bg**- Connected Spaces

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Abstract
In this paper, we define the bg**-connected space and study the relation between this space and other kinds of connected spaces .Also we study some types of continuous functions and study the relation among (connected space, b-connected space, bg-connected space and bg**-connected space) under these types of continuous functions.

Key words: bg**-closed set, bg**-connected space.

1.Introduction

The notion of b-open set was introduced in 1996 [1], since then it has been widely investigated in the literature (see [1], [8]) .A.M. [7] introduce the concepts (bg**-open set, bg**-continuous function and bg**-irresolute function). The concepts (b-connected space and bg-connected space) were introduced in [9] and [6] respectively. In this work,we introduce the concept of bg**-connected space and study its relations with (b-connected and bg-connected space) .Also we study some types of continuous function which are: (b-continuous function, bg-continuous function and bg**-continuous function), and study the image of (connected space, b-connected space, bg-connected space and bg**-connected space) under these types of functions.

2. Preliminaries

Throughout the paper X and Y represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. We recall the following definitions, which are useful in the sequel.

Definition 2.1: [1] A subset A of a topological space X is said to be **b-open** if $A \subseteq cl(int(A)) \cup int(cl(A))$. And A is said to be **b-closed** set if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

Definition 2.2 : [5] A subset A of a topological space X is said to be **bg-closed** if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. A will be called **bg-open** if its complement is bg-closed.

Definition 2.3 : [2] A subset A of a topological space X is said to be **g**-open** if and only if there exists an open set U of X such that $U \subseteq A \subseteq cl^{**}(U)$, and A is said to be g**- closed if its complement is g**-open set, where $cl^{**}(U) = \cap\{ F: F \text{ is g-closed and } U \subseteq F\}$.

Definition 2.4 : [7] A subset A of a topological space X is said to be **bg**-closed** if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g**-open. The set of all bg**-closed sets of X denoted by bG**C(X).

A subset A of X is called **bg**-open** if X – A is bg**-closed in X.

Example : If $X= \{a, b, c\}$,$\tau =\{\phi, X, \{a\}\}$, then the set of all bg**-closed sets of X bG**C(X) are $\{\phi, X,\{b\},\{c\},\{b, c\}\}$.

Definition 2.5: A map $f : X \to Y$ from a topological space X into a topological space Y is called:

1) a **b-continuous** if $f^{-1}(V)$ is b-closed set in X for every closed set V of Y. [3]

2) a **bg-continuous** if $f^{-1}(V)$ is bg-closed in X for every closed set V of Y . [6]

3) a **bg**-continuous** if $f^{-1}(V)$ is a bg**-closed set in X for every closed set V in Y.[7]

4) a **bg**-irresolute** if $f^{-1}(V)$ is a bg**-closed set in X for every bg**-closed set V in Y.[7]

Remark 2.6 :

1- Every open set is b-open (bg-open) [5].
2- Every bg-open set is b-open [6].
3- Every bg-open set is bg**-open [7].
4- Every bg**-open set is b-open [7].

3. On bg**- Connected Space

In this section we introduce the concept of bg**-connected space, and study some of their properties .Also we study the relation between it and (b-connected and bg-connected space).

Definition 3.1: [1] A topological space X is said to be **b-connected** if X can not be expressed as a disjoint union of two non-empty b-open sets. A subset of X is b-connected if it is b-connected as a subspace.

Definition 3.2: [6] A topological space X is said to be **bg-connected** if X can not be expressed as a disjoint union of two non-empty bg-open sets. A subset of X is bg-connected if it is bg-connected as a subspace.

Definition 3.3: A topological space X is said to be **bg**-connected** if X can not be expressed as a disjoint union of two non-empty bg**-open sets, otherwise X is called (bg**-disconnected space).

A subset of X is bg**-connected if it is bg**-connected as a subspace.

Example : Let X = {a, b ,c} and let τ = {X, φ, {a}}. Then X is bg**-connected.

Remark 3.4:

1- Every b-connected space is connected.
2- Every bg-connected space is connected.

Proof: By remark 2.6.

Theorem 3.5:

(i) Every b-connected space is bg**-connected.
(ii) Every bg**-connected space is bg-connected.

Proof : (i) Let X be b-connected space .Suppose that X is not bg**-connected. Then there exist disjoint non-empty bg**-open sets A and B such that X= A∪B . By Remark 2.6(4), A and B are b-open sets . This is a contradiction with X is b-connected .Therefore X is bg**-connected.
(ii) Its clear from Remark 2.6(3), and by the same way of proof (i).

Remark 3.6. From Theorems 3.5 and Remarks 3.4, we have diagram (1) .

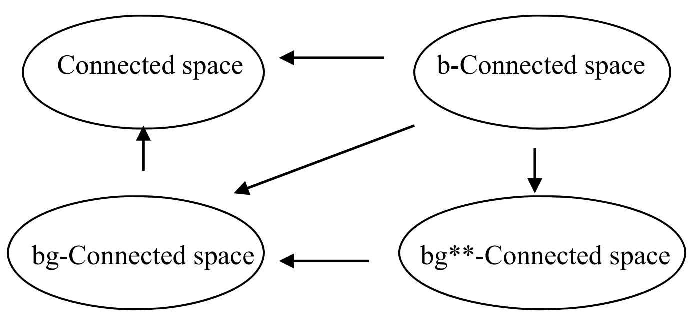

Diagram (1): The relationships between connected space ,b-connected space ,bg-connected space and bg-connected space .**

Theorem 3.7: For a topological space X, the following statements are equivalent.

1- X is bg**-connected
2- The only subsets of X which are both bg**-open and bg**-closed are the empty set and X.
3- Each bg**-continuous map of X into a discrete space Y with at least two points is a constant map

Proof: (1)→ (2) Let U be a bg** -open and bg** closed subset of X. then X- U is both bg** -open and bg** -closed . Since X is the disjoint union of bg** - open sets U and X-U, then one of these must be empty, that is U = ϕ or X – U = ϕ.
(2)→ (1) Suppose that X = A∪ B where A and B are disjoint non empty bg** - open sets of X, then A is both bg**-open and bg**-closed subset of X. By assumption, A=ϕ or A = X. This implies X is bg**-connected.
(2)→(3)Let f:X → Y be a bg**-continuous map, then X is covered by bg**-open and bg**–closed covering $\{f^{-1}(y):y\in Y\}$. By assumption $f^{-1}(y) = \phi$ then f fails to be bg**-continuous. Therefore $f^{-1}(y) = X$. This implies f is a constant map.
(3)→ (2) Let U be both bg**-open and bg**-closed in X. Suppose U ≠ ϕ .Let f: X → Y be bg**-continuous map defined by f(U) = {y} and f(X-U) = {w} for some distinct points y and w in Y. By assumption, f is a constant map. Therefore we have U = X

Theorem 3.8:

(i)If f:X→Y is a bg**-continuous surjection map and X is bg**-connected, then Y is connected.
(ii)If f:X→ Y is a bg**-irresolute surjection map and X is bg**-connected, then Y is bg**–connected.

Proof:(i) Suppose that Y is not connected , then Y= A∪ B where A and B are disjoint non-empty open sets in Y. Since f is bg**-continuous and onto , $X = f^{-1}(A)\cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty bg**-open sets which is a contradiction to our assumption that X is bg**-connected . Hence Y is connected.
(ii) It follows from the definition of bg**-irresolute map.

4. On Some Types of Continuous Functions & bg**-Connected Space

In this section we study some types of continuous functions, and study the relations between (connected space, b-connected space, bg-connected space and bg**-connected space) under these types of continuous functions.

Theorem 4.1: [8] Continuous image of connected space is connected.

Theorem 4.2:
(i) Continuous image of b-connected space is connected.
(ii) Continuous image of bg-connected space is connected.

Proof: (i) Let f : X→ Y be continuous function ,and let X be b-connected space. To prove Y is connected.Suppose that Y is disconnected space ,then Y= A∪B, where A and B are disjoint non-empty open sets in Y.Since f is continuous $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty open sets in X ,and $f^{-1}(A)$ and $f^{-1}(B)$ are b-open sets in X such that $X = f^{-1}(A)\cup f^{-1}(B)$ (by Remark 2.6(1)). This contradicts the fact that X is b-connected. Hence Y is connected.
(ii) Its clear from Remark 2.6(1), and by the same way of proof (i).

Remark 4.3: Diagram (2) shows the relationships between (connected space, b-connected space, bg-connected space and bg**-connected space) under the continuous function.

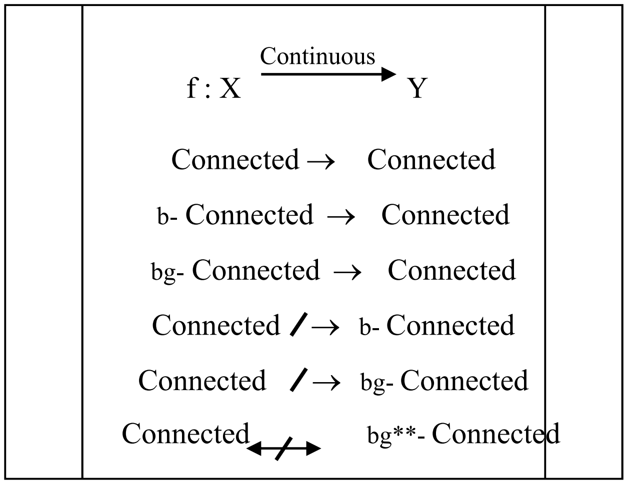

Diagram (2):The relationships between (connected space, b-connected space, bg-connected space and bg-connected space) under the continuous function.**

Theorem 4.4.: b-continuous image of b-connected space is connected.

Proof: Let f : X→ Y be b-continuous function ,and let X be b-connected space. To prove Y is connected. Suppose that Y is disconnected space ,then Y= A∪B, where A and B are disjoint non-empty open sets in Y.Since f is b-continuous $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty b-open sets in X such that $X = f^{-1}(A)\cup f^{-1}(B)$. This contradicts the fact that X is b-connected. Hence Y is connected.

Remark 4.5: Diagram (3) shows the relationships between (connected space, b-connected space, bg-connected space and bg**-connected space) under the b-continuous function.

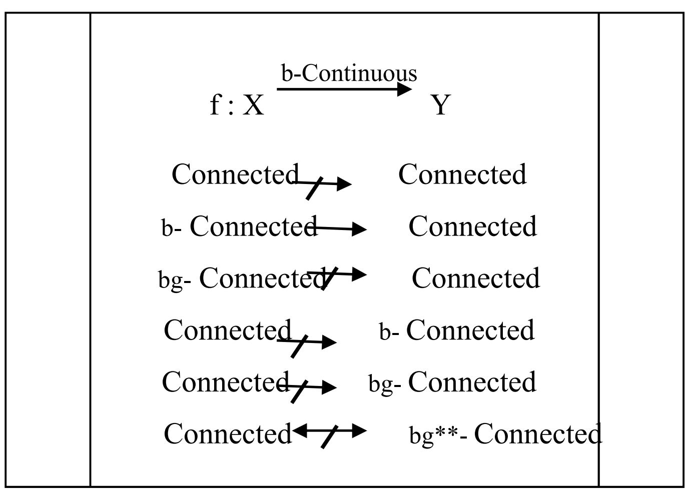

Diagram (3):The relationships between (connected space, b-connected space, bg-connected space and bg-connected space) under the b-continuous function.**

Theorem 4.6:

(i) bg-Continuous image of b-connected space is connected.
(ii) bg-Continuous image of bg-connected space is connected.
(iii) bg-Continuous image of bg**-connected space is connected.
Proof: (i) Let $f : X \to Y$ be bg-continuous function ,and let X be b-connected space. To prove Y is connected.Suppose that Y is disconnected space ,then $Y = A \cup B$, where A and B are disjoint non-empty open sets in Y.Since f is bg-continuous $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty bg-open sets in X ,and $f^{-1}(A)$ and $f^{-1}(B)$ are b-open by Remark 2.6(2) , such that $X = f^{-1}(A) \cup f^{-1}(B)$. This contradicts the fact that X is b-connected. Hence Y is connected.
(ii) and (iii) by the same way of proof (i) ,and Remark 2.6(3) .

Remark 4.7: Diagram (4) shows the relationship between (connected space, b-connected space, bg-connected space and bg**-connected space) under the bg-continuous function.

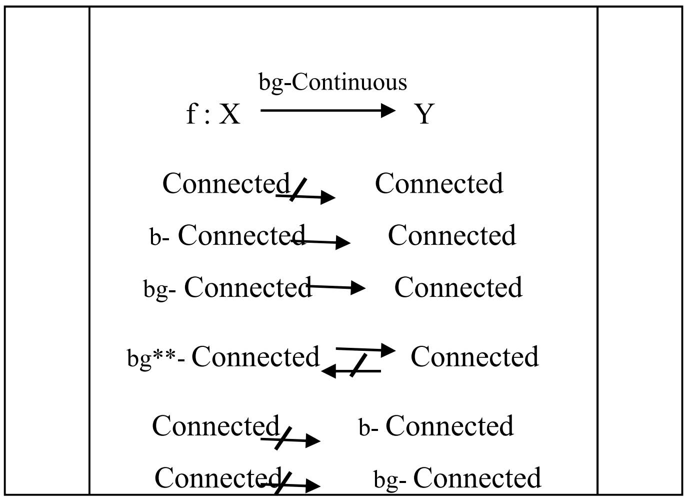

Diagram (4):The relationships between (connected space, b-connected space, bg-connected space and bg-connected space) under the bg-continuous function.**

Theorem 4.8:

(i) bg**-Continuous image of b-connected space is connected.
(ii) bg**-Continuous image of bg**-connected space is connected.
Proof: (i) Let $f : X \to Y$ be bg**-continuous ,and let X be b-connected space. To prove Y is connected.
Suppose that Y is disconnected space, then $Y = A \cup B$, where A and B are disjoint non-empty open sets in Y.Since f is bg**-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty bg**-open sets in X, and $f^{-1}(A)$ and $f^{-1}(B)$ are b-open by remark 2.6(4), such that $X = f^{-1}(A) \cup f^{-1}(B)$. This contradicts the fact that X is b-connected. Hence Y is connected.
(ii) By Theorem 3.8(i).

Remark 4.9: Diagram (5) shows the relationships between (connected space, b-connected space, bg-connected space and bg**-connected space) under the bg**-continuous function.

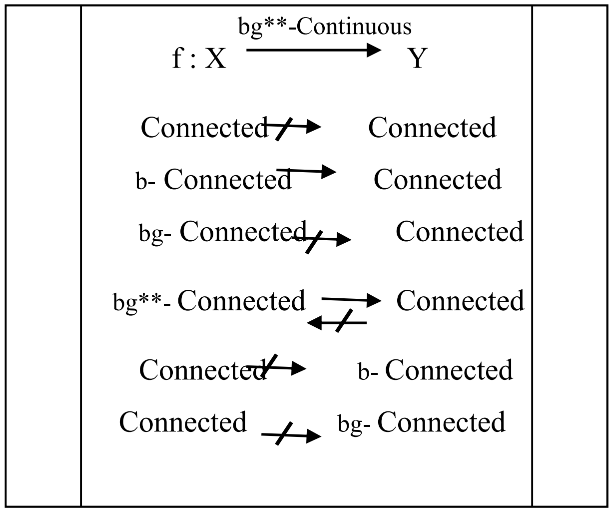

Diagram (5):The relationships between (connected space, b-connected space, bg-connected space and bg-connected space) under the bg**-continuous function.**

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الفضاءات المترابطة-**bg

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الخلاصة

في هذا البحث قمنا بتعريف الفضاء المترابط – **bg ، ودرسنا العلاقة بينه وبين انواع اخرى من الفضاءات . ودرسنا بعض الانواع من الدوال المستمرة ايضا ودرسنا العلاقة بين (الفضاءات المترابطة ،الفضاءات المترابطة- b ، الفضاءات المترابطة- bg والفضاءات المترابطة-**bg) تحت تأثير تلك الانواع من الدوال المستمرة .

الكلمات المفتاحية: المجموعة المغلقة-**bg ، الفضاء المترابط-**bg .