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Some Properties of Connectedness in Grill Topological Spaces

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Abstract

We use the idea of Grill, this study generalized a new sort of linked space like –connected –hyperconnected and investigated its features, as well as the relationship between it and previously described notation. It also developed new sorts of functions, such as hyperconnected space, and identifying their relationship, by offering numerous instance and attributes that belong to this set. This set will serve as a starting point for further research into the sets many future possibilities. Also, we use some of the theorems and observations previously studied and relate them to the grill and the Alpha group, and benefit from them in order to obtain new results in this research. We applied the concept of Connected to them and obtained results related to Connected. The sources related to the Connected and Alpha were considered as starting points and an important basis in this research.

Keywords: Grill, $₲^*\alpha$-connected, $₲^*\alpha$-disconnected, $₲^*\alpha o(Ꝡ)$, $₲^*\ \alpha$ -open.

1. Introduction

[1], [2] established a grill notion in a topological space, and the grill has shown to be an effective tool for learning a variety of topological concerns. A subset of a topological space $(Ꝡ, \tau)$ which is a non-empty collection ₲ is referred to be a grill whenever (a) Ⱥ ∈₲ and Ⱥ ⊆ Ɱ implying Ɓ∈₲. (b) Ꝡ has a subset Ⱥ and Ɱ also Ⱥ ∪ Ɓ∈₲ lead to Ⱥ ∈₲ or Ɓ∈₲. A triple $(Ꝡ, \tau, ₲)$ topological space with grills is one type of topological space.

Mukherjee and Roy [3] created a distinctive topology with a grill and investigated topological notions. For every topological space $(Ꝡ, \tau)$ point Ꝡ, Neighborhoods are open of Ꝡ embodied by τ(Ɏ). A mapping Ψ:ℙ(Ɏ) → ℙ(Ɏ) is referred to as ∲(Ⱥ) = { Ɏ ∈ Ɏ: Ⱥ ∩ Ũ ∈ 𝔊 apiece Ũ ∈ τ(Ɏ)} for every Ⱥ ∈ ℙ(Ɏ). A mapping Ψ: ℙ(Ɏ) →ℙ(Ɏ) is referred to as Ψ (Ⱥ) = Ⱥ ∪ ∲ (Ⱥ) for every Ⱥ ∈ ℙ(Ɏ). The map Ψ Kuratowski closure axioms are met:

(a) Ψ (∅) = ∅,
(b) $when$ Ⱥ ⊆ Ɓ, so Ψ (Ⱥ) ⊆ Ψ (Ɓ),
(c) $when$ Ⱥ ⊆ Ɏ, so Ψ (Ψ (Ⱥ)) = Ψ (Ⱥ),
(d) $when$ Ⱥ, Ɓ ⊆ Ɏ, so Ψ (Ⱥ ∪ Ɓ) = Ψ (Ⱥ) ∪ Ψ (Ɓ).

Grill topological spaces come in a variety of shapes and size as a discrete topology and a complement finite topology.

[4] [5] On a space (Ɏ, τ), this agrees to a grill 𝔊 a topology exists $\tau_{\mathfrak{G}}$ on Ɏ that is given by no one else by $\tau_{\mathfrak{G}}$ = {Ũ ⊆Ɏ: Ψ (Ɏ – Ũ) = Ɏ – Ũ}, Consequently, Ⱥ ⊆ Ɏ, Ψ (Ⱥ) = Ⱥ ∪ ∲(Ⱥ). τ ⊆ $\tau_{\mathfrak{G}}$ and Ψ (Ⱥ) = $c\iota$(Ⱥ). Using the following as a basis, we are able to locate $\tau_{\mathfrak{G}}$ on Ɏ by supplying $\tau_{\mathfrak{G}}$ through using the following as a basis β ($\tau_{\mathfrak{G}}$, Ɏ) = {Ƴ − Ⱥ; Ƴ ∈ τ, Ⱥ ∉ 𝔊}.

[6]. On a space (Ɏ, τ), there is a grill 𝔊, $\tau \subseteq \beta(\mathfrak{G}, \tau) \subseteq \tau_{\mathfrak{G}}$, where $\beta(\mathfrak{G}, \tau)$ basis for $\tau_{\mathfrak{G}}$.

As an example, [7] to exist in a space (Ɏ, τ) , $\tau_{\mathfrak{G}} = \tau$ whenever 𝔊 = ℙ(Ɏ) \ {∅} implying $\tau_{\mathfrak{G}}$ = τ. The family of all α -open set is showed by τ_s.

α *-open is a subset* Ⱥ *of a space* (Ɏ, τ), [8] *if* Ⱥ ⊆ ςl(ꞁnt(Ⱥ)),

Let (Ɏ, τ, 𝔊) be topological space. The subset Ⱥ in Ɏ is known as 𝔊 α -open if Ⱥ ⊆ Ψ(ꞁnt(Ⱥ)), and every Ψ α -open is an α -open. Many researchers have generalized using these [9] [10]. The symbol ꞁnt(Ⱥ) is the interior set of Ⱥ and ςl(Ⱥ) denotes the closure of **Ⱥ.**

combinations

The space (Ɏ, τ) is disconnected if and only if there exist two open disjoint nonempty sets Ⱥ and Ɓ such that Ⱥ∪Ɓ = Ɏ. i.e., Ɏ is disconnected *if and only if* Ɏ = ∪Ɓ ; Ⱥ, Ɓ ∈ τ, Ⱥ ∩ Ɓ = ϕ, Ⱥ, Ɓ ≠ ϕ.

The sets Ⱥ and Ɓ form a separation of Ɏ. The space (Ɏ, τ) is connected if and only if it is not disconnectaaed. Ɏ is connected if and only if Ɏ ≠ Ⱥ∪Ɓ ; Ⱥ, Ɓ ∈ τ, Ⱥ ∩Ɓ = ϕ, Ⱥ, Ɓ ≠ ϕ.

2. Grill α -open sets

Definition 2.1.

In [10], the set Ⱥ is referred to as Grill α-open if ȿ ∈ τ ; ȿ − Ⱥ ∉ 𝔊 and Ⱥ − ꞁ$nt\varsigma l_{\mathfrak{G}}$(ȿ) ∉ 𝔊. As stated by $\mathfrak{G}^*\alpha$ − open, the complment of $\mathfrak{G}^*$ α − open is $\mathfrak{G}^*\alpha$ − closed. The set of all $\mathfrak{G}^*\alpha$ − open is represented by $\mathfrak{G}^*\alpha o(y)$, and $\mathfrak{G}^*\alpha$ − closed is denoted by $\mathfrak{G}^*\alpha c(y)$. a

Example 2.2. [10]

Let (Ɏ,τ, 𝔊) is a topological space of the grill, and let Ɏ = $\{y_1, y_2, y_3\}$, τ = {Ɏ, ∅, $\{y_1\}$, $\{y_1, y_2\}$}, $\mathcal{F}$ = {Ɏ, ∅, $\{y_3\}$, $\{y_2, y_3\}$}, 𝔊 = {ȿ ⊆ Ɏ; y_2 ∈ ȿ} , ∅: P(Ɏ) → P(Ɏ), ∅(Ⱥ) = {y ∈ Ɏ; ∀ȿ ∈ τ_x ; ȿ ∩ Ⱥ ∈ 𝔊}, Ψ (Ⱥ) = Ⱥ ∪ ∅, $\tau_{\mathfrak{G}}$ ={ Ɏ, ∅,{ y_2, y_1},{y_3, y_2},{y_2}} ,$\mathcal{F}_{\mathfrak{G}}$ = {Ɏ, ∅,{ y_3},{y_1},{y_2, y_3}},so $\mathfrak{G}^*\alpha o(y)$ = {Ɏ, ∅, $\{y_3\}$, $\{y_2\}$, $\{y_1\}$, $\{y_2, y_3\}$, $\{y_1, y_2\}$, $\{y_3, y_1\}$}.

Theorem 2.3. [10]

The union of any family of $\mathfrak{G}^* - \alpha$ open set is a $\mathfrak{G}^* - \alpha$ open set.

Remark 2.4. [10]

Both ideas are related $\mathfrak{G}^*\alpha$ $-$ open set , α $-$ open set are independent.

Remark 2.5. [10]

Supra topology refers to the collection of all $\mathfrak{G}^*\alpha$-open sets.

Remark 2.6 [10]

i. The open set leads to $\mathfrak{G}^*\alpha$- open set.

ii. The closed set leads to $\mathfrak{G}^*\alpha$- closed set.

Theorem 2.7. [10]

The $\mathfrak{G}$ α-open leads to $\mathfrak{G}^*\alpha$-open.

Proposition 2.8. [10]

For any Grill topology $(\mathcal{Y}, \tau, \mathfrak{G})$, $\mathcal{A}$ is a $\mathfrak{G}$ α-open set if and only if $\mathcal{A}$ *is a* $\mathfrak{G}^*\alpha$-open set whenever $\mathfrak{G} = \mathbb{P}(\mathcal{Y}) \setminus \{\emptyset\}$.

Definition 2.9. [10]

The function $\mathfrak{f}: (\mathcal{Y}, \tau, \mathfrak{G})) \to (\mathcal{Y}, \tau', \mathfrak{G})$ is referred to as:

1. $\mathfrak{G}^*\alpha$-open function, shortly "$\mathfrak{G}^*\alpha$-o function" if $\mathfrak{f}(\varsigma) \in \mathfrak{G}^*\alpha o(y)$ when $\varsigma \in \mathfrak{G}^*\alpha o(\mathcal{Y})$.
2. $\mathfrak{G}^{**}\alpha$- open function, shortly "$\mathfrak{G}^{**}\alpha$-o function " if $\mathfrak{f}(\varsigma) \in \mathfrak{G}^*\alpha o(y)$ when $\varsigma \in \tau$.
3. $\mathfrak{G}^{***}\alpha$- open function, shortly "$\mathfrak{G}^{***}\alpha$-o function " if $\mathfrak{f}(\varsigma) \in \tau'$ whenever $\varsigma \in \mathfrak{G}^*\alpha o(\mathcal{Y})$.

Definition 2.10

The function $\mathfrak{f} : (\mathcal{X}, \tau, \mathfrak{G}) \to (\mathcal{Y}, \tau', \mathfrak{G})$ is called ;

1. $\mathfrak{G}^*\alpha$ $-$continuose function, shortly "$\mathfrak{G}^*\alpha$-"continuous function\" " if $\mathfrak{f}^{-1}(\varsigma) \in \mathfrak{G}^*\alpha O(\mathcal{X})$ for all $\varsigma \in \tau'$.
2. Strongly $\mathfrak{G}^*\alpha - c$ontinuose function shortly "strongly $\mathfrak{G}^*\alpha$-continuous function" if $\mathfrak{f}^{-1}(\varsigma) \in \tau$, fore every $\varsigma \in \mathfrak{G}^*\alpha O(\mathcal{Y})$.
3. $\mathfrak{G}^*\alpha$ $-$irresolute function, shortly "$\mathfrak{G}^*\alpha$-irresolute function" if $\mathfrak{f}^{-1}(\varsigma) \in \mathfrak{G}^*\alpha O(\mathcal{X})$, for every $\varsigma \in \mathfrak{G}^*\alpha O(\mathcal{Y})$

3. Grill α -open sets in grill connected space

Definition 3.1:

The space $(\mathcal{Y}, \tau, \mathfrak{G})$ is a $\mathfrak{G}^*\alpha o$- disconnected if and only if there exist two $\mathfrak{G}^*\alpha$-open disjoint nonempty sets $\mathcal{A}$ and $\mathcal{B}$ such that $\mathcal{A} \cup \mathcal{B} = \mathcal{Y}$. i.e.,$\mathcal{Y}$ is a $\mathfrak{G}^*\alpha o$- disconnected if and only if $\mathcal{Y} = \mathcal{A} \cup \mathcal{B}$; $\mathcal{A}, \mathcal{B} \in \mathfrak{G}^* so(\mathcal{Y})$, $\mathcal{A} \cap \mathcal{B} = \phi$.

The sets $\mathcal{A}$ and $\mathcal{B}$ form a $\mathfrak{G}^* so$-separation of $\mathcal{Y}$. The space $(\mathcal{Y}, \tau, \mathfrak{G})$ is a $\mathfrak{G}^* so$- connected if and only if it is not $\mathfrak{G}^*\alpha o$- disconnected. Ɏ is a $\mathfrak{G}^*\alpha o$- connected if and only if Ɏ $\neq$ ȺՄƁ ; Ⱥ, Ɓ $\in$ $\mathfrak{G}^*\alpha o$(Ɏ), Ⱥ $\cap$ Ɓ = ϕ, Ⱥ, Ɓ$\neq\phi$.

Example 3.2.

Let (Ɏ, τ, $\mathfrak{G}$) topological space be a grill and Ɏ $= \{$Ɏ$_1$, Ɏ$_2$, Ɏ$_3\}$, $\tau = \tau_{\mathfrak{G}} =$ $\{$Ɏ$, \emptyset, \{$Ɏ$_2\}, \{$Ɏ$_3\}\}$, $\mathfrak{G} = \mathbb{P}$(Ɏ) \ $\{\emptyset\}$ is a $\mathfrak{G}^*\alpha o$- connected space.

Remark 3.3

Every disconnected set is a $\mathfrak{G}^*\alpha o$-disconnected.

Proof.

Let Ɏ is disconnected set, then the exist $\mathcal{W} \neq \emptyset, \mathcal{Z} \neq \emptyset$ and $\mathcal{W}, \mathcal{Z} \in \tau$; $\mathcal{W} \cap \mathcal{Z} = \emptyset$ *and* $\mathcal{W} \cup \mathcal{Z} =$ Ɏ since every open set in $\mathfrak{G}^*\alpha$-open set. Therefore, Ɏ is $\mathfrak{G}^*\alpha o$-disconnected.

Remark 3.4.

The space (Ɏ, τ, $\mathfrak{G}$) is a $\mathfrak{G}^*\alpha o$- disconnected with any grill and $\mathfrak{G}^*\alpha o$(Ɏ) = $\mathbb{P}$(Ɏ) if Ɏ contained more than one element since there exist Ⱥ and Ⱥ^c $\in$ $\mathfrak{G}^*\alpha o$(Ɏ); Ⱥ ՄƁ = Ɏ, Ⱥ $\cap$ Ɓ = ϕ, Ⱥ, Ɓ$\neq\phi$.

Remark 3.5.

The space (Ɏ, τ, $\mathfrak{G}$) is a $\mathfrak{G}^*\alpha o$- disconnected and $\mathfrak{G} = \emptyset$ and $\tau_{\mathfrak{G}} = \{$Ɏ$, \emptyset\}$, so $\mathfrak{G}^*\alpha o$- connected space.

Remark 3.6.

If $\tau_{\mathfrak{G}} = \mathcal{F}_{\mathfrak{G}}$ and $\tau_{\mathfrak{G}} \neq$ I*ndiscret* , when $\mathfrak{G} = \mathbb{P}$(Ɏ) \ $\{\emptyset\}$. This means that (Ɏ, τ, $\mathfrak{G}$) is a $\mathfrak{G}^*\alpha o$- disconnected .

Theorem 3.7.

The space (Ɏ, τ, $\mathfrak{G}$) is a $\mathfrak{G}^*\alpha o$-connected if and only Ɏ cannot be written as a union of two non-empty disjoint closed sets.

Proof.

Let Ɏ be a $\mathfrak{G}^*\alpha o$-connected if Ɏ = Ⱥ ՄƁ, such that; Ⱥ and Ɓ $\in$ $\mathfrak{G}^*\alpha c$(Ɏ) , Ⱥ $\cap$ Ɓ = ϕ, Ⱥ, Ɓ$\neq\phi$, so. Ⱥ = Ɓ^c *and* Ɓ = Ⱥ^c , then Ɏ = ȺՄƁ; Ⱥ and Ɓ $\in$ $\mathfrak{G}^*\alpha o$(Ɏ), Ⱥ $\cap$ Ɓ = ϕ, Ⱥ, Ɓ$\neq\phi$, that is mean that Ɏ is a $\mathfrak{G}^*\alpha o$- disconnected. That is contradiction. Therefore, Ɏ cannot be written as a union of two non-empty disjoint closed set. Now, if Ɏ is a $\mathfrak{G}^*\alpha o$-disconnected space ,so. Ɏ= Ⱥ ՄƁ; Ⱥ and Ɓ $\in$ $\mathfrak{G}^*\alpha o$(Ɏ), Ⱥ $\cap$ Ɓ = ϕ, Ⱥ, Ɓ$\neq\phi$, so Ⱥ = Ɓ^c *and* Ɓ = Ⱥ^c,then Ⱥ and Ɓ $\in$ $\mathfrak{G}^*\alpha c(x)$, but that is a contradiction. Therefore, Ɏ is a $\mathfrak{G}^*\alpha o$-connected space.

Theorem 3.8.

The space (Ɏ, τ, $\mathfrak{G}$) is a $\mathfrak{G}^*\alpha o$-connected if and only if the only the subsets of the space Ɏ which are $\mathfrak{G}^*\alpha$-open and $\mathfrak{G}^*\alpha$-closed are Ɏ and $\emptyset$.

Proof.

Let $\mathbb{A}$ and $\mathbb{A}^c \in \mathfrak{G}^*\alpha o(x)$ and $\mathbb{A} \neq \mathbb{W}$, $\mathbb{A} \neq \emptyset$,so $\mathbb{W} = \mathbb{A} \cup \mathbb{A}^c$; $\mathbb{A} \cap \mathbb{A}^c = \phi$, $\mathbb{A}, \mathbb{A}^c \neq \phi$. Then, $\mathbb{W}$ is a $\mathfrak{G}^*\alpha o$-disconnected, and that is a contradiction. So, where $\mathbb{A} \subseteq \mathbb{W}$, $\mathbb{A}$, $\mathbb{A}^c \in \mathfrak{G}^*\alpha o(x)$, then $\mathbb{A} = \mathbb{W}$ or $\mathbb{A} = \emptyset$, let $\mathbb{W}$ be a $\mathfrak{G}^*\alpha o$-disconnected space. This means that $\mathbb{W} = \mathcal{W} \cup \mathcal{Z}$; $\mathcal{W}, \mathcal{Z} \in \mathfrak{G}^*\alpha o(x)$, $\mathcal{W} \cap \mathcal{Z} = \phi$ and $\mathcal{W}, \mathcal{Z} \neq \emptyset$ implies that $\mathcal{W} = \mathcal{Z}^c$ and $\mathcal{Z} = \mathcal{W}^c$. So $\mathcal{W}, \mathcal{Z} \in \mathfrak{G}^*\alpha c(x)$ (that is contradiction) Therefore, $\mathbb{W}$ is a $\mathfrak{G}^*\alpha o$-connected space.

Remark 3.9.

If $\mathfrak{f} : (\mathbb{W}, \tau, \mathfrak{G}) \to (\Upsilon, \tau', \mathfrak{G})$ is a $\mathfrak{G}^*\alpha o$-irresolute and onto function and Υ is a $\mathfrak{G}^*\alpha o$-connected space then $\mathbb{W}$ is not necessary $\mathfrak{G}^*\alpha o$-connected space.

Example 3.10.

Let $\mathfrak{f} : (\mathbb{R}, \mathrm{D}, \mathfrak{G}) \to (\mathbb{R}, \mathrm{I}, \mathfrak{G})$; $\dot{\mathfrak{f}}(\mathbb{W}) = \mathbb{W}$ for all $\mathbb{W} \in \mathbb{R}$ and $\mathfrak{G} = \mathbb{P}(\mathbb{W}) \setminus \{\emptyset\}$,so $\dot{\mathfrak{f}}$ is a $\mathfrak{G}^*\alpha o$-*irresolute* and onto function and $(\mathbb{R}, \mathrm{I}, \mathfrak{G})$ is a $\mathfrak{G}^*\alpha o$-connected space and $(\mathbb{R}, \mathrm{D}, \mathfrak{G})$ is a $\mathfrak{G}^*\alpha o$-disconnected space.

Theorem 3.11.

If $\mathbb{A}$ and B are a $\mathfrak{G}^*\alpha o$-connected spaces of $(\mathbb{W}, \tau, \mathfrak{G})$ and $\mathbb{A} \cap \mathrm{B} \neq \phi$, then is a $\mathfrak{G}^*\alpha o$-connected space.$\mathbb{A} \cup \mathrm{B}$

Proof.

Let $(\mathbb{W}, \tau, \mathfrak{G})$ be a grill topological space and $\mathbb{A}, \mathrm{B} \subseteq \mathbb{W}$; $\mathbb{A}, \mathrm{B}$ be a $\mathfrak{G}^*\alpha o$-connected space. Now, If $\mathbb{A} \cup \mathrm{B}$ is a $\mathfrak{G}^*\alpha o$-disconnected space, so $\mathbb{A} \cup \mathrm{B} = \mathcal{W} \cup \mathcal{Z}$; $\mathcal{W}, \mathcal{Z} \in \mathfrak{G}^*\alpha o(x)_{(\mathbb{A} \cup \mathrm{B})}$, $\mathcal{W} \cap \mathcal{Z} = \phi$ and $\mathcal{W}, \mathcal{Z} \neq \emptyset$,then $\mathbb{A} \subseteq \mathbb{A} \cup \mathrm{B}$, $\mathbb{A} \subseteq \mathcal{W} \cup \mathcal{Z}, \mathbb{A} \subseteq \mathcal{W}$ *or* $\mathbb{A} \subseteq \mathcal{Z}$. Similarly, B leads to either $\mathbb{A} \subseteq \mathcal{W}$ and $\mathrm{B} \subseteq \mathcal{W}$, then $\mathbb{A} \cup \mathrm{B} \subseteq \mathcal{W}$, then $\mathcal{Z} = \emptyset$ contradiction. Or, $\mathbb{A} \subseteq \mathcal{Z}$ and $\mathrm{B} \subseteq \mathcal{Z}$, then $\mathbb{A} \cup \mathrm{B} \subseteq \mathcal{Z}$, then $\mathcal{W} = \emptyset$ C! Or $\mathbb{A} \subseteq \mathcal{W}$ and $\mathrm{B} \subseteq \mathcal{Z}$, then $\mathbb{A} \cap \mathrm{B} \subseteq \mathcal{W} \cap \mathcal{Z}$, then $\mathbb{A} \cap \mathrm{B} = \emptyset$ C! Or $\mathbb{A} \subseteq \mathcal{Z}$ and $\mathrm{B} \subseteq \mathcal{W}$, then $\mathbb{A} \cap \mathrm{B} \subseteq \mathcal{W} \cap \mathcal{Z}$, then $\mathbb{A} \cap \mathrm{B} = \emptyset$. This is a contradiction. So, $\mathbb{A} \cup \mathrm{B}$ is a $\mathfrak{G}^*\alpha o$-connected space.

Remark 3.12.

We can generalize **Theorem 3.11** to a family of a $\mathfrak{G}^*\alpha o$-connected sets as follows: let $\{\mathbb{A}_\propto\}_{\propto \in \Lambda}$ be a family of a $\mathfrak{G}^*\alpha o$-connected subsets of a space $(\mathbb{W}, \tau, \mathfrak{G})$ and $\bigcap_{\propto \in \Lambda} \mathbb{A}_\propto \neq \emptyset$, then $\bigcup_{\propto \in \Lambda} \mathbb{A}_\propto$ is a $\mathfrak{G}^*\alpha o$-connected set.

4. Grill α-open sets in grill connected space hyperconnected

Definition 4.1.

In any grill topological space, $(\mathbb{W}, \tau, \mathfrak{G})$ is said to be:

1. $*$-hyperconnected if $\mathbb{A}$ is $\tau_{\mathfrak{G}}$-dense ($\varsigma l_{\mathfrak{G}}(\mathbb{A}) = \mathbb{W}$) for every non-empty open subset $\mathbb{A}$ of $\mathbb{W}$.
2. $\mathfrak{G}^*$-hyperconnected if $\mathbb{W} - \varsigma l_{\mathfrak{G}}(\mathbb{A}) \notin \mathfrak{G}$ for every non-empty open subset $\mathbb{A}$ of $\mathbb{W}$.
3. $\mathfrak{G}^*\alpha$-hyperconnected if $\mathbb{W} - \varsigma l_{\mathfrak{G}}(\mathbb{A}) \notin \mathfrak{G}$ for every non-empty $\mathfrak{G}^*\alpha$-*open* subset $\mathbb{A}$ of $\mathbb{W}$.
4. $\mathfrak{G}^*\alpha o$-hyperconnected if $\varsigma l_{\mathfrak{G}}(\mathbb{A}) = \mathbb{W}$ for all $\mathbb{A} \in \mathfrak{G}^*\alpha o(\mathbb{W})$.

Proposition 4.2.

1.Every $\mathfrak{G}^*\alpha o$-hyperconnected is $\ast$-hyperconnected.
2. Every $\ast$-hyperconnected is $\mathfrak{G}^*$-hyperconnected.
3. Every $\mathfrak{G}^*\alpha$-hyperconnected is $\mathfrak{G}^*$-hyperconnected.
4. Every $\mathfrak{G}^*\alpha o$-hyperconnected is $\mathfrak{G}^*\alpha$-hyperconnected.

Proof.

1. Let Ⱥ is a $\mathfrak{G}^*\alpha o$-hyperconnected, then $c\iota_{\mathfrak{G}}(Ⱥ) = Ꝡ$ then Ⱥ is a $\ast$-hyperconnected (since $Ⱥ \in \mathfrak{G}^*\alpha o(Ꝡ)$. then $Ⱥ \in \tau$).
2. Let Ⱥ be a $\ast$-hyperconnected. This means that $c\iota_{\mathfrak{G}}(Ⱥ) = Ꝡ$. So, $Ꝡ - c\iota_{\mathfrak{G}}(Ⱥ) = \emptyset \notin \mathfrak{G}$. Therefore, Ⱥ is a $\mathfrak{G}^*$-hyperconnected.
3. Let Ⱥ be an open in $\mathfrak{G}^*\alpha$-hyperconnected that's mean that $Ꝡ - c\iota_{\mathfrak{G}}(Ⱥ) \notin \mathfrak{G}^*\alpha$-hyperconnected (since every open set in $\mathfrak{G}^*\alpha$-hyperconnected is an open set in $\mathfrak{G}^*$-hyperconnected. So, $Ꝡ - c\iota_{\mathfrak{G}}(Ⱥ) \notin \mathfrak{G}$ .Therefore, Ⱥ is a $\mathfrak{G}^*$-hyperconnected.
4. Let Ⱥ be an open in $\mathfrak{G}^*\alpha o$-hyperconnected. This means that $c\iota_{\mathfrak{G}}(Ⱥ) = Ꝡ$. So, $Ꝡ - c\iota_{\mathfrak{G}}(Ⱥ) \notin \mathfrak{G}$. Therefore, Ⱥ is a $\mathfrak{G}^*s$-hyperconnected.

The following diagram shows the relationship between the types of hyperconnected.

$$
\begin{tikzcd}
 & \ast\text{-hyperconnected} \arrow[dr] & \\
\mathfrak{G}^*\alpha o\text{-hyperconnected} \arrow[ur] \arrow[dr] & & \mathfrak{G}^*\text{-hyperconnected} \\
 & \text{-hyperconnected} \quad \mathfrak{G}^*\alpha \arrow[ur] &
\end{tikzcd}
$$

Diagram 1
hyperconnected space via grill space

4. Conclusion

In this research we studied a connect space in the grill α -open topological spaces, we showed some examples and applied some theorems for this new sets. We also found some new properties of these sets.

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